The Italian Interbank Network: statistical properties and a simple model

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ABSTRACT

We use the theory of complex networks in order to quantitatively characterize the structure of reciprocal expositions of Italian banks in the interbank money market market. We observe two main different strategies of banks: small banks tend to be the lender of the system, while large banks are borrowers. We propose a model to reproduce the main statistical features of this market. Moreover the network analysis allows us to investigate properties of robustness of this system.

1. SOCIO-ECONOMIC NETWORKS

In the recent years we observe an increasing interest in complex networks theory. Looking at the recent literature, we observe two parallel lines of research: on the one side many researcher from different scientific fields (Biology, Computer Science, Physics) focus on a description of the complex structure of interactions in many complex systems (ranging from Internet to the WWW, from the protein interaction to the food web) and on their scaling properties;[1] on the other side, Social Scientists and Economists focus on application of networks in order to understand the role of heterogeneous interactions in the formation of aggregate behavior of systems composed by many agents. In this paper we want to consider both lines of research and use the recently developed statistical tools for complex networks in order to get some insights in the most crucial economics issues.

In particular we study the credit-debt relationships of banks in the Italian Interbank market focusing on the amount, maturity and interest rate of reciprocal exposures. The investigation of this system is very important not only to have a statistical description of the overall structure of relationships, but also to better understand possible mechanisms of systemic risk in the Italian bank system.

2. A SURVEY OF THE MONEY MARKET

The interbank market emerges as a consequence of the need of banks to manage their liquidity. Liquidity management in the banking system is essential for a smooth operate of payment systems, and in particular real time gross settlement (RTGS) systems. For example, in the Euro area the European Central Bank (ECB) normally aims to satisfy the liquidity needs of the banking system via its open market operations (main and long-term re-financing operations, fine-tuning and structural interventions) the most relevant of which are the weekly auctions. Auctions were executed at a fix term rate until June 2000 and since then are conducted with variable rate. The ECB decides in advance the minimum bid rate and the fixed amount of liquidity to be supplied through the auction. On Mondays afternoon credit institutions present their bids to the respective national central banks (NCB). Banks submit the amount of money they want to deal and interest rate they are ready to pay for it. The ECB collects bids on Tuesdays morning and executes the auctions. The allocations are settled on the bank's account to the NCB on Wednesdays. The Euro-system also offers credit institutions two standing facilities: the marginal lending facility in order to obtain overnight liquidity from the central bank, against the presentation of sufficient eligible assets; and the deposit facility in order to make overnight deposits with the central bank.

Credit institutions in the Euro area are required to hold minimum reserve balances with NCBs (set at 2% of all deposits and debts issued with a maturity of less than two years, excluding repos and interbank liabilities, but with a minimum threshold applied). Reserves provide a buffer against unexpected liquidity shocks, mitigating the related fluctuations of market rates. They have to be fulfilled only on average over a *one-month maintenance period* that runs from the 24th of a month to the 23rd of the following month (when this is not a holiday in which case is anticipated to the previous working day).

3. THE SPECIAL CASE OF ITALIAN MONEY MARKET

The interbank markets are basically managed by each European country.[4] These markets are in almost all case phone-based, that means that each bank has some brokers doing their transactions by phone. The only exception is the Italian market, which is totally screen-based, implying that each banks operator can see real time quotes of all other banks and do its transaction.

The recent paper by Boss et al[5] investigate the network of overall credit relationships in the Austrian Interbank market. In their study the authors analyze all the liabilities for ten quarterly single months periods, between 2000 and 2003, among 900 banks. They find a power-law distribution of contract sizes, and a power-law decay of the distribution of incoming and outgoing links (a link between two banks exists if the banks have an overall exposure with each other). Furthermore they show that the most vulnerable vertices are those with the highest centrality (measured by the number of paths that go through them).

A different issue has been explored by Cocco et al.[6] who have investigated the nature of lending relationships in the fragmented Portuguese interbank market over the period 1997-2001. In fragmented markets the amount and the interest rate on each loan are agreed on a one-to-one basis between borrowing and lending institutions. Other banks do not have access to the same terms, and no public information regarding the loan is available. The authors showed that frequent and repeated interactions between the same banks appear with a probability higher than those expected for random matching. In addition they found that during illiquid periods, and in particular during the Russian financial crisis preferential lending relationships increased.

Our paper focuses on the network analysis of the overnight maturity on the e-MID interbank market.[?] This market is unique in the Euro area in being screen based and fully electronic: outside Italy interbank trades are largely bilateral or undertaken via voice brokers. While banks can still choose with whom to trade, the information about the rates and the trades are public.

