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# Optimal Political Institutions in the Shadow of Conflict\*

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## Abstract

Two groups choose common political institutions, here modeled as an abstract mechanism aimed at generating investments and then distributing the resulting output. For example, two independent regions may consider creating a federation. Importantly, political institutions are *in the shadow of conflict*: each group may, ex post, unilaterally withdraw from the common institution and trigger a non-cooperative game. The payoff in case of conflict depends on the players' investment. It follows that the optimal political institutions may distort each player's investment away from the first best level, as a way to discourage the other player from deviating. Distorting the players' investment, however, also reduces the welfare in case of peace. It is possible that there is no political institution that can prevent deviations, and hence the only possible outcome is conflict.

**JEL classification:** D72, D74, F55, H77.

**Keywords:** Endogenous political institutions, conflict, international institutions, international agreements.

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## 1 Introduction

This paper considers the creation of common political institutions by two competing, selfish groups. Inspired by the literature studying contractual solutions to hold up problems, we model political institutions as a mechanism that allocates the available social surplus to the different groups as a function of their investments, and is chosen optimally at the beginning of the game.

Our main contribution is to explicitly consider the possibility that, after common institutions are created, each group can unilaterally dissolve them by triggering a non-cooperative game, which we call a conflict. The payoffs in case of conflict are determined by the players' investments. Hence, political institutions need to satisfy two endogenous, ex-post participation constraints, which depend on the equilibrium investment of the non-deviating player. As we will see, this feature will play a key role in the design of the optimal mechanism, which might distort each player's investment profile as a way to discourage the opponent from deviating. Beside these ex-post participation constraints, we do not impose any additional limitation on the political institutions that can be chosen. We assume full information and full "contractibility" (that is, the mechanism can specify payoffs contingent on all types of investments), as well as full commitment (that is, the mechanism can credibly threaten to destroy welfare). Furthermore, we assume that conflict is inefficient.

Our results are quite negative. There may no mechanism that achieves the first best. Intuitively, if each group expects the other group to choose the first-best investment profile, the benefit of deviating and triggering a conflict may be very large, possibly exceeding the available social surplus. In this case, the optimal mechanism may need to distort the players' investment profile away from the first best, so to reduce the incentive to trigger a conflict. Hence, first-best efficiency may be incompatible with peace, but an inefficient peace may be able to prevent conflict. However, distorting the investment profiles to discourage deviations also reduces the total surplus to be shared in case of peace. It follows that a mechanism that achieves peace may not exist. Conflict casts a shadow on political institutions, and generates inefficiencies also in case of peace. These inefficiencies may be so large that an inefficient conflict is the only possible outcome of the game.

We illustrate our point via a general model and an example. The example is a version of the "guns and butter" model in Skaperdas (1992), in which two players first invest in guns (i.e. weapons) and butter (i.e. productive activities), and then

decide whether to trigger a conflict. Unlike Skaperdas (1992), we allow the players to create common political institutions before investing.<sup>1</sup> This allows the players to maintain peace also when, under Skaperdas (1992)'s assumptions the outcome should be conflict. More interestingly, in our model, to maintain peace the optimal political institutions may require the players to invest in guns. This discourages each player from triggering a conflict, because he now anticipates that he will fight an opponent who is armed. Of course, by mandating a positive investment in arms, the optimal political institutions generate an inefficiency. This inefficiency is a function of the destructiveness of the conflict—with more destructive conflict requiring lower investment in guns to prevent deviations and hence generating lower inefficiency. If the destructiveness of the conflict is sufficiently low, it is possible that conflict is the unique equilibrium of the game.

Hence, similarly to other theories of conflict, whether peace can be maintained is a function of the destructiveness of the conflict. However, and specific to our context, there is a relationship between the destructiveness of the conflict and welfare under peace—because less destructive conflict implies that higher distortions imposed by the political institutions. This type of “armed peace” is the optimal outcome given the players' ex-post participation constraints. We then modify the example by introducing a second type of productive investment: eggs. Eggs are more costly to produce than butter, but they are more easily destroyed in case of conflict. Hence, in the first best, the players only invest in butter. To prevent deviations, however, the optimal political institutions may require the players to invest both in eggs and in guns. Interestingly, we show that if the optimal political institutions mandate positive investment in eggs, they also mandate positive investment in guns. Otherwise, a player may deviate by switching 100% of his investment to butter without fear of being attached—which can be a profitable deviation but, clearly, not an equilibrium of the conflict game.

Finally, two comments on the methodology. We model political institutions as a very abstract mechanism to induce a level of investment and then share the resulting social surplus. We do not worry about how a specific political institution may achieve this. Our results therefore provide an upper bound to what more realistic political institutions that face additional constraints (such as informational constraints or commitment issues) can achieve, both in terms of social welfare in case of peace

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<sup>1</sup> Other similar model of cooperation and conflict in the absence of institutions are Taylor (1987), Grossman and Kim (1995), and Hirshleifer (1995).

and in terms of preventing inefficient conflict. Second, for ease of exposition we only consider a finite-time game in which first the players invest, and then there is either conflict or peace. But the model can also be interpreted as a reduced form of an infinitely-repeated game. In this case, the payoff from conflict is the expected present discounted value of deviating one period and then playing conflict in every subsequent period (as in a grim-trigger strategy). The payoff from peace is the expected present discounted value of maintaining peace in every period.

## Related literature

The idea that political institutions operate “in the shadow of conflict” is well known in political philosophy, and is central to most theories of the social contract. In particular, in Thomas Hobbes’ view, absent political institutions people would live in “the state of nature”: the outcome of non-cooperative, violent, rule-free interactions. Hence the role of political institutions is to provide security and peace. Note that Hobbes’ argument readily extends beyond security and peace to all forms of collective action problems, such as for example the provision of public goods (see Taylor, 1987, chapter 1). The possibility of reverting to the state of nature, however, imposes a constraint on the allocations that can be implemented by the political institutions (see Taylor, 1987, chapter 6).

This paper is motivated by the observation that social surplus to be shared in case of peace and the payoffs in case of conflict (i.e., in the state of nature) depend, at least in part, from prior investments made by the different individual/groups who participate in these political institutions. The endogeneity of these payoffs distinguishes our theory from the existing economic analysis of Hobbes’ political philosophy (for example that of Esteban and Sákovics, 2008, Bester and Wärneryd, 2006) and connects us with the literature studying contractual arrangements. In particular we are related to the literature studying contracts with endogenous ex-post outside options.<sup>2</sup> Importantly, the novelty of our paper is that, here, the ex-post

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<sup>2</sup> The most famous model of contracting with endogenous outside option is that of Gibbons and Murphy (1992), in which after signing a labor contract, a worker can take actions that increase his/her outside option. See also Edlin and Reichelstein (1996), Che and Hausch (1999), and Chatterjee and Chiu (2013), in which an agent can make a productive investment that affects both the value of transacting with the other player and the value of transacting with third parties. Also related are Kranton and Minehart (2000), Kranton and Minehart (2001) and Elliott (2015), who consider a network of buyers and sellers, in which each player can spend resources to link

outside option is a conflict, which implies that a player's incentive to deviate (i.e., choose his ex-post outside option) depends on the investments made by *both* players.

