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Essays on Conflict, Aggressivity and Punishment

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"The best thesis defense, is a good thesis offence".

To the Patesa.

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Publications

1. P. Marcos-Prieto and J. Perote, “Agents’ Behavior in Market Bubbles: Herding and Information Effects.” (2017), *Economics*, 5(1), 44-51.
2. E. Bilancini, L. Boncinelli and P. Marcos-Prieto, “Conflict initiation function shapes the evolution of persistent outcomes in group conflict.” (2024), *European Economic Review*, 161, 104648.
3. E. Ordali, P. Marcos-Prieto, G. Avvenuti, E. Ricciardi, L. Boncinelli, P. Pietrini, G. Bernardi and E. Bilancini, “Prolonged exertion of self-control causes increased sleep-like frontal brain activity and changes in aggressivity and punishment”, *PNAS (forthcoming).*
4. P. Marcos-Prieto, E. Ordali, S. Palumbo, S. Vellucci, E. Ricciardi, P. Pietrini, S. Pellegrini, L. Boncinelli and E. Bilancini, “Dopaminergic and serotonergic genetic variants and their relationships with classical economic games”, *under review*.

Presentations

1. P. Marcos-Prieto, "Conflict Initiation Function Shapes the Evolution of Persistent Outcomes in Group Conflict" at *SAEe2020, Spanish Economic Association Conference*, Salamanca, Spain, December 10^{th}, 2020.
2. P. Marcos-Prieto, "Conflict Initiation Function Shapes the Evolution of Persistent Outcomes in Group Conflict" at *GAMES2020, 6th World Congress of the Game Theory Society*, online, July 19^{th}, 2021.
3. P. Marcos-Prieto, "Más allá de los sesgos en expertos: el efecto de "extreme Ego Depletion" en la práctica jurídica" at *Centro de Investigación para la Gobernanza Global (CIGG)*, Salamanca, Spain, May 26^{th}, 2021.
4. P. Marcos-Prieto, "Extreme Ego Depletion and Prosocial Behavior" at 7^{th} *Meeting of the Behavioral and Experimental Economics Network (BEEN)*, Lucca, Italy, February 18^{th}, 2022.
5. P. Marcos-Prieto, "Conflict Initiation Function Shapes the Evolution of Persistent Outcomes in Group Conflict" at *Learning, Evolution and Games (LEG) Conference*, Lucca, Italy, July 9^{th}, 2022.
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7. P. Marcos-Prieto, "Experience-dependent induction of low-frequency sleep-like activity (extreme ego depletion) can affect socially relevant choices" at *Young Economists of Tuscan Institutions (YETI) Meeting*, Lucca, Italy, December 18^{th}, 2022.
8. P. Marcos-Prieto, "Experience-dependent induction of low-frequency sleep-like activity (extreme ego depletion) can affect socially relevant choices" at *Jornadas de Economía Experimental y Sostenibilidad, Universidad de Salamanca*, Salamanca, Spain, June 9^{th}, 2023.

Abstract

Conflict is a crucial force that shapes the world. If, at a group level, conflict is able to shape allocations of power and resources, often changing the equilibrium between two groups, at the individual level, conflict affects social interactions, priming aggressive and hostile behaviors that disrupt cooperation. In this Doctoral Thesis, we take a theoretical evolutionary perspective, as well as an experimental one, to investigate how conflict affects different aspects of socially relevant behaviors. In the first chapter, using an evolutionary perspective, we explore the implications of different relationships between power and initiation of conflicts for the long-run distribution of power between groups. The second chapter describes two experiments in which we demonstrate that aggressive behavior is more likely to happen after extreme extension of self-control, with parallel appearances of signs of functional fatigue in areas of the prefrontal cortex implied in emotional and impulsive regulation. Finally, the last chapter describes a theoretical model in which a community enforcement system that uses social norms supports both pro-norm and anti-norm punishment as evolutionary stable.

Chapter 1

Introduction

As researchers in Economics, we usually find peace of mind by thinking that parsimonious market forces drive most changes. Notwithstanding that, it is fair to say that conflict is a powerful force in many individual, social, and group interactions. Since many implications of different types of conflictual situations, intending not only open confrontation but any context that involves the opposition of different forces, norms, or systems, are somehow understudied, I believed that I could focus the present thesis precisely on those instances.

In recent years, there has been an important increase in multidisciplinary research. The desire to let out discipline-specific practices to merge different branches of scientific literature has proven to be an interesting new avenue in which science stops being divided according to the deontological costumes of the different groups that practice it but instead is judged purely based on its quality. As I strongly agree with the idea of flexibilizing the frontiers between methodologies in order to achieve results that would not be possible under single-specialization groups of researchers, I had very clear from the beginning that I would look for a multidisciplinary approach in my work. Merging the previous two concepts, conflict and multidisciplinarity, results in the present work.

In the Second Chapter, I concentrate on conflict between groups while achieving multidisciplinarity by looking at ideas of economic, evolutionary, and international relationships. The principal aim of this chapter is to build an economic model that explains which distributions of power are most likely to emerge in the long run whenever groups engage in conflict. The classical debate has identified two main distributions that are thought to be more resilient and likely to survive over time: hegemony, in which power is concentrated, and balance of power, in which power is evenly distributed. In order to identify which of these options is more persistent, the analysis relies on the *conflict initiation function*, which characterizes the likelihood of initiating a conflict against another group, instead of the *conflict resolution function*, which characterizes probabilities and shares that result after the breaking of conflict. Using tools from *stochastic stability* analysis, it is shown that hegemony is more likely to appear in the long run whenever the relatively stronger groups attack more often; this is, it is stochastically stable in the sense that we should observe it more often in the long run whenever we allow for mistakes in the system. Conversely, balance of power is stochastically stable whenever the relatively weaker groups attack more often. The results rely on the relatively mild assumption that the conflict resolution function is bounded away from zero; hence, victory is never certain. We complete the chapter by looking at historical examples and discussing different factors that might influence conflict initiation and conflict resolution functions, adding another incident layer of multidisciplinary by glancing at historical and military literature.

In the Third Chapter, I go from the lowest granularity of conflict between groups to the highest one of conflict within individuals. This requires a look at the behavioral economic, psychological, and neuroscientific literature, crystallized in recent years in the term Neuroeconomics. Specifically, this chapter aims to study what occurs whenever we induce specific states of fatigue in human individuals who subsequently have to perform a decision-making task. In Psychology, the idea of executive control as a muscle that gets tired whenever is overused has given rise to the extensive and polemic literature on *ego depletion*. At the same time, Neuroscience looked at fatigue protocols as a possible manipulation to create sleep-like activity in several brain areas that, when considering self-control and decision-making, call for the possibility of inducing functional fatigue in frontal brain areas. Finally, in Economics, the dual process theories of decision-making, along with the treatments used in the previous disciplines, prompted researchers to try to design cognitive manipulations to observe the decision-making in individuals under different states, usually proxied through economic games. The obvious question is, can we design a behavioral task that can appeal to psychologists by using self-control depletion, appeal to neuroscientists by looking at sleep-like episodes in brain activity, and appeal to economists by looking at differences in decision-making induced by cognitive manipulation? If the answer is yes, then what if the result actually expresses an internal conflict within individuals that prone them towards aggressive acts?

To achieve the stated goals, the Third Chapter describes two experiments where the role of self-control depletion is crucial in favoring the expression of aggressive acts in social situations. Specifically, with the aid of an extended ego depletion paradigm (~ 45 min of fatigue induction), electroencephalographic measures and economic games, we explored the relationship between self-control depletion, mental fatigue and hostile reactions (Study 1, $N = 44$). A second, behavioral-only, and larger experiment (Study 2, $N = 403$) was conducted to replicate the findings in a more classical and aseptic Experimental Economics setting. The results showed that, after extended practice with tasks requiring continuous exertion of self-control to be performed correctly, slow waves typical of NREM sleep (i.e., delta waves, 1-4 Hz) appeared in the local EEG within the pre-frontal cortex. Moreover, participants that experienced this *frontal fatigue* were more likely to display aggressive-like behavior, as measured with a Hawk and Dove Game ($p < 0.001$). At the same time, the use of prosocial punishment in a Public Goods Game with Punishment is reduced in the group of depleted participants ($p < 0.001$), while the use of spiteful punishment, which we define as punishing an individual at random, increases ($p = 0.002$). Overall, I believe this chapter opens an interesting debate in which researchers should think carefully about the specific cognitive manipulation used in an experiment and the specific behavioral measure or game to test the hypothesis at hand.

In the Fourth Chapter, I look at conflict within groups with the aim of conciliating the observable spread of cooperation in groups with the existence of such a harmful (but not that uncommon) phenomenon as antisocial punishment, punishment directed at cooperators in social interactions. I merge economic and biological ideas condensed in an Evolutionary Game Theory framework to do so. The model features a population that is randomly matched in pairs to play a Prisoner's Dilemma Game with Punishment, with the possibility of subsequently updating the strategies or genotypes using a myopic best reply rule with the random appearance of mistakes in the process. The key ingredient that articulates the possibility of observing different modalities of punishment is the introduction of a *community enforcement* mechanism that assigns the green color to the modal strategies and the red color to the non-modal strategies, with the possibility of conditioning both the cooperative and punishment action to the color of the strategy of your pair in the game. As usual in community enforcement models, the space supporting equilibria is enlarged, with up to seven strategies being evolutionary stable. When applying the more stringent techniques of stochastic stability, it turns out that the margin for equilibrium selection is severely reduced by our color assignment mechanism, with the possibility of several different states to be stochastically stable whenever they manage to reach modal status, but with each simultaneously stochastically stable state being very resilient to the appearence of a few or even rather large number of mistakes. Overall, this helps us explain the polemic pervasiveness of cooperation even in the presence of antisocial punishment while also hinting at the multiplicity of stochastically stable states proxying the multiplicity of social norms about punishment that have been observed in the experimental literature. Incidentally, the community enforcement mechanism creates a paradoxical situation that allows for the coexistence of stochastically stable states but also makes each of them very difficult to exit from. Finally, the brief fifth chapter concludes the present work.

Chapter 2

Conflict Initiation Function Shapes the Evolution of Persistent Outcomes in Group Conflict

2.1 Introduction

Group conflict is a crucial force of change in the world, shaping allocations of power and resources. In this paper, we study the evolution of group conflict, trying to shed light on the general conditions under which different distributions of power are persistent over time.

The literature on group conflict and international relations has focused on two main distributions of power that are observed to be persistent, in the sense that they have proved on several occasions to survive over long periods of time: *hegemony* and *balance of power*.

Hegemony is a situation in which power is extremely concentrated in the hands of a single group. We can think of power in several ways, from the possession of lands to the market share or military predominance. The persistence of hegemonies has been justified on the basis that, once a group has obtained all the power and becomes the hegemon, it is fully satisfied, since there is no more gain to be pursued. The only thing that can worry a hegemonic power is to lose its position, but, at the same time, the other groups are too weak to try to overcome the hegemon. In this case, other groups have to accept a subordinate role and end up supporting the stability that the presence of a hegemon can provide. Several examples of hegemonies over history have been justified with similar arguments: Old, Middle and New Egyptian Kingdoms, the city-state of Sparta in the Peloponnesian League, the Roman Empire, the Persian Empire, several Chinese Dynasties (such as Song, Ming and Qing), the Carolingian Empire, or the British Empire.

Balance of power is a situation in which the power is evenly distributed among all groups, societies or agents. There are solid arguments also in favor of it. Power parity can serve as a suitable device for preventing conflict, given that no single state concentrates enough power to expect a sound victory against others, making warfare much riskier. To destabilize the balance of power, it is not enough that a change occurs within a society, such as the adoption of new military organizations and technologies or the relaxation of incentive compatibility constraints of the individuals due to, for example, nationalism or fanaticism of any kind. Given the likelihood of those situations in which war could be actually looked for, once a decent parity of power is established, the different states will seek a policy of alliances and counter-alliances so that the equilibrium is not distorted, building up the stability of the system and making balance of power highly persistent over time. Some historical examples of this (in the sense that relatively stable periods of peace have been observed) can be found, for instance, in the Italic League (1454), the Peace of Westfalia (1648), the Congress of Vienna (1814), and the modern European Union.

It is reasonable to conjecture that both hegemony and balance of power may be persistent under appropriate favorable conditions. A fundamental concept used to study such conditions is the conflict success function, otherwise called conflict resolution function, introduced by Hirshleifer (1988) and Hirshleifer (2000). This function takes as input the strength of the contending forces, usually measured by military power, and returns the outcome of the conflict. In a more naive interpretation, the conflict resolution function yields the share of resources retained by every side after the conflict, which is like saying that, for instance, the result of a battle depends solely on the proportion and quality of soldiers in each army. Another frequent and more realistic interpretation is that this function results in a probability of winning a conflict for each of the factions at play. This is the approach we adopt here, recognizing the intrinsic stochastic nature of conflict.

Our crucial innovation is the introduction, besides the conflict resolution function, of a *conflict initiation function*, which takes as input the current power of a given group and the power of its potential target, and gives as output the likelihood that the group initiates a conflict with the target group. We stress that the initiation of a conflict is stochastic and not necessarily the result of best-replying, capturing the fact that initiating a conflict is often not the result of a forward-looking decision but, rather, the outcome of many concurrent determinants that go beyond a rational decision process. One might ask why the separation of conflict resolution and conflict initiation is sensible. Our results show that the shape of the conflict initiation function directly impacts long-run outcomes.

