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ORIGINAL RESEARCH

A game-theoretical pricing model for green/non-green products

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Abstract
Environmental pollution represents a significant negative externality for society by imposing substantial costs on it. By incentivizing eco-friendly production, one can mitigate these costs and promote sustainable development. Given this framework, the present article formulates and solves a multi-stage game-theoretical pricing model involving two firms: one commercializes a non-green product; the other one is involved in the production of a green product. The model also involves the State, which can influence the two firms through a dual mechanism based on imposing an excise duty and promoting an advertising campaign. Firms' profits and social welfare at the subgame perfect equilibrium are obtained in closed form, and their dependence on exogenous parameters is analyzed both theoretically and numerically. An interesting outcome of our analysis is that, in certain cases, the excise duty is found to be negative at the equilibrium and therefore corresponds to a subsidy rather than a green tax. Finally, possible future developments of the proposed model are discussed.

Keywords Non-cooperative game theory · Pricing · Green/non-green products · State intervention · Profits/social welfare optimization

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1 Introduction

Nowadays, concerns about ecological issues – such as reduction of pollution sources, unrecyclable waste, and harmful substances – are considered one of the most important priorities for public opinion and are taken in strong consideration also at the international level [Ghosh et al. (2020); Biancalani et al. (2024)]. This promotes the development of ecologically-friendly economic and industrial systems [Compernolle and Thijssen (2022)], an objective that can be achieved in several non-mutually exclusive ways. For instance, as proposed by the economist Pigou [Baumol (1972)], to compensate the environmental negative externality due to production/consumption (e.g., one related to pollution), one could introduce a tax, which equals such negative externality. A second way to create a greener economic system could consist of persuading people to purchase eco-friendly products instead of traditional ones [Dorfman and Steiner (1954); Barbarossa and De Pelsmacker (2016)].

Given this framework, the aim of this article is to combine these two mechanisms[1] by introducing a multi-stage game-theoretical pricing model with green/non-green products. The game overlays several stages involving the non-cooperative interaction of three players: two firms (one commercializing a non-green product; the other involved in the production of the green product), and the State. The model is solved by finding its Subgame Perfect Equilibrium (SPE),[2] where all the players make optimal choices at each stage [Osborne and Rubinstein (1994)].

This leads to analyzing the role of the externalities on the SPE, including costs, societal impact of the technologies, and transfer between products due to their interdependency [Basiri and Heydari (2017); Compernolle et al. (2022)]. In the end, we obtain a closed-form derivation, under relatively mild assumptions, of multiple metrics of interest, such as the utilities of the players (the individual profits for the manufacturers, and the societal welfare for the State) and the optimal excise duty that can be applied by the State to penalize non-green products [Yang et al. (2021); Ling et al. (2022)]. In certain cases, this is actually found to be negative in our model at the equilibrium and therefore corresponds to a subsidy rather than a green tax, motivated by the greater benefit that the green technology causes, which is indirectly even promoted by a more intense consumption of the non-green product.[3] As a result, high-level governmental policies of pollution mitigation may be impacted as future extensions [El Ouardighi et al. (2020)].

[1] In our case, from a technical point of view, we do not consider a Pigouvian tax here, because it can be different from the external marginal cost of the negative externality. But the proposed tax is something related to the non-green production. For more details, see the description of the proposed model in Section 3.

[2] For a better reading, the following concepts from non-cooperative game theory are briefly recalled here in an informal way. A Nash Equilibrium (NE) for a non-cooperative game with a finite set of n players is a n-tuple of strategies, one strategy for each player, where each player's strategy is a best response of that player to the strategies of all the other players. An SPE is a refinement of the NE concept, where one further assumes that the n-tuple of players' strategies forms a NE for every subgame of the original game, i.e., for any game that starts at a generic decision stage of the original game, with the only constraint that the given decision stage represents a singleton information set (i.e., every player knows exactly where it is in the game when the subgame begins).

[3] Indeed, it can make sense for a government to subsidize a non-green (or less-green) product to create a market and infrastructure that will eventually support the purchase of a truly green product. A recent example is the US government's adoption of subsidies (by means of the Inflation Reduction Act of 2022) for plug-in hybrid electric vehicles, to pave the way for a transition to fully electric cars.

The article is organized as follows. Section 2 provides a short literature review. Section 3 describes the model and its equilibrium analysis. Section 4 investigates effects on profits and social welfare of changes in the exogenous parameters of the model. Section 5 provides numerical results. Finally, Section 6 concludes the article and discusses possible future developments. Some technical details and additional numerical results are reported in Appendices and B, respectively.

2 Literature review

In the literature, several theoretical models, based on non-cooperative game theory, were proposed to investigate equilibrium in a market with green and non-green products, with firms' choices representing, e.g., prices, quantities, and/or greenness levels of their products. One recent representative work in this stream is [Strandholm et al. (2023)], which develops a three-stage model wherein two firms decide how to invest in a green technology and compete based on quantity choice. The model presented in [Liu et al. (2012)] involves manufacturers/retailers with a continuum of eco-friendly levels for their products. Other theoretical works investigate price competition between producers of green/non-green products. For instance, firms can decide prices/greenness levels of their products [Sana (2022)]. In both [Buccella et al. (2021); Cappelletti et al. (2021)], the choice between a traditional (non-green) production or a green one is a preliminary decision made by players that after this initial stage compete in Cournot or Bertrand duopolies, whereas [Zhang et al. (2020)] takes a similar approach but the competition is based on the quality of the product.

The research made in [Ülkü and Hsuan (2017)] explores the impact of product modularity and consumer sensitivity to sustainability on the pricing decisions of two competing firms. The model investigated in [Sana (2020)] analyzes the impact of corporate social responsibility on the pricing decisions of two firms, one selling a green product and the other one a non-green product. The work [Ho et al. (2018)] analyzes a firm characterized by a hybrid production system, able to generate both new and remanufactured products. Its pricing analysis incorporates elements such as the presence of various consumer types, and both internal and external competition.

In the model considered in [Basiri and Heydari (2017)], a two-stage supply chain is considered, wherein the first stage involves selling a non-green traditional product, and the second stage involves releasing a novel green product beside the non-green traditional one. This is reminiscent of our approach, but that interaction is entirely deregulated, whereas we instead consider a further intervention by the State, e.g., to impose an excise duty.

The models investigated in [Zhu and He (2017)] involve multiple supply chains, with manufacturers producing respectively green/non-green products, then selling such products to corresponding retailers that sell in a competitive market. This is further expanded in [Mondal et al. (2022)] by involving a reverse supply chain, allowing for the possibility of replacing defecting non-green items. Moreover, the manufacturer that produces the green product has the possibility to decide its greenness level.

Finally, in some models, the government can act with subsidies and/or tax differentiation: e.g., lower tax for green products/producers, higher tax for non-green products/producers [Fang and Zhao (2023)]. More in general, the role of green taxes or green subsidies is a subject of multiple studies [Yang et al. (2021); Ling et al. (2022); Barros and Pádua (2019); Yi et al. (2021); Zhang et al. (2023); Zhu et al. (2023); Zolfagharinia et al. (2023); Liang and Zhang (2025)], even within a game theoretical perspective: for example, [Fu et al. (2023)] discusses how a government intervention can be used to obtain Pareto efficient outcomes by breaking free from a Prisoner's dilemma, whereas [Hu and Wang (2022)] explores an evolutionary game to track the impact of carbon taxes together with the customers' preferences for environmental-friendly technologies.

Given this framework, a peculiar feature of our proposed model is that the green/non-green manufacturers do not make their decisions simultaneously, which in our opinion better reflects that the green product might become available at a later stage than the non-green product. Moreover, their interaction arises from the fact that the demand met by the non-green manufacturer influences the demand curve associated with the green product. Additionally, the model includes, among the decision variables of the government, an excise duty for the production of each unit of the non-green product. Depending on the government's decision, this can potentially become a subsidy to the production of the non-green product, motivated by the fact that an increase in the demand met by the non-green manufacturer shifts upwards the demand curve associated with the green product.

3 Model

In this work, we present a multi-stage model with two firms, which maximize their profits, and the State, which maximizes social welfare. Precisely, the first firm (M_1) makes a traditional non-green product ω_1, whereas the second firm (M_2) develops a green product ω_2. We assume that the green product ω_2 provides a positive externality $\alpha_2 > 0$ per unit to the general population well-being. Conversely, the traditional non-green product ω_1 provides a negative externality $\alpha_1 < 0$ per unit. Moreover, the State (*S*) maximizes a social welfare function, which includes the weight of (negative and positive) environmental externalities in monetary terms, the financial resources provided by an environmental excise duty, and *S*'s expenditure to persuade consumers to purchase the green product. In accordance with a large part of the literature presented in Sect. 2, also in our model the two manufacturers M_1 and M_2 interact by choosing the prices of their own products. We assume that initially, only the traditional non-green product ω_1 exists because the technology needs time to provide an upgraded green product; when the green product ω_2 becomes available, a certain amount of consumers, positively correlated with the previous consumption of ω_1, purchases ω_2 [Ricci et al. (2018)]. The positive effect of the consumption of ω_1 on that of ω_2 relies on the reasonable hypothesis that the average consumer purchases ω_2 when this becomes available, even if s/he has already consumed ω_1. In our model, *S* has two instruments to influence M_1 and M_2, with the aim of increasing social welfare: (i) levying an excise duty to M_1 for the production of ω_1, or even providing a subsidy to it (the latter case being motivated by the positive correlation between the consumptions of ω_1 and ω_2); (ii) promoting ω_2 with its own resources (e.g., through advertising).