We are also related to the literature on self-enforcing (or relational) contracts, studying infinitely repeated games in which each player can always deviate to an outside option. The vast majority of papers in this literature assume that this outside option is exogenous.<sup>3</sup> An exception is Thomas and Worrall (2018), in which each player's outside option is increasing in the action (that is, productive effort) of the other player. They also show that a player's action may be distorted so to discourage the other player from deviating. The main difference with our model is the scope of the focus of the analysis. Thomas and Worrall (2018) study the long run-evolution of this distortion, and establish condition under which it converges, and its limit. Here instead we consider multiple types of investments and are interested in understanding precisely what margins will be distorted by the political institutions.<sup>4</sup>

By aiming at connecting the theory of social contract with modern contract theory, our paper ends up connecting the literature studying inefficient conflicts and secession, with the literature studying endogenous political institutions.<sup>5</sup> In order to reconcile rationality with conflict, the first strand of literature proposes a number of explanations such as asymmetric information between the players, conflict over large indivisibles, or the inability to commit to initial agreements. Lack of commitment is also at the base of our model, because the players cannot commit not to trigger a

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with an additional buyer/seller and therefore increase his bargaining power. Also relevant is Cole, Mailath, and Postlewaite (2001), who study bargaining protocols leading to efficient non-contractible investments prior to matching.

<sup>3</sup> See, for example, Ligon, Thomas, and Worrall (2002) and Ray (2002). Interestingly, in these papers if the outside option is sufficiently high (but not too high) then only "second best" cooperative outcomes can be sustained in equilibrium.

<sup>4</sup> There is also a second, methodological difference: in Thomas and Worrall (2018) contracts cannot be enforced. The incentive to act cooperatively (i.e., choose a positive action and then not trigger the outside option) exclusively comes from the infinite horizon of the game. In our framework instead, agreements (which here take the form of institutions) are enforceable up to the players' endogenous outside options. This implies, for example, that cooperation is possible also in a finite-horizon game.

<sup>5</sup> On inefficient conflicts, see the survey article by Jackson and Morelli, 2011. On inefficient secession, see a recent paper by Esteban, Flamand, Morelli, and Rohner, 2018. On endogenous political institutions, the most closely related papers are the ones studying optimal design of constitutions (see the seminal work of Buchanan and Tullock, 1962 as well as the more recent Aghion, Alesina, and Trebbi, 2004).

conflict ex-post. However, contrary to existing theories (such as Fearon, 1995, and Powell, 2006), in our model the players' ability to commit is endogenous and depends on the investment profiles mandated by the political institutions. To prevent conflict, these investment profiles may be inefficient. As a consequence, we also provide a novel rationale why inefficient political institutions may emerge.<sup>6</sup>

Finally, some of the issues discussed in this paper were also discussed in Canidio and Esteban (2018), in which two players can make investments before negotiating an agreement. In that paper we consider a very specific family of arbitration procedures and derive the welfare maximizing one. Here instead we do not impose any structure (and therefore any constraint) on the ability of the mechanism to affect the ex-post payoffs.

## 2 General model

We start by presenting a general model that illustrates our main results, albeit on a somewhat abstract level. In the next sections we consider more specific models and derive additional results.

There are two players, 1 and 2, which we interpret as two independent regions. In the first period of the game the two players agree to set up common political institutions. These institutions are chosen cooperatively (e.g., via Nash bargaining) and players can perform side transfers. It follows that the common political institutions set up by the players maximize the sum of their continuation utilities.

In the following period, both players simultaneously choose a vector of investments  $x_i \in X_i \subset \mathbb{R}_+^L$  for  $i \in \{1, 2\}$  (e.g., transport infrastructures, universities, R&D labs, weapons, military bases, ...), where  $X$  represents the set of feasible investment vectors and is assumed to be a compact set.<sup>7</sup> After investing, there can be either a conflict or peace. Conflict occurs in two cases:

1. if the players did not set up common political institutions. Because the com-

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<sup>6</sup> A similar argument is provided by Acemoglu (2003) and Acemoglu (2006), who show that if elites cannot commit to a given set of transfers, then the political outcome may be inefficient. In both papers, however, there is no possibility of conflict. Our argument is therefore related but not identical to that in Acemoglu (2003) and Acemoglu (2006).

<sup>7</sup> Assuming that  $X$  is a compact set is the simplest way to guarantee the existence of the first-best level of investment, of the best responses of the conflict game, and of the Nash equilibrium. Of course, other assumptions are possible.

mon political institutions are chosen cooperatively, this will happen only if welfare in case of conflict is greater than welfare in case of peace under all possible political institutions. As we will see, this is the only type of conflict that emerges in the equilibrium of the game.

2. if, after setting up common political institutions, a player unilaterally decides to trigger one. That is, after common political institutions are set up, the players simultaneously choose their investment levels and whether to trigger a conflict or not. As we will see, this type of conflict never happens in equilibrium, but its possibility will nonetheless impose a constraint on the type of political institutions that can be chosen initially.

In case of conflict the players' utilities are  $\alpha \cdot u_1(x_1, x_2)$ ,  $\alpha \cdot u_2(x_1, x_2)$ , where  $u_1(x_1, x_2)$ ,  $u_2(x_1, x_2)$  are continuous and positive for all  $x_1, x_2$ . The parameter  $\alpha > 0$  measures the destructiveness of the conflict, with lower values corresponding to a more destructive conflict, and is a useful way to parameterize the benefit of maintaining peace. If there is no conflict, then there is peace, in which case the common political institutions acquire the control of  $x_1, x_2$ , use them to produce  $W(x_1, x_2)$ , and then transfers  $\pi_1(x_1, x_2)$  to player 1 and  $\pi_2(x_1, x_2)$  to player 2.<sup>8</sup> The choice of political institutions at the beginning of the game therefore amounts to the choice of the functions  $\pi_1(x_1, x_2)$  and  $\pi_2(x_1, x_2)$  subject to the feasibility constraint  $\pi_2(x_1, x_2) \leq W(x_1, x_2) - \pi_1(x_1, x_2)$ . See Figure 1 for the timeline.

Define:

$$\{x_1^{**}, x_2^{**}\} \equiv \operatorname{argmax}_{x_1 \in X, x_2 \in X} W(x_1, x_2)$$

as the surplus maximizing investment levels in case no conflict occurs (note that  $\{x_1^{**}, x_2^{**}\}$  could be a set). Define  $x_1^{BR}(x_2)$  and  $x_2^{BR}(x_1)$  as the two players' best responses in case of conflict

$$x_i^{BR}(x_{-i}) = \operatorname{argmax}_{x_i \in X} u_i(x_1, x_2).$$

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<sup>8</sup> Note that we are abstracting away from the cost of investing. Without loss of generality, we can assume that these costs are embedded in the functions  $u_1(x_1, x_2)$ ,  $u_2(x_1, x_2)$  and  $W(x_1, x_2)$ .

