Conflict Initiation Function Shapes the Evolution of Persistent Outcomes in Group Conflict

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Abstract

We take an evolutionary perspective to explore the implications of different relationships between power and initiation of conflicts (i.e., conflict initiation function) for the long-run distribution of power between groups. So far, attention has focused on how the role played by the relationship between power and success in conflicts (i.e., conflict success function) affects the long-run distribution of power. We find conditions under which hegemony is a long-run outcome, as well as analogous conditions for balance of power. Specifically, hegemony prevails when conflicts are more likely to be initiated by stronger groups against weaker groups, while balance of power prevails when the opposite holds. Interestingly, the conflict success function plays a minor role in our setting, where victory or defeat are always outcomes that occur with non-negligible probability.

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1 Introduction

Group conflict is a crucial force of change in the world, shaping allocations of power and resources. In this paper, we study the evolution of group conflict, trying to shed light on the general conditions under which different distributions of power are persistent over time.

The literature on group conflict and international relations has focused on two main distributions of power that are observed to be persistent, in the sense that they have proved on several occasions to survive over long periods of time: *hegemony* and *balance of power*.

Hegemony is a situation in which power is extremely concentrated in the hands of a single group. We can think of power in several ways, from the possession of lands to the market share or military predominance. The persistence of hegemonies has been justified on the basis that, once a group has obtained all the power and becomes the hegemon, it is fully satisfied, since there is no more gain to be pursued. The only thing that can worry a hegemonic power is to lose its position, but, at the same time, the other groups are too weak to try to overcome the hegemon. In this case, other groups have to accept a subordinate role and end up supporting the stability that the presence of a hegemon can provide. Several examples of hegemonies over history have been justified with similar arguments: Old, Middle and New Egyptian Kingdoms, the city-state of Sparta in the Peloponnesian League, the Roman Empire, the Persian Empire, several Chinese Dynasties (such as Song, Ming and Qing), the Carolingian Empire, or the British Empire.

Balance of power is a situation in which the power is evenly distributed among all groups, societies or agents. There are solid arguments also in favor of it. Power parity can serve as a suitable device for preventing conflict, given that no single state concentrates enough power to expect a sound victory against others, making warfare much riskier. To destabilize the balance of power, it is not enough that a change occurs within a society, such as the adoption of new military organizations and technologies or the relaxation of incentive compatibility constraints of the individuals due to, for example, nationalism or fanaticism of any kind. Given the likelihood of those situations in which war could be actually looked for, once a decent parity of power is established, the different states will seek a policy of alliances and counter-alliances so that the equilibrium is not distorted, building up the stability of the system and making balance of power highly persistent over time. Some historical examples of this (in the sense that relatively stable periods of peace have been observed) can be found, for instance, in the Italic League (1454), the Peace of Westfalia (1648), the Congress of Vienna (1814), and the modern European Union.

It is reasonable to conjecture that both hegemony and balance of power may be persistent under appropriate favorable conditions. A fundamental concept used to study such conditions is the conflict success function, otherwise called conflict resolution function, introduced by Hirshleifer (1988, 2000). This function takes as input the strength of the contending forces, usually measured by military power, and returns the outcome of the conflict. In a more naive interpretation, the conflict resolution function yields the share of resources retained by every side after the conflict, which is like saying that, for instance, the result of a battle depends solely on the proportion and quality of soldiers in each army. Another frequent and more realistic interpretation is that this function results in a probability of winning a conflict for each of the factions at play. This is the approach we adopt here, recognizing the intrinsic stochastic nature of conflict.

Our crucial innovation is the introduction, besides the conflict resolution function, of a *conflict initiation function*, which takes as input the current power of a given group and the power of its potential target, and gives as output the likelihood that the group initiates a conflict with the target group. We stress that the initiation of a conflict is stochastic and not necessarily the result of best-replying, capturing the fact that initiating a conflict is often not the result of a forward-looking decision but, rather, the outcome of many concurrent determinants that go beyond a rational decision process. One might ask why the separation of conflict resolution and conflict initiation is sensible. Our results show that the shape of the conflict initiation function directly impacts long-run outcomes.

Our main contribution is the demonstration that, under the rather mild assumptions that nobody can secure complete victory with certainty (no matter how relatively stronger) and that a conflict can be initiated only if some gain can be obtained in terms of the possession of resources, the conditions that sustain hegemony and balance of power in the long run only depend on the characteristics of the conflict initiation function. In contrast, the characteristics of the conflict success function can, at most, affect convergence times. More specifically, we show that hegemony is more persistent whenever the stronger groups are more likely to initiate a conflict, whereas balance of power is more persistent whenever the weaker groups are more likely to initiate a conflict. Formally, we characterize conditions for hegemonies and balance of power to be observed almost always in the long run as the probability of war initiation goes to zero (i.e., they are *stochastically stable* as defined in Foster and Young, 1990), and we show that such conditions hold for every conflict resolution function that satisfies the property that any group has a probability bounded away from zero of avoiding defeat, no matter the power of the conflicting groups.

We stress that an essential characteristic of conflict success functions plays an important role in our analysis: the impossibility of securing complete victory with certainty. While one might assume that a party with zero power will never have a chance to avoid defeat in a conflict (as it happens when the contest success function is modeled as a ratio between contenders' forces, see Hirshleifer, 1989), it seems more reasonable to assume that even a negligible amount of power allows having a non-negligible chance to avoid total defeat. This latter assumption embeds the stochastic nature of the outcome of a conflict, which can be the consequence of a model that is non-fully fledged, leaving aside the possibility of external intervention in the conflict, or even that other determinants can affect the probability of victory, like the morale of the soldiers and the spirit of sacrifice to the cause.