The Italian electronic broker market MID (Market for Interbank Deposits)[*] covers the entire existing domestic overnight deposit market in Italy. Both Italian banks and foreign banks can exchange funds on the e-MID. The participating banks was 215 in 1999, 196 in 2000, 183 in 2001 and 177 in 2002. Rates can be expressed in basis points or 1/16ths for the Euro and in basis points or 1/32nds for the US Dollar. The minimum quote is 1.5 million Euros and 5 Million US dollars. Each quote is identified as an offer or a bid. An offer indicates that the transaction has been concluded at the selling price of the quoting bank while a bid indicates that a transaction has been concluded at the buying price of the quoting bank. The names of the banks are visible next to their quotes to facilitate credit line checking. Quotes can be submitted and a transaction is finalized if the ordering bank accepts a listed bid/offer. When a bid rate is hit, the transaction can be executed automatically or manually within 90 seconds, if a bank prefers to first check the lending counterpart name. Whilst in the case of a hit on an offer rate, this always needs to be accepted manually within 90 seconds, to allow for credit line checking. The market also permits bilateral trades with a specific counterpart of choice[†].

4. DATA SET

Our data set is composed by banks operating on the Italian market for which we have the complete record of all transactions. The data set used to construct the interbank network includes only overnight transactions and consists of encoded name of first bank, encoded name of second bank, amount of transaction, rate applied to transactions, rate. To de-codify the name of banks, a list of banks has been provided: the first number is the label of the bank, the second one is the group of classification of that bank provided by Italian National Bank

[*]e-MID is run by e-MID S.p.A. Società Interbancaria per l'Automazione (SIA), Milan. The central system is located in the office of the SIA and the peripherals on the premises of the member participants.

[†]For more details see http://www.e-mid.it/index.php/article/articleview/85/0/29/.

(1= foreign banks, 2= big Italian banks, 3= medium Italian banks, 4= small Italian banks, 5= cooperative credit banks).

For every day of trading, we compute the network of debts/loans.

Our data set consists of all the overnight transactions concluded on the e-MID from January 1999 to December 2002 for a total of 586,007 transactions. For each contract we have information about the date and time of the trade, the quantity, the interest rate and the name of the quoting and ordering bank. The information about the parties involved in a transaction allows us to perform an accurate daily analysis of the connectivity among banks and its change over time.

5. NETWORK ANALYSIS

Given our dataset, we can define three daily matrices: the adjacency matrix A, the connectivity matrix C and the weighted connectivity matrix W.

The element of the adjacency matrix a_{ij} indicate if a transaction between bank i and bank j has occurred during a given day, i.e. $a_{ij} = 0$ if no transaction has occurred and $a_{ij} = 1$ if at least one transaction has occurred.

The elements of the connectivity matrix c_{ij} denote the number of transactions between bank i and bank j in a given day.

The elements of the weighted connectivity matrix w_{ij} denote the overall volume exchanged between bank i and bank j in a given day. The number of active links in the network is defined as $N_l = \sum_{ij} a_{ij}/2$, the number of transactions as $N_T = \sum_{ij} c_{ij}/2$ and the overall trading volume as $V = \sum_{ij} w_{ij}/2$. We denote the number of active banks as N_b.

The three matrices, A, C, W, define non-directed graphs in the sense that the links are bi-directional with $a_{ij} = a_{ji}$, $c_{ij} = c_{ji}$ and $w_{ij} = w_{ji}$. Our dataset also allow us to construct matrices associated to directed graphs. We can make links directional by allowing them to follow the flow of money, so that a link is incoming to the buyer and outgoing from the seller. A directed graph may be more relevant if one was interested in assessing the risk of contagion[‡] and systemic default in the system. Hence we define six more matrices A^b, A^l, C^b, C^l and W^b, W^l. The elements a^b_{ij} (a^l_{ij}) indicate if at least one transaction has occurred on a given day between bank i and bank j with bank i as the borrowing (lending) bank. The elements of the connectivity matrix c^b_{ij} (c^l_{ij}) denote the number of transactions on a given day between bank i and bank j with bank i as the borrowing (lending) bank. The elements of the weighted connectivity matrix w^b_{ij} (w^l_{ij}) denote the overall volume exchanged on a given day between bank i and bank j with bank i as the borrowing (lending) bank. Obviously $w^l_{ij} = w^b_{ji}$. We define the flow between two banks as $f_{ij} = w^l_{ij} - w^b_{ij}$. The flow is positive if the bank is a net lender. With this convention we define a weighted graph, whose plot is in figure 1

Highly interconnected systems have been the focus of a great body of research in Computer Science, Physics and the Social Sciences. As stated in the Chapter 1, recently the focus has shifted to weighted networks. A set of metrics combining weighted and topological observable have been proposed to characterize the statistical properties of the strength of edges and vertices and to investigate the correlations among weighted quantities and the underlying topological structure. We remember the commonly used metrics with some more general definitions:

- *Degree*

  The degree of a node is defined as

  $$k_i = \sum_{j \in \mathcal{V}(i)} a_{ij}, \tag{1}$$

  where the sum runs over the set $\mathcal{V}(i)$ of neighbors of i. The in-degree k^b and out-degree k^l are defined as

  $$k^b_i = \sum_{j \in \mathcal{V}(i)} a^b_{ij}, \qquad k^l_i = \sum_{j \in \mathcal{V}(i)} a^l_{ij}, \tag{2}$$

[‡]a shortage of liquidity can propagate from a bank to the ones that have a relation of credit to it