Our main contribution is the demonstration that, under the rather mild assumptions that nobody can secure complete victory with certainty (no matter how relatively stronger) and that a conflict can be initiated only if some gain can be obtained in terms of the possession of resources, the conditions that sustain hegemony and balance of power in the long run only depend on the characteristics of the conflict initiation function. In contrast, the characteristics of the conflict success function can, at most, affect convergence times. More specifically, we show that hegemony is more persistent whenever the stronger groups are more likely to initiate a conflict, whereas balance of power is more persistent whenever the weaker groups are more likely to initiate a conflict. Formally, we characterize conditions for hegemonies and balance of power to be observed almost always in the long run as the probability of war initiation goes to zero (i.e., they are *stochastically stable* as defined in Foster and Young, 1990), and we show that such conditions hold for every conflict resolution function that satisfies the property that any group has a probability bounded away from zero of avoiding defeat, no matter the power of the conflicting groups.

We stress that an essential characteristic of conflict success functions plays an important role in our analysis: the impossibility of securing complete victory with certainty. While one might assume that a party with zero power will never have a chance to avoid defeat in a conflict (as it happens when the contest success function is modeled as a ratio between contenders' forces, see Hirshleifer, 1989), it seems more reasonable to assume that even a negligible amount of power allows having a non-negligible chance to avoid total defeat. This latter assumption embeds the stochastic nature of the outcome of a conflict, which can be the consequence of a model that is non-fully fledged, leaving aside the possibility of external intervention in the conflict, or even that other determinants can affect the probability of victory, like the morale of the soldiers and the spirit of sacrifice to the cause.

The rest of the paper is structured as follows. Section 2.2 discusses the relevant literature. Section 2.3 presents the model and states our assumptions. Section 2.4 provides results about the long-run emergence of hegemonies and balance of power. Section 2.5 provides some historical interpretation of the results. Finally, section 2.6 concludes.

2.2 Related literature

An essential insight by Hirshleifer (2001) is that the allocation of resources within a society matters, and whether a group decides to invest in military or productive activities (or leisure, or any other kind of) may be crucial for the final result. Recent evolutionary literature on group conflict has been focusing on this issue. Levine and Modica (2013b) explores the rise of hegemonies in a setting in which outsiders can exert influence, and each society devotes resources to state power (which is a determinant of conflict success) and productive activities. They find that the development of a hegemony is crucially connected to the level of state power in a society. However, they do not provide conditions under which a balance of power between societies is achieved, even when they do mention that more inclusive states are less likely to become a hegemon. Very related to this paper is Levine and Modica (2013a), in which they find that hegemony is likely to appear whenever a *strongest* society emerges that also has expansionary desires. Finally, Levine and Modica (2016) finds that hegemonies are more likely to survive if external intervention is limited, a concept already introduced in Levine and Modica (2013b). In contrast to these papers, we do not explicitly model the in-society game, taking a reduced perspective.

So far, no results about the emergence of balance of power had been found in the evolutionary literature on group conflict. To address that, Levine and Modica (2018) shows that external intervention may not only be a source of the fall of hegemonies, but actually, it can help to achieve balance of power. Depending on the strength of the intervention, different outcomes are possible, including *hot peace* (a low-intensity but long-lasting conflict) or *prolonged war* (a high-intensity conflict with many casualties). These preliminary ideas are extended in Levine and Modica (2022), where they distinguish between extractive and inclusive balance of power and consider not only the strength of outsiders but also the quality of fortifications as possible sources of balance of power. It is also worth noticing that in this paper, different active groups (commercial elites or military elites) decide whether or not to initiate a conflict. Finally, all the previous elements are summarized and confronted with empirical evidence in Levine and Modica (2021).

Going beyond the issue of the long-run selection of hegemony as opposed to balance of power, the evolutionary literature has focused on social conflict, i.e., conflict within a society, such as in Naidu, Hwang, and Bowles (2017) and Hwang et al. (2018). Further, the modeling of the conflict initiation function is, in fact, a way to model transitions across states in a Markovian stochastic evolutionary model, quite similar to what is done in the literature on stochastic behavioral rules as myopic best reply with uniform mistakes (P. Young, 1993; Kandori, Mailath, and Rob, 1993), intentional mistakes (Naidu, Hwang, and Bowles, 2010), coalitional mistakes (Newton, 2012), condition-dependent mistakes (Bilancini and Boncinelli, 2020), or break-easier-than-build mistakes (Boncinelli and Pin, 2012; Boncinelli and Pin, 2018), and the logit model (Blume, 1993) or similar stochastic behavioral rules implying that likelihood of selection depends negatively on the payoff loss associated to it.

Finally, there is a vast game theoretic literature about conflict in different settings, mainly relying on sophisticated strategic thinking and adopting traditional solution concepts for sequential games, e.g., Baliga and Sjöström (2004), Jackson and Morelli (2007), Morelli and Rohner (2015), Caselli, Morelli, and Rohner (2015), Jackson and Nei (2015), and Canidio, Esteban, et al. (2018). For a book-length treatment of early contributions, see Coyne and Mathers (2011), while for a recent, comprehensive review of the literature about war in economics, see Kimbrough, Laughren, and Sheremeta (2020). Similar reviews with an evolutionary focus can be found in Rusch and Gavrilets (2020), from the theoretical biology perspective, and Glowacki, Wilson, and Wrangham (2020), from the anthropological perspective.

2.3 Model

In this section, we craft the elements of our model to create a simple, straightforward theory of the evolution of societies in conflict.

Groups and resources. There is a finite set of groups $G = \{1, \ldots, |G|\}$ that compete over an also finite number of resources $R \in \mathbb{N} > 0$. Time is discrete and denoted by $t = 0, 1, 2, \ldots$ A state at time t is an allocation of resources among groups in G, and it is denoted by

$$z_t = (z_{t,1}, ..., z_{t,g}, ...z_{t,|G|}),$$

where $z_{t,g} \in [0, R]$ is the non-negative integer number of resources allocated to group $g \in G$ at time t. In the following, we will refer to groups as societies, states or nations, but groups can be any sub-population acting coordinately within a larger population. We denote the state space with

Z,

$$Z = \{z_t \colon \sum_{g \in G} z_{t,g} = R, z_{t,g} \geq 0 \text{ for all } g \in G\}$$

Finally, say that a group is *inactive* whenever it has zero resources, i.e., g is inactive at time t if $z_{t,g} = 0$.

Power distribution. *Balance of power* is a state where all the resources are evenly distributed among groups, i.e., $z_{t,i} = z_{t,j} \; \forall\, i, j \in G$. For this to exist, we assume that $\frac{R}{|G|} \in \mathbb{N}$.

Hegemony is a state where a single group possesses all the resources, i.e., $\exists\, i \in G$ such that $z_{t,i} = R$. Hegemony can be seen as a state corresponding to a vertex of a $|G| - 1$-dimensional simplex, while the balance of power is the state located at the center of the simplex.

Conflict initiation. We consider conflict initiation as a rare and stochastic event. At any time t, a single pair of groups (i, j) is randomly selected and the probability of a conflict being initiated by group i against group j is assumed to depend only on $(z_{t,i}, z_{t,j})$.

Our main object of analysis is the *conflict initiation function*, denoted by $\iota(z_{t,i}, z_{t,j})$, mapping pairs of resource stocks $(z_{t,i}, z_{t,j})$, one for group i and one for group j, into probabilities of i initiating conflict with j at time t.

Conflict success. We model the outcome of a conflict that has been initiated by group i against group j by means of the *conflict success function* that maps the current resources of the two groups involved in the conflict into the probability that $l \in \mathbb{Z}$ resources are won ($l > 0$) or lost ($l < 0$) by group i and, respectively, lost or won by group j – i.e., a transfer of l resources between group i and group j. We denote the conflict success function at time t with $\sigma(l|z_{i,t}, z_{j,t})$ with the resulting probability distribution over l summing up to 1, namely $\sum_{l=-z_{i,t}}^{z_{j,t}} \sigma(l|z_{i,t}, z_{j,t}) = 1$, and allowing for the possibility that no amount of resources is looted by any group $\sigma(0|z_{i,t}, z_{j,t}) > 0$.

Assumptions. We further consider the following assumptions.

ASSUMPTION A1 (Hegemon's satisfaction).

$$\iota(R, 0) = 0$$

Assumption A1 states that a hegemon group never attacks. The justification is that conflict aims to gain resources, and a hegemon has nothing to gain from conflict.

ASSUMPTION A2 (Conflict initiation function).

$$\iota(z_{t,i}, z_{t,j}) = \epsilon^{r(z_{t,i}, z_{t,j})} < 1, \quad \forall\ (z_{t,i}, z_{t,j}) \neq (R, 0)$$

In A2 $\epsilon \in (0, 1)$ is typically assumed to be very small and $r(z_{t,i}, z_{t,j})$, which is often referred to as "resistance", is an explicit measure of the unlikeliness that group i initiates a conflict towards group j.

ASSUMPTION A3 (Victory is always possible).

$$\sigma(1|z_{t,i}, z_{t,j}) > 0, \quad \forall\ (z_{t,i}, z_{t,j}), \text{ with } z_{t,j} \geq 1$$

Assumption A3 means that a group i who initiates a conflict always has some positive probability of subtracting a unit of resources from the target group j, even if the distribution of resources is highly unequal, e.g., even if $z_{t,i} = 0$ and $z_{t,j} = R$. This implies that the attacked group can never be sure to secure victory.

The main reason for having that the probability of winning at least one resource is bounded away from zero for the attacker is that, in determining the outcome of a conflict, not only material resources matter. In fact, there are plenty of historical examples in which a weaker side wins because they have better human resources, are more clever or simply had luck. David against Goliath does not lose with probability one.

From a technical point of view, assumption A3 can be regarded as a rather mild one in that it is consistent with many different specifications on the conflict success function. In particular, if we take as reference the Constant Elasticity of Augmentation (CEA) contest success functions proposed by Hwang (2012), namely, the outcome of a conflict initiated by group i against group j is:

$$\frac{\exp\{\kappa\frac{1}{1-\rho}z_{t,i}^{1-\lambda}\}}{\exp\{\kappa\frac{1}{1-\rho}z_{t,i}^{1-\rho}\}+\exp\{\kappa\frac{1}{1-\rho}z_{t,j}^{1-\lambda}\}} \tag{2.1}$$

where $\kappa > 0$ measures the scaling of the decisiveness of power disparities and λ is the elasticity of augmentation, possibly taking any real value. It is worth noting that the CEA contest success function particularizes to the difference contest success function for $\lambda = 0$, i.e., the probability of winning depends on the difference in resources. In contrast, it particularizes to totally random contest success function for $\lambda = 1$, i.e., the probability of winning is always one-half no matter the differences between the attacker's and defendant's resources (see Hirshleifer, 1989, for a comparison of different shapes of the contest success function). It is straightforward to recognize that, if $\sigma(l|z_{t,i}, z_{t,j})$ scales with the CEA function, then assumption A3 is satisfied for any parameter value. However, assumption A3 is not satisfied for the exact specification of the ratio contest success functions, while it holds for any specification of the difference contest success functions.

A key aspect of our formulation is that we allow $\sigma(\cdot)$ to have any structure as long as it satisfies A3. In the literature, it is frequently established that the side with more resources is the one with more probability to win. However, we are allowing even for the possibility that the weaker side has actually better chances to win. This could be rationalized along the lines of Levine and Modica (2013b), where the resources devoted to build military power are determined endogenously within a society. So, a society with few resources can devote much of its energy to build military power, for example, if the preferences of its population change through the relaxation of the incentive compatibility constraints (what Levine and Modica call *barbarian hordes* or *zealots*).

Another concern regarding assumption A3 could be that it implies that inactive groups maintain some positive probability of winning a resource by attacking another group, even if they target a hegemon. This possibility does not seem unreasonable, at least when total annihilation of the inactive group is not feasible. Think of a country that has been conquered and left without resources, which are all managed by the invader;

in this case, the oppressed population could still rise a revolt to recover a piece of land; success might be unlikely, but not with zero probability.

In the subsequent analysis, we will assume that A1, A2 and A3 always hold. In addition, we will make further alternative assumptions about the conflict initiation function, which are meant to capture alternative ways in which the likelihood of conflict initiation depends on the distribution of groups' resources.

ASSUMPTION A4 (Uniform conflict initiation likelihood). *Function* $r(z_{t,i}, z_{t,j})$ *is constant on* $(z_{t,i}, z_{t,j})$, $\forall\ (z_{t,i}, z_{t,j}) \neq (R, 0)$.

Assumption A4 implies that the probability of conflict initiation is independent of the distribution of resources among groups. It is the counterpart, in this setup, of the uniform error model widely applied in the literature on stochastic stability analysis (see Newton, 2018, for a recent survey), and which in fact is implicitly applied in Levine and Modica (2013a) and Levine and Modica (2013b) where perturbations are introduced in the conflict success function (and where the conflict initiation function is not explicitly modeled).

ASSUMPTION A5 (Increasing-in-relative-power conflict initiation likelihood). *The resistance function* $r(z_{t,i}, z_{t,j})$ *is either decreasing in* $z_{t,i}$ *and non-decreasing in* $z_{t,j}$ *or non-increasing in* $z_{t,i}$ *and increasing in* $z_{t,j}$, $\forall\ (z_{t,i}, z_{t,j}) \neq (R, 0)$.

Assumption A5 implies that the probability that group i initiates a conflict with group j increases in the relative number of resources possessed by i with respect to j. This captures the case where the side with more resources has a greater chance of winning, or the case where having more resources makes losing less troublesome, which in turn may increase the inclination to pursue conquests. Another case is where a society with more resources comes to believe that it is entitled to expand because of its superiority.

ASSUMPTION A6 (Decreasing-in-relative-power conflict initiation likelihood). *The resistance function* $r(z_{t,i}, z_{t,j})$ *is either increasing in* $z_{t,i}$ *and non-increasing in* $z_{t,j}$, *or non-decreasing in* $z_{t,i}$ *and decreasing in* $z_{t,j}$, $\forall\ (z_{t,i}, z_{t,j}) \neq (R, 0)$.