The rest of the paper is structured as follows. Section 2 discusses the relevant literature. Section 3 presents the model and states our assumptions. Section 4 provides results about the long-run emergence of hegemonies and balance of power. Section 5 provides some historical interpretation of the results. Finally, section 6 concludes.

2 Related literature

An essential insight by Hirshleifer (2001) is that the allocation of resources within a society matters, and whether a group decides to invest in military or productive activities (or leisure, or any other kind of) may be crucial for the final result. Recent evolutionary literature on group conflict has been focusing on this issue. Levine and Modica (2013b) explores the rise of hegemonies in a setting in which outsiders can exert influence, and each society devotes resources to state power (which is a determinant of conflict success) and productive activities. They find that the development of a hegemony is crucially connected to the level of state power in a society. However, they do not provide conditions under which a balance of power between societies is achieved, even when they do mention that more inclusive states are less likely to become a hegemon. Very related to this paper is Levine and Modica (2013a), in which they find that hegemony is likely to appear whenever a *strongest* society emerges that also has expansionary desires. Finally, Levine and Modica (2016) finds that hegemonies are more likely to survive if external intervention is limited, a concept already introduced in Levine and Modica (2013b). In contrast to these papers, we do not explicitly model the in-society game, taking a reduced perspective.

So far, no results about the emergence of balance of power had been found in the evolutionary literature on group conflict. To address that, Levine and Modica (2018) shows that external intervention may not only be a source of the fall of hegemonies, but actually, it can help to achieve balance of power. Depending on the strength of the intervention, different outcomes are possible, including *hot peace* (a low-intensity but long-lasting conflict) or *prolonged war* (a high-intensity conflict with many casualties). These preliminary ideas are extended in Levine et al. (2022), where they distinguish between extractive and inclusive balance of power and consider not only the strength of outsiders but also the quality of fortifications as possible sources of balance of power. It is also worth noticing that in this paper, different active groups (commercial elites or military elites) decide whether or not to initiate a conflict. Finally, all the previous elements are summarized and confronted with empirical evidence in Levine and Modica (2021).

Going beyond the issue of the long-run selection of hegemony as opposed to balance of power, the evolutionary literature has focused on social conflict, i.e., conflict within a society, such as in Naidu et al. (2017) and Hwang et al. (2018). Further, the modeling of the conflict initiation function is, in fact, a way to model transitions across states in a Markovian stochastic evolutionary model, quite similar to what is done in the literature on stochastic behavioral rules as myopic best reply with uniform mistakes (Young, 1993; Kandori et al., 1993), intentional mistakes (Naidu et al., 2010), coalitional mistakes (Newton, 2012), condition-dependent mistakes (Bilancini and Boncinelli, 2019), or break-easier-than-build mistakes (Boncinelli and Pin, 2012, 2018), and the logit model (Blume, 1993) or similar stochastic behavioral rules implying that likelihood of selection depends negatively on the payoff loss associated to it.

Finally, there is a vast game theoretic literature about conflict in different settings, mainly relying on sophisticated strategic thinking and adopting traditional solution concepts for sequential games, e.g., Baliga and Sjöström (2004), Jackson and Morelli (2007a), Jackson and Morelli (2007b), Morelli and Rohner (2014), Caselli et al. (2015), Jackson (2015), Canidio and Esteban (2018). For a book-length treatment of early contributions, see Coyne et al. (2011), while for a recent, comprehensive review of the literature about war in economics, see Kimbrough et al. (2020). Similar reviews with an evolutionary focus can be found in Rusch and Gavrilets (2020), from the theoretical biology perspective, and Glowacki et al. (2020), from the anthropological perspective.

3 Model

In this section, we craft the elements of our model to create a simple, straightforward theory of the evolution of societies in conflict.

Groups and resources. There is a finite set of groups $G = \{1, \ldots, |G|\}$ that compete over an also finite number of resources $R \in \mathbb{N} > 0$. Time is discrete and denoted by $t = 0, 1, 2, \ldots$

A state at time t is an allocation of resources among groups in G, and it is denoted by $z_t = (z_{t,1}, ..., z_{t,g}, ...z_{t,|G|})$ where $z_{t,g} \in [0, R]$ is the non-negative integer number of resources allocated to group $g \in G$ at time t. In the following, we will refer to groups as societies, states or nations, but groups can be any sub-population acting coordinately within a larger population. We denote the state space with Z,

$$Z = \{z_t \colon \sum_{g \in G} z_{t,g} = R, z_{t,g} \geq 0 \,\text{for all}\, g \in G\}$$

Finally, say that a group is *inactive* whenever it has zero resources, i.e., g is inactive at time t if $z_{t,g} = 0$.

Power distribution. *Balance of power* is a state where all the resources are evenly distributed among groups, i.e., $z_{t,i} = z_{t,j} \,\forall\, i, j \in G$. For this to exist, we assume that $\frac{R}{|G|} \in \mathbb{N}$.

Hegemony is a state where a single group possesses all the resources, i.e., $\exists\, i \in G$ such that $z_{t,i} = R$. Hegemony can be seen as a state corresponding to a vertex of a $|G| - 1$-dimensional simplex, while the balance of power is the state located at the center of the simplex.

Conflict initiation. We consider conflict initiation as a rare and stochastic event. At any time t, a single pair of groups (i, j) is randomly selected and the probability of a conflict being initiated by group i against group j is assumed to depend only on $(z_{t,i}, z_{t,j})$.

Our main object of analysis is the *conflict initiation function*, denoted by $\iota(z_{t,i}, z_{t,j})$, mapping pairs of resource stocks $(z_{t,i}, z_{t,j})$, one for group i and one for group j, into probabilities of i initiating conflict with j at time t.