3.1 Mathematical formulation

We provide a mathematical formalization of the ideas exposed above. Firstly, *S* has to maximize the social welfare function

$$W = W_1 + W_2, \tag{1}$$

expressed in terms of the two components

$$W_1 = \alpha_1 D_1 + \tau D_1,$$
$$W_2 = \alpha_2 D_2 - \mu\theta^2,$$

where

- $\alpha_1 < 0$ is the value in monetary terms of the negative externality provided by the consumption of one unit of ω_1;
- D_1 is the total quantity of ω_1 consumed by the economic system;
- τ, chosen by *S*, is the excise duty (or subsidy, when $\tau < 0$), expressed in monetary terms, that *S* charges to M_1 for the production of one unit of ω_1;
- $\alpha_2 > 0$ is the value in monetary terms of the positive externality provided by the consumption of one unit of ω_2;
- D_2 is the total quantity of ω_2 consumed by the economic system;
- $\theta \geq 0$ is the level of advertisement, chosen by *S*, that *S* can impose on the total demand function of ω_2, whose expression is reported later in (3);
- $\mu > 0$ is an exogenous parameter associated with the total cost of social advertisement campaign $\mu\theta^2$ incurred by *S* to increase the demand of ω_2.

The profit to be maximized by the first firm M_1 is expressed as

$$\Pi_1 = (p_1 - (c_1 + \tau))D_1 \geq 0,$$

where

- $p_1 \geq \max(c_1 + \tau, 0)$ is the price (chosen by M_1) of one unit of ω_1;
- c_1 is the exogenous constant marginal cost (incurred by M_1), to produce (and sell) one unit of ω_1.

In other words, the term $(c_1 + \tau)$ represents the "total marginal cost" of production of ω_1 and is decomposed into two components: c_1, depending exogenously on the technology; τ, selected by *S* to modify the general consumption level of ω_1.

Similarly, the profit to be maximized by the second firm is expressed as

$$\Pi_2 = (p_2 - c_2)D_2 \geq 0,$$

where

- $p_2 \geq c_2$ is the price (chosen by M_2) of one unit of ω_2;
- c_2 is the exogenous constant marginal cost (incurred by M_2) to produce (and sell) one unit of ω_2.

In the work, we do not make assumptions on the relationship between c_1 and c_2, thereby any case among $c_1 < c_2$, $c_1 = c_2$, $c_1 > c_2$ may occur.

The demand D_1 for the quantity of ω_1 is decreasing with respect to p_1, and is written as

$$D_1 = \max(d_1 - \beta_1 p_1, 0), \tag{2}$$

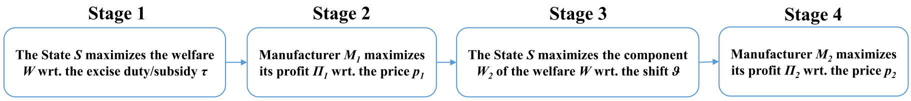

Fig. 1 Timing of the model

where

- $d_1 > 0$ is the intercept level of the demanded quantity of ω_1 in case its price is zero, i.e., it is the theoretical maximum quantity of requests of ω_1 within the economic system. In this work, it is modeled as an exogenous constant;
- $\beta_1 > 0$ represents minus the slope of the demanded quantity of ω_1 as a function of its price p_1.

Finally, the demand D_2 for the quantity of ω_2 is decreasing with respect to p_2, and it is is written as

$$D_2 = \max(d_2 - \beta_2 p_2 + \theta + \lambda D_1, 0), \tag{3}$$

where

- $d_2 > 0$ is the intercept level of the demanded quantity of ω_2 in case its price is zero. In this work, it is modeled as an exogenous constant;
- $\beta_2 > 0$ represents minus the slope of the demanded quantity of ω_2 as a function of its price p_2;
- $\lambda > 0$ is an exogenous constant, which captures the effect of the total consumption of ω_1 on the total consumption of ω_2. In the following, we will refer to λ as the *transfer parameter*. The idea behind this is that some people who purchased ω_1, later would purchase ω_2, too.

In other words, when it is larger than 0, the demand D_2 for ω_2 is given by the "classical term" $d_2 - \beta_2 p_2$, plus the intervention of S through the selected level of θ, and the effect of a previous consumption of ω_1.

Regarding the timing of the model, we consider four stages (see Fig. 1), which are described as follows. Everyone knows what happened in the previous stage(s):

1. S chooses τ to maximize the social welfare function W, anticipating the optimal players' behavior in the successive stages [Yi et al. (2021); Zolfagharinia et al. (2023)]. Initially, there exists only ω_1, but the value $\alpha_2 > 0$ represents public information regarding the positive externality per unit associated with the future green product.
2. M_1 maximizes its profit Π_1 by selecting p_1.
3. The technology makes (exogenously) an anticipated leap to the new green good ω_2, then S maximizes the second component $W_2 = \alpha_2 D_2 - \mu\theta^2$ of the social welfare function W with respect to θ.
4. M_2, which owns the technology to produce ω_2, maximizes its profit Π_2 by selecting p_2 optimally.

In the following Sects. 3.2-3.5, we solve the game above by backward induction[4] (from Stage 4 to Stage 1). Then, in Sect. 3.6, we find its SPE (as defined, e.g., in [Osborne and Rubinstein (1994)]). We make the following assumptions on the exogenous parameters.

Assumption A1 The following holds:

$$d_2 - \beta_2 c_2 > 0; \tag{4}$$

Assumption A2 The following holds:

$$\frac{2d_1 - 2\beta_1 c_1 + \alpha_2 \beta_1 \lambda}{2\beta_1} > |\alpha_1|, \text{ and } \frac{6d_1 + 2\beta_1 c_1 - \alpha_2 \beta_1 \lambda}{2\beta_1} > \alpha_1. \tag{5}$$

Assumptions A1 and A2 simplify the equilibrium analysis made in the following by guaranteeing, for each stage, the validity of the next expressions (8), (12), (18), regardless of the players' choices in the previous stages (when present). In particular, Assumption A1 (respectively, Assumption A2) guarantees that the green (respectively, non-green) product is sold at the equilibrium path of the SPE (i.e., in the case for which all the players apply the respective decision strategies associated with the SPE). The reader is referred to Sect. 3.7 for further comments about these two assumptions, i.e., about the possibility of extending the analysis by removing a subset of such assumptions.

3.2 Analysis of stage 4

In Stage 4, the manufacturer M_2 chooses the price p_2 of the green product ω_2 with the aim of maximizing its profit Π_2, knowing all the decisions taken in the previous stages. Hence, the optimization problem in Stage 4 is

$$\max_{p_2 \geq c_2} \ \Pi_2 = (p_2 - c_2) D_2. \tag{6}$$

Using the superscript "$\circ$" to denote optimality, for this problem, $\Pi_2^\circ \geq 0$ holds automatically (since $\Pi_2 = 0$ is obtained for p_2 large enough, in such a way as to make the demand D_2 equal to 0).

We first consider the case $D_2 = d_2 - \beta_2 p_2 + \theta + \lambda D_1$ (neglecting for a moment the max in (3)). Then, the optimization problem (6) becomes

$$\max_{p_2 \geq c_2} \ \Pi_2 = (p_2 - c_2)(d_2 - \beta_2 p_2 + \theta + \lambda D_1),$$

provided one gets $\Pi_2^\circ \geq 0$.

The First Order necessary Condition (FOC) for an interior optimal solution is

$$\frac{\partial \Pi_2}{\partial p_2} = 0. \tag{7}$$

[4] This term is commonly used in the game-theoretical literature, and refers to the application of Bellman's principle of optimality to the specific case in which each subproblem corresponds to a subgame.

To ensure that this FOC really provides a maximum, we check the negativeness of the second partial derivative: $\frac{\partial^2 \Pi_2}{\partial p_2^2} = -2\,\beta_2 < 0$. Then, by solving (7), the optimal level of p_2 is

$$p_2^\circ = \frac{\beta_2 c_2 + d_2 + \theta + \lambda D_1}{2\,\beta_2}. \tag{8}$$

Due to Assumption A1 (see Equation (4)), one gets $p_2^\circ > c_2$ (hence, the optimal solution is really interior) and

$$D_2^\circ = d_2 - \beta_2 p_2^\circ + \theta + \lambda D_1 > 0, \tag{9}$$

independently of the specific values assumed by $\theta \geq 0$ and $D_1 \geq 0$. Hence, $\Pi_2^\circ > 0$. This makes (8) valid also under the more general expression (3) for D_2. Moreover, one gets

$$\frac{\partial D_2^\circ}{\partial D_1} = \frac{\partial\,(d_2 - \beta_2 p_2^\circ + \theta + \lambda D_1)}{\partial D_1} = \frac{\lambda}{2} > 0, \tag{10}$$

hence D_2° increases strictly with respect to D_1.