Assumption A6 implies that the probability that group i initiates a conflict with group j decreases in the relative number of resources possessed by i with respect to j. This is the case where a group with few resources has not much to lose and therefore fears less the conflict. This is also the case where a group that does not control as many resources as others is envious and, hence, more likely to initiate a conflict. Another case is that in which military technology is such that the group with fewer resources finds it more efficient to invest in military power than economic activity.

2.4 Results

Before starting to deliver the results of our model, it is worth mentioning how the transition probabilities between two states are computed. For any two states z_t and z'_{t+1} such that $z'_{t+1,i} = z_{t,i} + l$, $z'_{t+1,j} = z_{t,j} - l$ and $z'_{t+1,k} = z_{t,k}\ \forall k \neq i, j$, the transition probability from z_t to z'_{t+1} can be computed as follows:

$$\begin{aligned}\mathbb{T}(z_t, z'_{t+1}) = &\frac{1}{|G|(|G|-1)}\iota(z_{t,i}, z_{t,j})\sigma(l|z_{t,i}, z_{t,j}) + \\ &+ \frac{1}{|G|(|G|-1)}\iota(z_{t,j}, z_{t,i})\sigma(-l|z_{t,j}, z_{t,i}), \quad (2.2)\end{aligned}$$

which is basically the sum of two products of probabilities: (i) the probability that i and j are paired, multiplied by the probability of i attacking j, multiplied by the probability of i winning l units of resources, and (ii) the probability that j and i are paired, multiplied by the probability of j attacking i, multiplied by the probability of j losing l units of resources.

In the following, we focus on the *long-run evolutionary equilibrium* of the model. Namely, on the set of states that turns out to be stochastically stable according to any stochastic dynamics consistent with (2.2). Given that the stochastic dynamics in (2.2) is ergodic, meaning that it is possible to get from every state to every other state with positive probability, a unique invariant distribution exists which describes the time average behavior of the system, thanks to the fundamental theorem of Markov chains. As the amount of noise ϵ tends to zero, the invariant distribution varies and (under mild assumptions that are here satisfied) approaches a limiting distribution, called stochastically stable distribution (Foster and Young, 1990). Technically, we rely on the results by P. Young (1993), which are built on Freidlin, Wentzell, et al. (n.d.). A long-run evolutionary equilibrium is a state that receives a positive probability in the stochastically stable distribution.

For simplicity, within the following discussion and proofs, we drop the time index t of the states z_t. In the present setup, given two states z' and z'', we say that the *resistance* between state z' and state z'' is a measure of how difficult the transition from state z' to state z'' is in terms of mistakes.

For any conceivable tree (i.e., a graph such that any two vertices are connected by exactly one path) having the state z' as root and all other states as nodes, consider the sum of resistances assigned to each edge of the tree, and take the minimum over trees of such a sum. This number represents the *stochastic potential* of state z'. Intuitively, the stochastic potential tells us how difficult is to reach a state starting from other states. Using a fundamental result on stochastic stability, we know that in the present setup, a state is stochastically stable if and only if it has minimum stochastic potential. Intuitively, this amounts to looking for the distributions of power that are in place most of the time when the probability of initiating a conflict becomes very small.

We first study the case of uniform conflict initiation likelihood, formalized in Assumption A4, where differences in resources between countries do not translate into a significant difference in the tendency to initiate a conflict. In terms of interpretation, in this case, there is no discernible intention behind the outbreak of war, rendering it a mistake in the truest sense. This first case is intended to serve as a benchmark to be contrasted with the results obtained under alternative assumptions (i.e., replacing A4 with A5 or A6). We denote with $\tilde{r}(z, z')$ the resistance of moving from state z to state z' in one unit of time. We note that, in our setting, the transition from z to z' can be the outcome of a conflict initiated by each of the two countries involved, i.e., $\tilde{r}(z, z') = \min\{r(z_i, z_j), r(z_j, z_i)\}$ whenever z and z' are accessible one from the other, meaning that $\exists\, i, j \in G$ with $z_i \neq z'_i$ and $z_j \neq z'_j$, while $z_k = z'_k$ for any $k \in G$, with $k \neq i, j$. If, instead, z and z' are not accessible one from the other, then $\tilde{r}(z, z') = +\infty$, which means that z' cannot be reached from z as the result of a single conflict.

PROPOSITION **1.** *If Assumptions A1, A2, A3 and A4 hold, then every state is a long-run evolutionary equilibrium.*

Proof. We denote with h the constant level of function $\iota(z_i, z_j)\ \forall\, i, j \in G$ implied by Assumption A4. We show that, for any pair of states z'', z', with $z'' \neq z'$, there exists a path $(z^0, \ldots, z^w, \ldots, z^W)$, with $z^0 = z'$ and $z^W = z''$, such that $\tilde{r}(z^w, z^{w+1}) = h$ for $w = 0, \ldots, W-1$.

We define the distance between z' and z'': $d(z', z'') = \sum_{g \in G} |z'_g - z''_g|$. We take groups i and j such that $z_i > z'_i$ and $z_j < z'_j$ (two such groups always exist, if $z'' \neq z'$). We set $z^0 = z'$ and $z^W = z''$. Starting from z^0, we observe that the resistance for group i to initiate a conflict with group j is h (by Assumption A4 after observing that group i is not a hegemon) and, conditional on that, there is a positive probability (by Assumption A3) that state z^1 is reached next period, with $z^1_i = z'_i + 1$, $z^1_j = z'_j - 1$, and $z^1_k = z'_k$ for all $k \in G$, $k \neq i, j$. Hence, $\tilde{r}(z^0, z^1) = h$. We note that $d(z^1, z^W) = d(z^0, z^W) - 2$. All other steps in the sequence are constructed in the same way, with W being equal to $d(z^0, z^W)/2$. Since every state is absorbing, and the minimum resistance to moving from any two states is h, the stochastic potential of any state z cannot be lower than h times the number of transitions needed to reach z. Therefore, the stochastic potential of any state $z \in Z$ is $\rho(z) = (|Z| - 1)h$, since any tree is comprised of a number of links equal to the number of nodes decreased by 1. □

Proposition 1 tells us that if the likelihood of conflict initiation is independent of the current power allocation, then every power distribution can arise in the long run. Two things are worth noting: first, not only hegemony and balance of power are stochastically stable, but all possible distributions of power, and second, this holds for a rather wide class of contest success functions.

In order to consider the long-run outcomes of the other two alternative assumptions on the conflict initiation function, we have first to establish two preliminary results.

LEMMA **1.** *If Assumption A5 holds, then, for any $z' \in Z$ which is not a hegemony, there exists a hegemony $z'' \in Z$ and a path $(z^0, \dots, z^w, \dots, z^W)$, with $z^0 = z'$ and $z^W = z''$, such that $\tilde{r}(z^w, z^{w+1}) = \min_{z \in Z} \tilde{r}(z^w, z)$ for $w = 0, \dots, W-1$.*

Proof. We define the distance between z' and z'': $d(z', z'') = \sum_{g \in G} |z'_g - z''_g|$. We also define $\widehat{G}(z) = \{g \in G : z_g > 0\}$. We take groups i and j such that $z'_i = \max_{g \in G} z'_g$ and $z'_j = \min_{g \in \widehat{G}(z')} z'_g$. We consider the hegemony z'' where $z''_i = R$. We set $z^0 = z'$ and $z^W = z''$. Starting from z^0, and conditional on i initiating a conflict with j, there is a positive probability (by Assumption A3) that state z^1 is reached in the next period, with $z^1_i = z'_i + 1$, $z^1_j = z'_j - 1$, and $z^1_k = z'_k$ for all $k \in G$, $k \neq i, j$. Moreover, under Assumption A5 we have that $\tilde{r}(z^0, z^1) = \min_{z \in Z} \tilde{r}(z_i, z)$, due to $z^0_i = \max_{g \in G} z^0_g$ and $z^0_j = \min_{g \in \widehat{G(z^0)}} z^0_g$. Finally, we note that z^0_i is smaller than R, while z^0_j is larger than 0; hence, $d(z^1, z^W) = d(z^0, z^W) - 2$. All other steps in the sequence are constructed in the same way, with W being equal to $d(z^0, z^W)/2$. □

LEMMA **2.** *If z'' is the unique balance of power and Assumption A6 holds, then for any $z' \in Z$, $z' \neq z''$, there exists a path $(z^0, \dots, z^w, \dots, z^W)$, with $z^0 = z'$ and $z^W = z''$, such that $\tilde{r}(z^w, z^{w+1}) = \min_{z \in Z} \tilde{r}(z^w, z)$ for $w = 0, \dots, W-1$.*

Proof. The proof unfolds along the lines of the proof of Lemma 1, with a few adjustments. We define the distance between z' and z'': $d(z', z'') = \sum_{g \in G} |z'_g - z''_g|$. We take groups i and j such that $z'_i = \min_{g \in G} z'_g$ and $z'_j = \max_{g \in G} z'_g$. We set $z^0 = z'$ and $z^W = z''$. Starting from z^0, and conditional on i initiating a conflict with j, there is a positive probability (by Assumption A3) that state z^1 is reached in the next period, with $z^1_i = z'_i + 1$, $z^1_j = z'_j - 1$, and $z^1_k = z'_k$ for all $k \in G$, $k \neq i, j$. Moreover, under Assumption A6 we have that $\tilde{r}(z^0, z^1) = \min_{z \in Z} \tilde{r}(z_i, z)$, due to $z^0_i = \min_{g \in G} z^0_g$ and $z^0_j = \max_{g \in G} z^0_g$. We observe that $d(z^1, z^W) = d(z^0, z^W) - 2$, since passing from z^0 to z^1 one resource has been moved from a group having more than $R/|G|$ resources to a group having less $R/|G|$ resources. All other steps in the sequence are constructed in the same way, with W being equal to $d(z^0, z^W)/2$. □

Lemma 1 establishes that whenever the groups with more resources are more belligerent, i.e., if A5 holds, then a hegemony can be reached from any other state with a path that has minimum resistance at each step. Conversely, Lemma 2 establishes that whenever the groups with fewer resources are more belligerent, i.e., if A6 holds, then the state with balance of power can be reached from any other state with a path that has minimum resistance at each step. Thanks to these findings, we can now characterize the long-run evolutionary equilibria.

PROPOSITION **2.** *If Assumptions A1, A2, A3 and A5 holds, then a state z is a long-run evolutionary equilibrium if and only if z is a hegemony.*

Proof. We take a state z' that is not a hegemony. By Lemma 1, there exists a hegemony z'' and a path $(z^0, \dots, z^w, \dots, z^W)$, with $z^0 = z'$ and $z^W = z''$, such that $\tilde{r}(z^w, z^{w+1}) = \min_{z \in Z} \tilde{r}(z^w, z)$ for $w = 0, \dots, W-1$. We consider a tree T', rooted at z' and with Z as set of nodes, which minimizes the sum of the resistances over its links among all trees rooted at z'. Starting from T' we add all links connecting consecutive states in such path from z' to z'', and we remove all links in T' which were outgoing from any state in the path. By doing so, we obtain a tree T'' rooted at z''. We now compare the sum of the resistances over links in T' and T''.

Firstly, any state that is not in the path has a resistance associated with its outgoing link that is the same in T'' and in T'. Secondly, any state z^w in the path, with $w = 1, \dots, W-1$, has a resistance associated to its outgoing link that cannot be larger in T'' than in T', due to Lemma 1. Thirdly, we observe that in T' there is an outgoing link from z'' but no outgoing link from z' (since z' is the root of T'), while in T'' there is an outgoing link from z' but no outgoing link from z'' (since z'' is the root of T''). As a result of this, and knowing by Lemma 1 that $\tilde{r}(z', z^1) = \min_{z \neq z'} \tilde{r}(z', z)$, we can write the following lower bound for the difference between the stochastic potentials of z' and z'':

$$\rho(z') - \rho(z'') \geq \min_{z \neq z''} \tilde{r}(z'', z) - \min_{z \neq z'} \tilde{r}(z', z) > 0.$$

The latter inequality holds because $\min_{z \neq z''} \tilde{r}(z'', z) = r(0, R)$, since a hegemon never initiates a conflict (by Assumption A1), and $r(0, R) > \min_{z \neq z'} \tilde{r}(z', z)$, due to Assumption A5, since z' is not a hegemony.

Therefore, z' cannot have minimum stochastic potential; also, since a state with minimum stochastic potential must exist, such a state must be a hegemony. We conclude by noting that all hegemonies must have the same stochastic potential by symmetry: if z'' is a hegemony where all resources are assigned to group i, and z' is a hegemony where all resources are assigned to group j, then every tree rooted at z'' can become a tree rooted at z' after exchanging labels between i and j. $\square$

Proposition 2 establishes that, when Assumption $A5$ holds, the states entailing hegemony are the only stochastically stable states in our setup, meaning that hegemony should be expected in the long-run whenever stronger groups are more likely to initiate a conflict against weaker groups.

PROPOSITION **3.** *If Assumptions A1, A2, A3 and A6 holds, then a state z is a long-run evolutionary equilibrium if and only if z is the balance of power.*

Proof. The proof is similar to the one of Proposition 2, and to some extent, simpler due to the fact that a single balance of power exists instead of multiple hegemonies. We denote with z'' the balance of power. By virtue of Lemma 2, a tree can be constructed that is rooted at z'' and has Z as the set of nodes, such that if a link from state $\hat{z}$ to state $\tilde{z}$ belongs to the tree, then $\tilde{r}(\hat{z}, \tilde{z}) = \min_{z \in Z} \tilde{r}(\hat{z}, z)$; therefore, the stochastic potential of z'' is $\rho(z'') = \sum_{\hat{z} \in Z \setminus \{z\}} \min_{z \in Z} \tilde{r}(\hat{z}, z)$.