Conflict success. We model the outcome of a conflict that has been initiated by group i against group j by means of the *conflict success function* that maps the current resources of the two groups involved in the conflict into the probability that $l \in \mathbb{Z}$ resources are won ($l > 0$) or lost ($l < 0$) by group i and, respectively, lost or won by group j – i.e., a transfer of l resources between group i and group j. We denote the conflict success function at time t with $\sigma(l|z_{i,t}, z_{j,t})$ with the resulting probability distribution over l summing up to 1, namely $\sum_{l=-z_{i,t}}^{z_{j,t}} \sigma(l|z_{i,t}, z_{j,t}) = 1$, and allowing for the possibility that no amount of resources is looted by any group $\sigma(0|z_{i,t}, z_{j,t}) > 0$.

Assumptions. We further consider the following assumptions.

Assumption A1 (Hegemon's satisfaction).

$$\iota(R, 0) = 0$$

Assumption A1 states that a hegemon group never attacks. The justification is that conflict aims to gain resources, and a hegemon has nothing to gain from conflict.

Assumption A2 (Conflict initiation function).

$$\iota(z_{t,i}, z_{t,j}) = \epsilon^{r(z_{t,i}, z_{t,j})} < 1, \quad \forall\, (z_{t,i}, z_{t,j}) \neq (R, 0)$$

In A2 $\epsilon \in (0, 1)$ is typically assumed to be very small and $r(z_{t,i}, z_{t,j})$, which is often referred to as "resistance", is an explicit measure of the unlikeliness that group i initiates a conflict towards group j.

Assumption A3 (Victory is always possible).

$$\sigma(1|z_{t,i}, z_{t,j}) > 0, \quad \forall\, (z_{t,i}, z_{t,j}), \text{ with } z_{t,j} \geq 1$$

Assumption A3 means that a group i who initiates a conflict always has some positive probability of subtracting a unit of resources from the target group j, even if the distribution of resources is highly unequal, e.g., even if $z_{t,i} = 0$ and $z_{t,j} = R$. This implies that the attacked group can never be sure to secure victory.

The main reason for having that the probability of winning at least one resource is bounded away from zero for the attacker is that, in determining the outcome of a conflict, not only material resources matter. In fact, there are plenty of historical examples in which a weaker side wins because they have better human resources, are more clever or simply had luck. David against Goliath does not lose with probability one.

From a technical point of view, assumption A3 can be regarded as a rather mild one in that it is consistent with many different specifications on the conflict success function. In particular, if we take as reference the Constant Elasticity of Augmentation (CEA) contest success functions proposed by Hwang (2012), namely, the outcome of a conflict initiated by group i against group j is:

$$\frac{\exp\{\kappa \frac{1}{1-\rho} z_{t,i}^{1-\lambda}\}}{\exp\{\kappa \frac{1}{1-\rho} z_{t,i}^{1-\rho}\} + \exp\{\kappa \frac{1}{1-\rho} z_{t,j}^{1-\lambda}\}} \tag{1}$$

where $\kappa > 0$ measures the scaling of the decisiveness of power disparities and λ is the elasticity of augmentation, possibly taking any real value. It is worth noting that the CEA contest success function particularizes to the difference contest success function for $\lambda = 0$, i.e., the probability of winning depends on the difference in resources. In contrast, it particularizes to totally random contest success function for $\lambda = 1$, i.e., the probability of winning is always one-half no matter the differences between the attacker's and defendant's resources (see Hirshleifer, 1989, for a comparison of different shapes of the contest success function). It is straightforward to recognize that, if $\sigma(l|z_{t,i}, z_{t,j})$ scales with the CEA function, then assumption A3 is satisfied for any parameter value. However, assumption A3 is not satisfied for the exact specification of the ratio contest success functions, while it holds for any specification of the difference contest success functions.

A key aspect of our formulation is that we allow $\sigma(\cdot)$ to have any structure as long as it satisfies A3. In the literature, it is frequently established that the side with more resources is the one with more probability to win. However, we are allowing even for the possibility that the weaker side has actually better chances to win. This could be rationalized along the lines of Levine and Modica (2013b), where the resources devoted to build military power are determined endogenously within a society. So, a society with few resources can devote much of its energy to build military power, for example, if the preferences of its population change through the relaxation of the incentive compatibility constraints (what Levine and Modica call *barbarian hordes* or *zealots*).

Another concern regarding assumption A3 could be that it implies that inactive groups maintain some positive probability of winning a resource by attacking another group, even if they target a hegemon. This possibility does not seem unreasonable, at least when total annihilation of the inactive group is not feasible. Think of a country that has been conquered and left without resources, which are all managed by the invader; in this case, the oppressed population could still rise a revolt to recover a piece of land; success might be unlikely, but not with zero probability.

In the subsequent analysis, we will assume that A1, A2 and A3 always hold. In addition, we will make further alternative assumptions about the conflict initiation function, which are meant to capture alternative ways in which the likelihood of conflict initiation depends on the distribution of groups' resources.

Assumption A4 (Uniform conflict initiation likelihood). *Function* $r(z_{t,i}, z_{t,j})$ *is constant on* $(z_{t,i}, z_{t,j})$, $\forall\ (z_{t,i}, z_{t,j}) \neq (R, 0)$.

Assumption A4 implies that the probability of conflict initiation is independent of the distribution of resources among groups. It is the counterpart, in this setup, of the uniform error model widely applied in the literature on stochastic stability analysis (see Newton, 2018, for a recent survey), and which in fact is implicitly applied in Levine and Modica (2013a,b) where perturbations are introduced in the conflict success function (and where the conflict initiation function is not explicitly modeled).