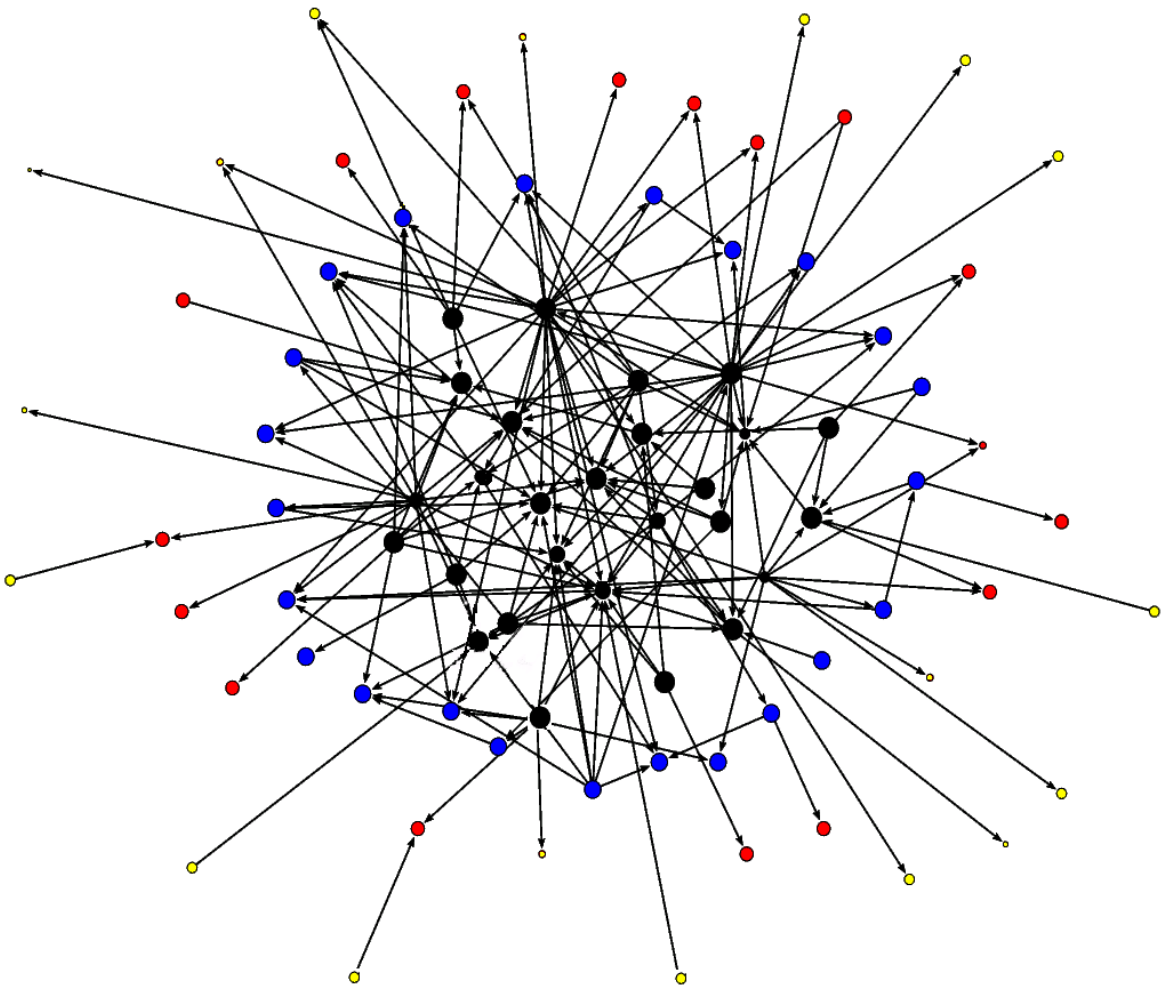

Figure 1. A plot of the inter bank network. The color codes for the various groups are the following: 1=yellow, 2=red, 3=blue, 4=black. Note that the black vertices (bank of group 4) form the core of the system.

- *Strength*

  Another measure of the network properties in terms of the actual weight of each link (i.e. the size of the trade on that link, or the number of times the link has been used) is obtained by looking at the vertex *strength* that can be defined in two ways: as

$$s_i^w = \sum_{j \in \mathcal{V}(i)} w_{ij}, \tag{3}$$

  or as

$$s_i^c = \sum_{j \in \mathcal{V}(i)} c_{ij}. \tag{4}$$

  Another measure of the vertex strength can be given in terms of the vertex net flow as

$$f_i = \sum_{j \in \mathcal{V}(i)} f_{ij} \tag{5}$$

  Similarly we can define the borrowing and lending strength as

$$s_i^{w,b} = \sum_{j \in \mathcal{V}(i)} w_{ij}^b \qquad\qquad s_i^{w,l} = \sum_{j \in \mathcal{V}(i)} w_{ij}^l \tag{6}$$

  and the equivalent expressions for $s_i^{c,b}$ and $s_i^{c,l}$.