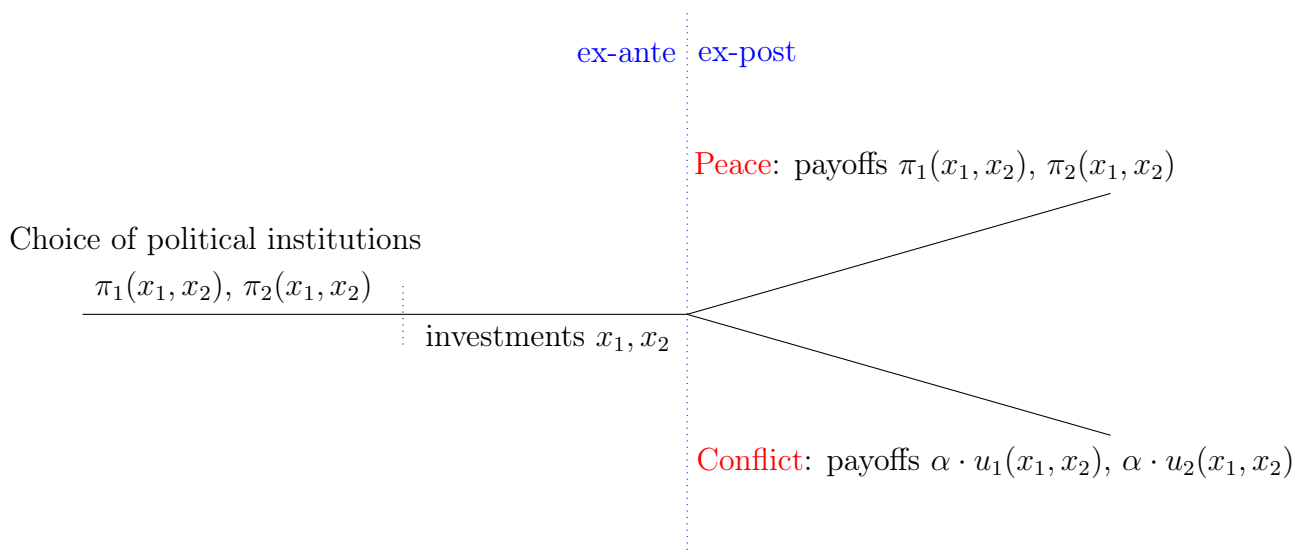


Fig. 1: Timeline

Call  $x_1^*$  and  $x_2^*$  the Nash equilibrium of the conflict game<sup>9</sup>

$$x_1^* = x_1^{BR}(x_2^*) \quad x_2^* = x_2^{BR}(x_1^*),$$

Finally, we call  $\tilde{\alpha}$  as the value of  $\alpha$  such that, from the ex-ante viewpoint (that is, before the investments are made), the sum of the players' utilities in the equilibrium of the conflict game is equal to the maximum surplus in case of peace, that is

$$\tilde{\alpha} \equiv \frac{W(x_1^{**}, x_2^{**})}{u_1(x_1^*, x_2^*) + u_2(x_1^*, x_2^*)} \quad (1)$$

Our main assumption is that conflict is inefficient:

$$\alpha < \tilde{\alpha} \quad (\text{A1})$$

**Discussion.** A few aspects of the model deserves to be discussed in some details. To start, as we will see, the optimal political institutions will impose some legally binding investment levels. Therefore, on the equilibrium path, the model is observationally equivalent to a situation in which the investment levels  $x_1, x_2$  are chosen

<sup>9</sup> If the Nash equilibrium of the conflict game is not unique, we restrict our attention to the Pareto preferred one. Our notation implicitly assumes that this equilibrium is in pure strategy, but our argument holds identical if the equilibrium instead is in mixed strategy (the only difference is that the utility in the equilibrium of the conflict game is now an expectation).

directly by the common political institutions. It is however key for our results that players could deviate to a different investment level (at the cost, of course, of either being punished by the common institutions, or of triggering a conflict).

Also, it is important to clarify that conflict is inefficient from the ex-ante viewpoint: that is, taking into consideration the fact that the equilibrium level of investment in case of conflict may be different from the first best one. However, we do not assume that conflict is inefficient ex-post (that is, for a given investment level). This will depend on the details of the conflict game. For example, in a public good game, for given investment levels social surplus in case of conflict is the same as social welfare in case of peace. If instead the non-cooperative game is a tournament in which the player exert fighting effort, then it is possible that conflict is less efficient than peace for every investment levels. Under our assumptions, it is also possible that, for some investment profiles, conflict is preferred to peace.

Finally, an implicit assumption is that the conflict payoffs depend only on the investment profiles and not on how the conflict came about. This is *with* loss of generality because, for given  $x_1, x_2$ , a conflict emerging because no common political institutions were set up may generate very different payoffs from a conflict started by a player while the other player expected the common political institution to survive. However, keeping track of how the conflict came about would significantly increase the complexity of the notation, for no additional insights. The utility in case a conflict emerging because no common political institutions were set up matters exclusively in determining whether conflict is efficient or inefficient (that is, whether A1 holds). What matters for the analysis of the optimal political institutions is the utility that each player can earn if he triggers a conflict while the other player invests as prescribed by the political institutions.

## Optimal political institutions

Without loss of generality, assume that the initial agreement between the two players has the following form:

- The political institutions will mandate two investment profiles  $\bar{x}_1$  and  $\bar{x}_2$  and will guarantee utility levels  $\bar{U}_1$  and  $\bar{U}_2 \leq W(\bar{x}_1, \bar{x}_2) - \bar{U}_1$  to the two players.
- The political institutions will impose the largest possible punishment to any player deviating from their prescribed investment level. Because, ex-post,

each player can unilaterally trigger a conflict, the largest punishment that the political institutions can impose is to keep the deviating player to the utility she would achieve in case of conflict.

Mathematically, such political institutions have the form:<sup>10</sup>

$$\pi_i(x_1, x_2) = \begin{cases} \bar{U}_i & \text{if } x_i = \bar{x}_i \\ \alpha \cdot u_i(x_1, x_2) & \text{otherwise} \end{cases} \quad (2)$$

Suppose player  $i$  expects player  $-i$  to follow the prescribed investment level. If player  $i$  decides to deviate from  $\bar{x}_i$ , by (2) it should deviate to  $x_i^{BR}(\bar{x}_{-i})$ . It follows that there is no profitable deviation from investment levels  $\bar{x}_1$  and  $\bar{x}_2$  if and only if:

$$\bar{U}_1 \geq \alpha \cdot u_1(x_1^{BR}(\bar{x}_2), \bar{x}_2) \quad \text{and} \quad \bar{U}_2 \geq \alpha \cdot u_2(\bar{x}_1, x_2^{BR}(\bar{x}_1)).$$

Note, therefore, that the game is similar to prisoner's dilemma: the players are jointly better off by maintaining peace, but if a player expects the opponent to invest the prescribed amount under peace, this player may want to trigger a conflict. Crucially, however, here the incentive to deviate is determined by investment levels prescribed by the political institution, which are endogenous.