Assumption A5 (Increasing-in-relative-power conflict initiation likelihood). *The resistance function $r(z_{t,i}, z_{t,j})$ is either decreasing in $z_{t,i}$ and non-decreasing in $z_{t,j}$ or non-increasing in $z_{t,i}$ and increasing in $z_{t,j}$, $\forall\ (z_{t,i}, z_{t,j}) \neq (R, 0)$.*

Assumption A5 implies that the probability that group i initiates a conflict with group j increases in the relative number of resources possessed by i with respect to j. This captures the case where the side with more resources has a greater chance of winning, or the case where having more resources makes losing less troublesome, which in turn may increase the inclination to pursue conquests. Another case is where a society with more resources comes to believe that it is entitled to expand because of its superiority.

Assumption A6 (Decreasing-in-relative-power conflict initiation likelihood). *The resistance function $r(z_{t,i}, z_{t,j})$ is either increasing in $z_{t,i}$ and non-increasing in $z_{t,j}$, or non-decreasing in $z_{t,i}$ and decreasing in $z_{t,j}$, $\forall\ (z_{t,i}, z_{t,j}) \neq (R, 0)$.*

Assumption A6 implies that the probability that group i initiates a conflict with group j decreases in the relative number of resources possessed by i with respect to j. This is the case where a group with few resources has not much to lose and therefore fears less the conflict. This is also the case where a group that does not control as many resources as others is envious and, hence, more likely to initiate a conflict. Another case is that in which military technology is such that the group with fewer resources finds it more efficient to invest in military power than economic activity.

4 Results

Before starting to deliver the results of our model, it is worth mentioning how the transition probabilities between two states are computed. For any two states z_t and z'_{t+1} such that $z'_{t+1,i} = z_{t,i} + l$, $z'_{t+1,j} = z_{t,j} - l$ and $z'_{t+1,k} = z_{t,k}\ \forall k \neq i, j$, the transition probability from z_t to z'_{t+1} can be computed as follows:

$$\mathbb{T}(z_t, z'_{t+1}) = \frac{1}{|G|(|G|-1)}\iota(z_{t,i}, z_{t,j})\sigma(l|z_{t,i}, z_{t,j}) + \frac{1}{|G|(|G|-1)}\iota(z_{t,j}, z_{t,i})\sigma(-l|z_{t,j}, z_{t,i}), \quad (2)$$

which is basically the sum of two products of probabilities: (i) the probability that i and j are paired, multiplied by the probability of i attacking j, multiplied by the probability of i winning l units of resources, and (ii) the probability that j and i are paired, multiplied by the probability of j attacking i, multiplied by the probability of j losing l units of resources.

In the following, we focus on the *long-run evolutionary equilibrium* of the model. Namely, on the set of states that turns out to be stochastically stable according to any stochastic dynamics consistent with (2). Given that the stochastic dynamics in (2) is ergodic, meaning that it is possible to get from every state to every other state with positive probability, a unique invariant distribution exists which describes the time average behavior of the system, thanks to the fundamental theorem of Markov chains. As the amount of noise ϵ tends to zero, the invariant distribution varies and (under mild assumptions that are here satisfied) approaches a limiting distribution, called stochastically stable distribution (Foster and Young, 1990). Technically, we rely on the results by Young (1993), which are built on Freidlin and Wentzell (1984). A long-run evolutionary equilibrium is a state that receives a positive probability in the stochastically stable distribution.

For simplicity, within the following discussion and proofs, we drop the time index t of the states z_t. In the present setup, given two states z' and z'', we say that the *resistance* between state z' and state z'' is a measure of how difficult the transition from state z' to state z'' is in terms of mistakes.

For any conceivable tree (i.e., a graph such that any two vertices are connected by exactly one path) having the state z' as root and all other states as nodes, consider the sum of resistances assigned to each edge of the tree, and take the minimum over trees of such a sum. This number represents the *stochastic potential* of state z'. Intuitively, the stochastic potential tells us how difficult is to reach a state starting from other states. Using a fundamental result on stochastic stability, we know that in the present setup, a state is stochastically stable if and only if it has minimum stochastic potential. Intuitively, this amounts to looking for the distributions of power that are in place most of the time when the probability of initiating a conflict becomes very small.

We first study the case of uniform conflict initiation likelihood, formalized in Assumption A4, where differences in resources between countries do not translate into a significant difference in the tendency to initiate a conflict. In terms of interpretation, in this case, there is no discernible intention behind the outbreak of war, rendering it a mistake in the truest sense. This first case is intended to serve as a benchmark to be contrasted with the results obtained under alternative assumptions (i.e., replacing A4 with A5 or A6). We denote with $\tilde{r}(z, z')$ the resistance of moving from state z to state z' in one unit of time. We note that, in our setting, the transition from z to z' can be the outcome of a conflict initiated by each of the two countries involved, i.e., $\tilde{r}(z, z') = \min\{r(z_i, z_j), r(z_j, z_i)\}$ whenever z and z' are accessible one from the other, meaning that $\exists\, i, j \in G$ with $z_i \neq z'_i$ and $z_j \neq z'_j$, while $z_k = z'_k$ for any $k \in G$, with $k \neq i, j$. If, instead, z and z' are not accessible one from the other, then $\tilde{r}(z, z') = +\infty$, which means that z' cannot be reached from z as the result of a single conflict.

PROPOSITION **1.** *If Assumptions A1, A2, A3 and A4 hold, then every state is a long-run evolutionary equilibrium.*

Proof. We denote with h the constant level of function $\iota(z_i, z_j)$ $\forall$ $i, j \in G$ implied by Assumption A4. We show that, for any pair of states z'', z', with $z'' \neq z'$, there exists a path $(z^0, \ldots, z^w, \ldots, z^W)$, with $z^0 = z'$ and $z^W = z''$, such that $\tilde{r}(z^w, z^{w+1}) = h$ for $w = 0, \ldots, W-1$.