- *Assortativity*

  Assortativity is a measure of similarity among nodes and is defined as

$$k_{nn}(i) = \frac{1}{k_i} \sum_{j \in \mathcal{V}(i)} k_j. \tag{7}$$

- *Clustering*

  The clustering coefficient c_i is a measure of the density of connections around a vertex and is defined as

$$c_i = \frac{2}{k_i(k_i - 1)} \sum_{j,h} a_{ij} a_{ih} a_{jh}. \tag{8}$$

  Hence, the clustering coefficient allows to calculate the proportions of nearest neighbors of a node that are linked to each others (in our contest, if there exist a link between two banks who have a common trading partner bank). The average clustering coefficient,

$$C = \frac{1}{N} \sum_i c_i$$

  expresses the statistical level of cohesiveness measuring the global density of interconnected vertex triplets in the network.

- *Diameter*

  In a graph the distance between two vertices is given by the length of the shortest path joining them (if it exists). In a connected graph the average distance is the average over all distances. If the graph is not connected, the average distance is defined as the average among all distances for pairs both belonging to the same connected component. The diameter of a graph is given by the maximum of all distances between pairs.

- *Participation ratio*

  For a given node i with connectivity k_i and strength s_i the weights of the edges can either be of the same order of magnitude s_i/k_i, or can be heterogeneously distributed, with some edges dominating others. The participation ratio is defined as

$$Y_2^w(i) = \sum_{j \in \mathcal{V}(i)} \left[\frac{w_{i,j}}{s_i^w} \right]^2 , \tag{9}$$

  or equivalently

$$Y_2^c(i) = \sum_{j \in \mathcal{V}(i)} \left[\frac{c_{i,j}}{s_i^c} \right]^2 . \tag{10}$$

  If all weights are of the same order then $Y_2 \sim 1/k_i$ while if a small number of weights dominate Y_2 is close to 1. A value of the participation ratio close to one would then indicates preferential relationships among banks[§].

  Similarly we can define the participation rates $Y_2^{w,b}(i)$ and $Y_2^{w,l)}(i)$ separating incoming and outgoing links. The average participation ratio is then computed as

$$Y_2^w = \frac{1}{N} \sum_i Y_2^w(i) \qquad\qquad Y_2^c = \frac{1}{N} \sum_i Y_2^c(i)$$

6. STATIC PROPERTIES

The distribution of banks' degree and strength indicates that banks have an highly heterogeneous behavior, since the number of their partners varies very widely. In Fig. 2 the distribution of degree for a particular day is plotted: we observe high skewness, proving that some banks have many partners, but most of the banks have few partners. Moreover in same figure the assortativity and clustering distributions show a peculiar architecture of the network. The system shows disassortative mixing, i.e. banks with higher degree are more likely connected to banks with lower degree.

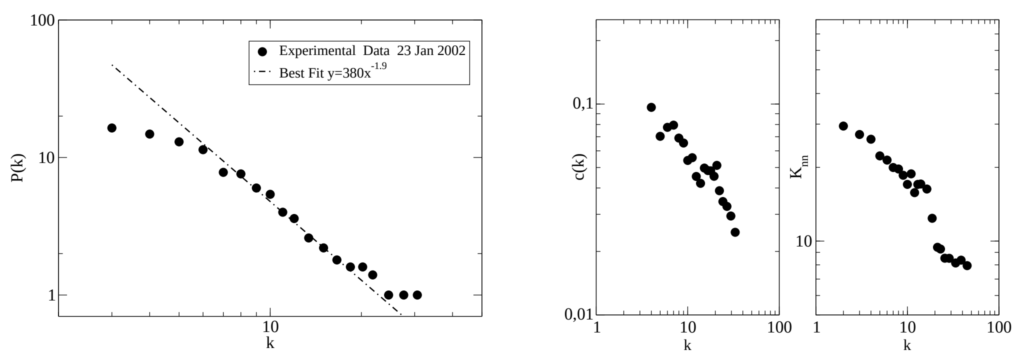

Figure 2. Cumulative distribution of banks degree (left); distribution of clustering coefficient c and assortativity $k_{nn}(k)$ versus k.

We note that the degree and the volume are correlated,[9] since $v(k) = k^{1.1}$ (see Fig.5) and therefore is statistically significant to consider the degree and the volume as different informations.

[§]Note that the statistics used by Cocco et al. (2003) is the lender preference index (LPI) (and the corresponding borrower preference index) defined as $LPI = \frac{\sum_{t=1}^{30} w_{i,j}^l(t)}{\sum_{t=1}^{30} s_i^{w,l}(t)}$

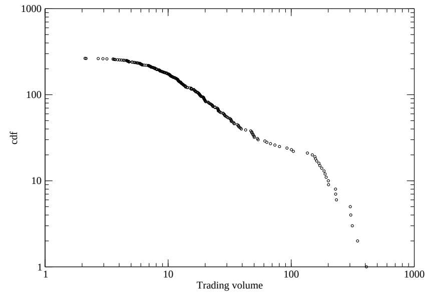

Figure 3. Distribution of weights w_{ij}, that represent the contract volumes

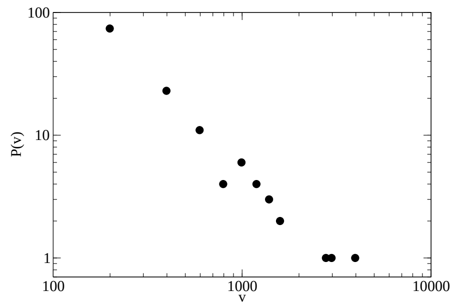

Figure 4. Distribution of tota ... sacted by each bank (v)