3.3 Analysis of stage 3

In Stage 3, the State S chooses the level of advertisement θ to push people to consume the green product ω_2, with the aim of maximizing the social welfare function W. In doing this, S anticipates the optimal choice p_2° of p_2 by the manufacturer M_2 (and the corresponding optimal value D_2° of the demand D_2) in Stage 4, and knows all the decisions taken in the previous stages. Since the value of the only decision variable (the price p_1 of the non-green product ω_1) that influences the first component W_1 of the social welfare function was already decided in the previous Stage 2, the term W_1 represents an additive constant in S's objective function in Stage 3. Hence, S is actually maximizing only the second component W_2 of the social welfare function in this stage. Concluding, the optimization problem in Stage 3 is

$$\max\nolimits_{\theta \geq 0} \;\; W_2 = \alpha_2 D_2^\circ - \mu\theta^2. \tag{11}$$

By expressing D_2° as a function of θ (see Equation (9)), the optimization problem (11) becomes

$$\max\nolimits_{\theta \geq 0} \;\; W_2 = \alpha_2(d_2 - \beta_2 p_2^\circ + \theta + \lambda D_1) - \mu\theta^2.$$

The FOC is

$$\frac{\partial W_2}{\partial \theta} = 0.$$

Again, we check the negativeness of the second partial derivative: $\frac{\partial^2 W}{\partial \theta^2} = -2\,\mu < 0$. The optimal value of θ is interior and is expressed as

$$\theta^{\circ} = \frac{\alpha_2}{4\mu}. \tag{12}$$

Indeed, $\theta^{\circ} > 0$, because $\alpha_2 > 0$ and $\mu > 0$.

3.4 Analysis of stage 2

In Stage 2, the manufacturer M_1 sets the price p_1 of the non-green product ω_1 with the aim of maximizing its profit Π_1. In doing this, M_1 knows what happened in Stage 1, namely the choice of the level of the excise duty tax (or, if negative, the subsidy) τ by S. M_1 also anticipates the optimal choices θ° and p_2° made in the successive stages, although they do not influence its objective function (however, its choice of the price p_1 made in Stage 2 influences later stages – particularly, Stage 4 – through the associated value of the demand D_1). Hence, the optimization problem in Stage 2 is

$$\max_{p_1 \geq 0} \ \Pi_1 = (p_1 - (c_1 + \tau))D_1. \tag{13}$$

For this problem, $\Pi_1^{\circ} \geq 0$ holds automatically (since $\Pi_1 = 0$ is obtained for p_1 large enough, in such a way as to make the demand D_1 equal to 0).

We first consider the case $D_1 = d_1 - \beta_1 p_1$ (neglecting for a moment the max in (2)). Then, the optimization problem (13) becomes

$$\max_{p_1 \geq 0} \ \Pi_1 = (p_1 - (c_1 + \tau))(d_1 - \beta_1 p_1),$$

provided one gets $\Pi_1^{\circ} \geq 0$. The FOC is

$$\frac{\partial \Pi_1}{\partial p_1} = 0.$$

Again, we verify the negativeness of the second partial derivative: $\frac{\partial^2 W}{\partial \theta^2} = -2\,\beta_1 < 0$. Then, the optimal value of p_1 is

$$p_1^{\circ} = \frac{\beta_1\,(c_1 + \tau) + d_1}{2\,\beta_1}, \tag{14}$$

when $\frac{-d_1 - \beta_1 c_1}{\beta_1} \leq \tau \leq \frac{d_1 - \beta_1 c_1}{\beta_1}$. These two inequalities guarantee, respectively, $p_1^{\circ} \geq 0$, and $d_1 - \beta_1 p_1^{\circ} \geq 0$. They also imply $D_1^{\circ} = \frac{d_1 - \beta_1\,(c_1 + \tau)}{2} \geq 0$ and $\Pi_1^{\circ} \geq 0$. Instead, when $\tau < \frac{-d_1 - \beta_1 c_1}{\beta_1}$ or $\tau > \frac{d_1 - \beta_1 c_1}{\beta_1}$, (14) is not valid anymore, and one has to consider the more general expression (2) for D_1, getting, respectively,

$$p_1^{\circ} = 0, \tag{15}$$

when $\tau < \frac{-d_1 - \beta_1 c_1}{\beta_1}$, and

$$p_1^{\circ} = \text{any real number} \geq \frac{d_1}{\beta_1}, \tag{16}$$

when $\tau > \frac{d_1 - \beta_1 c_1}{\beta_1}$, i.e., in this second case there is no demand for the product ω_1 (because in that case $D_1^\circ = \max(d_1 - \beta_1 p_1^\circ, 0) = 0$).

We remark here that the two latter expressions of the optimal price for the product sold by the first manufacturer actually hold at the finite extremes of the respective intervals, too. In these cases, respectively, they coincide with/include the former expression, and provide the same value of the demand for the first product as that expression. Hence, they are equivalent from the point of view of the optimization problem solved in Stage 1 (see the next subsection). For this reason, for simplicity and without any loss of generality, in the analysis reported in the following subsection we take into account, at the finite extremes of these two intervals, only the former expression of the optimal price for the product sold by the first manufacturer.

3.5 Analysis of stage 1

In Stage 1, the State S tries to maximize the social welfare function W by choosing the excise duty tax (or subsidy) τ, anticipating the optimal choices p_1°, θ°, and p_2° made in the successive stages. Hence, the optimization problem in Stage 1 is

$$\max_\tau \; W = \alpha_1 D_1^\circ + \alpha_2 D_2^\circ - \mu(\theta^\circ)^2 + \tau D_1^\circ. \tag{17}$$

Without loss of generality (i.e., without discarding the possibility of solving the problem (17) optimally), the State can anticipate that the expression (14) for p_1° (corresponding to $\frac{-d_1 - \beta_1 c_1}{\beta_1} \leq \tau \leq \frac{d_1 - \beta_1 c_1}{\beta_1}$) holds in Stage 2, because otherwise, if $\tau > \frac{d_1 - \beta_1 c_1}{\beta_1}$ held (corresponding to the expression (16) for p_1°), one would get $D_1^\circ = 0$, a case already covered by $\tau = \frac{d_1 - \beta_1 c_1}{\beta_1}$. In other words, for $\tau > \frac{d_1 - \beta_1 c_1}{\beta_1}$, one would get feasible solutions to the optimization problem (17) not better than its optimal solution. Similarly, the case $\tau < \frac{-d_1 - \beta_1 c_1}{\beta_1}$ (which leads to the expression (15) for p_1°) can be excluded from the analysis, too, since, for $\tau \leq \frac{-d_1 - \beta_1 c_1}{\beta_1}$, one would get $D_1^\circ = d_1$ (a positive constant) and the only term of the objective function W in (17) showing a dependence on τ would be $\tau D_1^\circ = \tau d_1$, which would be maximized by $\tau = \frac{-d_1 - \beta_1 c_1}{\beta_1}$ (a case already covered by the analysis made for $\frac{-d_1 - \beta_1 c_1}{\beta_1} \leq \tau \leq \frac{d_1 - \beta_1 c_1}{\beta_1}$).

Based on the considerations above, by expressing D_1° and D_2° in terms of the optimal values of p_2, θ, p_1 coming from the analyses made in Stages 4, 3, 2, respectively, the optimization problem (17) becomes

$$\begin{aligned}\max_{\frac{-d_1 - \beta_1 c_1}{\beta_1} \leq \tau \leq \frac{d_1 - \beta_1 c_1}{\beta_1}} \; W = & \frac{\alpha_1 d_1}{2} + \frac{\alpha_2 d_2}{2} + \frac{d_1 \tau}{2} - \frac{\beta_1 \tau^2}{2} - \frac{\alpha_1 \beta_1 c_1}{2} \\ & - \frac{\alpha_2 \beta_2 c_2}{2} + \frac{\alpha_2 d_1 \lambda}{4} - \frac{\alpha_1 \beta_1 \tau}{2} - \frac{\beta_1 c_1 \tau}{2} \\ & - \frac{\alpha_2 \beta_1 \lambda \tau}{4} - \frac{\alpha_2 \beta_1 c_1 \lambda}{4} + \frac{\alpha_2^2}{16\mu}.\end{aligned}$$

Due to both parts of Assumption A2 (see Equation (5)), one gets that the optimal value τ° of τ is interior, i.e., that $\frac{-d_1 - \beta_1 c_1}{\beta_1} < \tau^\circ < \frac{d_1 - \beta_1 c_1}{\beta_1}$ (actually, one can easily check that

Assumption A2 is equivalent to $\frac{-d_1-\beta_1 c_1}{\beta_1} < \tau^\circ < \frac{d_1-\beta_1 c_1}{\beta_1}$, when τ° is replaced by the next expression in (18)). The FOC is

$$\frac{\partial W}{\partial \tau} = 0.$$

Again, we check the negativeness of the second partial derivative: $\frac{\partial^2 W}{\partial \tau^2} = -\beta_1 < 0$. Then, τ° has the expression

$$\tau^\circ = -\frac{\beta_1 \alpha_2 \lambda + 2\beta_1 c_1 + 2\beta_1 \alpha_1 - 2d_1}{4\beta_1}. \tag{18}$$

Interestingly, one cannot exclude the case $\tau^\circ < 0$, interpreted as a subsidy to production per unit [Barros and Pádua (2019)]. Taking into account that the analysis requires the validity of both Assumptions A1 and A2, this case occurs, e.g., when Assumption A1 holds and $\frac{\alpha_2 \lambda}{2} = -\alpha_1 + \frac{3d_1}{\beta_1}$ (since this equality implies both the validity of Assumption A2 and the negativity of the expression (18) for τ°).

3.6 Decisions at the subgame perfect equilibrium (SPE)

The following theorem summarizes the results of the analyses made in Sect. 3.2-3.5.