We define the distance between z' and z'': $d(z', z'') = \sum_{g \in G} |z'_g - z''_g|$. We take groups i and j such that $z_i > z'_i$ and $z_j < z'_j$ (two such groups always exist, if $z'' \neq z'$). We set $z^0 = z'$ and $z^W = z''$. Starting from z^0, we observe that the resistance for group i to initiate a conflict with group j is h (by Assumption A4 after observing that group i is not a hegemon) and, conditional on that, there is a positive probability (by Assumption A3) that state z^1 is reached next period, with $z^1_i = z'_i + 1$, $z^1_j = z'_j - 1$, and $z^1_k = z'_k$ for all $k \in G$, $k \neq i, j$. Hence, $\tilde{r}(z^0, z^1) = h$. We note that $d(z^1, z^W) = d(z^0, z^W) - 2$. All other steps in the sequence are constructed in the same way, with W being equal to $d(z^0, z^W)/2$. Since every state is absorbing, and the minimum resistance to moving from any two states is h, the stochastic potential of any state z cannot be lower than h times the number of transitions needed to reach z. Therefore, the stochastic potential of any state $z \in Z$ is $\rho(z) = (|Z| - 1)h$, since any tree is comprised of a number of links equal to the number of nodes decreased by 1. □

Proposition 1 tells us that if the likelihood of conflict initiation is independent of the current power allocation, then every power distribution can arise in the long run. Two things are worth noting: first, not only hegemony and balance of power are stochastically stable, but all possible distributions of power, and second, this holds for a rather wide class of contest success functions.

In order to consider the long-run outcomes of the other two alternative assumptions on the conflict initiation function, we have first to establish two preliminary results.

LEMMA **1.** *If Assumption A5 holds, then, for any $z' \in Z$ which is not a hegemony, there exists a hegemony $z'' \in Z$ and a path $(z^0, \ldots, z^w, \ldots, z^W)$, with $z^0 = z'$ and $z^W = z''$, such that $\tilde{r}(z^w, z^{w+1}) = \min_{z \in Z} \tilde{r}(z^w, z)$ for $w = 0, \ldots, W-1$.*

Proof. We define the distance between z' and z'': $d(z', z'') = \sum_{g \in G} |z'_g - z''_g|$. We also define $\widehat{G}(z) = \{g \in G : z_g > 0\}$. We take groups i and j such that $z'_i = \max_{g \in G} z'_g$ and $z'_j = \min_{g \in \widehat{G}(z')} z'_g$. We consider the hegemony z'' where $z''_i = R$. We set $z^0 = z'$ and $z^W = z''$. Starting from z^0, and conditional on i initiating a conflict with j, there is a positive probability (by Assumption A3) that state z^1 is reached in the next period, with $z^1_i = z'_i + 1$, $z^1_j = z'_j - 1$, and $z^1_k = z'_k$ for all $k \in G$, $k \neq i, j$. Moreover, under Assumption A5 we have that $\tilde{r}(z^0, z^1) = \min_{z \in Z} \tilde{r}(z_i, z)$, due to $z^0_i = \max_{g \in G} z^0_g$ and $z^0_j = \min_{g \in \widehat{G(z^0)}} z^0_g$. Finally, we note that z^0_i is smaller than R, while z^0_j is larger than 0; hence, $d(z^1, z^W) = d(z^0, z^W) - 2$. All other steps in the sequence are constructed in the same way, with W being equal to $d(z^0, z^W)/2$. □

LEMMA **2.** *If z'' is the unique balance of power and Assumption A6 holds, then for any $z' \in Z$, $z' \neq z''$, there exists a path $(z^0, \ldots, z^w, \ldots, z^W)$, with $z^0 = z'$ and $z^W = z''$, such that $\tilde{r}(z^w, z^{w+1}) = \min_{z \in Z} \tilde{r}(z^w, z)$ for $w = 0, \ldots, W - 1$.*

Proof. The proof unfolds along the lines of the proof of Lemma 1, with a few adjustments. We define the distance between z' and z'': $d(z', z'') = \sum_{g \in G} |z'_g - z''_g|$. We take groups i and j such that $z'_i = \min_{g \in G} z'_g$ and $z'_j = \max_{g \in G} z'_g$. We set $z^0 = z'$ and $z^W = z''$. Starting from z^0, and conditional on i initiating a conflict with j, there is a positive probability (by Assumption A3) that state z^1 is reached in the next period, with $z^1_i = z'_i + 1$, $z^1_j = z'_j - 1$, and $z^1_k = z'_k$ for all $k \in G$, $k \neq i, j$. Moreover, under Assumption A6 we have that $\tilde{r}(z^0, z^1) = \min_{z \in Z} \tilde{r}(z_i, z)$, due to $z^0_i = \min_{g \in G} z^0_g$ and $z^0_j = \max_{g \in G} z^0_g$. We observe that $d(z^1, z^W) = d(z^0, z^W) - 2$, since passing from z^0 to z^1 one resource has been moved from a group having more than $R/|G|$ resources to a group having less $R/|G|$ resources. All other steps in the sequence are constructed in the same way, with W being equal to $d(z^0, z^W)/2$. □

Lemma 1 establishes that whenever the groups with more resources are more belligerent, i.e., if A5 holds, then a hegemony can be reached from any other state with a path that has minimum resistance at each step. Conversely, Lemma 2 establishes that whenever the groups with fewer resources are more belligerent, i.e., if A6 holds, then the state with balance of power can be reached from any other state with a path that has minimum resistance at each step. Thanks to these findings, we can now characterize the long-run evolutionary equilibria.