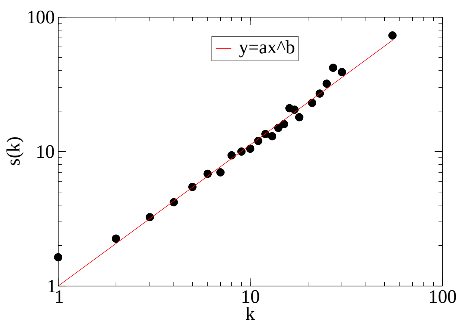

Figure 5. Scaling of the strength versus the degree $s(k)$

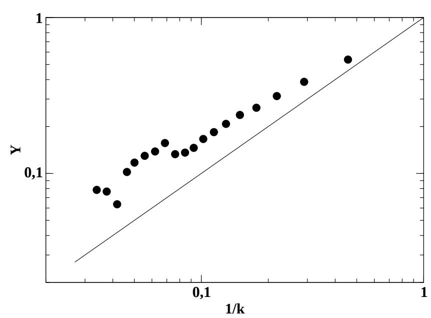

Figure 6. Participation ratio $Y_c(k)$

In figure 3 and 4 we show that the banking system is highly heterogeneous with both the daily cumulative distribution of banks' weights (volume of single contract) and strengths (trading volumes) fat tailed. Banks with more partners transact bigger amounts of liquidity. The total volume transacted by each bank is not equally distributed of its links as we observe from the plot of the participation ratio $Y_2^c(i)$ (Fig.6): real participation ratio (dots) differ from a random case of equally distributed weights (straight line); banks with an high number of connections have heterogeneous volumes of contracts with their partners.

Interestingly, regardless the change in volumes all the above topological measurements remain similar when computed in different days of month.

7. EMERGENCE OF DIFFERENT BEHAVIORS IN THE INTERBANK MARKET

Detecting common behaviors of communities of vertices would help in providing insights to understand the of hidden organization principles. We try to understand if there are some banks with similar behavior and if they have some properties in common.[2] We are able to identify specific features for banks of different size. In fact for each bank we know its category (small, medium, large, very large) based on the capital of the banks (as recorded by Bank of Italy). We observe that this classification is strongly correlated with the total amount of daily volume of transactions. Using the latter quantity we can divide banks in four groups (same number of classes of the Bank of Italy classification). *Group 1* with volume in the range $0 - 23$ million Euro per day, *Group 2* in the range $23 - 70$ million Euro per day, *Group 3* in the range $70 - 165$ million Euro per day, *Group 4* over 165 million Euro per day. In this way we find an overlap of more than 90% between the two classifications.

Using this information we realized the picture 1 of the system as an oriented network whose size and color of the vertices represent the different groups that play the role of communities when described by means of a network. We used the Kamada-Kaway algorithm: this algorithm makes more connected nodes to be nearer to each other. As evident from Figure 1 we find that the core of the structure is composed by banks of the last groups (very large). The edges in Figure 1 represent the net amount of money exchanged in a whole day. As mentioned above, the measurements in different days give similar results. A more quantitative measure of the different behavior of banks from different groups is given in the Table 8, where for every pair of groups we reported the mean percentage of the total number of transactions between banks of those groups. This result is confirmed by the first two plots of Figure 7, where we represented in-degree frequency distribution (number of borrowing edges) and the out-degree frequency distribution (number of lending edges) in the network. It is possible to compute the group of the banks whose degree is k. We represented this information by coloring accordingly the plot.

With respect to the scale of colors in Figure 1, we also added some intermediate colors to account for the values between one group and another. The tail of the two distributions is black, i.e. it is mainly composed by banks of group 4. We again find that banks of groups 1 and 2 are the leaves of the network, staying at periphery of the structure and not interacting each other. This particularity together with the experimental evidence that they are more lenders on average means that banks of this group are the lenders for the whole system (in fact the 70% of the total volume transacted pass through lenders).

8. A MODEL TO REPRODUCE THE TOPOLOGICAL FEATURES

In order to reproduce the topological properties we define a model whose only assumption is that a vertex is solely determined by its size (as measured by its capital or equivalently by its group).[2] Therefore, the idea is that the vertices representing the banks are defined by means of an intrinsic character corresponding to the size of the bank.[10,11] Since this information is not available we use the total daily volume of transactions as a good measure of the size of banks (we stated above that this is a good approximation).

Following the Pareto's law (confirmed in this data analysis) we assume that the distribution of sizes v in the model is a power-law $P(v) \propto v^{-2}$, where the value of the exponent correspond to that of the data, as we see in Fig. 4.