Theorem 1 *Under Assumptions A1 and A2, the decisions of the players at the equilibrium path of the SPE are uniquely expressed as follows:*

- Stage 1, player *S*: $\tau^\circ = -\frac{\beta_1 \alpha_2 \lambda + 2\beta_1 c_1 + 2\beta_1 \alpha_1 - 2d_1}{4\beta_1}$.
- Stage 2, player M_1: $p_1^\circ = \frac{\beta_1 \left(c_1 - \frac{\beta_1 \alpha_2 \lambda + 2\beta_1 c_1 + 2\beta_1 \alpha_1 - 2d_1}{4\beta_1}\right) + d_1}{2\,\beta_1}$.
- Stage 3, player *S*: $\theta^\circ = \frac{\alpha_2}{4\mu}$.
- Stage 4, player M_2: $p_2^\circ = \frac{\left(\beta_1\,\alpha_2\,\lambda^2 + ((-2\,c_1 + 2\,\alpha_1)\beta_1 + 2\,d_1)\lambda + 8\,c_2\,\beta_2 + 8\,d_2\right)\mu + 2\,\alpha_2}{16\,\mu\,\beta_2}$.

Proof The proof follows by combining the results of the several analyses made in Sect. 3.2–3.5, proceeding this time in a forward way from Stage 1 to Stage 4, making an optimal decisional choice in each stage. During this forward process, a unique optimal decision is obtained in each stage. □

3.7 Comments

Three variations of the main analysis can be considered by removing one or both of Assumptions A1 and A2. In the following, only the first such variation is analyzed in detail, because it leads to a minor change in the outcome of the main analysis, presented in Sect. 3.2–3.5.

1. If Assumption A1 holds but Assumption A2 does not, then the decision maker in Stage 1 – which is the State – anticipates a possibly different functional form for D_1° (corresponding to one of the two cases in (2)), depending on the current decision taken by

the State in Stage 1. In this case, the only change in the outcome of the main analysis is that an optimal choice for τ is either

$$\tau^\circ = \frac{-d_1 - \beta_1 c_1}{\beta_1}, \tag{19}$$

or any value of τ for which D_1° is equal to 0, i.e.,

$$\tau^\circ = \text{any real number} \geq \frac{d_1 - \beta_1 c_1}{\beta_1}. \tag{20}$$

Of course, the choice of one of the two expressions (19) and (20) for τ° depends on which of the two maximizes the associated value of the objective function in (17).

2. If Assumption A2 holds but Assumption A1 does not hold, then the decision maker in Stage 3 (the State) anticipates a possibly different functional form for D_2° (corresponding to one of the two cases in (3)), depending on the previous decisions made by the State itself in Stage 1 and by the first firm in Stage 2, and on the current decision taken by the State in Stage 3. This may also change the expression of the objective function of the optimization problem in Stage 1 (by possibly changing the expressions of θ° and D_2° anticipated by the State in Stage 1). Compared to item 1 above, a number of additional subcases in the analysis of the SPE may be needed, making it unlikely to find closed-expressions for all the decisions of the players at the SPE.
3. If neither Assumption A1 nor Assumption A2 holds, then the decision maker in Stage 1 (the State) anticipates a possibly different functional form for D_1°, corresponding to one of the two cases in (2), D_2°, corresponding to one of the two cases in (3), and θ°, corresponding to the outcome of a suitably-modified analysis of Stage 3, depending on the current decision taken by the State in Stage 1. Again, compared to item 1 above, a number of additional subcases may be needed in the analysis of the SPE, making it unlikely to solve the SPE in closed form.

4 Effects on profits and social welfare of changes in the exogenous parameters

In this section, we analyze the effects of changes in the exogenous parameters α_1, α_2, c_1, and c_2 on the profits and social welfare, namely on Π_1, Π_2, and W, evaluated at the equilibrium path of the SPE (i.e., obtained by replacing τ, p_1, θ, and p_2 with their optimal values at the equilibrium path). Their expressions are reported in the Appendix A, together with the expressions of W_1, W_2, D_1, and D_2, also evaluated at the equilibrium path of the SPE.

Proposition 2 An exogenous increase of $\alpha_1 < 0$ (i.e., its exogenous decrease in absolute value) represents a strong Pareto's improvement for the players (namely M_1, M_2, and S), in the sense that it leads to a strict increase of Π_1°, Π_2°, and W°.

Proof To prove the statement, it suffices to check the positivity of the following first partial derivatives:

- $\frac{\partial \Pi_1^\circ}{\partial \alpha_1} = \frac{(\alpha_2\,\lambda - 2\,c_1 + 2\,\alpha_1)\beta_1}{16} + \frac{d_1}{8} > 0$, due to the first part of Assumption A2;

- $\frac{\partial \Pi_2^\circ}{\partial \alpha_1} = -\frac{\beta_1 \left(\left(-\frac{\beta_1\,\alpha_2\,\lambda^2}{2}+\lambda\,((c_1-\alpha_1)\beta_1-d_1)+4\,c_2\,\beta_2-4\,d_2\right)\mu-\alpha_2\right)\lambda}{32\,\mu\,\beta_2} > 0$, due to Assumption A1 and the first part of Assumption A2, $\alpha_2 > 0$, $\beta_1 > 0$, $\beta_2 > 0$, $\lambda > 0$, and $\mu > 0$;
- $\frac{\partial W^\circ}{\partial \alpha_1} = \frac{(\alpha_2\,\lambda-2\,c_1+2\,\alpha_1)\beta_1}{8} + \frac{d_1}{4} > 0$, due to the first part of Assumption A2. □

Proposition 3 An exogenous increase of $\alpha_2 > 0$ represents a strong Pareto's improvement for the players.

Proof Again, to prove the statement it suffices to check the positivity of the following first partial derivatives:

- $\frac{\partial \Pi_1^\circ}{\partial \alpha_2} = -\frac{\left(\left(-\frac{\alpha_2\,\lambda}{2}+c_1-\alpha_1\right)\beta_1-d_1\right)\lambda}{16} > 0$, due to the first part of Assumption A2, and $\lambda > 0$;
- $\frac{\partial \Pi_2^\circ}{\partial \alpha_2} = -\frac{\left(\beta_1\,\mu\,\lambda^2+2\right)\left(\left(-\frac{\beta_1\,\alpha_2\,\lambda^2}{2}+\lambda_2\,((c_1-\alpha_1)\beta_1-d_1)+4\,c_2\,\beta_2-4\,d_2\right)\mu-\alpha_2\right)}{64\,\mu^2\beta_2} > 0$, due to Assumption A1 and the first part of Assumption A2, $\alpha_2 > 0$, $\beta_1 > 0$, $\beta_2 > 0$, $\lambda > 0$, and $\mu > 0$;
- $\frac{\partial W^\circ}{\partial \alpha_2} = \frac{\left(\beta_1\,\alpha_2\,\lambda^2+\lambda((-2\,c_1+2\,\alpha_1)\beta_1+2\,d_1)-8\,c_2\,\beta_2+8\,d_2\right)\mu+2\,\alpha_2}{16\,\mu} > 0$, due to Assumption A1 and the first part of Assumption A2, $\alpha_2 > 0$, $\lambda > 0$, and $\mu > 0$. □

Proposition 4 An exogenous decrease of $c_1 > 0$ represents a strong Pareto's improvement for the players.

Proof In this case, to prove the statement it suffices to check the negativity of the following first partial derivatives:

- $\frac{\partial \Pi_1^\circ}{\partial c_1} = \frac{(-\alpha_2\,\lambda+2\,c_1-2\,\alpha_1)\beta_1}{16} - \frac{d_1}{8} < 0$, due to the first part of Assumption A2;
- $\frac{\partial \Pi_2^\circ}{\partial c_1} = \frac{\beta_1 \left(\left(-\frac{\beta_1\,\alpha_2\,\lambda^2}{2}+\lambda\,((c_1-\alpha_1)\beta_1-d_1)+4\,c_2\,\beta_2-4\,d_2\right)\mu-\alpha_2\right)\lambda}{32\,\mu\,\beta_2} < 0$, due to Assumption A1 and the first part of Assumption A2, $\alpha_2 > 0$, $\beta_1 > 0$, $\beta_2 > 0$, $\lambda > 0$, and $\mu > 0$;
- $\frac{\partial W^\circ}{\partial c_1} = \frac{(-\alpha_2\,\lambda+2\,c_1-2\,\alpha_1)\beta_1}{8} - \frac{d_1}{4} < 0$, due to the first part of Assumption A2. □

Proposition 5 An exogenous decrease of $c_2 > 0$ represents a weak Pareto's improvement for the players, in the sense that it leads to a weak increase of Π_1°, Π_2°, and W°, and at least one of these increases is strict.