PROPOSITION **2.** *If Assumptions A1, A2, A3 and A5 holds, then a state z is a long-run evolutionary equilibrium if and only if z is a hegemony.*

Proof. We take a state z' that is not a hegemony. By Lemma 1, there exists a hegemony z'' and a path $(z^0, \ldots, z^w, \ldots, z^W)$, with $z^0 = z'$ and $z^W = z''$, such that $\tilde{r}(z^w, z^{w+1}) =$

$\min_{z \in Z} \tilde{r}(z^w, z)$ for $w = 0, \ldots, W-1$. We consider a tree T', rooted at z' and with Z as set of nodes, which minimizes the sum of the resistances over its links among all trees rooted at z'. Starting from T' we add all links connecting consecutive states in such path from z' to z'', and we remove all links in T' which were outgoing from any state in the path. By doing so, we obtain a tree T'' rooted at z''. We now compare the sum of the resistances over links in T' and T''.

Firstly, any state that is not in the path has a resistance associated with its outgoing link that is the same in T'' and in T'. Secondly, any state z^w in the path, with $w = 1, \ldots, W-1$, has a resistance associated to its outgoing link that cannot be larger in T'' than in T', due to Lemma 1. Thirdly, we observe that in T' there is an outgoing link from z'' but no outgoing link from z' (since z' is the root of T'), while in T'' there is an outgoing link from z' but no outgoing link from z'' (since z'' is the root of T''). As a result of this, and knowing by Lemma 1 that $\tilde{r}(z', z^1) = \min_{z \neq z'} \tilde{r}(z', z)$, we can write the following lower bound for the difference between the stochastic potentials of z' and z'':

$$\rho(z') - \rho(z'') \geq \min_{z \neq z''} \tilde{r}(z'', z) - \min_{z \neq z'} \tilde{r}(z', z) > 0.$$

The latter inequality holds because $\min_{z \neq z''} \tilde{r}(z'', z) = r(0, R)$, since a hegemon never initiates a conflict (by Assumption A1), and $r(0, R) > \min_{z \neq z'} \tilde{r}(z', z)$, due to Assumption A5, since z' is not a hegemony.

Therefore, z' cannot have minimum stochastic potential; also, since a state with minimum stochastic potential must exist, such a state must be a hegemony. We conclude by noting that all hegemonies must have the same stochastic potential by symmetry: if z'' is a hegemony where all resources are assigned to group i, and z' is a hegemony where all resources are assigned to group j, then every tree rooted at z'' can become a tree rooted at z' after exchanging labels between i and j. □

Proposition 2 establishes that, when Assumption *A5* holds, the states entailing hegemony are the only stochastically stable states in our setup, meaning that hegemony should be expected in the long-run whenever stronger groups are more likely to initiate a conflict against weaker groups.

Proposition 3. *If Assumptions A1, A2, A3 and A6 holds, then a state z is a long-run evolutionary equilibrium if and only if z is the balance of power.*

Proof. The proof is similar to the one of Proposition 2, and to some extent, simpler due to the fact that a single balance of power exists instead of multiple hegemonies. We denote with z'' the balance of power. By virtue of Lemma 2, a tree can be constructed that is rooted at z'' and has Z as the set of nodes, such that if a link from state $\hat{z}$ to state $\tilde{z}$ belongs to the tree, then $\tilde{r}(\hat{z}, \tilde{z}) = \min_{z \in Z} \tilde{r}(\hat{z}, z)$; therefore, the stochastic potential of z'' is $\rho(z'') = \sum_{\hat{z} \in Z \setminus \{z\}} \min_{z \in Z} \tilde{r}(\hat{z}, z)$.

We now consider a state $z' \neq z''$. Clearly, $\rho(z') \geq \sum_{\hat{z} \in Z \setminus \{z'\}} \min_{z \in Z} \tilde{r}(\hat{z}, z)$. If we take the difference between $\rho(z')$ and $\rho(z'')$ we obtain:

$$\rho(z') - \rho(z'') \geq \min_{z \neq z''} \tilde{r}(z'', z) - \min_{z \neq z'} \tilde{r}(z', z) > 0.$$

The latter inequality holds due to Assumption A6, after observing that the group with the fewest resources has more resources in z'' than in any other state, and that the group with most resources has fewer resources in z'' than in any other state.

Therefore, z is the unique state with minimum stochastic potential, and as such, it is the unique stochastically stable state. $\square$

Proposition 3 establishes that when Assumption *A6* holds, the unique state entailing balance of power is the only stochastically stable state in our setup, meaning that balance of power should be expected in the long-run whenever weaker groups are more likely to initiate a conflict against stronger groups.

It might seem very surprising that the actual shape of the conflict success function does not substantially affect the results provided in Propositions 1 to 3 (the only requirement being that A3 holds). However, it may become less surprising if one considers that in our setup, conflicts are modeled as rare events which, once occurring, can entail any redistribution of resources between conflicting groups. This implies that the likelihood of being successful in a conflict has effects on the dynamics which are of second-order importance with respect to the effects of the likelihood of being involved in a conflict.

5 Interpretation and historical facts

As previewed in the Introduction, the evolutionary literature on conflict has mainly focused on the conflict resolution function (CRF), frequently incorporating several factors that are believed to contribute crucially to the likelihood of success in a confrontation (Levine and Modica, 2013a,b, 2018; Levine et al., 2022; Levine and Modica, 2021). These same features could also influence the conflict initiation function (CIF). In this section, we aim to review some traditional determinants that arguably affect the CRF and illustrate situations where they can also, or alternatively, affect the CIF. We will also discuss the role of geographical barriers, third-party involvement in the conflict, technological advancements, the presence of a charismatic leader, and culture.