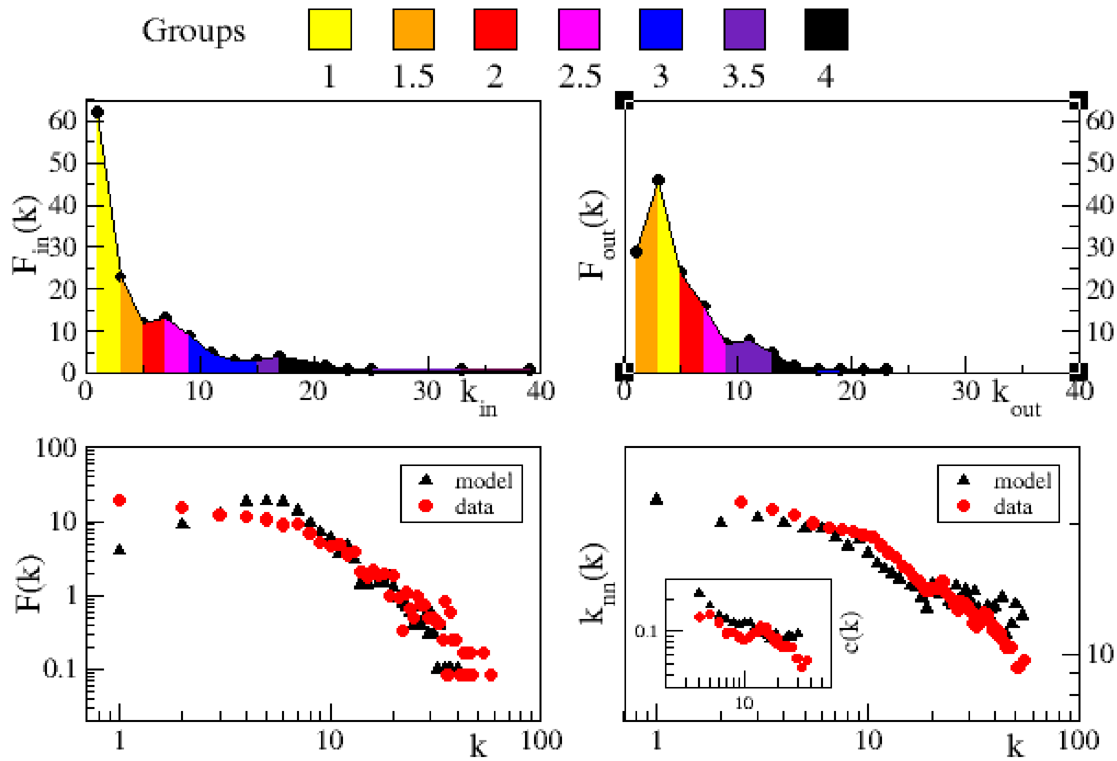

Figure 7. Top left and top right: a plot of the in-degree and out-degree distributions respectively. As already noticed, the contribution to the tail of frequency distribution emerges from the banks of group 4. Using the division in 4 groups, i.e. in 4 colors, mentioned in the text, we also colored each bin of $F(k)$ with the average color of vertices which are in that bin. For example the average color of banks with degree 10 is blue. For non integer value of this average we introduced intermediate colors. Bottom left: frequency distribution $F(k)$ for a certain degree k. Comparison between experiment (red dots) and results obtained with simulation of our model (black dots). Bottom right: comparison between experiment (red dots) and results obtained with simulation of our model (black dots) for the assortativity $\langle k_{nn}(k)\rangle$ and in the inset for the clustering coefficient $c(k)$.

We assign to the N nodes (N is the size of the system) a value drawn from the previous distribution. Vertices origin and destination for one edge are chosen with a probability p_{ij} proportional to the sum of respective sizes v_i and v_j. In formulas

$$\begin{aligned} p_{ij} &= \frac{(v_i+v_j)}{\sum_{i,j>i}(v_i+v_j)}. & (11)\\ \sum_{i,j>i}(v_i+v_j) &= \frac{1}{2}\sum_{i,j\neq i}(v_i+v_j) = 2(N-1)V_{tot} & (12)\\ V_{tot} &= \frac{1}{2}\sum_j v_j & (13) \end{aligned}$$

We obtain in this way $p_{ij} = \frac{v_i+v_j}{2(N-1)V_{tot}}$ This choice of probability reproduces the fact that big banks are privileged in transactions among themselves while two little banks are very unlikely to interact. We produce an ensemble of 100 equiprobable statistical realizations of the model and then we calculate average statistical distributions. In Fig. 7 we compare experimental and simulated $P(k)$, $c(k)$ and $k_{nn}(k)$: here the distributions are also averaged on all EOM days of 2002. The simulation of the model reproduces remarkably well the considered topological properties of the inter-bank market $P(k)$, $c(k)$ and $k_{nn}(k)$. To quantify the agreement between experimental and simulated networks we also define an overlap parameter m specifying how good is the behavior of the model in reproducing the observed clustering.