We now consider a state $z' \neq z''$. Clearly,

$$\rho(z') \geq \sum_{\hat{z} \in Z \setminus \{z'\}} \min_{z \in Z} \tilde{r}(\hat{z}, z)$$

If we take the difference between $\rho(z')$ and $\rho(z'')$ we obtain:

$$\rho(z') - \rho(z'') \geq \min_{z \neq z''} \tilde{r}(z'', z) - \min_{z \neq z'} \tilde{r}(z', z) > 0.$$

The latter inequality holds due to Assumption A6, after observing that the group with the fewest resources has more resources in z'' than in any other state, and that the group with most resources has fewer resources in z'' than in any other state.

Therefore, z is the unique state with minimum stochastic potential, and as such, it is the unique stochastically stable state. $\square$

Proposition 3 establishes that when Assumption $A6$ holds, the unique state entailing balance of power is the only stochastically stable state in our setup, meaning that balance of power should be expected in the long-run whenever weaker groups are more likely to initiate a conflict against stronger groups.

It might seem very surprising that the actual shape of the conflict success function does not substantially affect the results provided in Propositions 1 to 3 (the only requirement being that A3 holds). However, it may become less surprising if one considers that in our setup, conflicts are modeled as rare events which, once occurring, can entail any redistribution of resources between conflicting groups. This implies that the likelihood of being successful in a conflict has effects on the dynamics which are of second-order importance with respect to the effects of the likelihood of being involved in a conflict.

2.5 Interpretation and historical facts

As previewed in the Introduction, the evolutionary literature on conflict has mainly focused on the conflict resolution function, frequently incorporating several factors that are believed to contribute crucially to the likelihood of success in a confrontation (Levine and Modica, 2013a; Levine and Modica, 2013b; Levine and Modica, 2018; Levine and Modica, 2022; Levine and Modica, 2021). These same features could also influence the conflict initiation function (CIF). In this section, we aim to review some traditional determinants arguably affecting the conflict resolution function (CRF), and we illustrate situations where they can also, or alternatively, affect the CIF.

To begin with, we can briefly discuss the case in which we look at A5, in which the relatively stronger is more likely to attack, predicting the emergence of a hegemony.

One might argue that this assumption crucially depends on the fact that the nature of the CRF is more frequently likely to be "strong favored" (using the terminology of Dziubiński, Goyal, and Minarsch, 2021), meaning that the side with more resources has more chances to prevail in a conflict. Given the amount of group conflict situations in which it is sensible to assume so, it could very well be that the temptation to attack minimizes the resistance of the stronger towards attacking the weak. This would explain the relatively higher frequency of hegemonies in history, especially in settings without barriers (geographical, political, economic, or any other type of deterrent) to compensate for the apparently easy gain. It is worth noting that, from the fact that the CRF is strong favored, it does not follow that warfare or any other type of conflict is riskless, which is exemplified in our model by assuming a CRF bounded away from zero. A strong favored CRF could lead to a CIF where the stronger is more likely to initiate a conflict. However, this can be the case even if the CRF is not strong favored when, for instance, the stronger has internal reasons – i.e., different from obtaining resources – for generating external conflict, such as maintaining power.

The cases that support A6, i.e., in which the relatively weaker is more likely to attack, are trickier. In order to identify factors supporting those cases, one has to look for potential causes of conflict initiation that are either independent of actual power in conflict resolution or that make attacking more probable when possessing relatively fewer resources. According to our long-run analysis, all these factors are conducive to balance of power.

A characteristic typically considered to prevent hegemony is the presence of geographical barriers. Barriers work at a basic level by increasing the logistic cost of attacking, deterring the attacker's seizing of resources. However, it could also be that the existence of geographical barriers, particularly in the case in which they are asymmetric, creates situations in which the weaker side is more likely to attack the stronger one. An asymmetric geographical barrier might prevent the stronger from attacking effectively, while allowing the weaker to engage in low-risk attempts to extract resources from the stronger, or even have more probability of success. At the same time, the benefits of many barriers can be capitalized on only as long as the attacker is weak since, if the group gets more resources, and hence maybe expands its area of influence, it could lose the advantage of the barrier. This may work as well in local markets, where weak incumbents can successfully fight bigger competitors via reputation or proximity with the client. At low scale, these arguments explain the common use of guerrilla tactics across history, which involve situations in which the weaker side attacks more often precisely because the geography helps to reduce the aggregated risk of any given conflict (one could take the example of Afghanistan, which has been an absolute hell to conquer and maintain all across history Kaye, 2021; Girardet, 2012; Holt, 2012; Malkasian, 2021). Guerrilla tactics are employed because they have been very effective in various situations. However, the possibility of relying on them crucially depends on the asymmetric use of geography and the advantage that keeping a low profile concedes to guerrilla fighters, making it more likely for the weaker side to initiate a conflict.

On a bigger scale, one can look at historical regularities such as the balancing role attributed to England in Europe (Paul, Wirtz, and Fortmann, 2004). The argument is related to the fact that the presence of strong external outsiders is considered to reduce the possibility of an hegemony emerging (see below). However, the presence of the Channel has been a critical factor in allowing the influence of England first and the British Empire later on continental Europe. While the aggregate military power of the British has been inferior to those of their traditional rivals, such as France or the Spanish Empire, the existence of the water barrier has allowed England to successfully sustain conflicts with both of them, by relying on naval specialization, and often as an attacker, hence exemplifying a situation in which geographical barriers can lead a weaker side to initiate conflict. However, England's footholds in Europe have not been successfully maintained, precisely due to the fact that the advantage provided by the barrier is removed when becoming stronger (think of Aquitania or Britain during the Hundred Years War).

Another example regarding water barriers is the Kingdom of Sicily during the 12th century, safely protected by its naval power but losing the advantage when expanding to the North of Africa, (Matthew, 1992). In conclusion, it seems that geographical barriers, no matter the relative strength of the groups protected by them, seem to affect the CRF and the CIF roughly in the same direction by providing both a ceteris paribus advantage in the probability of success and incentivizing the initiation of conflict in the case the barrier is asymmetric.

Another extensively studied feature that prevents hegemony and favors balance of power is the presence of strong outsiders that can intervene in a conflict between groups, acting as a balancing force. The leading example in the literature, as previewed below, is the balancing effect of first the Vikings and then England in continental Europe from the early Middle Ages to the present (Levine and Modica, 2013b). However, the intervention of strong outsiders has been frequently modeled as a factor directly influencing the CRF, with the introduction of a parameter that increases the chances of victory of one side, usually the weaker. As exposed in Levine and Modica (2018), external intervention can lead to a hot peace, a low-intensity conflict with few casualties (like the conflict between Israel and Arab countries), or a prolonged war, a high-intensity, very bloody conflict in which the outside intervention restores the balance between forces, therefore prolonging the conflict.

These arguments, although correct, consider only the actual involvement of the outsiders in the conflict, while it may very well be that the presence of outsiders by itself influences the CIF. At the same time, a peaceful balance of power, for example, with intervention as a deterrent, should also be considered. However, it gets outside of the scope of our model since, in our case, the dynamics are always driven by conflict. The presence of strong outsiders, and the possibility of their intervention, can, in practice, create a situation in which weaker groups might be pushed to risk the possibility of attack, also considering the fact that they could ask for help or arbitrage in the case of an unfavorable outcome. A third party, strong outsider, might have an effect just through the CIF, without affecting the CRF, simply by modifying the risk perception of the side it supports. In this case, the intervention by itself is not necessary, since it is the presence of outsiders influencing the boldness of weaker groups that drives the CIF without affecting the CRF by itself. An example is the situation of Anatolia at the beginning of the first century BCE, with the presence of two big powers at each side of the peninsula, Rome at the west and Pontus at the east, and many little kingdoms that were often in a warring state, but with no big overall changes due to the eventual diplomatic intervention of one of the two outside local hegemonies (Glew, 1977). In this case, the balance of power in Anatolia was due to the fact that each of the outside powers had no individual incentive to initiate conflict due to the presence of the other. Hence, the stronger was unlikely to attack the weak. It was only with the beginning of the Social War in Italy that the balance was broken, with Rome unable to devote military resources in the east, eventually leading to the invasion of Anatolia by Pontus and the beginning of the First Mithridatic War. Rome's presence as an implicit factor causing weaker groups to attack, in fact, is an empirical regularity at the frontiers of the Roman Empire.

Technology is another essential factor in group conflict initiation and resolution. As it is developed in Levine and Modica (2022), defensive technology, such as castles, favors inclusive institutions, which in turn supports balance of power. In contrast, offensive technology, such as cannons, favors extractive institutions, which could lead either to hegemony or balance of power. The argument for the former claim is straightforward, given that defensive technology provides an advantage to the weaker side in the CRF (we could think in the wall of a castle, but also in antitrust or patent legislation). Putting an example, it would have been challenging for crusader forces to maintain a footprint in Asia Minor without heavily relying upon the castle systems in the Levant (Kennedy, 2001). This incidentally strengthens our claim about geographic barriers since, by similar arguments as the ones used above, the advantageous use of fortifications is only exploitable as long as the resources do not grow too much, due to the fact that crowding effects might appear.

On the other hand, the development of offensive technology increases the marginal contribution of resources, independently of the group's relative strength. However, in order to clarify the effects of technology on the CIF, we have to distinguish between the cases where a new technology adds to existing resources available or just affects CIF without affecting resources. For example, the development of oblique phalanx tactics by Epaminondas (Hanson, 1988) effectively helped Thebes overcome the Spartan hegemony in the Peloponnese, but can we claim that the new tactic affected the likelihood for the conflict to arise, via increasing confidence, or it just directly increased the strength of Thebes over Sparta? Putting an example with defensive technology, the Battle of Cape Matapan during the Second World War Johnman and Murphy (2005), in which the British fleet had radars in their ships while the Italian fleet did not, resulted in a decisive ally victory (incidentally, this battle was possible thanks to the deciphering machine Ultra, an even greater example of asymmetric defensive technology). However, it is difficult to argue that the effect of the radar does not work just by increasing the likelihood of success through the CRF. In any case, we should be aware of the possible existence of non-innocuous interactions between technological developments and the CIF. For example, using drones in war might increase the probability of conflict by reducing the political costs of initiating a war, and this would work independently of the group's relative strength. To conclude, it seems clear that further research should be pursued in order to clarify the relationship between technological progress and CIF.

Charismatic leaders are essentially random shocks within a group, but they can have a decisive effect both in the likelihood of initiating conflict and in the result of it. Charismatic leaders may have the ability to assemble their groups towards warfare, which would result in a higher likelihood to conflict, no matter the groups' relative strength. Since the direction of the effect seems to be that a charismatic leader can serve as a means to attack in general (it is worth mentioning that many leaders have used external warfare as a means of preserving power, as in Lopez, 2020), we should consider if a charismatic leader is more likely to arise in a weaker group. Intuitively, a weaker group may be both more sensitive to populist arguments and smaller in size, so that the relative marginal effect of a shock yielding a charismatic leader is larger. In addition, a weaker group might have less strict democratic controls, making it easier for a charismatic leader to establish an authoritarian regime. Notice that a leader could, for example, attack totally on purpose, but the emergence of such a leader in a society is by itself stochastic and could be considered a mistake affecting conflict initiation, i.e., the shape of the CIF. Nevertheless, the emergence of a charismatic leader in a group does not necessarily imply higher success in conflict, as the correlation between charisma and martial progress is ambiguous – there are many examples of both great success (e.g., Alexander the Great) and great disasters (e.g., Spartacus). It is true, however, that a charismatic leader can work by concentrating resources within a group towards conflict, and hence directly affecting the CRF.

This argument is similar to the one provided by Levine and Modica (2013a) regarding barbarian hordes and zealots, in which aggressive groups with relatively lower incentive compatibility constraints can effectively seize a substantial amount of resources but do not have particularly stable institutions. Charismatic leaders, hence, through their capacity to convince a group to weaken its constraints, would work both by increasing the likelihood of success in the CRF, via concentration of resources towards military power, and by directing the group more often towards conflict, hence affecting the CIF. Sometimes, the demise of a charismatic leader can shake the form of the CIF for a considerably long period of time, as we can see in the case of Alexander the Great proving the possibility of success of a weaker group, resulting in a relative balance of power both in Anatolia (up to the Roman Empire) and Persia (up to the Parthian Empire), having both regions traditionally tended towards hegemony.

As a final factor that may be overlooked, we can consider cultural differences and their effects on CIF and CRF. Consider warrior honor codes (Pressfield, 2011) that take rational weighting of forces in a conflict out of the equation. Examples of codes of conduct are widespread across history, with popular examples including Chivalry in Europe or Bushidō in Japan. Some of these codes have relied on characteristics that are considered honorable. However, they do not relate to an actual evaluation of the CRF, with concepts like "no retreat", "no surrender" or "fight until the end". In a conflict, warrior honor codes might lead a side to endorse irrational stubbornness, which should show up in the CRF. In particular, the per capita effect of relentless warriors is higher in any situation favoring attrition and lower in settings where conservative approaches are preferred. As an example of the first effect, we can mention the Battle of Empel, in which Spanish soldiers, when offered an opportunity to surrender, answered "Spanish soldiers prefer death to dishonor. We will talk about surrender after death". Later, they ended up winning the battle thanks to the freezing of the Meuse river during the night. The best example of the second effect is the invention of Fabian tactics by Romans during the Second Punic War (Erdkamp, 1992; Carr and Walsh, 2022). Although previous generals had promoted direct confrontation against Hannibal and the Punics, the realization of dictator Quintus Fabius that directly facing Hannibal was almost suicidal, and hence avoiding combat, was essential to prevent more catastrophic defeats. In summary, the effect of warrior honor codes in the CRF is very much context-dependent.

