Advanced Reduced Order Methods and Applications in Computational Fluid Dynamics

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To our families, to our younger collaborators,
to little Petra and Clara representing the future,
and to all people who contributed to and inspired this book

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Preface

Reduced order modeling is an important and fast-growing research field in computational science and engineering, motivated by several reasons, of which we mention just a few: parametric computing, repetitive computational environments, real-time computing, increasing complexity of scenarios with uncertainties, and better computational performance in optimization, control, and inverse problems. This book deals with recent developments in model order reduction in fluid dynamics, including challenging topics such as turbulence, optimal control, flow stability, aerodynamics, shape optimization, inverse problems, multiphysics, and uncertainty quantification. We present here an integrated, reduced, parametric computational framework characterized by a wide portability to be embedded in the most modern computational pipelines, including automatic learning and digital twin developments.

After this preface and an overview and motivation in Chapter 1, the first part is concerned with finite element–based reduced order modeling with a focus on laminar computational fluid dynamics (Chapter 2), then with the introduction of a simple turbulent pattern (Chapter 3), and then with the optimal flow control framework (Chapter 4). In Chapter 5, bifurcation problems are studied, while in Chapter 6 the focus is on transport-dominated problems.

The second part of the book deals with finite volume and spectral element methods and discontinuous Galerkin-based reduced order modeling. Chapters 7 and 8 deal with finite volume–based reduced order modeling in computational fluid dynamics (CFD) from laminar to turbulent flows. In Chapter 9 we deal with nonintrusive data-driven reduced order models. Chapter 10 introduces spectral element method–based reduced order modeling, while Chapter 11 deals with discontinuous Galerkin–based reduced order modeling.

The third part deals with advanced reduced order modeling in CFD. Chapter 12 deals with weighted reduced order modeling for uncertainty quantification, Chapter 13 with model order reduction for embedded methods and level-set geometries, Chapter 14 with multiphysics problems (fluid-structure interaction), and Chapter 15 with bifurcations in parametric multiphysics settings.

The fourth part is concerned with perspectives and applications. In Chapter 16 we deal with reduction in parameter space, and Chapter 17 deals with geometrical parametrization and applications. Hemodynamics applications are introduced as examples in Chapter 18. In Chapter 19 we introduce our scientific computing open-source libraries and Python tools. Last, but not least, Chapter 20 provides perspectives and current preliminary development to improve model reduction by automatic learning, concluding with the digital twin concept.

We accompany this book with our open-source software collection and worked problems, available at `mathlab.sissa.it/cse-software`.

We acknowledge all the chapter contributors (SISSA mathLab current members and past members) as listed in the frontmatter and at the beginning of each chapter. We acknowledge our long-lasting national and international research collaborations, without which this work would not exist.

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The book is based on preliminary lecture notes and slides used at the first Summer School on Advanced Reduced Order Methods and Applications in CFD (AROMA-CFD), held in Trieste at SISSA in July 2019; see `https://indico.sissa.it/event/34/`.

This project has been carried out during lockdowns and the pandemic spread of COVID-19, and it has represented for all of us a good way to remain optimistic about the future. This is the reason the project's "secret" name was "Decameron," based on Boccaccio's manuscript.

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We hope that you take some inspiration from this book and that this work will be known as the AROMA book.

Trieste, Italy, 20 April 2022 Gianluigi Rozza, Giovanni Stabile, Francesco Ballarin