For given  $\bar{x}_1$  and  $\bar{x}_2$ , there exist  $\bar{U}_1, \bar{U}_2 \leq W(\bar{x}_1, \bar{x}_2) - \bar{U}_1$  that satisfy both constraints if and only if:

$$W(\bar{x}_1, \bar{x}_2) \geq \alpha \cdot u_1(x_1^{BR}(\bar{x}_2), \bar{x}_2) + \alpha \cdot u_2(\bar{x}_1, x_2^{BR}(\bar{x}_1)). \quad (3)$$

Given this, to solve for the optimal political institutions we need to find the  $\bar{x}_1$  and  $\bar{x}_2$  that maximize  $W(\bar{x}_1, \bar{x}_2)$  subject to (3). If such political institutions exist and generates higher welfare than conflict, then the optimal political institutions are the solution to this problem. Otherwise, the equilibrium of the game is conflict.

**Proposition 1.** *If*

$$\begin{aligned} u_1(x_1^{BR}(x_2^{**}), x_2^{**}) &\leq u_1(x_1^*, x_2^*) \\ u_2(x_2^{BR}(x_1^{**}), x_1^{**}) &\leq u_2(x_1^*, x_2^*) \end{aligned} \quad (4)$$

<sup>10</sup> The reason why it is without loss of generality to restrict our attention to such political institutions is because, by definition, they impose the largest possible punishment on any deviation. That is, if a given  $\pi_1(x_1, x_2), \pi_2(x_1, x_2)$  generates an equilibrium level of investments  $\bar{x}_1$  and  $\bar{x}_2$  then also  $\pi_1(x_1, x_2), \pi_2(x_1, x_2)$  as in (2) generates the equilibrium level of investments  $\bar{x}_1$  and  $\bar{x}_2$ .

then the optimal political institutions always achieves the first best. If instead

$$\begin{aligned} u_1(x_1^{BR}(x_2^{**}), x_2^{**}) &> u_1(x_1^*, x_2^*) \\ u_2(x_2^{BR}(x_1^{**}), x_1^{**}) &> u_2(x_1^*, x_2^*) \end{aligned} \tag{5}$$

there exists an  $\hat{\alpha} < \tilde{\alpha}$  (where  $\tilde{\alpha}$  is defined in 1) such that for  $\alpha \leq \hat{\alpha}$  the optimal political institutions achieve the first best, while for  $\hat{\alpha} < \alpha < \tilde{\alpha}$  no political institutions achieve the first best.<sup>11</sup>

To understand the above proposition, note that  $\alpha \cdot u_1(x_1^*, x_2^*)$  and  $\alpha \cdot u_2(x_1^*, x_2^*)$  are the utilities in case of conflict and hence they are the *ex-ante* outside options: the players' best alternative to setting up common political institutions. Instead  $\alpha \cdot u_1(x_1^{BR}(x_2^{**}), x_2^{**})$  and  $\alpha \cdot u_2(x_1^{**}, x_2^{BR}(x_1^{**}))$  are the utility that each player earns by triggering a conflict against an opponent that invested the first best level. They are the players' *ex-post* outside option (assuming the first best level of investment).

The proposition therefore makes clear that when the ex-ante outside option is greater than the ex post outside option the first best is always achievable. This would be the case if, for example, the payoff in case of conflict is independent from the players' investment. It would also be the case if the first-best level of investment is, for the most part, not appropriable in case of conflict (see Section 3.1 for an example). It corresponds to the “textbook” hold up problem, in which the fact that the ex-post outside option is endogenous is irrelevant, and hence because of full observability and full contractibility the first best is always achieved. When the ex-post outside option is above the ex-ante outside option, instead, whether the first best is achievable depends on how large is the benefit of peace (as measured by  $\alpha$ ).

If the first best is not achievable, the optimal political institutions will need to distort the investment levels so that (3) is satisfied with equality. This can only be achieved by reducing the RHS of (3). That is, in order to maximize welfare, the political institutions will need to distort the investment mix so to make conflict more costly. Doing so increases the punishment that the political institutions can impose on each player in case of deviation (the RHS of 3). At the same time, it also reduces the peace payoff below the first best, and with it the benefit of maintaining

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<sup>11</sup> Note that the proposition implicitly assumes that  $\{x_2^{**}, x_2^{**}\}$  is unique. If it is not unique, then the optimal political institutions always achieve the first best if there exists a  $x_1, x_2 \in \{x_2^{**}, x_2^{**}\}$  such that 4 holds. If instead 5 holds for all  $x_1, x_2 \in \{x_2^{**}, x_2^{**}\}$ , then the first best is achievable for low  $\alpha$  but not for high  $\alpha$ .

peace (the LHS of 3). As a consequence, at the  $\bar{x}_1, \bar{x}_2$  such that (3) is satisfied with equality, we may have

$$W(\bar{x}_1, \bar{x}_2) < \alpha \cdot u(x_1^*, x_2^*) + \alpha \cdot u(x_2^*, x_1^*),$$

that is, the distortion in the investment mix required to maintain peace is so severe, that conflict is preferred to such peace. It is also possible that there is no value of  $\bar{x}_1, \bar{x}_2$  that satisfies (3), in which case the only possible outcome is conflict. The next lemma provides sufficient condition for conflict to emerge, either because it is the most efficient outcome, or because it is the only possible outcome.

**Lemma 1.** *If  $\alpha$  is sufficiently close to  $\tilde{\alpha}$  and (5) holds, then the unique outcome is conflict.*

Hence, despite the fact that peace is efficient, achieving peace may not be desirable or even possible if the investment mix needs to be distorted in order to prevent the players from triggering conflict. Finally, note that whenever neither (4) nor (5) hold, then whether the first best is achievable depends not only on the conflict function  $u(., .)$ , but also on the payoff in case of peace  $W(x_1^{**}, x_2^{**})$ .

### 3 Examl: guns and butter.

We now consider a conflict game that is a special case of the guns and butter model in Skaperdas (1992). Unlike in our paper, in Skaperdas (1992) the players are not allowed to set up political institutions and hence, if they are symmetric the outcome of the game is conflict.<sup>12</sup> One of our goal here is, therefore, to make transparent how the possibility of choosing political insitutions changes Skaperdas (1992)'s results. Our second goal is to generate more detailed predictions with respect to the distortions that the optimal political institutions may need to impose in order to maintain peace.

The players' investment levels are here  $x_i = \{g_i, b_i\}$ , where  $g_i \geq 0$  are guns and  $b_i \geq 0$  is butter, with  $g_i + b_i = 1$ . In case of peace, total surplus to be shared is  $b_1 + b_2$ . The first-best level of welfare is equal to 2, which is achieved by investing all resources in butter. In case of conflict, instead, player  $i$  earns  $\alpha(b_1 + b_2)$  with probability

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<sup>12</sup> If the players are asymmetric, then Skaperdas (1992) shows that conflict may be avoided also in the absence of political institutions.