Context-dependency of culture is also likely for the CIF. A weaker side embracing codes that promote retaliation and the necessity to amend affronts might actually attack (and continue attacking) more often, and such non-consequential morality would favor the attainment of a balance of power. An example can be found when looking at a map of pre-Roman Gaul, in which we can observe many clusters of little tribes with no hegemony and relative balance of power, which can be explained both by the fact that warrior honor codes would prescribe immediate counterattack even from a position of weakness (and, also, that the defensive – fortifications – technology was much more advanced than the siege technology at that time and place). In fact, the extraordinary stubbornness of the Gallic was frequently documented by Caesar himself. On the other hand, honor codes that promote mercy with the enemy could prevent stronger sides from attacking, which should also favor balance of power. In conclusion, we should be aware of possible interactions between culture, CRF and CIF, and recognize that sometimes culture might affect either function differently. We have explicitly discussed warrior honor codes, but we could consider the effects of nationalism, religious beliefs, or any other cultural dimension.

2.6 Conclusions

In this paper, we have studied an evolutionary model in which the possession of resources, which measures the distribution of power, evolves over time as the result of conflicts among groups. Differently from the previous literature, where the likelihood of conflicts and the distribution of their outcome are modeled jointly, we have provided a model where the likelihood of conflict initiation and the likelihood of conflict outcomes are determined by distinct functions which depend on the current power of the groups involved. Importantly, we assume that conflicts are initiated with a vanishing probability, while the probability of winning or losing resources – once a conflict is initiated – remains non-negligible. We believe that these assumptions are reasonable if: (i) conflict is a rare event resulting from loss of control, miscalculation, or rule-following, i.e., something that depends on the resources possessed but that, anyway, happens rarely; (ii) once a conflict breaks out, many factors can affect the outcome of the conflict beyond the factors that triggered the conflict, e.g., greater knowledge of the conflict area, higher intrinsic motivation, internal turmoil, help from an external power, and innovations.

Our model allows us to uncover the relationship between the shape of the conflict initiation function and the long-run distribution of power. Moreover, we have clarified that the conflict success function, which has received the most attention from the literature so far, does not have a substantial impact on the long-run distribution of power provided that the attacked group cannot avoid the risk of losing some resources, even if it is very powerful.

Importantly, our results show that both hegemony and balance of power can be observed as long-run evolutionary outcomes of conflicts among groups, and which of them will be observed most often crucially depends on the determinants of the initiation of conflicts. Specifically, when conflicts are more likely to be initiated by stronger groups against weaker groups, then hegemony will prevail most of the time in the long run. In contrast, the balance of power will prevail most of the time in the long run if conflicts are more likely to be initiated by weaker groups against stronger ones.

A natural step for future research would be to model the stage game within a group in order to explore which of our assumptions is more likely to hold depending on the details of the strategic setup within the group. This line of research would also allow exploring the role of complementarities and substitutabilities betweeen different types of resources as, for instance, in the case where resources must be alternatively allocated to productive or military purposes (as in, e.g., Hwang, 2012; Levine and Modica, 2013b).

Another promising avenue for future research is the introduction of some topological structure (e.g., geography) determining accessibility between groups, so that conflicts may be assumed to initiate with a larger probability when two groups have direct access to each other. In some cases, it might also be reasonable to assume that conflicts are less likely to arise between neighboring groups because of alliances or commonalities in history and culture. A network structure may hence allow embedding geographic accessibility in the model, as well as alliances or historical rivalries. For a recent examination of the role of different conflict resolution functions in networks, see Dziubiński, Goyal, and Minarsch (2021).

Chapter 3

Prolonged Impulse Control Prompts Sleep-like Brain Activity and Aggressivity

3.1 Introduction

When making decisions in everyday life, we are inevitably shaped by contingent physiological, psychological, and cognitive factors. As a result, sometimes we make decisions that may appear inconsistent with our usual social behavior, even leading to unexpected aggressive behavior instead of a more desirable cooperative one (Loewenstein, Lerner, et al., 2003; Lerner and Keltner, 2001). Among the cognitive factors underpinning social behavior, the tension between intuition and deliberation plays a major role in determining cooperative or antisocial choices. Studies that investigated the role of this factor typically did so by combining cognitive manipulations and economic games (Belloc et al., 2019; Achtziger, Alós-Ferrer, and Wagner, 2015; Strack and Deutsch, 2004; Duffy and Smith, 2014; Schulz et al., 2014). Many of such studies advocated a positive effect of intuition on cooperation (Rand, Peysakhovich, et al., 2014; Rubinstein, 2007; Kieslich and Hilbig, 2014), while others supported a central role of deliberation in cooperative behavior (Achtziger, Alós-Ferrer, and Wagner, 2016; Capraro and Cococcioni, 2016), with antisocial acts (including criminal acts) assumed to have a more impulsive nature instead (Gottfredson and Hirschi, 1990).

Among the cognitive manipulations used to affect the balance between intuition and deliberation, ego depletion is one of the most popular and controversial (Baumeister, Bratslavsky, et al., 2018). In a typical ego depletion paradigm, cognitively demanding tasks requiring participants to use self-control and decision-making (so-called 'executive') functions are employed under the assumption that such an effort will deplete willpower resources (Baumeister, Bratslavsky, et al., 2018; Mischel, Cantor, and Feldman, 1996; T. Heatherton, Tice, et al., 1994). Tasks adopted in the literature include for instance, Stroop tasks, emotion suppression, and other various high-conflict decision-making tasks (Baumeister, Bratslavsky, et al., 2018; Xu, Bègue, and Bushman, 2012; Muraven, Pogarsky, and Shmueli, 2006). After such manipulations, individuals asked to complete a subsequent task requiring the same cognitive resources showed impaired executive functions and greater reliance on a more automatic and intuitive course of action (Vohs and T. F. Heatherton, 2000; Dang, Liu, et al., 2017; Boksem, Meijman, and Lorist, 2005; Van Der Linden and Eling, 2006).

However, results concerning the ego depletion effect have been strongly criticized due to inconsistent replication across studies (Friese et al., 2019). Indeed, while some investigations, including a large multi-site replication study, failed to obtain solid evidence in support of an ego depletion effect (Hagger et al., 2016), a re-analysis of the same data and other works indicated that some behavioral manipulations may be more efficient than others at inducing ego depletion (Dang, 2016; Dang, Barker, et al., 2021). Moreover, some authors noted that the literature includes little or no evidence in net contrast with the ego depletion hypothesis (i.e., reporting that manipulations improved self-control; Friese et al., 2019). Overall, these observations may be taken to indicate ego depletion as lying along a spectrum in which the strength of the effect depends on the efficacy and/or duration of the applied manipulation.

Extended task practice leads to the appearance of slow electroencephalographic (EEG) waves in task-related areas (Hung et al., 2013; Bernardi et al., 2015; Quercia et al., 2018). These waves occur in the delta (1-4 Hz) and/or theta (4-8 Hz) frequency ranges and closely resemble the slow waves that characterize human non-rapid-eye-movement (NREM) sleep. In addition to increasing locally in task-related areas, sleep-like slow waves also increase globally following sleep restriction or deprivation (Hung et al., 2013; Bernardi et al., 2015). In animal models, these waves have been shown to reflect neuronal off-periods resembling those underlying the emergence of NREM slow waves (Vyazovskiy, Olcese, et al., 2011). Crucially, sleep-like slow waves are associated with performance impairment and loss of task focus when occurring in task-related areas (Bernardi et al., 2015; Thomas Andrillon et al., 2021). Given these observations, sleep-like slow waves during wakefulness have been suggested to represent a sign of neuronal (and cognitive) fatigue.

Interestingly, frontal brain areas involved in executive functions seem to be particularly vulnerable to fatigue and are among the first to show increases in the occurrence of local, sleep-like slow waves during extended wakefulness (Bernardi et al., 2015). In light of this, observations commonly attributed to the ego depletion phenomenon may actually reflect a form of task-dependent neural fatigue associated with sleep-like slow waves in frontal brain areas. Based on this view, the high variability in findings among previous ego-depletion studies could be due to the short duration of the tasks used in the literature, which may fail to induce sufficiently strong and consistent levels of functional fatigue in study participants.

In the present pre-registered research, we investigated whether an extended period of practice with tasks requiring decision-making and self-control could lead to local sleep-like activity within frontal areas and alterations in socially relevant behaviors, as measured by economic games. The employed tasks were selected from a previous work demonstrating their efficacy and specificity at inducing changes in self-control and frontal brain activity, in contrast to other tasks relying on different cognitive functions and brain areas (Hung et al., 2013; Bernardi et al., 2015). Here, we used two versions of the same tasks requiring or not the exertion of executive functions, to obtain different levels of efficacy in inducing task-specific slow waves and behavioral impairment.

In contrast to previous studies in the literature, we used longer fatigue-inducing tasks (~ 45 min vs. ~ 10 min). Furthermore, we collected brain activity (high-density EEG) measures that allowed us to identify the connection between ego depletion manipulations and changes in sleep-like activity. Finally, a battery of one-shot, anonymous, and incentivized economic games was administered to assess the potential effects of prolonged task practice on behavior. We conducted two studies on a total of 447 healthy adults who were assigned to one of two experimental conditions, the No Fatigue (NF) or Frontal Fatigue (FF) groups. Study 1 included both behavioral and EEG measures ($N = 44$) to assess changes in brain activity associated with the applied behavioral manipulation, while Study 2 replicated behavioral findings on a larger participant sample ($N = 403$).

3.2 Materials and Methods

3.2.1 Ethics Approval

The whole study was conducted under two protocols defined in accordance with the ethical standards of the 2013 Declaration of Helsinki. Study 1 protocol (n.1485/2017) was approved by the Ethical Committee Area Vasta Nord Ovest, while Study 2 protocol (n.03/2022) was approved by the Joint Ethical Committee for Research of the Scuola Normale Superiore and the Scuola Superiore Sant'Anna. A fixed reimbursement ($40 euros$ for Study 1 and $10 euros$ for Study 2) was provided to each participant upon completion of the experimental procedures. In addition, participants received a bonus payment relative to their performance in the incentivized economic games presented to them, as detailed below. The experimental procedures and expected results for both studies were pre-registered on AsPredicted (Study 1 and Study 2).

3.2.2 Participants

As explained in the pre-registrations of both studies, for Study 1, the sample size was pre-determined based on similar previous investigations (Quercia et al., 2018; Avvenuti et al., 2021), assessing the effect of task-related mental fatigue on behavioral performance. Based on these studies, we expected a large effect size (> 0.9). A power calculation based on this assumption, with two independent samples, $beta = 0.8$ and $alpha = 0.05$, led to a minimum sample size of 21 subjects per group. For Study 2, performed in less controlled experimental conditions, a smaller sample size was assumed. In particular, we estimated that a sample size of 400 subjects would allow us to identify medium-size effects of around 0.25 for the continuous variables (one tail) using a non-parametric Wilcoxon-Mann-Whitney test with two groups.

Participants were recruited through flyers, social media, and by word of mouth within the Tuscany areas of Lucca and Pisa for Study 1 and Florence for Study 2. Potential volunteers were pre-screened with an online questionnaire to exclude any medical, neurological, or psychiatric condition potentially affecting brain function and behavior. We also assessed the presence of relevant sleep-related issues (Pittsburgh Sleep Quality Index – PSQI (Buysse et al., 1989), cut-off score¡10) and extreme chronotypes (Horne-Östberg Morningness-Eveningness Questionnaire – MEQ (Horne and Ostberg, 1976), scores ranging between 30 and 70). Individual preferences were measured in the pre-screening questionnaire through the "Global Preferences Survey", designed to assess a wide range of personal characteristics, such as time preferences (e.g., short-term or long-term orientation), risk preferences, positive and negative reciprocity, altruism, and trust (Falk et al., 2018). The admitted participants were subsequently contacted to schedule the experimental sessions. All volunteers signed a written informed consent form before taking part in the studies and retained the faculty to drop from the study at any time.

Together with biological sex, age, and level of education, the indexes derived from the Global Preferences Survey (positive reciprocity, negative reciprocity, altruism, patience, risk, trust) were used to divide participants into matched groups according to the mean and variance of the variables (see Table S1): the Frontal Fatigue group (FF) and the No Fatigue group (NF).

3.2.3 Structure of the Session

The experimental procedures are detailed in Figure 1. All experiments took place in the morning, from 9-10 AM to 12 PM, and involved a single participant at a time for Study 1 and up to 20 simultaneous participants for Study 2. The time window of each session was kept fixed to avoid possible confounding factors related to time-of-day effects or the influence of daily activities (e.g., work-related fatigue). Subjects were instructed to keep their usual sleep schedule the night before the experiment and to abstain from caffeinated beverages in the morning.

Additionally, the day after the experimental session, all participants received an online set of self-report questionnaires by email, measuring impulsivity-related behaviors and beliefs (Barratt Impulsiveness Scale - BIS-11 Patton, Stanford, and Barratt, 1995), individual tendencies toward aggressive behavior (Buss-Perry Aggression Questionnaire – BPAQ Buss and Perry, 1992) and empathy (Interpersonal Reactivity Index – IRIDavis et al., 1980). Questionnaires were sent to the subjects the day after the experiment to avoid potential confounds related to priming effects or effects related to the different experimental protocols (FF, NF).

3.2.4 Test Block

Questionnaires

Participants were asked to self-report their mood, sleepiness, fatigue, and motivation using 10-point Likert scales and to complete the Positive Affect, Negative Affect Scale – PANAS (Watson, Clark, and Tellegen, 1988). The scores obtained from the questionnaires were compared across experimental conditions to identify potential effects induced by the different cognitive engagements in FF and NF (Appendix A Tables 2 to 7).