To begin with, we can briefly discuss the case in which we look at A5, in which the relatively stronger is more likely to attack, predicting the emergence of a hegemony. One might argue that this assumption crucially depends on the fact that the nature of the CRF is more frequently likely to be "strong favored" (using the terminology of Dziubiński et al., 2021), meaning that the side with more resources has more chances to prevail in a conflict. Given this, it could very well be that the temptation to attack minimizes the resistance of the stronger towards attacking the weak. This would explain the relatively higher frequency of hegemonies in history, especially in settings without barriers (geographical, political, economic, or any other type of deterrent) to compensate for the apparently easy gain. It is worth noting that, from the fact that the CRF is strong favored, it does not follow that warfare, or any other type of conflict, is riskless, which is exemplified in our model by assuming a CRF bounded away from zero. A strong favored CRF could lead to a CIF where the stronger is more likely to initiate a conflict. However, this can be the case even if the CRF is not strong favored when, for instance, the stronger has internal reasons – i.e., different from obtaining resources – for generating external conflict, such as maintaining power.

The cases that support A6, i.e., in which the relatively weaker is more likely to attack, are trickier. To identify factors supporting those cases, one has to look for potential causes of conflict initiation that are either independent of actual power in conflict resolution or that make attacking more probable when possessing relatively fewer resources. According to our long-run analysis, all these factors are conducive to balance of power.

A characteristic typically considered to prevent hegemony is the presence of geographical barriers. Barriers work by increasing the logistic cost of attacking, deterring the attacker's resource seizing. However, it could also be that geographical barriers, particularly in the case in which they are asymmetric, create situations in which the weaker side is more likely to attack the stronger one. At a low scale, these arguments explain the common use of guerrilla tactics across history, which involve situations in which the weaker side attacks more often precisely because geography helps to reduce the aggregated risk of any given conflict (one could take the example of Afghanistan, which has been an absolute hell to conquer and maintain all across history Kaye, 1851; Girardet, 1981; Holt, 2005; Malkasian, 2021).

On a bigger scale, one can look at historical regularities such as the balancing role attributed to England in Europe (Paul et al., 2004), thanks to the presence of the Channel. While the aggregate military power of the British has been inferior to those of their traditional rivals, such as France or the Spanish Empire, the water barrier allowed England to successfully sustain conflicts with both of them by relying on naval specialization. In conclusion, it seems that geographical barriers, no matter the relative strength of the groups protected by them, seem to affect the CRF and the CIF roughly in the same direction by providing both a ceteris paribus advantage in the probability of success and incentivizing the initiation of conflict in the case the barrier is asymmetric. However, England's footholds in Europe have not been successfully maintained, precisely due to the fact that the advantage provided by the barrier is removed when becoming stronger (think of Aquitania or Britain during the Hundred Years War).

Another extensively studied feature that prevents hegemony and favors balance of power is the presence of strong outsiders that can intervene in a conflict between groups, acting as a balancing force. The leading example in the literature is the balancing effect of the Vikings and then England in continental Europe from the early Middle Ages to the present (Levine and Modica, 2013b).

However, the intervention of strong outsiders has been frequently modeled as a factor directly influencing the CRF, with the introduction of a parameter that increases the chances of victory of one side, usually the weaker. As exposed in Levine and Modica (2018), external intervention can lead different typologies of conflict, such as hot peace or prolonged war. Although correct, these arguments consider only the actual involvement of the outsiders in the conflict, while, a third party, strong outsider, might have an effect just through the CIF, without affecting the CRF, simply by modifying the risk perception of the side it supports. An example is the situation of Anatolia at the beginning of the first century BCE, with the presence of two big powers at each side of the peninsula, Rome at the west and Pontus at the east, and many little kingdoms that were often in a warring state, but with no big overall changes due to the eventual diplomatic intervention of one of the two outside local hegemonies (Glew, 2015).

Technology is another essential factor in group conflict initiation and resolution. As it is developed in Levine et al. (2022), defensive technology, such as castles, favors inclusive institutions, supporting balance of power. In contrast, offensive technology, such as cannons, favors extractive institutions, which could lead to hegemony or balance of power. The argument for the former claim is straightforward, given that defensive technology provides an advantage to the weaker side in the CRF (we could think of the wall of a castle, but also antitrust or patent legislation).

On the other hand, the development of offensive technology increases the marginal contribution of resources, independently of the group's relative strength. However, to clarify the effects of technology on the CIF, we have to distinguish between the cases where a new technology adds to existing resources available or just affects CIF without affecting resources, such as the development of oblique phalanx tactics by Epaminondas (Hanson, 1988). This, in fact, helped Thebes overcome the Spartan hegemony in the Peloponnese, but we cannot claim that the new tactic affected the likelihood of the conflict to arise. An example of defensive technology is the Battle of Cape Matapan during the Second World War Johnman and Murphy (2005), in which the British fleet had radars in their ships while the Italian fleet did not, resulting in a decisive Ally victory. However, it is difficult to argue that the effect of the radar does not work just by increasing the likelihood of success through the CRF. In any case, we should be aware of the possible existence of non-innocuous interactions between technological developments and the CIF. For example, using drones in war might increase the probability of conflict by reducing the political costs of initiating a war, and this would work independently of the group's relative strength. To conclude, it seems clear that further research should be pursued in order to clarify the relationship between technological progress and CIF.