$g_i/(g_1 + g_2)$ , where  $\alpha \geq 0$ . If no player invests in guns and a conflict occurs, each player probability of winning is  $1/2$ . Butter therefore represents investments that are productive both in peace and in case of conflict (but possibly differently so depending on  $\alpha$ ). Guns instead are non-productive investments that increase the probability of winning a conflict.

Again, the parameter  $\alpha$  measures the inefficiency of conflict. For example, the use of guns during a conflict may destroy part of the investment in butter, which implies  $\alpha < 1$ . If instead  $\alpha > 1$ , then a given investment in butter generates higher utility in conflict than in peace. We do not think that this last case is particularly realistic,<sup>13</sup> but we will nonetheless consider it in our analysis to illustrate the theoretical possibility that an inefficient conflict is the unique outcome of the game.

**Conflict.** We start by solving the conflict game. The two best responses are:

$$g_i^{BR}(x_{-i}) = \sqrt{2g_{-i}} - g_{-i}, \quad b_i^{BR}(x_{-i}) = 1 - g_i^{BR}(x_{-i})$$

The Nash equilibrium is  $g_1^* = g_2^* = \frac{1}{2}$ ,  $b_1^* = b_2^* = \frac{1}{2}$ ,  $e_1^* = e_2^* = 0$ . Social surplus in case of conflict is equal to  $\alpha$ . Assumption (A1) holds as long as  $\alpha < 2$ , which we assume.

**Optimal political institutions.** To start, note that, here (5) holds and therefore, by Proposition 1 and Lemma 1, we should expect that for low values of  $\alpha$  the first best is achievable, for intermediate values of  $\alpha$  the first best is not achievable but may be possible to achieve peace, for high values of  $\alpha$  conflict is the unique outcome.

The political institutions set mandatory investment levels  $\bar{b}_1 \geq 0, \bar{b}_2 \geq 0, \bar{g}_1 = 1 - \bar{b}_1 \geq 0, \bar{g}_2 = 1 - \bar{b}_2 \geq 0$  under the threat of a punishment that cannot exceed a player's outside option. Equation (3) here is equivalent to:

$$2 - (\bar{g}_1 + \bar{g}_2) \geq \alpha \left( \sqrt{2} - \sqrt{\bar{g}_1} \right)^2 + \alpha \left( \sqrt{2} - \sqrt{\bar{g}_2} \right)^2 \quad (6)$$

The important thing to note is that mandating a given investment in guns decreases each player's incentive to deviate, because each player anticipates that, if he deviates, he will fight against a stronger opponent. Investing in guns, however, generates a welfare loss and makes maintaining peace less valuable.

<sup>13</sup> This is not to say that it is completely unreasonable. For example, it is a known fact that the marginal utility of consumption of some goods increases with the level of stress.

**Proposition 2.** *The first best is achievable if and only if  $\alpha \leq \frac{1}{2}$ .*

The above proposition follows by simple inspection of (6). Quite intuitively, at the first-best level of investment each player trigger a conflict and capture the entire surplus by investing arbitrarily little in guns. It is possible to prevent both players from deviating only if conflict destroys at least half of the surplus, so that the sum of the utilities from deviating is below the first-best level of welfare.

If, instead  $\alpha > \frac{1}{2}$ , then conflict is not sufficiently destructive and, as a consequence, the first best is not achievable. Hence the optimal political process will need to impose positive investment in guns, so to make (7) binding.

**Proposition 3.** *Whenever  $1/2 < \alpha \leq 1$ , the optimal political institutions maintain peace by imposing*

$$\bar{g}_1 = \bar{g}_2 = \frac{1}{2} \left( \frac{2\alpha - \sqrt{2(1-\alpha)}}{\alpha + 1} \right)^2$$

*Social welfare is strictly decreasing in  $\alpha$ , and equal to social welfare in case of conflict for  $\alpha = 1$ .*

*Whenever  $\alpha > 1$  then it is not possible to satisfy (6) and the unique outcome of the game is conflict.*

Figure 2 plots total investment in guns and social welfare in equilibrium. The bottom line is that conflict is avoided by setting up political institutions that require both players to make a positive investment in guns. This investment in guns decreases with the destructiveness of the conflict (as measured by  $\alpha$ ) because, as  $\alpha$  decreases, a smaller investment in guns by a player is required in order to “punish” the other player in case of deviation. It follows that if  $\alpha$  is sufficiently small welfare in case of peace achieves the first best level. If instead  $\alpha$  is sufficiently large the required punishment is so large that it is not possible to maintain peace, and an inefficient conflict is the only outcome. Contrast this result with Skaperdas (1992), who does not allow the players to agree on the political institutions ex ante and therefore finds that conflict is the unique equilibrium of the game for every  $\alpha$ . Note also that, here, the equilibrium level of welfare is non-monotonic in  $\alpha$ : it decreases in  $\alpha$  whenever the political institutions can maintain peace, and increase in  $\alpha$  in case of conflict. In Skaperdas (1992), instead, because conflict is the unique outcome social welfare is always increasing in  $\alpha$ .

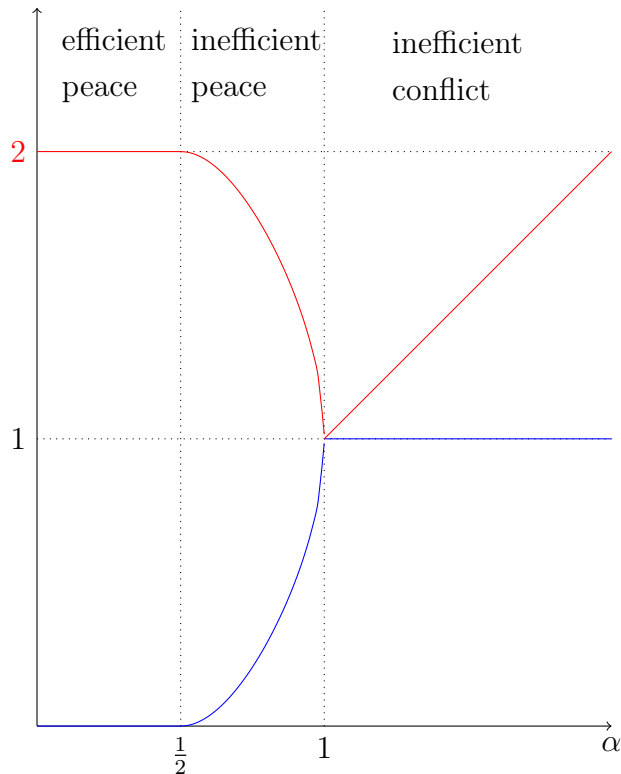


Fig. 2: Equilibrium investment in guns (blue line) and social welfare (red line)

### 3.1 Extension: guns, butter and eggs

The general model shows that, in order to maintain peace, the political institutions may distort the players' investment away from the first best level. In the simple model above, this distortion takes the form of requiring players to invest in arms. But, in general, other distortion may emerge. For example, there could be multiple types of productive investments, some more appropriable than others in case of conflict. In this case, it is possible that the political institutions will tilt the investment mix toward the least appropriable productive investment in order to reduce the players' payoff in case of conflict.