Proof In this case, to prove the statement one needs to study the signs of the following first partial derivatives:

- $\frac{\partial \Pi_1^\circ}{\partial c_2} = 0$ identically, being Π_1° independent of c_2;
- $\frac{\partial \Pi_2^\circ}{\partial c_2} = \frac{\left(-\beta_1\,\alpha_2\,\lambda^2+\lambda((2\,c_1-2\,\alpha_1)\beta_1-2\,d_1)+8\,c_2\,\beta_2-8\,d_2\right)\mu-2\,\alpha_2}{16\,\mu} < 0$, due to Assumption A1 and the first part of Assumption A2, $\alpha_2 > 0$, $\lambda > 0$, and $\mu > 0$;

- $\frac{\partial W^{\circ}}{\partial c_2} = -\frac{\beta_2 \alpha_2}{2} < 0$, due to $\alpha_2 > 0$, and $\beta_2 > 0$. □

5 Numerical results

We report some numerical evaluations of the performance at the (equilibrium path of the) SPE of the game (to avoid excessive repetitions and without any risk of ambiguity, in the rest of this section and in the two Appendices, the expression "equilibrium path of the SPE" is replaced by "SPE", apart from the captions of the figures, which specify it). Unless otherwise reported, we consider the following choices for the values of the parameters. The negative externality (per-unit) of the non-green product ω_1 is set to $\alpha_1 = -5$. The value of the exogenous parameter associated with the social advertisement campaign is set to $\mu = 1$. The values of the exogenous marginal cost of the green product ω_2 and the consumption transfer parameter from ω_1 to ω_2 are set, respectively, to $c_2 = 8$ and $\lambda = 0.5$. We also remark that we will choose different values for the exogenous marginal cost c_1 of the non-green product ω_1 and the positive externality (per-unit) α_2 of the green product, to essentially allow for a comparison of different cases, i.e., where $c_1 > c_2$ or $c_1 < c_2$, and similarly with the extent of the positive externality to be, in absolute value, greater than or less than the negative one, i.e., comparing the absolute values of α_1 and α_2.

For what concerns the demands D_1 and D_2, they are governed by the pairs of parameters (d_1, β_1) and (d_2, β_2) respectively. For these, we consider the following two scenarios: (A) $(d_1, \beta_1) = (180, 10)$, $(d_2, \beta_2) = (100, 10)$; (B) $(d_1, \beta_1) = (100, 10)$, $(d_2, \beta_2) = (100, 6)$. These choices are also summarized in Table 1. The idea behind these values is that, in Scenario A, the technology behind ω_1 and ω_2 is not particularly advanced, which implies that customers are not aware of the difference between green and non-green products and modify the demand according to the price of the product in the same way, i.e., $\beta_1 = \beta_2$; also, the maximum demand d_2 of the green product is slightly lower than d_1, because it may be less available [Angrist and Krueger (2001)]. In contrast, Scenario B is supposed to describe a scenario in which the technology is more mature and impacting, making the demands of the two products identical if the price is zero, thus $d_1 = d_2$. However, as the price increases, the demand for the green product is stronger, as customers tolerate a slightly higher price of a more environmentally friendly technology, therefore $\beta_2 < \beta_1$.

We now discuss some numerical results. A more in-depth analysis of the quantities involved is reported in Appendix B. Fig. 2 shows the SPE total welfare W° as a function of the marginal cost c_1, for different scenarios of demand relationships and positive externality α_2. Three different values of α_2 are considered, $\alpha_2 \in \{1, 15, 50\}$, chosen to represent situations where α_2 is lower, higher, or extremely higher than the absolute value of α_1. The trends shown are intended to show that Propositions 3 and 4 hold, which can be explored in more detail by referring to the individual SPE profits of M_1 and M_2, see Appendix B.

Also, one can notice that some curves stop, which is due to Assumption A2 no longer being met. The stopping point can be shown to correspond to the case in which M_1 obtains zero profit due to the demand D_1 being nullified. This means that if c_1 further increases, D_1 will actually stay at zero and the analysis will no longer hold because there is no demand for the non-green product ω_1 that can be transferred to the green product ω_2.

Table 1 Values of the parameters for the two scenarios considered in Sect. 5

Scenario	d_1	β_1	d_2	β_2
A	180	10	100	10
B	100	10	100	6

We note that different demand scenarios achieve similar performance, while the main difference is due to the positive externality α_2. In particular, the total welfare is contained when α_2 is limited. This is consistent with the intuition that if the positive impact of green technology is marginal, then the social benefit is also limited. In contrast, welfare becomes very high for high α_2, that is, when green technology is extremely beneficial in the societal sense.

In Fig. 3, we plot the excise duty τ° applied by the State at the SPE versus the marginal cost c_1. In addition to the plots with the same conventions as in the previous figure, we also show two lines (one for each demand scenario) representing the upper bound $\tau^\circ \leq d_1/\beta_1 - c_1$ that arises as a consequence of Assumption A2, see the analysis made in Sect. 3.5. This justifies why some curves in the previous figures terminate at an intermediate point, which is where τ° is intercepted by the upper limit. Additionally, it is shown that the excise duty can be negative, even though this happens when the positive externality α_2 is high and also the cost c_1 is heavy. As argued in the discussion of Equation (18), the underlying reason behind this phenomenon is that the production of ω_1, despite its negative impact reflected by α_1, can also lead to an increased production of ω_2, which acts as a positive externality for the system. Thus, if α_2 is very high, then the State may be willing to incentivize the production of ω_1, not because it is itself desirable, but since it increases the green transition towards ω_2. As such, if the cost of ω_1 is high, the State may even consider subsidizing it.

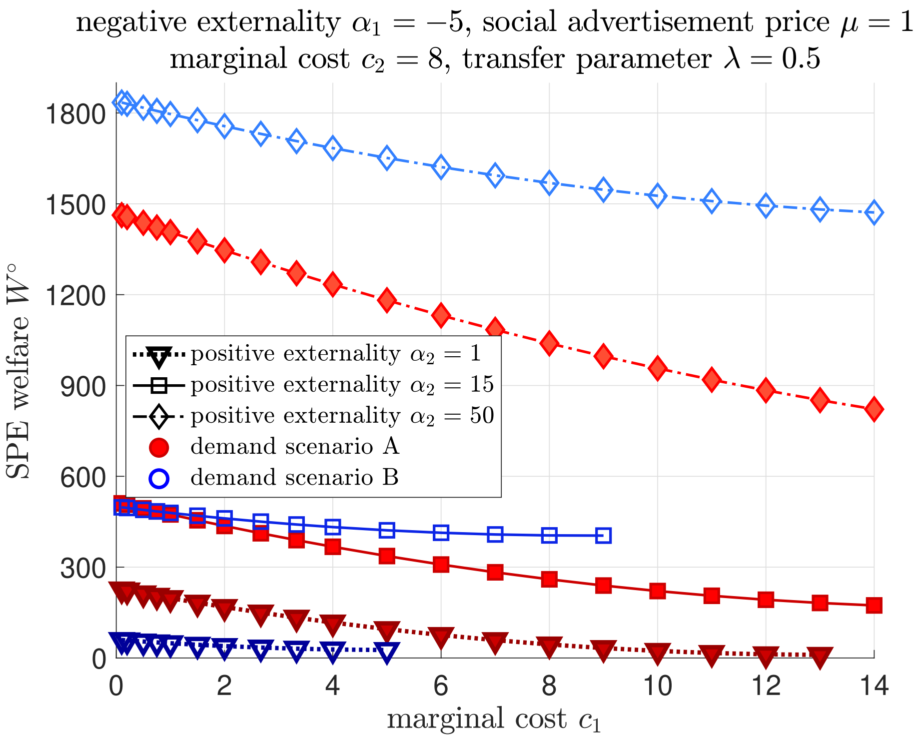

Fig. 2 SPE welfare W° (at the equilibrium path) vs. c_1

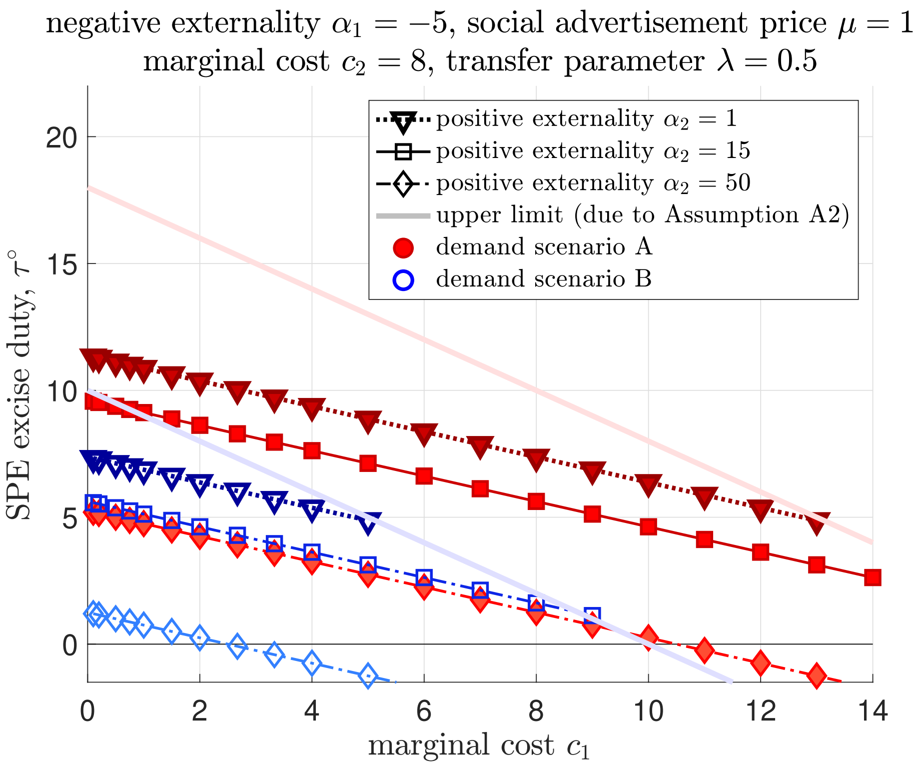

Fig. 3 SPE excise duty τ° (at the equilibrium path) vs. c_1

6 Conclusions and possible future developments

This work shows that the sale of a green product can be aided by an indirect intervention from the State providing an advertisement campaign addressed to the consumer, without the need to provide classical direct contributions to the green product. Often, such subsidies are vulnerable to fraud against the State [Drew and Drew (2010)]: e.g., people may falsely declare to purchase the green product just to get the monetary contribution. Moreover, persuasion through advertisement may have a better impact than taxation and also more long-lasting effects, persisting after the end of the campaign [Hassan et al. (2007)]. At the same time, the presence of an excise duty (when non-negative) on the non-green product has two effects: reducing the consumption of that product (which is associated with a negative externality); getting tax revenues to fund the advertising campaign. The theoretical findings have been complemented by numerical results, which have provided additional insights.