To quantify the agreement between experimental and simulated networks, we proceed in the following way. We define a matrix E, that is a weighted matrix 4×4, where the weights represent the number of connections between groups. In order to measure the overlap between the matrices obtained by data and by computer model, we define a distance based on the differences between the elements of the matrices.

$$d = \sum_{g,k\geq g} |E^{exp}_{g,k} - E^{num}_{g,k}| \qquad (14)$$

The sum of all elements, $\sum_{g,k\geq g} E^{exp}_{g,k}$ and $\sum_{g,k\geq g} E^{num}_{g,k}$, is equal to E_{tot} in both cases. Therefore the maximum possible difference is $2E_{tot}$. This happens when all the links are between two groups in one case and in other two groups in the other. We use this maximum value to normalize the above expression and we than define the overlap parameter m: $m = 1 - d/2E_{tot}$

A natural way to define groups in the model is to obtain a similar number c of banks for each class i.e. $c = N_{banks}/N_{classes}$. It is useful nevertheless to pass to continuous form. Using the previously introduced $P(v)$ giving the probability distribution of the size v of one bank. Banks of the same group g are in the range $[v_g, v_g + \triangle v_g]$ so that by taking a continuous distribution for v we get

$$\int_{v_g}^{v_g+\triangle v_g} P(v')dv' = c \qquad (15)$$

where $P(v')$ is defined above. In our case, since the average number of banks is 140, we get $c = 35$. Then $\triangle v = cv^2/(N - cv)$ We now compute the number $E_{g,k}$ of links going from one group of banks g_g to another one g_k, for every possible pair of banks.

$$E_{g,k} = \sum_{i,j} a_{i,j}\delta(g_g - g(i))\delta(g_k - g(j)) \qquad (16)$$

where $g(i)$ represent the group of bank i. In the continuous approximation, defining $E_{v'v''}$ the number of edges from vertices of fitness v' to vertices of fitness v'', $E_{g,k}$ is given by

$$\begin{aligned} E_{g,k} &= \int_{v_g}^{v_g+\triangle v_g}\int_{v_k}^{v_k+\triangle v_k} E_{v'v''}dv'dv'' = \\ &= (N/2)\int\int P(v')P(v'')p(v',v'')dv'dv'' & (17) \end{aligned}$$

Group	1	2	3	4
1	0	6	4	8
2	6	3	8	17
3	4	8	5	27
4	8	17	27	22

Table 1. The number of daily interactions between the banks of different groups. Data have been averaged during one month.

where N is the number of vertices, $p(v', v'')$ is the linking probability, $P(v)$ is the fitness distribution and the formula is obtained integrating the expression for the average degree[11] (the integration domains are the ranges of volumes of groups g and k respectively). To evaluate the relevance of division in classes, we have to compare the value of $E_{g,k}$ with the corresponding quantity $E_{g,k}^{null}$ for a network where there is not a division in classes (null hypothesis). The analytical expression for the null case is $E_{g,k}^{null} = E_{tot}/10$ where 10 is the number of possible couplings between the 4 groups. The comparison between the two networks evidences that in the real case emerges the division in groups: in Tab. 1 for each possible combination of groups is reported the value $E_{g,k}/E_{tot}$. In the null case, each element of the same matrix should be equal to 10. In our case the overlap m is very good (98%).

The role of the different groups is shown in the Figure 8.

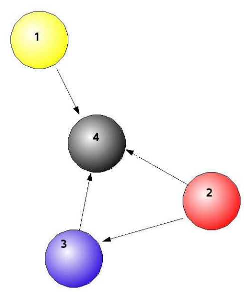

Figure 8. The division on classes of vertices allows to represent in a very easy way the organizational principles of the network. Following results of Table 1 we draw a link among two groups when the number of links between banks belonging to them is bigger then the average value (10). Using the net volumes as weight of links, we can represent the directed interactions among classes of nodes: class 4 appear to be clearly a borrower and class 1 lender.

9. EFFICIENCY AND STABILITY OF THE INTERBANK NETWORK

The main objectives of economical scientific research on interbank liquidity markets are to assess the efficiency of the interbank market and second to understand potential implications of the current institutional arrangements on the stability of the banking system.

About the first topic, namely *efficiency*, the market is efficient if each bank can provide the liquidity it needs in short time. We show that preferential lending in our system is limited and in fact banks interact with banks randomly chosen and cash flows directly from the lender to the borrower without intermediaries. These observations suggest that the interbank market is relatively efficient.

The *stability* of an economic system is very important field of research. Financial crises, i.e. the incapacity to finance businesses and industries have recently hit several countries all around the world. These events have triggered a boom of papers on banking-crises, financial risk analysis and numerous policy initiatives to find and understand the weak points of the financial system. One of the major concerns in these debates is the so called *systemic risk*: the large scale breakdown of financial intermediation due to domino effects of insolvency. This phenomenon is favored by the small world property: in this network the maximum topological distance between any two banks is 3 . From figure 9 we can see that almost all nodes are far away from any other by at most three links.