To illustrate this possibility, we consider a variation of first example in which there are three types of investments: guns  $g_i$ , butter  $b_i$  and eggs  $e_i$  with  $g_i + b_i + e_i = 1$ . Eggs are valuable in case of peace, but may be less valuable than butter: total surplus in case of peace is  $b_1 + b_2 + \tau(e_1 + e_2)$  for  $\tau \in [0, 1]$ , where  $\tau$  is a parameter measuring the marginal rate of technical substitution between butter and eggs in case of peace.

Again, the first-best social surplus is 2. In case of conflict, player  $i$  earns  $\alpha(b_1 + b_2)$  with probability  $g_i/(g_1 + g_2)$ .

Hence, whereas butter is valuable both in peace and in conflict, eggs are valuable only in peace (because they easily break). For example, butter could represent physical capital while eggs could represent human capital. Furthermore, producing butter is always more efficient than producing eggs (strictly so if  $\tau < 1$ ), but more so in case of conflict than in case of peace. As we will see, this implies that, to discourage conflict, the optimal political institutions may mandate positive investment in eggs even if  $\tau < 1$ .

**Conflict.** The fact that eggs are not valuable in case of conflict implies that the conflict game is a standard “guns and butter” game as in Skaperdas (1992). The two best responses are:

$$g_i^{BR}(x_{-i}) = \sqrt{(2 - e_{-i})g_{-i} - g_{-i}}, \quad b_i^{BR}(x_{-i}) = 1 - g_i^{BR}(x_{-i}), \quad e_i^{BR}(x_{-i}) = 0.$$

The Nash equilibrium is, again,  $g_1^* = g_2^* = \frac{1}{2}$ ,  $b_1^* = b_2^* = \frac{1}{2}$ ,  $e_1^* = e_2^* = 0$ . Social surplus in case of conflict is, again, equal to  $\alpha$ , and therefore (A1) holds.

**Optimal political institutions.** In case  $\tau = 1$ , condition (4) applies and, by Proposition 1, the first best is achievable. If  $\tau < 1$ , condition (5) applies and, again by Proposition 1 and Lemma 1, for low values of  $\alpha$  the first best is achievable, for intermediate values of  $\alpha$  the first best is not achievable but peace may be possible, for high values of  $\alpha$  conflict is the unique outcome. Of course, the difference with the “guns and butter” model presented earlier is that the thresholds determining what case emerges here will depend on  $\tau$ .

Again, the political institutions set mandatory investment levels  $\bar{b}_1, \bar{b}_2, \bar{e}_1, \bar{e}_2, \bar{g}_1, \bar{g}_2$  under the threat of a punishment that cannot exceed a player’s outside option. Equation (3) here is equivalent to:

$$2 - (1 - \tau)(\bar{e}_1 + \bar{e}_2) - (\bar{g}_1 + \bar{g}_2) \geq \alpha (\sqrt{2 - \bar{e}_1} - \sqrt{\bar{g}_1})^2 + \alpha (\sqrt{2 - \bar{e}_2} - \sqrt{\bar{g}_2})^2 \quad (7)$$

Plus two feasibility constraints:  $0 \leq \bar{e}_i \leq 1$  and  $0 \leq \bar{g}_i \leq 1 - e_i$ .

Similarly to mandating a given investment in guns, also mandating a given investment in eggs decreases each player’s incentive to deviate. Investing in eggs implies that social surplus is less appropriable by the other player in case of conflict. Also in the case of eggs, however, preventing conflict may come at the cost of reducing the surplus in case of peace.

The next proposition shows that introducing the possibility of investing in eggs expands the range of  $\alpha$  for which it is possible to prevent conflict.

**Proposition 4.** *if  $\alpha \leq 1$  or  $\tau = 1$  it is always possible to maintain peace (that is, to satisfy 10). If  $\alpha \geq \frac{\sqrt{5}+1}{2}$  instead it is never possible to maintain peace. If  $1 < \alpha < \frac{\sqrt{5}+1}{2}$  it is possible to maintain peace for  $\tau \geq \frac{\alpha}{1-\alpha^2+2\alpha}$  but not otherwise.*

Note that the above proposition does not address the question of when the political process will want to maintain peace. That is, it is possible that peace can be maintained but the distortion required is so large that conflict is preferred to peace. We return to this point later (see Corollary 1).

The next proposition provides the full solution for the case  $\alpha = 1$ .

**Proposition 5.** *Assume  $\alpha = 1$ . If  $\tau < 4/7$  then the solution is again the one derived in Proposition 3:  $\bar{g}_1 = \bar{g}_2 = \bar{b}_1 = \bar{b}_2 = 1/2$ ,  $\bar{e}_1 = \bar{e}_2 = 0$ , welfare in case of peace is equal to welfare in case of conflict.*

*If instead  $\tau \geq 4/7$ , then*

$$\bar{e}_1 = \bar{e}_2 = \frac{3\tau + 2\sqrt{\tau(2\tau - 1)}}{\tau(\tau + 4)} \quad \bar{g}_1 = \bar{g}_2 = \frac{\tau(\tau + 1) - 2\sqrt{\tau(2\tau - 1)}}{\tau(\tau + 4)} \quad \bar{b}_1 = \bar{b}_2 = 0$$

*welfare in case of peace is strictly greater than welfare in case of conflict, increasing in  $\tau$  and converging to its first-best level for  $\tau \rightarrow 1$ .*

Figure 3 provides a graphical representation of the solution. For low  $\tau$  the optimal political process will impose positive investment in butter and guns, but no investment in eggs. The solution is therefore the same derived in the previous section. For higher  $\tau$  instead the investment in eggs will be positive, and will be used to maintain peace also for  $\alpha$  such that, absent eggs, there would be conflict.

Perhaps surprisingly, a positive investment in guns is always required in order to maintain peace, even when the investment in eggs is positive. The reason is that, if all resources are invested in eggs and there is peace, total surplus is  $2\tau$  and each player receives  $\tau$ . A deviating player could instead invest almost all his resources in butter and very little resources in guns, then trigger a conflict and enjoy a utility equal to (approximately) 1. That is, because butter is more productive than eggs, when all resources are invested in eggs a player may deviate not to appropriate the other player's resources, but rather to switch from investing in eggs to investing in butter. As a consequence, if  $\tau < 1$  to prevent this deviation some resources will need to be invested in guns.