Possible future developments involve:

- analyzing long-lasting effects, to assess the robustness of the findings of the proposed model;
- validating its theoretical results with empirical data;
- investigating theoretically the sign of other partial derivatives with respect to the ones considered in Sect. 4;

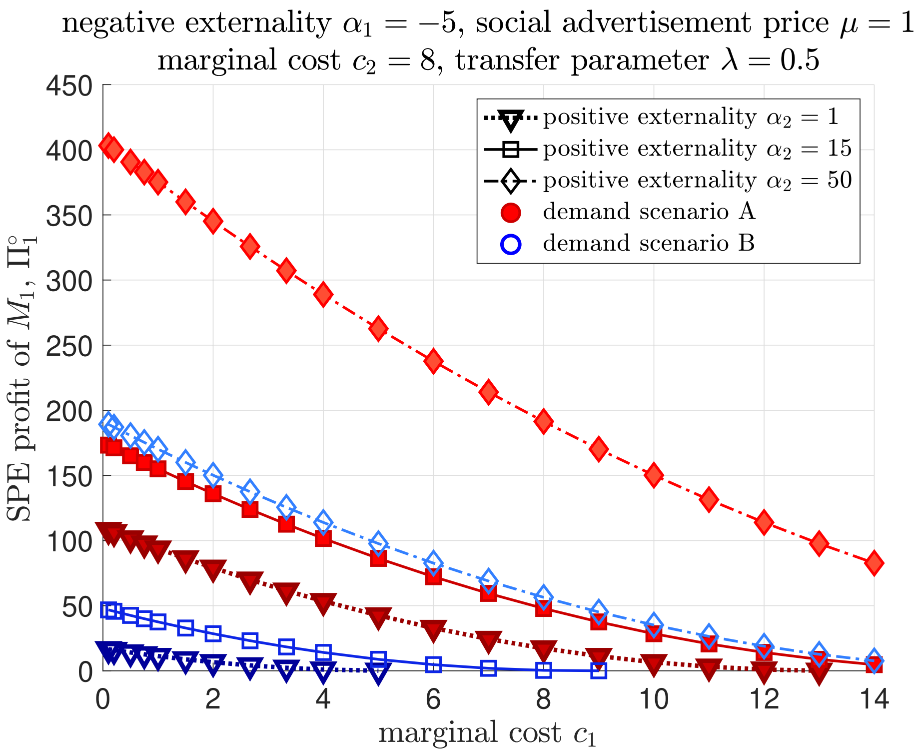

Fig. 4 SPE profit Π_1° (at the equilibrium path) of manufacturer M_1 vs. c_1

- increasing the number of decision stages in the model by including in it an additional green product ω_3, which is significantly better than ω_2, and becomes available later, due to further technological innovation;
- filling the details of the analysis for all the model variations introduced briefly in Sect. 3.7;
- investigating other model variations wherein the green/non-green producers play simultaneously, at least in one stage (competing, e.g., à la Cournot/à la Bertrand [Buccella et al. (2021); Gori et al. (2024)]), or wherein they act as a single player (in different stages), who optimizes the sum of their profits (modeling the integration of the two firms), then analyzing their corresponding changes on the equilibrium value of the social welfare function;
- formulating and analyzing suitable stochastic variations of the model, possibly examining other notions of equilibrium [Osborne and Rubinstein (1994)];
- extending the model to a principal/agent framework [Biancalani et al. (2022); Li et al. (2024)];
- integrating it with game-theoretical models of transboundary pollution mitigation [El Ouardighi et al. (2020, 2018)], by including, e.g., the interaction of governments of two different countries based on contingent mitigation strategies (which are related to the concept of Markov-Perfect Nash Equilibrium, or MPNE);
- or finally, exploring macroeconomic insights within the energy sector to understand the broader impact of contingent policy measures in mitigating transition risks [Ciola et al. (2023)].

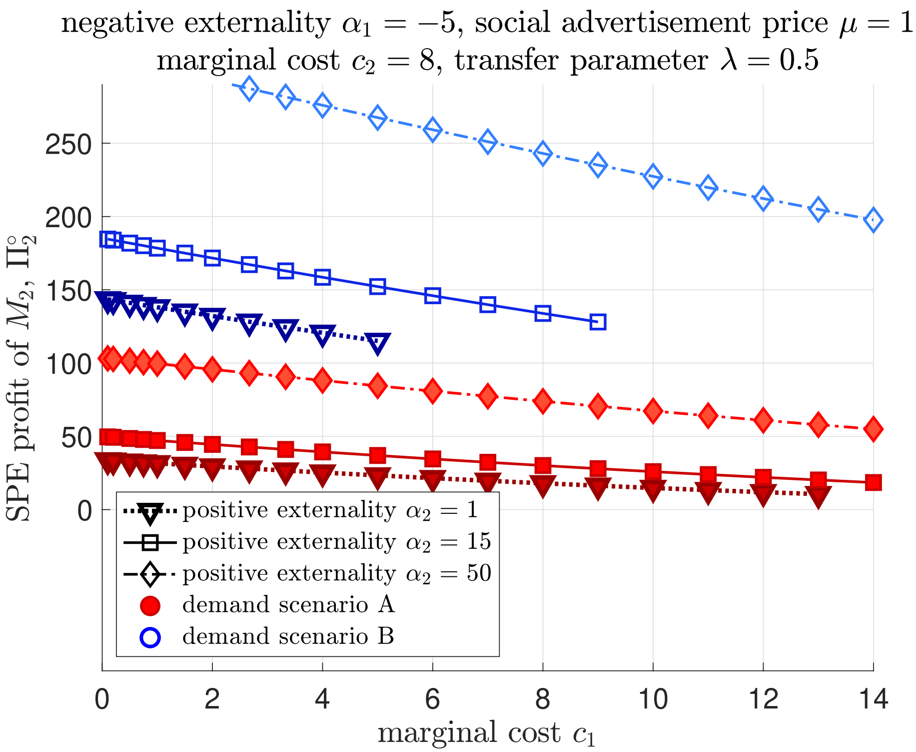

Fig. 5 SPE profit Π_2° (at the equilibrium path) of manufacturer M_2 vs. c_1

Appendix A

Here, we report the SPE values of the main variables, as functions of the exogenous parameters:

$$\Pi_1^\circ = \frac{\left(\left(-\frac{\alpha_2\lambda}{2} + c_1 - \alpha_1\right)\beta_1 - d_1\right)^2}{16\beta_1},$$

$$\Pi_2^\circ = \frac{\left(\left(-\frac{\beta_1\alpha_2\lambda^2}{2} + \lambda\left((c_1-\alpha_1)\beta_1 - d_1\right) + 4c_2\beta_2 - 4d_2\right)\mu - \alpha_2\right)^2}{64\mu^2\beta_2},$$

$$W^\circ = \frac{4\left(-\frac{\alpha_2\lambda}{2} + c_1 - \alpha_1\right)^2\mu\beta_1^2}{32\beta_1\mu} + \frac{\left(\left((-16c_2\beta_2 + 4\lambda d_1 + 16d_2)\alpha_2 - 8d_1(c_1-\alpha_1)\right)\mu + 2\alpha_2^2\right)\beta_1}{32\beta_1\mu} + \frac{4d_1^2\mu}{32\beta_1\mu},$$

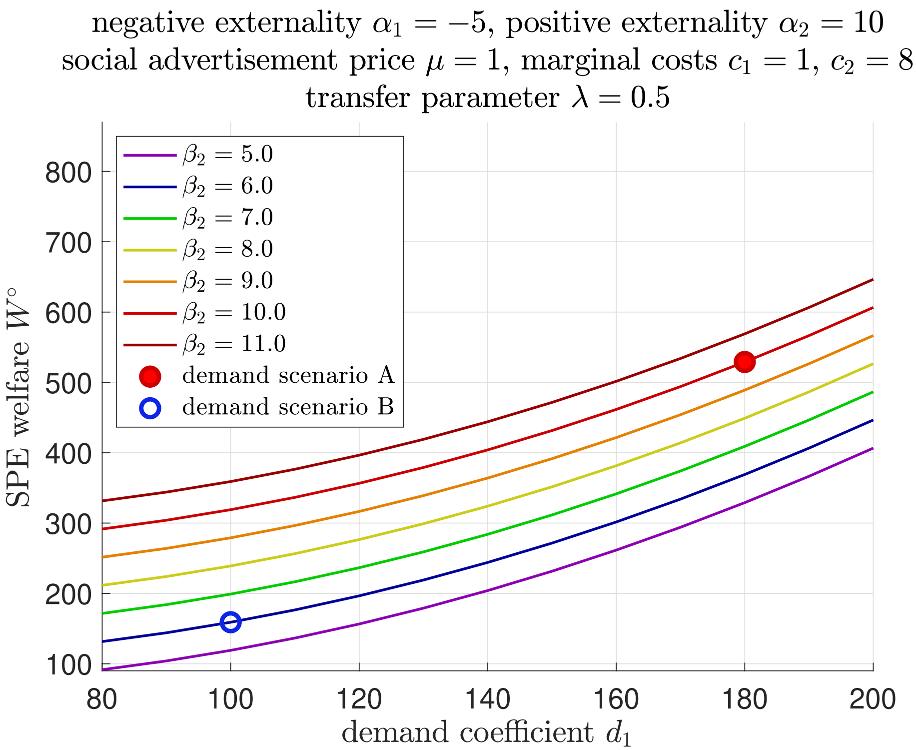

Fig. 6 SPE welfare W° (at the equilibrium path) for different demand scenarios

$$W_1^\circ = \frac{\left(\left(-\frac{\lambda\alpha_2}{2} + c_1 - \alpha_1\right)\beta_1 - d_1\right)\left(\left(\frac{\lambda\alpha_2}{2} + c_1 - \alpha_1\right)\beta_1 - d_1\right)}{8\beta_1},$$