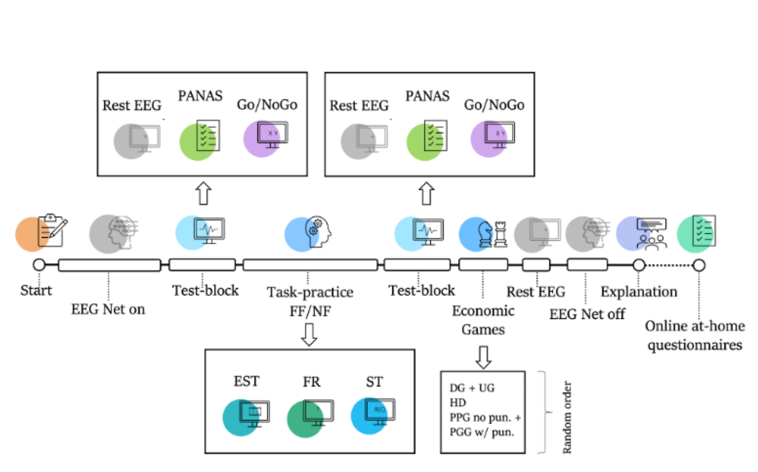

Figure 1: Visual articulation of an experimental session. Gray icons represent EEG-related operations present in Study 1 but not in Study 2. The length of the white bars reflects the relative length of each phase performed in the lab.

Rest EEG Recordings

Resting-state EEG activity was recorded for 6 minutes (500 Hz sampling frequency), divided into three epochs of two minutes each. Recordings were performed three times: once in the baseline test block (T0), then at the test block after task practice and before the economic games (T1), and finally at the end of the experimental session (T2).

Go/NoGo Task

During test blocks T0 and T1, participants also completed two 5-minute trials of a classical Go/NoGo task (Garavan, T. Ross, and Stein, 1999; Garavan, Thomas J Ross, et al., 2002; Roche et al., 2005; Chuah et al., 2006), as in a previous work Bernardi et al., 2015. Previous evidence indicated that extended exertion of self-control was associated with performance deterioration in this task. Here, we attempted to replicate this observation as a secondary aim of our study (see preregistration), as this would have been useful to demonstrate the efficacy of our experimental protocol in case no significant effects would emerge for the economic games. It is important to note, though, that our present study differs from previous investigations as a practice/learning session was not included to reduce experimental demands.

3.2.5 Task practice in FF and NF conditions

All participants completed three different tasks, each lasting about 15 minutes, for a total task practice time of $sim45$ minutes. In the FF condition, the tasks required high levels of impulse control, decision-making, and conflict resolution, while in the NF condition, a modified version requiring no exertion of self-control was presented (Avvenuti et al., 2021). The three tasks were always presented in the same order and included an emotion suppression task (Baumeister, Bratslavsky, et al., 2018), a false response task (Bernardi et al., 2015), and a classical Stroop task (Stroop, 1935). A detailed description of the tasks is provided below. The computerized tasks used in Study 1 were implemented using E-Prime 2 (Psychology Software Tools, Pittsburgh, PA), while a PsychoPy implementation (Peirce, Gray, et al., 2019; Peirce, Hirst, and MacAskill, 2022) was used for Study 2.

Emotion Suppression Task

In the emotion suppression task (Baumeister, Bratslavsky, et al., 2018; Dang, 2018), participants watched a series of brief video clips showing humans and/or animals in amusing situations. Participants in the FF group were explicitly requested to completely suppress their facial reactions, whereas volunteers in the NF group were left free to express their emotional responses. Compliance with the task was assessed using a camera pointing at the participant's face in Study 1 or through direct monitoring by the experimenters of the participant's computer stations in Study 2.

False Response Task

During the false response task (Bernardi et al., 2015), study participants were presented with 180 simple short questions (e.g., *What is the color of a banana?*) and two possible responses (e.g., *Yellow or Blue*). After a 300 ms delay, a green or a red sign appeared below the question, indicating whether the participant had to provide the correct or wrong response, respectively. For participants in the FF group, the circle's color was randomly assigned for each stimulus, while it remained always green for participants in the NF group. Participants were required to respond as fast and accurately as possible. Adherence to the task requirements was verified through an a-posteriori assessment of task accuracy (50% chance level). Since some of the responses in Study 2 were not correctly recorded due to a technical issue, only the results from Study 1 are reported. Specifically, participants achieved accuracies of 60.3±18.3% (SD) and 89.9±6.2% in the FF and NF conditions, respectively ($p < 0.001$, vs. chance level, t-test).

Stroop Task

In the Stroop Task, participants were presented with color names written with the same ink color indicated by the name or a different color (Stroop, 1935). Four trials, including two different versions of the task, were presented in alternating order: in one version, participants had to indicate the color name represented by the word (ignoring the ink color), while in the other version, they had to indicate the ink color (ignoring the color name). Each trial included 64 stimuli. Responses were provided using different response buttons associated with each of the four used colors, which were blue, red, yellow, and green. For participants in the FF group, the color name and the ink color could be either congruent (i.e., color ink and color name were matched) or incongruent (i.e., the color ink and the color name did not match), while for participants in the NF group, stimuli were always congruent (i.e., color ink and color name were always matched). Adherence to the task requirements was verified through an a-posteriori assessment of task accuracy (25% chance level). Since some of the responses in Study 2 were not correctly recorded due to a technical issue, only the results from Study 1 are reported. Specifically, participants achieved accuracies of 85.7±14.3% (SD) and 98.3±1.3% in the FF and NF conditions, respectively ($p < 0.001$, vs. chance level, t-test).

3.2.6 Economic Games

All subjects completed three blocks of economic games presented in random order: Block 1 included a Dictator Game followed by an Ultimatum Game, Block 2 included a Hawk and Dove Game, and Block 3 contained a Public Goods Game without Punishment, followed by a Public Goods Game with Punishment. Instructions for the games can be found in our online repository. Participants were informed that their counterparts in the economic games were other study participants and that the pairs or groups were changed for each game so that they would not interact with the same person twice. After each decision, participants were asked about their beliefs about others' average behavior for the whole experiment in the case of Study 1 or their own experimental session in the case of Study 2. One decision in the game and one belief were picked up at random at the end, and subjects were paid according to their earned points at the ratio of €1 for each experimental point, plus a bonus of two points if the chosen belief was close enough to the actual behavior. In addition to the fixed payoffs, participants received, on average, a game payoff of €13.17 in Study 1 and of €10.74 for Study 2. The economic games used in these experiments were implemented using oTree (D. L. Chen, Schonger, and Wickens, 2016).

Dictator Game

Participants were matched in pairs and received the role of either dictator or recipient. The dictator had to decide how to split 20 points between himself and the recipient. The recipient has no action to take and just obtains the points decided by the dictator. Participants could earn two bonus points if they guessed the average split with an accuracy within one point of the actual value.

Ultimatum Game

Participants were matched in pairs and received the role of either proposer or responder. The proposer had to make an offer regarding how to split 20 points between himself and the responder. The responder was given the choice of accepting or rejecting offers. In case the responder accepts the offer, the proposed distribution is implemented. In case the responder rejects the offer, both the proposer and the responder obtain zero points. The responder stated the minimum acceptance threshold that he was willing to accept, as done using the strategy method (Brandts and Charness, 2011). Participants could earn two bonus points if they guessed the average proposal with an accuracy within one point of the actual value. Moreover, participants could earn two bonus points if they guessed the average minimum acceptance threshold with an accuracy within one point of the actual value.

Hawk and Dove Game

Participants were matched in pairs and were both given the option of taking action either as Hawk or Dove. When both players chose Dove, they both received 10 points. When one player chose Hawk and the other chose Dove, the Hawk received 15 points, while the Dove received 5 points. If both participants chose Hawk, both received zero points. Actions were labeled as A and B. In Study 1, the action Dove was always action A, while in Study 2, the label was randomly assigned. Participants could earn two bonus points if they guessed the proportion of Dove plays with accuracy within five percentage points of the actual value.

Public Goods Game without Punishment

Participants were matched in groups of four. Each participant received a personal endowment of 10 points and was given the option to put any amount in a common pool, keeping the rest. Contributions to the common pool were then doubled and divided equally between all members of the group. Participants could earn two bonus points if they guessed the average minimum contribution with accuracy within half a point of the actual value.

Public Goods Game with Punishment

The procedure was similar to the Public Goods Game without punishment. However, in this case, participants were given the option to spend two points to reduce the earnings of another player by four points. This decision was presented using the strategy method to reduce feedback negativity. Therefore, participants decided, without receiving any feedback, whether they would like to punish the person who contributed the most ("antisocial punishment"), the person who contributed the least ("prosocial punishment"), one person at random ("spiteful punishment"), or not punish at all ("second-order free riding"). Participants could earn two bonus points if they guessed the average minimum contribution with accuracy within half a point of the actual value. Moreover, they earned two bonus points if they guessed both the most and least frequent modalities of punishment.

3.2.7 EEG data analysis

Following the procedure described in Avvenuti et al. (2021), the resting-state EEG recordings were band-pass filtered between 0.5 and 45 Hz (Kaiser-windowed finite impulse response filter; −0.01 dB passband gain, −40 dB stopband gain, 0.49 Hz roll-off) and divided into non-overlapping 4-second epochs (Avvenuti et al., 2021). The resulting traces were visually inspected to identify and mark bad channels and epochs containing clear artifactual activity. Then, an independent component analysis (ICA) was performed in EEGLAB (Delorme and Makeig, 2004) to remove signal components reflecting ocular, muscular, and electrocardiograph artifacts. Rejected bad channels were subsequently interpolated using spherical splines. Finally, for each EEG derivation, the signal was average-referenced. Power spectral density (PSD) estimates were computed using Welch's method (pwelch function) in each 4-second epoch (2-s Hamming windows, 50% overlap). Finally, as delta (0.5-4 Hz) and theta (4-8 Hz) frequency bands are knowingly associated with sleep states, signal power was computed for those bands and averaged across epochs.

3.2.8 Source-modeling of EEG data

Source modeling of EEG data was performed using Brainstorm (Tadel et al., 2011). Specifically, as reported in Avvenuti et al. (2021), the conductive head volume was modeled using a three-layer symmetric boundary element method (OpenMEEG BEM Gramfort et al., 2010) and the default ICBM152 anatomical template. The forward model was constructed using a standard set of electrode positions (GSN HydroCel 64). The source space was constrained to the cerebral cortex, which was modeled as a three-dimensional grid of 15,002 vertices. The inverse matrix was computed using the standardized low-resolution brain electromagnetic tomography (sLORETA) constraint with a regularization parameter equal to $10^{-2}\lambda$. Finally, the signal power was computed for each vertex in the source space using Welch's method (1-s Hamming windows, 50% overlap).

3.2.9 Scalp and source level analyses of EEG data

Comparisons of EEG signal power across experimental conditions were performed using paired t-tests and a permutation-based cluster-mass correction (Nichols and Holmes, 2002). In brief, each contrast was repeated (10,000 permutations) after shuffling the labels of the experimental conditions, and all clusters of significant electrodes were identified ($p < 0.05$). Then, we computed the sum of test statistics across electrodes belonging to the same cluster, and the maximum obtained value was saved in a frequency table. A minimum cluster-mass threshold corresponding to the 95^{th} percentile of the resulting distribution was applied to correct for multiple comparisons. For comparisons performed at the scalp level, the analyses were restricted to 51 "internal" electrodes in order to minimize the possible impact of residual artifactual activity in channels located near the eyes or on the temporal/neck muscles (Hung et al., 2013). All statistical analyses were performed in MATLAB. At the source level, power maps were exported to MATLAB, and statistical comparisons were performed as described for scalp analyses. The obtained results were re-imported in Brainstorm for visualization.

3.2.10 Analysis Models

Supplementary Information Tables from S3 to S9 show regression analyses for each game using the pooled database in which three groups of control variables are added. Specifically, the demographic variables group includes age, gender, and education; the preference variables group includes our proxies of positive reciprocity, negative reciprocity, altruism, patience, risk, and trust; and the questionnaire variables group includes the Interpersonal Reactivity Index, Barratt's Impulsiveness Scale, and the Buss-Perry Aggression Questionnaire. Specifically, for the Hawk and Dove Game choice, probit models were used. For the split in the Dictator Game, the offer and the minimum acceptance threshold in the

Ultimatum Game, and the contributions in the Public Goods Game with and without Punishment, tobit models were used. Finally, for the Punishment modalities in the Public Goods Game, probit models were used for the individual categories and a multinomial logit for the combined category.

3.2.11 Additional Analysis

We performed an additional analysis for better investigating and verifying the association between brain activity and decision-making changes. Therefore, we performed a new analysis as described here. First, for each participant and EEG recording, we computed the mean delta power (1-4 Hz) across frontal electrodes (see topographic map in Appendix A Figure 15). Then, we computed the average across T1 and T2, and the resulting values were used to compute the ratio with respect to baseline power levels (T0). Finally, we performed a regression analysis using all participants of Study 1 ($N = 44$) to predict behavioral results based on delta-power variations in frontal electrodes, clustering standard errors at the individual level. Our probit analysis showed a positive association between increases in frontal delta power and the likelihood of playing aggressively in the Hawk and Dove game ($p = 0.0408$).

3.3 Results

3.3.1 Delta and Theta EEG Activity

To determine whether the administration of the fatigue-inducing paradigm would lead to increases in low-frequency (delta/theta) activity, we compared the relative change in signal power induced by the experimental procedures (from T0 to T1) across the two groups (Figure 2a). The increase in delta activity (1-4Hz) was significantly stronger in the FF group compared to the NF group in a frontal cluster of electrodes (Figure 2b). Source localization showed that the delta power difference was maximal within the left inferior frontal gyrus (IFG) and in the right middle/superior temporal gyri ($p < 0.01$, cluster-based correction, Figure

3). Instead, T0-T1 theta activity (4-8Hz) changes did not show significant differences across groups. However, a stronger increase in the FF relative to the NF condition was found after the economic games in temporoparietal and occipital electrodes (T2, corrected $p < 0.05$, Figure 10). Source-modeling identified the peak of theta power increase in the right temporal lobe ($p < 0.01$, cluster-based correction, Figure 11).