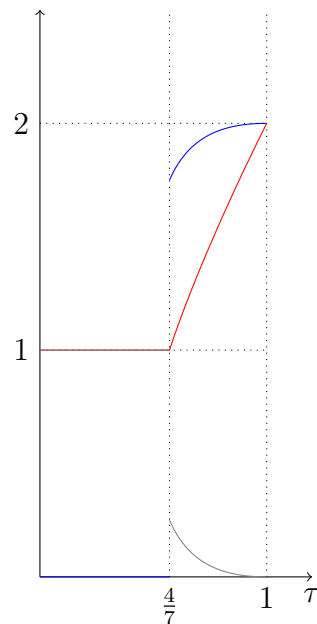


Fig. 3: Equilibrium investment in eggs (blue line), in guns (gray line) and social welfare (red line)

Finally, the above proposition serves to illustrate the fact that sometimes the political institutions could maintain peace, but at the cost of a distortions so large that conflict is preferred to peace.

**Corollary 1.** *Suppose  $\alpha > 1$  but approximately close to 1. Suppose  $\tau \in (1/2, 4/7)$ . Then peace could be maintained but conflict is preferred to peace.*

The fact that peace could be maintained follows directly from Proposition 4. The fact that conflict is preferred to peace follows by continuity to the case  $\alpha = 1$  considered in Proposition 5. The only difference is that when  $\alpha = 1$  it is possible to maintain peace without investing in eggs, leading to the same social welfare as conflict. If  $\tau \in (1/2, 4/7)$ , either peace without eggs or conflict are strictly preferred to a peace with eggs. If  $\alpha$  is just above 1, instead, it is not possible to maintain peace without eggs. Nonetheless, by continuity welfare in case of conflict is strictly preferred to a peace with eggs.

## 4 Conclusion

In this paper, we connect Hobbes' political philosophy with modern contract theory. We consider a model in which two groups set up common political institutions and then decide on a vector of investments. Political institutions are modeled as an abstract mechanism that allocate payoffs to the players as a function of their investment.

Each group can, ex-post, trigger a conflict that dissolves these political institutions. The political institutions are therefore "in the shadow of conflict": the payoff they allocate to the two groups cannot be below what these groups can obtain from conflict. We abstract away from all other forms of frictions and imperfections. Despite this, we find that the first best may not be achievable, in the sense that the optimal political institutions may need to distort the players' investment mix away from the first best. To better illustrate what these distortions may look like, we consider a guns and butter model á la Skaperdas (1992) and show that the political institutions may require the players to invest in guns. We also consider an extension in which there are multiple productive investments, and show that the optimal political institutions may distort the investment mix toward productive investments that are less appropriable in case of conflict. Finally, it is possible that an inefficient conflict is the unique outcome of the game. This will happen when the distortion required to maintain peace are too large.

## Mathematical Appendix

*Proof of 1.* By evaluating (3) at the first best level of investment, it is immediate to establish that the first best is achievable if and only if:

$$W(x_1^{**}, x_2^{**}) \geq \alpha \cdot u_1(x_1^{BR}(x_2^{**}), x_2^{**}) + \alpha \cdot u(x_1^{**}, x_2^{BR}(x_1^{**})). \quad (8)$$

By (A1), the above condition is always satisfied whenever 4 holds. This establishes the first part of the proposition.

For the second part, note that 5 implies

$$u_1(x_1^{BR}(x_2^{**}), x_2^{**}) + u_2(x_2^{BR}(x_1^{**}), x_1^{**}) > u_1(x_1^*, x_2^*) + u_2(x_1^*, x_2^*).$$

The second part of the proposition follows by defining  $\hat{a}$  as:

$$\hat{a} \equiv \frac{W(x_1^{**}, x_2^{**})}{u_1(x_1^{BR}(x_2^{**}), x_2^{**}) + u_2(x_2^{BR}(x_1^{**}), x_1^{**})}$$

so that for  $\alpha \leq \hat{\alpha}$  then (8) holds, but for  $\hat{\alpha} < \alpha < \tilde{\alpha}$  (A1) is satisfied but (8) is violated.

□

*Proof of Lemma 1.* Assume that  $\alpha = \tilde{\alpha}$ , so that conflict achieves the first best. If (3) has no solution, then it is not possible to achieve peace. If instead (3) has a solution, by the previous proposition it must be at some  $\bar{x}_1, \bar{x}_2$  different from  $x_1^{**}, x_2^{**}$ , which implies that peace at  $\{\bar{x}_1, \bar{x}_2\} \neq \{x_1^{**}, x_2^{**}\}$  is strictly worse than conflict. The proposition follows by continuity. □

*Proof of Proposition 3.* Call  $G$  the total investment in guns, with  $\beta$  the fraction of  $G$  invested by player 1, so that  $\bar{g}_1 = \beta G$ . Constraint (6) becomes

$$G(1 + \alpha) + 2(2\alpha - 1) \leq 2\alpha\sqrt{G}(\sqrt{2\beta} + \sqrt{2(1 - \beta)}) \quad (9)$$

If the LHS of the above inequality crosses its RHS, it will actually cross twice. The smallest  $G$  that satisfies (10) is the smallest of such intercepts, where the LHS of (10) crosses its RHS from below. This  $G$  is minimized whenever the RHS of (10) is maximized, which happens at  $\beta = 1/2$ . At this  $\beta$  (10) becomes

$$G(1 + \alpha) + 2(2\alpha - 1) \leq +4\alpha\sqrt{G}$$

with solution

$$G^* \equiv \left( \frac{2\alpha - \sqrt{2(1 - \alpha)}}{\alpha + 1} \right)^2$$

If  $\alpha \leq 1$ , the above solution always exists. It is also easy to check that social welfare in case of peace is strictly greater than social welfare in case of conflict for  $\alpha < 1$  and is equal to social welfare in case of peace for  $\alpha = 1$ . Social welfare in case of peace is also strictly decreasing (for a numerical solution, see Figure 2).

If instead  $\alpha > 1$ , then  $G^*$  does not exist. It is not possible to satisfy (10) and hence conflict is the only outcome. □

*Proof of Proposition 4.* Call  $G$  the total investment in guns, with  $\beta$  the fraction of  $G$  invested by player 1, so that  $\bar{g}_1 = \beta G$ . Call  $E$  the total expenditure in eggs, with  $\gamma$  the fraction invested by player 1, so that  $\bar{e}_1 = \gamma E$ . Constraint (7) becomes

$$G(1 + \alpha) + 2(2\alpha - 1) \leq (\tau + \alpha - 1)E + 2\alpha\sqrt{G}(\sqrt{(2 - \gamma E)\beta} + \sqrt{(2 - (1 - \gamma)E)(1 - \beta)}) \quad (10)$$

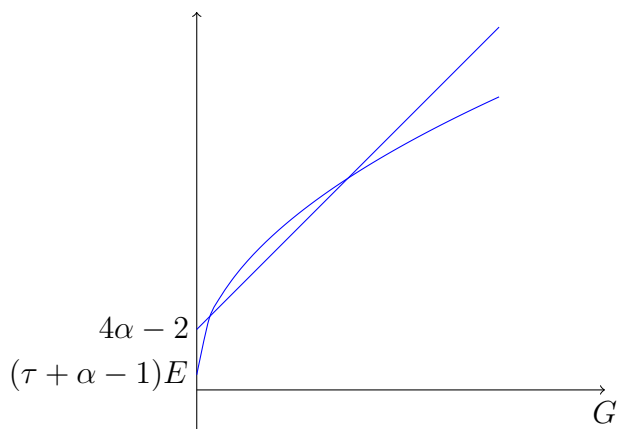


Fig. 4: LHS and RHS of 10, case 2.