$$W_2^\circ = \frac{-\alpha_2^2 - 2\left(\left(-\frac{\lambda^2\beta_1\alpha_2}{2} + \left(\left(c_1 - \alpha_1\right)\beta_1 - d_1\right)\lambda + 4c_2\beta_2 - 4d_2\right)\mu - \alpha_2\right)\alpha_2}{16\mu},$$

$$\tau^\circ = -\frac{\beta_1\alpha_2\lambda + 2\beta_1 c_1 + 2\beta_1\alpha_1 - 2d_1}{4\beta_1},$$

$$p_1^\circ = \frac{\left(-\alpha_2\lambda + 2c_1 - 2\alpha_1\right)\beta_1 + 6d_1}{8\beta_1},$$

$$\theta^\circ = \frac{\alpha_2}{4\mu},$$

$$p_2^\circ = \frac{\left(\beta_1\alpha_2\lambda^2 + \left(\left(-2c_1 + 2\alpha_1\right)\beta_1 + 2d_1\right)\lambda + 8c_2\beta_2 + 8d_2\right)\mu + 2\alpha_2}{16\mu\beta_2},$$

$$D_1^\circ = \frac{\left(\alpha_2\lambda - 2c_1 + 2\alpha_1\right)\beta_1}{8} + \frac{d_1}{4},$$

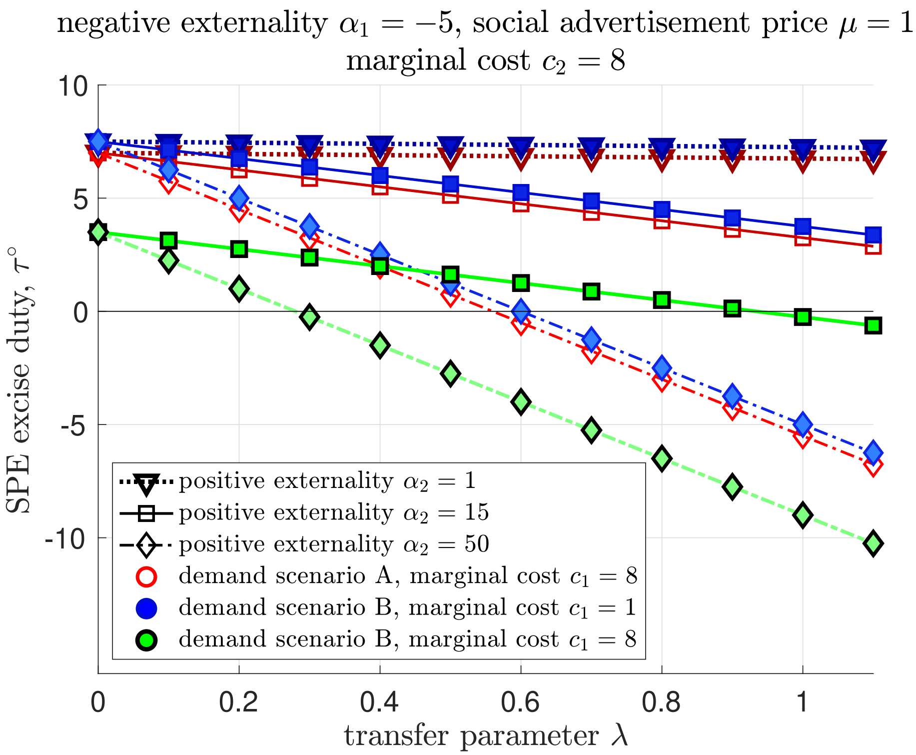

Fig. 7 SPE excise duty τ° (at the equilibrium path) vs. λ

$$D_2^\circ = \frac{\left(\beta_1 \alpha_2 \lambda^2 + \left(\left(-2c_1 + 2\alpha_1\right)\beta_1 + 2d_1\right)\lambda - 8c_2\beta_2 + 8d_2\right)\mu + 2\alpha_2}{16\mu}.$$

Appendix B

In this appendix, we report more detailed investigations of all the quantities involved in the analysis made in Section 5.

Fig. 4 reports the SPE profit Π_1° of manufacturer M_1 as a function of its marginal cost c_1. As discussed for the excise duty in Fig. 3, the curves stop when Assumption A2 is no longer met. However, as visible in the graph, the stopping point corresponds to zero profit for M_1 due to nullifying the demand D_1.

This evaluation confirms some theoretical results previously discussed, specifically in Propositions 3 and 4. In fact, M_1 considers any decrease in the marginal cost c_1 or increase in the positive externality α_2 as improving its profit. Moreover, if α_2 is lower than or com-

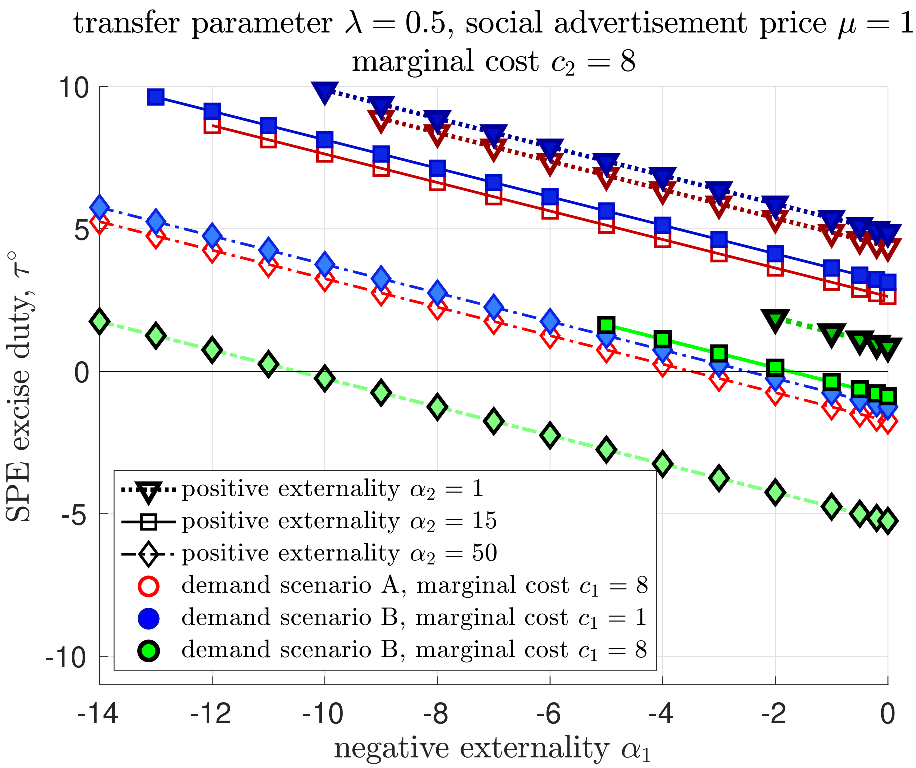

Fig. 8 SPE excise duty τ° (at the equilibrium path) vs. α_1

parable to the absolute value of α_1, then the profit obtained by M_1 at the SPE is relatively limited. This is due to the excise duty imposed by the State to counteract the negative effects (e.g., pollution) of producing ω_1. Thus, M_1 is able to make just a relatively low profit and only if its production costs are contained.

Fig. 5 reports instead the SPE profit Π_2° of manufacturer M_2 under the same conditions of Fig. 4. Propositions 3 and 4 still apply to M_2 as well, which increases its profit as c_1 decreases and/or α_2 increases. Also, demand scenario B is more favorable to this player. Overall, even though the trend of Π_2° is still decreasing in c_1, it is clearly less sensitive to it than Π_1° since the cost of ω_1 affects the profit of M_2 only indirectly.

We can remark that overall, similar conclusions can be drawn for both demand scenarios. Although scenario A is slightly advantageous for M_1 over scenario B, and the opposite happens for M_2, the considered demand scenarios exhibit similar trends, which confirms that our parametric choices in Table 1 are not restrictive.

As further confirmations of these findings, in Fig. 6, we give an in-depth exploration of the dependence on the demand scenarios for a specific numerical choice all the parameters.