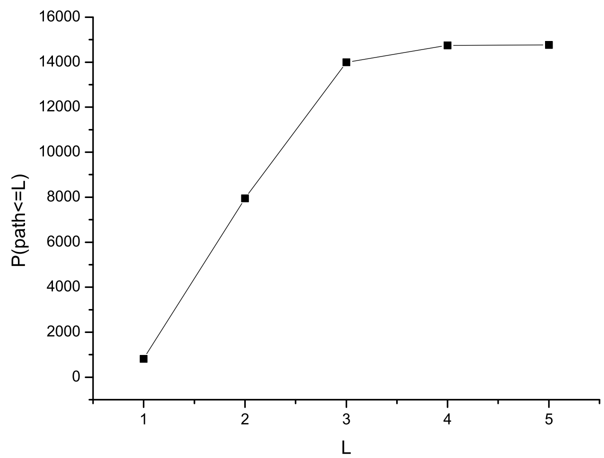

Figure 9. cumulative distribution of distances among all pairs of nodes: we observe that the maximum distance between any pair of nodes is $L = 3$

The network of mutual credit relations between financial institutions is supposed to play the key role in the risk of propagation of failures. The mutual credit network can be represented by the *liability or exposure*

The flow of payments is uniquely determined by the structure of the liability matrix. We are interested in the stability of financial system with respect to external shocks, in particular which banks are likely to default due to shocks and which banks will drag other banks into default due to their mutual credit relations.[?] These two different behaviors are related to the betweenness property of the node, that is the fraction of shortest paths among all possible pairs of nodes go trough that node.

$$b_i = \sum_{j,k} g_{jik}/g_{jk} \tag{18}$$

where g_{jk} is the number of shortest paths connecting nodes j and k and g_{jik} is the number of shortest paths connecting nodes j and k passing trough i.

The betweenness is a measure of flow of money passing trough the node.

Figure 10 discloses that nodes with high number of partners have also more flux of money trough them. This proves from the topological point of view that there is not a hierarchical structure in the network, but a configuration with highly connected nodes in the core, as we observed above. In fact big banks, which have more partners, manage also the most of liquidity of the market: we can understand that these banks may play a crucial role in the case of bankruptcy chains. Further will focus on the dynamics of these failure cascading.

10. CONCLUSIONS AND IMPLICATIONS ON ITALIAN INTERBANK MARKET

In this paper we focused on the micro structure of Italian Interbank Market, represented as a network and studied through topological measures.

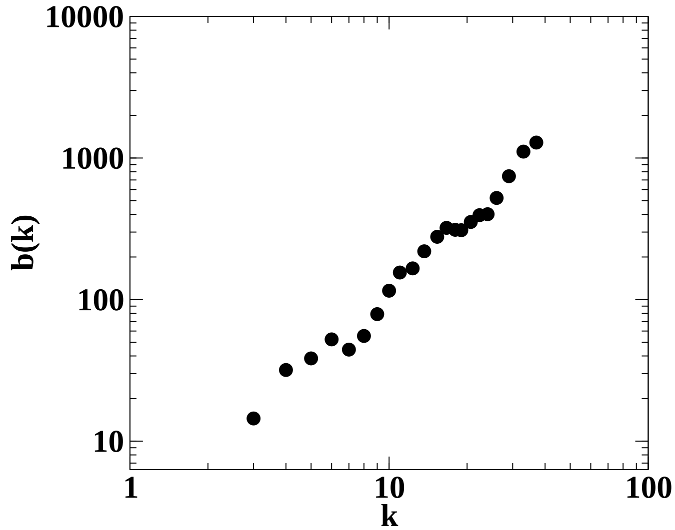

Figure 10. average betweenness b for nodes with degree k

We explored the network of interconnections among banks in the Italian overnight market and, by applying several metrics derived by computer science and physics, uncovered a number of microstructure characteristics.

The banking network is fairly random, preferential lending is limited and cash flows directly from the lender to the borrower without intermediaries. Banks also do not seem able to exploit short term profit opportunities by borrowing from some and lending to others on the same day. All these observations suggest that the interbank market is relatively efficient.[3]

The banking system is highly heterogeneous and is arranged in a configuration with large banks borrowing from a large number of small creditors. Iori et al.[?] showed, in artificial market models, that when banks are heterogeneous a high connectivity increases the risk of contagion and systemic failure. The current institutional settings push banks towards a even more connected configuration as the EoM date approaches, and doing it may increase the potentials for systemic risk. A policy implication of this work could be to encourage the design of a mechanism for reserve requirements that does not require banks to simultaneously fulfill their average reserve.

Finally we presented here a network representation of the interbank market that in a natural way allows to detect the presence of communities of banks: we observe that big banks are very interacting each other, while small banks always interact with the bigger ones.[2] Furthermore big banks act as borrower of the system and small banks as lenders. By means of a suitable chosen model of network formation we can also understand the mechanism driving the formation of such communities. The agreement between the model and experimental results is remarkably good; this seems to suggest that the network formation is not due to the growth mechanism of preferential attachment.

Since the effects of European Central Bank policies are under debate,[7] graph theory can help in understanding the system behavior under change of external conditions.

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