We fix  $E$  and look for the smallest  $G$  that satisfies the above constraint for some  $\beta$  and  $\gamma$ .

We distinguish between two cases. Case 1 is:

$$2(2\alpha - 1) \leq (\tau + \alpha - 1)E,$$

In this case (10) holds at  $G = 0$ , which is therefore the welfare maximizing  $G$  for given  $E$ , independently from  $\gamma$  and  $\beta$ . Note that if  $\alpha \leq 1/2$  the above inequality holds at  $E = 0$ , which implies that the first best is achievable. If instead  $\alpha > 1/2$  (which is what we assume here) then if  $\tau + \alpha \leq 1$  the above inequality never holds and hence we are never in this case. If  $\tau + \alpha > 1$  the above inequality holds for  $E \geq \frac{2(1-2\alpha)}{\alpha+\tau-1}$ . Note that because  $E$  is chosen optimally, then this case can emerge because  $E = \frac{2(1-2\alpha)}{\alpha+\tau-1}$ , which is also a subcase of case 2 (below).

Case 2 is:

$$2(2\alpha - 1) \geq (\tau + \alpha - 1)E.$$

In this case the LHS of (10) crosses its RHS twice (see Figure 4). The smallest  $G$  that satisfies (10) is the smallest of such intercepts, where the LHS of (10) crosses its RHS from below. This  $G$  is minimized whenever the RHS of (10) is maximized. For given  $\gamma$ , the RHS is maximized at  $\beta = (2 - \gamma E)/(4 - E)$ . By plugging this value of  $\beta$  into the RHS of 10, we see that the  $\gamma$  drops out. There are therefore multiple possible combinations of  $\gamma$  and  $\beta$  that maximize the RHS of (10). Among these solutions, the one at which the feasibility constraint is more likely to hold is

$\beta = \gamma = 1/2$ . At those  $\gamma$  and  $\beta$  (10) becomes

$$G(1 + \alpha) + 2(2\alpha - 1) \leq (\tau + \alpha - 1)E + 2\alpha\sqrt{G(4 - E)}$$

with solution

$$\begin{aligned} G \geq G(E) &\equiv \left( \frac{\alpha\sqrt{4 - E} - \sqrt{E(\tau(1 + \alpha) - 1) + 2(1 - \alpha)}}{\alpha + 1} \right)^2 \\ &= \frac{E((\alpha + 1)\tau - \alpha^2 - 1) + 2(2\alpha^2 - \alpha + 1) - 2\alpha\sqrt{A(E)}}{(\alpha + 1)^2} \end{aligned}$$

where

$$A(E) \equiv (4 - E) [E(\tau(1 + \alpha) - 1) + 2(1 - \alpha)]$$

The key observation is that  $G(E)$  may not exist. When this is the case, there is no political process that satisfies the no-deviation constraint. The existence of  $G(E)$  for some feasible  $E$  depends on cases:

- If  $\alpha \leq 1$ , then  $G(E)$  exists for some feasible  $E$ . To see this, just consider  $E = 0$ , so that  $G(0) = \left( \frac{2\alpha - \sqrt{2(1 - \alpha)}}{\alpha + 1} \right)^2$  which is feasible because simple algebra shows that  $G(0) < 2$ .
- If  $\alpha > 1$  and  $\tau \leq \frac{1}{1 + \alpha}$  then  $G(E)$  never exists. If instead  $\tau > \frac{1}{1 + \alpha}$  then  $G(E)$  exists for  $E$  sufficiently large. The  $E$  such that  $G(E)$  exists may, however, not be feasible. To see this, consider the smallest  $E$  such that  $G(E)$  exists (that is, such that  $A(E) = 0$ ):  $E = 2\frac{\alpha - 1}{\tau(\alpha + 1) - 1}$ . At this  $E$  the feasibility constraint holds if:

$$E + G(E) = 2\frac{\alpha - 1}{\tau(\alpha + 1) - 1} \left( 1 + \frac{(\alpha + 1)\tau - \alpha^2 - 1}{(1 + \alpha)^2} \right) + \frac{2(2\alpha^2 - \alpha + 1)}{(1 + \alpha)^2} \leq 2$$

or

$$\tau \geq \bar{\tau} \equiv \frac{\alpha}{1 - \alpha^2 + 2\alpha}$$

To conclude, note that  $\bar{\tau} > \frac{1}{1 + \alpha}$  whenever  $\alpha > 1$ . Hence, whenever  $\alpha > 1$ ,  $\tau \geq \bar{\tau}$  guarantees the existence of  $G(E)$  at some  $E$ . Also, because  $\tau$  must be below 1,  $\tau \geq \bar{\tau}$  never holds if  $\alpha \geq \frac{\sqrt{5} + 1}{2}$ .

□

*Proof of Proposition 5.* In the proof of Proposition 4 we derived  $G(E)$ , which, if  $\alpha = 1$ , becomes

$$G(E) = \frac{2 - E(1 - \tau) - \sqrt{(4 - E)E(2\tau - 1)}}{2}$$

If  $\tau < 1/2$ , then the only solution is  $E = 0$ ,  $G = 1$ , which is the same solution derived in the model without eggs. If instead  $\tau \geq 1/2$  then  $G(E)$  exists for all feasible values of  $E$ .

The value of  $E$  is chosen to minimize  $G(E) + (1 - \tau)E$ , which here becomes

$$1 + (1 - \tau)E - \frac{\sqrt{(4 - E)E(2\tau - 1)}}{2}$$

There are three possible solutions: a corner solution at  $E = 0$ , a corner solution at  $E + G(E) = 2$ , and an interior solution.

Taking first order conditions we get

$$E = 2 + \frac{1 - \tau}{\sqrt{4\tau^2 - 6\tau + 3}}$$

which is, however, not feasible.

At the corner solution  $E = 0$  we are back at the case without eggs, and social welfare is 1.

At the other corner solution we have  $E + G(E) = 2$  or

$$E = \frac{6\tau + 4\sqrt{\tau(2\tau - 1)}}{\tau(\tau + 4)} \quad G = \frac{2\tau(\tau + 1) - 4\sqrt{\tau(2\tau - 1)}}{\tau(\tau + 4)}$$

and no investment in butter. Social welfare is

$$\tau \frac{6\tau + 4\sqrt{\tau(2\tau - 1)}}{\tau(\tau + 4)}$$

which is greater than 1 only if  $\tau > 4/7$ .

□

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