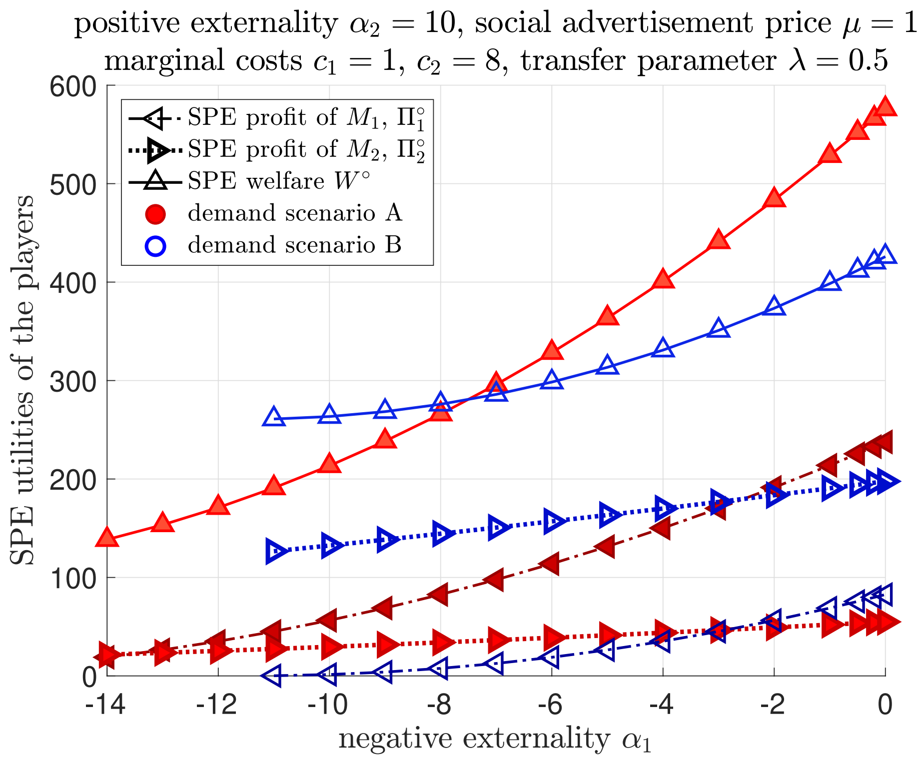

Fig. 9 SPE utilities (at the equilibrium path) vs. α_1

Here, α_1 is still -5 but $\alpha_2 = 10$, and the marginal costs are $c_1 = 1$ and $c_2 = 8$, while the other parameters are set to their defaults $\mu = 1$ and $\lambda = 0.5$. Finally, we change d_1 and β_2, while leaving d_2 and β_1 as constantly equal to 100 and 10, respectively. We tried multiple combinations of the pair (d_1,β_2) – in particular, the specific cases of Scenario A and Scenario B are highlighted – and the results were always found to be qualitatively similar. The plot just shows some variability due to the demand scenario, once again confirming the validity of our results for the entire range of demand parameters.

We can also explore further the dependence on λ, which describes the "transfer" between ω_1 and ω_2. Thus, in Fig. 7 we plot τ° vs. the transfer parameter λ. In this case, c_1 is set to specific different values. Once again, it is confirmed that a negative τ° can be obtained when α_2 is extremely high, whereas instead the trend is basically flat when α_2 is low. This is still consistent with the interpretation of the parameters.

So far, we have not explored the dependence on α_1. About this, we first of all plot the excise duty in Fig. 8 vs. the negative externality α_1. The curves stop at a certain value of τ°, corresponding to meeting Assumption A2 with equality, i.e., $\tau^\circ = \delta_1/\beta_1 - c_1$. Still, it is again shown that the excise duty can become negative, which happens when the positive externality α_2 is high, and only when the negative externality α_1 is relatively close to 0.

Fig. 9 explores the SPE values of the utility functions of all the players (M_1, M_2, and State) versus α_1. The scenarios are the same adopted in the other figures, and this result

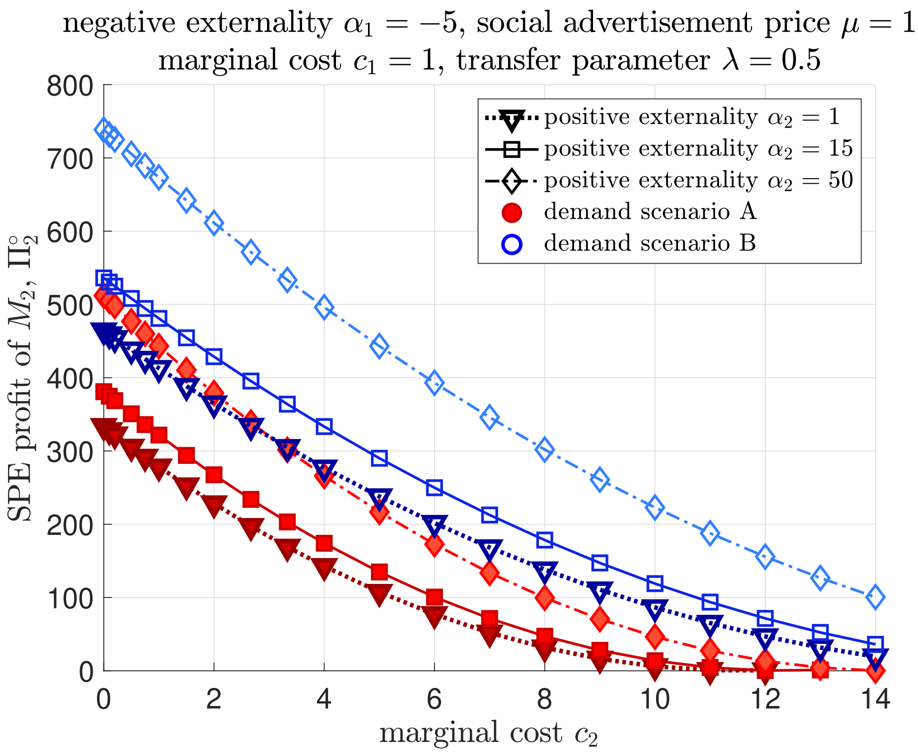

Fig. 10 SPE profit Π_2° (at the equilibrium path) of manufacturer M_2 vs. c_2

serves to validate Proposition 2, since it shows that increasing α_1 (i.e., decreasing its absolute value) represents a strong Pareto's improvement.

The last theoretical result to validate is Proposition 5, which implies that decreasing c_2 improves Π_2° and W°. These results are shown in Figs. 10 and 11, respectively. Conversely, neither Π_1° (coherently with what stated by Proposition 5) nor τ° are affected by c_2, and therefore their dependence is not shown. The trend in Fig. 10 is actually similar to that of Fig. 4, since c_2 directly influences the SPE profit Π_2° in a similar way to how c_1 impacts on Π_1°. For what concerns the SPE welfare W° in Fig. 11, instead, the behavior is similar to Fig. 2, with the SPE welfare being mostly determined by the positive externality α_2. However, it is also shown that the two demand scenarios, while having a similar descent trend depending on α_2, are different in that the SPE welfare is higher for demand scenario A at low values of cost c_2, but then scenario B takes over since its decrease in demand is less sensitive to the price.

To sum up, all of these results confirm the theoretical trends and give additional insight, for example on the case with negative excise duty and/or significant welfare. Moreover, it is suggested that even when assumptions like A2 no longer hold, the behavior can still be predicted from the previous values of the curve.

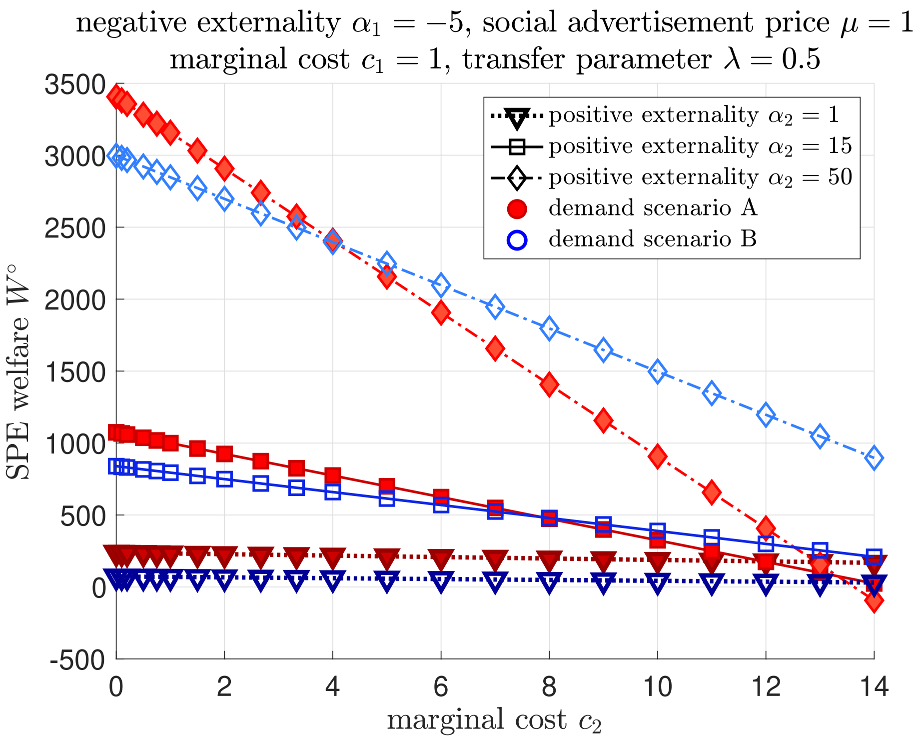

Fig. 11 SPE welfare W° (at the equilibrium path) vs. c_2

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Data Availability Numerical results for this article are based on artificial realistic values for the parameters, which are reported in the main text.

Declarations

Conflict of interest The authors declare that there is no Conflict of interest.

Generative AI and AI-assisted Technologies in the Writing Process Generative AI was not used to prepare the manuscript.

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