3.3.2 Economic Games

The following game decisions were collected: Dictator Game split, Ultimatum Game proposer offer, Ultimatum Game minimum acceptance threshold (MAT), contribution to the Public Goods Game without punishment, contribution to the Public Goods Game with punishment, and punishment decision in the Public Goods Game with punishment. Analyses were performed employing non-parametric tests and regression analyses. The specific test or model was selected depending on the nature of the dependent variable (binary, ordered, categorical, roughly continuous). Moreover, we performed individual tests for prosocial, antisocial, and spiteful modalities of punishment against the reference category of second-order free riding.

Study 1

Figure 4 shows the results of the Hawk and Dove Game and the punishment modalities in the Public Goods Game. The proportion of dovish behavior (i.e., peaceful cooperation) dropped from 86% to 41% ($p = 0.0017$, Pearson's chi-squared test), which is consistent with a tendency of the FF group to engage more often in hawkish behavior (i.e., aggressive seizing of resources). This result was robust to the use of a probit model, $p = 0.0020$. Of note, an additional probit analysis showed that delta-power variations from T0 to T1 in frontal electrodes (see Appendix A Figure 15) were positively associated with the likelihood of playing aggressively in the Hawk and Dove game ($p = 0.0408$; standard errors clustered at the individual level). No other significant result was found in the rest of the games (Figure 12). We performed a total of ten tests. Hence,

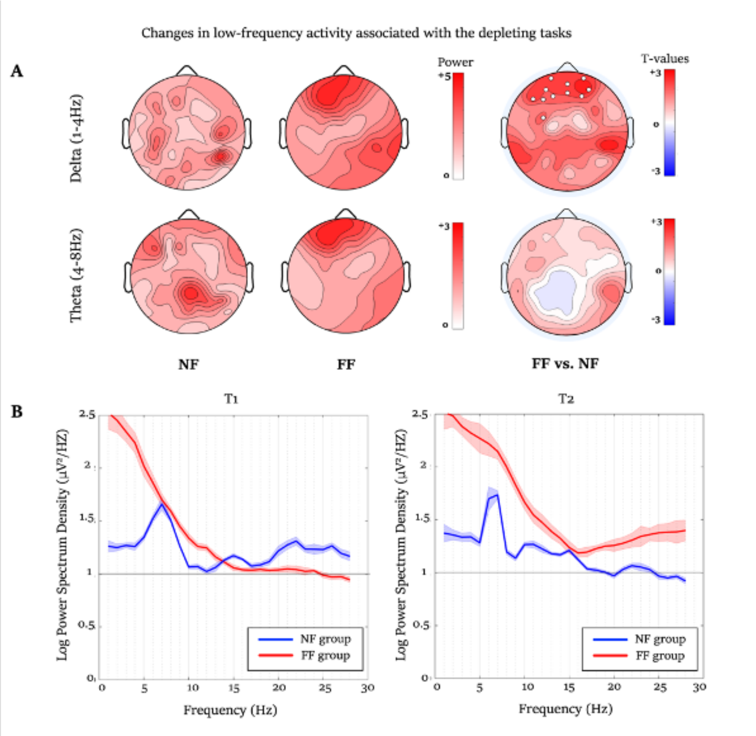

Figure 2: Changes in the low-frequency activity associated with the FF condition. Panel A shows low-frequency EEG activity changes associated with the FF and NF experimental conditions. Topographic plots on the left show T0 to T1 changes (ratio) in delta (top) and theta (bottom) power for the two experimental conditions. Topographic plots on the right show the statistical comparison between experimental conditions for the two frequency bands. White dots mark $p < 0.05$, cluster-based correction. Panel B shows power spectral density (PSD) changes computed in the significant frontal cluster highlighted in Panel A. The left plot shows the change in PSD in the NF group (blue) and in the FF group (red) after T1, while the right plot shows the PSD difference after subjects performed the economic games (T2).

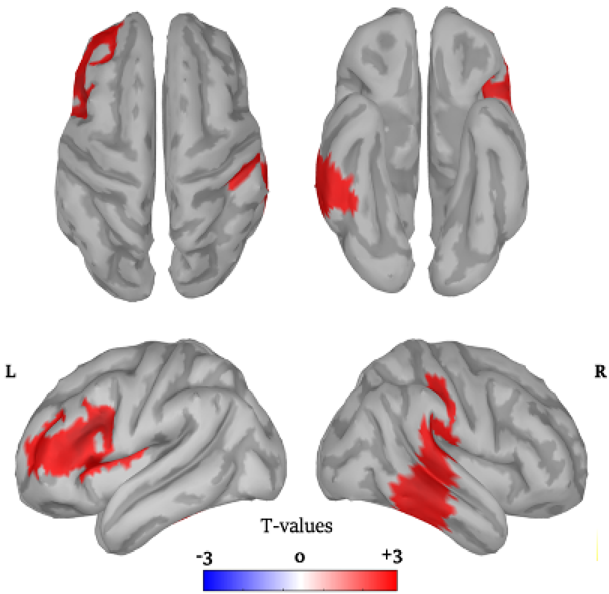

Figure 3: Source analysis of the delta EEG activity associated with the FF condition. Significant differences ($p < 0.01$, cluster-based correction) in T1/T0 delta ratios between FF and NF. Specifically, difference peaks were found in the left IFG (-36, 16, 28) and in the right Middle Temporal Lobe (70, -30, -2).

our results were robust to concerns regarding multiple testing and would survive any suitable correction. We did not find any results regarding differences in beliefs between groups (Figure 7, and a diff-in-diff analysis did not show differences in the average accuracy and response times of subjects in the Go/NoGo task (Figure 6a).

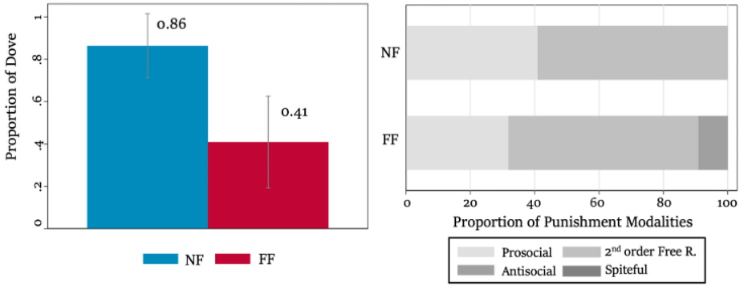

Figure 4: Results for the Hawk-Dove game and the punishment modalities for Study 1 (N= 44). The left panel shows the proportion of players choosing the peaceful option in the Hawk and Dove game. Players of the NF group (blue) chose the dove alternative 86% of the time, while players of the FF group (red) chose the same alternative significantly fewer times (41%, $p = 0.0017$). The right panel shows the distribution of punishment modalities between NF and FF groups. Prosocial punishment and 2nd order-free riding are the two most represented modalities in the NF group, while antisocial punishment is present only in the FF group. None of the players selected the option of spiteful punishment, which is absent in the graph.

Study 2

Figure 5 shows the results of the Hawk and Dove Game and the punishment modalities in the Public Goods Game. Consistent with the results of Study 1, behavioral choices significantly differed between the NF and FF groups ($p < 0.001$, Person's chi-squared test). The result also survived the use of a probit model ($p < 0.001$). Concerning punishment, Pearson's chi-squared test showed significant differences in the distribution of punishment modalities ($p < 0.001$). Specifically, pairwise Pearson chi-squared tests showed a significant reduction in prosocial punishment ($p < 0.001$) and a significant increase in spiteful punishment ($p = 0.0019$), while total punishment was unchanged ($p = 0.2961$). These results also survived the use of probit models ($p < 0.001$ and $p = 0.0022$, respectively) and a multinomial logit (Table 30). No other significant result was found in the remaining games (Figure 13). Diff-in-diff regressions did not show any difference in subjects' average accuracy and response times in the Go/NoGo task (Figure 6b).

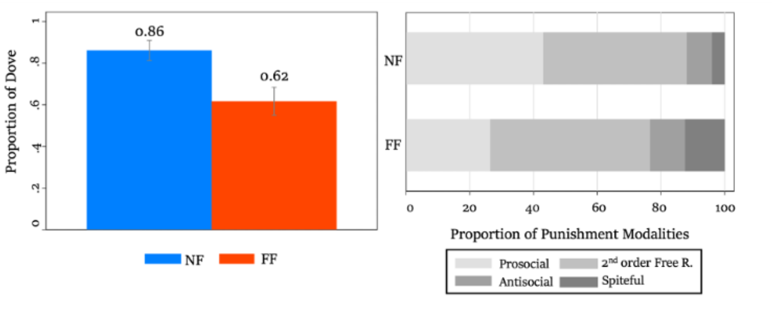

Figure 5: Results for the Hawk-Dove game and the punishment modalities for Study 2 (N= 403). The left panel shows the proportion of players choosing the peaceful option in the Hawk and Dove game. Players of the NF group (blue) chose the dove alternative 86% of the time, while players of the FF group (red) chose the same alternative significantly fewer times (62%, $p < 0.001$). The right panel shows the distribution of punishment modalities between NF and FF groups. Prosocial punishment and 2nd-order free riding are the two most represented modalities in the NF group, followed to a lesser extent by antisocial and spiteful punishment. The latter is instead more represented in the FF group, which also shows a reduction in the selection of prosocial punishment.

3.4 Discussion

Moral precepts frequently compel individuals to engage in actions seemingly detrimental to their immediate self-interests, including core objectives like survival and reproduction. Yet, actions that benefit others have been suggested to enhance survival and reproduction through indirect means, such as integration into social groups and cultural societies (Rand and Nowak, 2013; Baumeister and Alghamdi, 2015; Sigmund, 2010; Trivers, 1971). Nevertheless, fitness is also achieved through aggressive acts, which provide the individual with immediate benefits that are often able to outweigh the long-term consequences that such behaviors might hold (Georgiev et al., 2013), explaining why aggression is a basic form of human interaction (Rusch and Gavrilets, 2020).

Aggressive behavior plays a pivotal role in the animal kingdom, and research demonstrated that, in humans, it is modulated by distinct brain structures involved in emotion regulation and inhibitory control, and its impromptu expression depends on failures of self-regulation processes (Davidson, Putnam, and Larson, 2000; Pietrini et al., 2000). Consequently, self-control, functioning as the cognitive mechanism that facilitates the inhibition of instinctual responses in favor of actions endowed with a greater value, is intimately related to aggressive behavior and punishment activities.

Here, we demonstrated that the extended exertion of self-control leads to an increased propensity to engage in aggressive choices and a shift in punishment behavior. Furthermore, individuals who engaged in cognitive tasks requiring self-control presented an increase in sleep-like (delta) EEG activity within frontal areas involved in executive functions. Such a change is consistent with a use-dependent increase in local functional fatigue and may thus explain behavioral changes as reflecting the loss of efficient top-down executive regulation.

3.4.1 Extended exertion of self-control affects subsequent socially relevant choices

Previous research suggested that humans possess a finite reserve of self-regulatory resources, which may be depleted with extended use, a phenomenon defined as ego depletion (Baumeister, Bratslavsky, et al., 2018). According to this view, experimental manipulations expected to induce ego depletion have been widely employed to investigate the role of self-control in socially relevant human behaviors. However, research produced inconsistent results. Indeed, some studies showed that when individuals experience cognitive fatigue due to prior self-regulation, they exhibit diminished empathy, reduced willingness to engage in prosocial acts, and a lowered threshold for frustration and aggression control in social interactions (Muraven, Pogarsky, and Shmueli, 2006; DeWall et al., 2007; Stucke and Baumeister, 2006). Other studies, though, failed to detect any significant effects of ego depletion on social behavior (Achtziger, Alós-Ferrer, and Wagner, 2018). Here, we propose that a potential explanation for those discrepancies could lie in the relatively short duration of typical ego depletion paradigms, which may be insufficient to induce measurable effects in most individuals. In line with our hypothesis, we found that about 45 minutes of practice with tasks requiring self-control led to consistent behavioral changes in two independent samples studied using similar experimental paradigms and procedures. Specifically, individuals who completed tasks requiring extended exertion of self-control showed a higher propensity to fight in the Hawk and Dove game relative to those who engaged in similar tasks not requiring self-control. Thus, when confronted with a situation in which agents can decide either to resolve a conflicting situation peacefully or aggressively, depleted individuals are more likely to choose the aggressive alternative. In addition, fatigued participants showed a lower probability of selecting the option regarding prosocial punishment, that is, the act of punishing defectors, and an increased propensity in choosing to punish spitefully, that is, punishing another player at random. Nonetheless, the overall propensity for punishment remained unchanged. Interestingly, prosocial punishment has been suggested to represent one of the potential mechanisms for the evolution of cooperation (Boyd, Gintis, et al., 2003; Bowles and Gintis, 2004). According to this idea, one might speculate that differences in the distribution of punishment modalities across different social groups of individuals (Herrmann, Thoni, and Gachter, 2008) could be due to differences in vulnerability to task-dependent cognitive fatigue. However, studies about potential differences in the vulnerability to (frontal) local sleep across individuals or populations are currently missing. Overall, in line with previous research (Wiehler et al., 2022; Blain, Hollard, and Pessiglione, 2016), our results indicate that the tendency of individuals to engage in aggressive-like behaviors may increase following the extended use of self-control resources.