Charismatic leaders are essentially random shocks within a group, but they can have a decisive effect both in the likelihood of initiating conflict and in the result of it. Charismatic leaders may be able to assemble their groups towards warfare, which would result in a higher likelihood of conflict, no matter the groups' relative strength. Since the direction of the effect seems to be that a charismatic leader can serve as a means to attack in general (it is worth mentioning that many leaders have used external warfare as a means of preserving power, as in Lopez, 2020), we should consider if a charismatic leader is more likely to arise in a weaker group. Intuitively, a weaker group may be both more sensitive to populist arguments and smaller in size, so the relative marginal effect of a shock yielding a charismatic leader is larger. In addition, a weaker group might have less strict democratic controls, making it easier for a charismatic leader to establish an authoritarian regime. Nevertheless, the emergence of a charismatic leader in a group does not necessarily imply higher success in conflict, as the correlation between charisma and martial progress is ambiguous – there are many examples of both great success (e.g., Alexander the Great) and great disasters (e.g., Spartacus). It is true, however, that a charismatic leader can work by concentrating resources within a group towards conflict, and hence directly affecting the CRF.

This argument is similar to the one provided by Levine and Modica (2013a) regarding barbarian hordes and zealots, in which aggressive groups with relatively lower incentive compatibility constraints can effectively seize substantial resources but do not have particularly stable institutions. Charismatic leaders, hence, through their capacity to convince a group to weaken its constraints, would work both by increasing the likelihood of success in the CRF, via concentration of resources towards military power, and by directing the group more often towards conflict, hence affecting the CIF.

As a final factor that may be overlooked, we can consider cultural differences and their effects on CIF and CRF. Consider warrior honor codes (Pressfield, 2011) that take rational weighting of forces in a conflict out of the equation. Codes of conduct are widespread across history, with popular examples including Chivalry in Europe or Bushidō in Japan. Some of these codes have relied on characteristics that are considered honorable. In a conflict, warrior honor codes might lead a side to endorse irrational stubbornness, which should appear in the CRF. In particular, the per capita effect of relentless warriors is higher in any situation favoring attrition and lower in settings where conservative approaches are preferred. As an example of the first effect, we can mention the Battle of Empel, in which Spanish soldiers, when offered an opportunity to surrender, answered “Spanish soldiers prefer death to dishonor. We will talk about surrender after death”. Later, they won the battle thanks to the freezing of the Meuse river during the night. The best example of the second effect is the invention of Fabian tactics by Romans during the Second Punic War (Erdkamp, 1992; Carr and Walsh, 2022). Although previous generals had promoted direct confrontation against Hannibal and the Punics, the realization of dictator Quintus Fabius that directly facing Hannibal was almost suicidal, and hence avoiding combat, was essential to prevent more catastrophic defeats. Hence, the effect of warrior honor codes in the CRF is very much context-dependent. Context-dependency of culture is also likely for the CIF. A weaker side embracing codes that promote retaliation and the necessity to amend affronts might actually attack (and continue attacking) more often, and such non-consequential morality would favor the attainment of a balance of power. An example can be found when looking at a map of pre-Roman Gaul, in which we can observe many clusters of little tribes with no hegemony and relative balance of power, which can be explained both by the fact that warrior honor codes would prescribe immediate counter-attack even from a position of weakness. On the other hand, honor codes that promote mercy with the enemy could prevent stronger sides from attacking, which should also favor balance of power.

In conclusion, we should be aware of possible interactions between culture, CRF, and CIF and recognize that sometimes culture might affect either function differently. We have explicitly discussed warrior honor codes, but we could consider the effects of nationalism, religious beliefs, or any other cultural dimension.

6 Conclusions

In this paper, we have studied an evolutionary model in which the possession of resources, which measures the distribution of power, evolves over time as the result of conflicts among groups. Differently from the previous literature, where the likelihood of conflicts and the distribution of their outcome are modeled jointly, we have provided a model where the likelihood of conflict initiation and the likelihood of conflict outcomes are determined by distinct functions which depend on the current power of the groups involved. Importantly, we assume that conflicts are initiated with a vanishing probability, while the probability of winning or losing resources – once a conflict is initiated – remains non-negligible. We believe that these assumptions are reasonable if: (i) conflict is a rare event resulting from loss of control, miscalculation, or rule-following, i.e., something that depends on the resources possessed but that, anyway, happens rarely; (ii) once a conflict breaks out, many factors can affect the outcome of the conflict beyond the factors that triggered the conflict, e.g., greater knowledge of the conflict area, higher intrinsic motivation, internal turmoil, help from an external power, and innovations.

Our model allows us to uncover the relationship between the shape of the conflict initiation function and the long-run distribution of power. Moreover, we have clarified that the conflict success function, which has received the most attention from the literature so far, does not have a substantial impact on the long-run distribution of power provided that the attacked group cannot avoid the risk of losing some resources, even if it is very powerful.

Importantly, our results show that both hegemony and balance of power can be observed as long-run evolutionary outcomes of conflicts among groups, and which of them will be observed most often crucially depends on the determinants of the initiation of conflicts. Specifically, when conflicts are more likely to be initiated by stronger groups against weaker groups, then hegemony will prevail most of the time in the long run. In contrast, the balance of power will prevail most of the time in the long run if conflicts are more likely to be initiated by weaker groups against stronger ones.

A natural step for future research would be to model the stage game within a group in order to explore which of our assumptions is more likely to hold depending on the details of the strategic setup within the group. This line of research would also allow exploring the role of complementarities and substitutabilities betweeen different types of resources as, for instance, in the case where resources must be alternatively allocated to productive or military purposes (as in, e.g., Hwang, 2012; Levine and Modica, 2013b).

Another promising avenue for future research is the introduction of some topological structure (e.g., geography) determining accessibility between groups, so that conflicts may be assumed to initiate with a larger probability when two groups have direct access to each other. In some cases, it might also be reasonable to assume that conflicts are less likely to arise between neighboring groups because of alliances or commonalities in history and culture. A network structure may hence allow embedding geographic accessibility in the model, as well as alliances or historical rivalries. For a recent examination of the role of different conflict resolution functions in networks, see Dziubiński et al. (2021).

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