3.4.2 Local changes in brain fatigue and sleep-need may underlie the ego depletion effect

A growing body of evidence demonstrated that contrary to long-held assumptions, sleep and wakefulness are not mutually exclusive - instead, they are locally regulated and may often coexist across distinct brain areas. During wakefulness, increases in the number and magnitude of local, sleep-like episodes characterized by slow (delta/theta) EEG waves have been reported following extended task practice and prolonged wakefulness with sleep restriction or deprivation (Hung et al., 2013; Bernardi et al., 2015; Quercia et al., 2018; Avvenuti et al., 2021; Andrillon et al., 2022). A local, use-dependent increase in the number of sleep-like episodes has been observed after prolonged practice with a variety of tasks, including self-control, visuospatial, and motor tasks (Hung et al., 2013; Bernardi et al., 2015; Quercia et al., 2018). Importantly, sleep-like episodes appear to have a negative impact on behavioral performance when they occur during specific tasks and involve task-related brain areas (Vyazovskiy, Olcese, et al., 2011). In light of these considerations, sleep-like episodes observed during wakefulness have been suggested to represent the signature of use-dependent neuronal fatigue and sleep needs (Vyazovskiy and Harris, 2013). Therefore, we reasoned that frontal sleep-like episodes could offer a physiological explanation for behavioral changes observed after the exertion of self-control. Consistent with this, our present results showed that extended practice with tasks requiring self-control, but not practice with similar tasks not requiring self-control, is associated with a frontal increase in low-frequency, sleep-like EEG activity. A source modeling analysis confirmed that changes in sleep-like activity involved key areas strongly associated with the exertion of self-control, such as the left inferior frontal gyrus and the right middle temporal cortex (Mitchell, 2011; Brown et al., 2012).

Given these premises, we hypothesized that the effects attributed to ego depletion might actually depend on a use-dependent increase in the incidence of sleep-like episodes involving frontal brain areas associated with executive functions. Notably, though, the present findings did not allow us to demonstrate a direct causal link between sleep-like activity and decision-making, which should be the object of future research.

3.4.3 Limitations of the Study

Some limitations of the study should be acknowledged. The behavioral manipulation adopted in the present work was based on previous investigations showing that specific tasks induce changes in local brain activity and behavior consistent with self-control-related fatigue (Bernardi et al., 2015). However, we cannot entirely rule out the potential contribution of a general (non-task-specific) mental fatigue related to cognitive demands, as we did not include a control condition requiring similar cognitive efforts but distinct cognitive functions. Notably, though, no effects of the experimental condition (FF vs. NF) or interactions between condition and time (before vs. after task practice) were found for self-reported sleepiness, fatigue, mood and positive/negative affects (SI Table S2). A trend interaction was observed only for self-reported motivation but this effect did not survive correction for multiple testing ($p < 0.05$, uncorrected). Given these results, a non-specific effect of fatigue appears unlikely to explain the behavioral differences observed in our study. Moreover, while we tested several economic games, we found clear significant effects of task-dependent fatigue only for the Hawk and Dove game. The reason why choices in the other games, such as the Dictator or Ultimatum games, were not affected by the behavioral manipulation is unclear and in partial contrast with previous observations (Achtziger, Alós-Ferrer, and Wagner, 2015; Achtziger, Alós-Ferrer, and Wagner, 2016; Achtziger, Alós-Ferrer, and Wagner, 2018). Nevertheless, our present results were robust to corrections accounting for multiple testing and were replicated across two independent samples. We suggest that choices in distinct economic games may be differentially affected depending on the decision-making processes they reflect and the behavioral manipulation that is applied. The structure and complexity of the game may also play a role. For instance, the relative simplicity and clear-cut nature of the Hawk and Dove game may have facilitated the detection of cognitive alterations, which in contrast could be influenced by a variety of other mechanisms in more complex tasks (Mok and De Cremer, 2018).

Furthermore, while extensive research has been conducted on 'short' ego depletion and repeated games (Achtziger, Alós-Ferrer, and Wagner, 2015), our experimental protocol adopted a longer depletion strategy and relied exclusively on one-shot games. Exploring the interplay between ego depletion resulting from extended exertion of self-control and repeated interactions could unravel if, and to which extent, compensatory mechanisms could come into play to modulate individual choices that impact social life. Finally, the present study did not detect any differences in Go/NoGo performance across experimental conditions. We hypothesize that this could depend on the fact that a practice/learning period with the task was not included in the present study to reduce experimental demands. Therefore, learning and fatigue might have counteracted each other during the task, resulting in smaller and non-significant performance variation differences across experimental groups.

3.4.4 Conclusions

In conclusion, our study provides novel insights into the intricate relationship between self-control exertion, social behavior, and underlying neural mechanisms. We have demonstrated that extended exertion of self-control can lead to changes in neural activity within frontal cortical areas crucial for behavioral control and result in an increased propensity for aggressive choices in social interactions. This finding highlights the potential depletion of self-regulatory resources and its impact on prosocial behavior, shedding new light on the inconsistent results reported to date in ego depletion research. Indeed, our results support the idea of ego depletion as a spectrum in which the strength of the effect depends on the intensity and/or duration of the prior manipulation, as well as the nature of the behavior being tested. Moreover, our investigation into local changes in brain fatigue and sleep-like activity offers a novel perspective on the ego depletion phenomenon. Thus, we propose that the increased sleep-like activity in frontal brain areas associated with executive functions and self-control underlie the observed behavioral changes. This use-dependent neuronal fatigue may not only provide a physiological explanation for the effects of ego depletion but, for extension, also of the previously described effects of sleep deprivation and restriction on societally impactful behaviors (Dorrian et al., 2019). The paradigm shifts from sleep as a global phenomenon to a locally regulated one occurred in relatively recent years, and its full range of implications for our understanding of human behavior is far from being exhaustively explored. While this new perspective can offer a single and simple physiological explanation to a wide range of behavioral observations, future research will be necessary to understand the complexities of the interactions between local sleep regulation and other factors influencing human behavior.

Chapter 4

Community enforcement can help sustain cooperation even in the presence of antisocial punishment

4.1 Introduction

The evolution of cooperation is a theoretical and empirical puzzle that has driven the interests of different branches of research for decades. The intuitive reason is straightforward since a cooperator is always at risk of suffering defection from the cooperation of others, which in game theoretical terms means that a one-shot anonymous interaction dominates the cooperative action. Several explanations have been proposed in order to explain the substantial amount of cooperation that humans actually show in reality (see Rand and Nowak, 2013, for a review), and one of the most popular mechanisms that are thought to help sustain the evolution of cooperation is punishment. Punishment, that is, the act of imposing an undesirable outcome upon an individual (or a group), has been used extensively as a deterrent to norm behaviors. For example, prosocial punishment, which amounts to take a costly action in order to reduce the fitness of defectors or free riders, has being shown to be able to induce cooperation in experimental and theoretical settings (Boyd and Richerson, 1992; Boyd, Gintis, et al., 2003; Nakamaru and Iwasa, 2005). One common critique of punishment mechanisms' ability to sustain cooperation is the existence of a less pervasive but darker phenomenon: antisocial punishment. Antisocial punishment refers to a costly action that reduces the payoffs of cooperators, thus imposing an additional burden on those who decide to pay to increase the benefits of others. Moreover, experimental evidence shows that, although far from ubiquitous, and with wide heterogeneity across societies, antisocial punishment is indeed observed in humans (Herrmann, Thoni, and Gachter, 2008; Stavrova, Schlösser, and Fetchenhauer, 2013; Thöni, 2014; Bruhin, Janizzi, and Thöni, 2020). Finally, we will refer to spiteful punishment as a strategy that punishes both cooperators and defectors.

The coevolution of cooperation and punishment has been shown to be threatened when considering the presence of antisocial and spiteful modalities of punishment. Rand, Armao IV, et al., 2010, indeed, find that extending the strategy space so that punishment can be exerted in any direction, towards cooperators, defectors, or everybody, prevents cooperation from being evolutionary stable. Several attempts have been proposed to robustify the evolution of cooperation in the presence of antisocial punishment (see Section 4.2 below), with mixed but promising results. Ultimately, the issue is far from being closed, which prompts the questions, how can we reconcile the evolution of cooperation with the existence of antisocial punishment? How can we explain the huge variability in punishment attitudes? We propose that the answer might be found by taking tools from the community enforcement literature.

The community enforcement literature has studied that the space of options that can be supported as an equilibrium is enlarged by assigning labels to different strategies (Kandori, 1992). Recent papers have further studied the particular advantages that can arise when those labels are assigned using the modal actions and strategies as benchmark (Jindani, 2020). In this paper, we show that a community enforcement system that assigns a positive label to the modal strategy (the social norm) supports several combinations of both pro-norm and anti-norm cooperation and pro-norm and anti-norm punishment as evolutionary stable and, with additional parameter conditions, stochastically stable. In our model, on each period, a population is randomly matched in pairs to play a one-shot Prisonner's Dilemma in which the only information that the agents have is the "label" of their opponent. Specifically, modal strategies are assigned a green color, and non-modal strategies are assigned a red color. Upon observing the color or their match, players simultaneously decide whether to cooperate and/or punish their couples. After that, a fixed fraction of agents is randomly chosen for a revision opportunity, in which they can update their strategy based on a myopic best reply protocol. This is, they update their strategy towards the action that maximizes their fitness given the current distribution of strategies being played in the population. We stress that the information requirements for our color system are less stringent than those needed to myopically best reply. Hence, if you have the information necessary to update the strategy, you must observe the colors of all the strategies. After the strategies are updated, the colors assigned to each of them are reassigned in the case of changes in modality.

One thing must be noticed in order to properly interpret our results, which is the fact that, although the punishment behavior in any given pair of individuals, their strategies, and the color of their strategies, can be characterized as either prosocial, antisocial or spiteful, the strategies per se cannot be labeled on those terms ex-ante. In fact, we should refer to strategies in terms of pro- or anti-norm cooperation and punishment since the behavior prescribed by any strategy is conditional to the color of the others and not the actions of the others. For example, a strategy that punishes non-modal red strategies could engage in antisocial punishment whenever an individual using it is paired with an individual using a cooperative strategy that is, however, red at the moment. Since the colors of the strategies are state-dependent, so there are the payoffs arising from any given pair of subjects being formed, and hence, so it is our interpretation of the resulting punishment behavior as either prosocial, antisocial, or spiteful.

Using the stochastic stability techniques developed in Foster and Young, 1990, P. Young, 1993, Kandori, Mailath, and Rob, 1993 and Ellison, 2000, our results show that coexistence (in the sense of simultaneously stochastically stable) of different strategies of cooperation and punishment is possible. However, each of those coexisting norms has itself a basin of attraction big enough to make it very difficult to exit it once they are reached. The intuition for this seemingly paradoxical outcome, when we consider the existing results about selection and concurrence of equilibria, is actually straightforward. Once a strategy is modal and therefore green, we are very resilient to the non-modal red strategies, and hence you have a big basin of attraction, but it does not mean that the same strategy is strong whenever it is non-modal and red and another strategy is modal and green, thus creating room for coexistence. In terms of parameters, the results are also natural, with greater benefits of cooperation favoring pro-norm cooperation, i.e., cooperating with green strategies, which is just conditional cooperation, and greater differences between the cost of punishment and the harm inflicted favoring pro-norm punishment, i.e., punishing red strategies. At the same time, anti-norm and unconditional cooperation cannot be stochastically stable unless paired with pro-norm punishment, and anti-norm and unconditional (spiteful) punishment cannot be stochastically stable unless paired with pro-norm cooperation.

The consequences of the model are useful as long as we are able to accept the (we believe fair) assumption that individuals, although heterogeneous in behavior, more or less understand the modal behavior in their own society. If we drop this assumption, we should drop subjects' ability to myopically best reply, which would be problematic for many other conclusions in Evolutionary Game Theory.

Overall, the results help us better understand several mechanisms for the evolution of cooperation. First, the coevolution of cooperation and punishment is possible even in the presence of antisocial punishment. Second, the heterogeneity in norms about punishment observed across societies or groups is rationalized in our model by just looking at the coexistence of different strategies with different punishment prescriptions as stochastically stable, with the parameters of the game likely also different across groups, still having a lot to say in which norms will never be stochastically stable. Third, even in cooperative societies with well-established norms about prosocial punishment, a small amount of antisocial and spiteful punishment will be observed whenever small groups of mutants that follow cooperative but red strategies appear. For example, we could observe a conditional cooperator punishing an unconditional cooperator, rightfully labeling the behavior as antisocial punishment without realizing that it is just an instance of pro-norm punishment. All three of our main insights are achieved not by redefining the strategy space of the revision protocol but simply by introducing a state-dependent payoff rule for any given interaction, which, in our cases, is instrumentalized by the community enforcement mechanism.

The rest of the paper is structured as follows. Section 4.2 reviews the relevant literature, Section 4.3 outlines our model, Section 4.4 describes the results, and finally Section 4.5 discusses the results and concludes.

4.2 Related Literature

To begin with, we need to establish a picture of the evolutionary debate on antisocial punishment and its ability to hinder the evolution of cooperation. Rand, Armao IV, et al., 2010 argue that cooperation is no longer evolutionary stable whenever you allow punishment to be applied through all the strategy set, i.e., both towards cooperators and defectors. This result is extended in Rand and Nowak, 2011, where optional public games in which there is an outside option that allows the player to not engage in the cooperative stage of the game is added. Garcia and Traulsen, 2012 answer to this by argumenting that "leaving the loners alone", in the sense that those players who take the outside option cannot punish or be punished, restores the effectiveness of prosocial punishment in promoting cooperation, although this is no longer granted when mutation rates are high, as in Hauser, Nowak, and Rand, 2014. Furthermore, Hilbe and Traulsen, 2012 shows that relaxing anonymity can also help to dilute the negative force of antisocial punishment in a model that differs from ours, since in our case, anonymity is not relaxed at the individual level but at the strategy one, being the modal social norm the only thing that matters. Finally, our label of *antinorm cooperation* closely resembles the mechanism of Santos, 2015, in which defectors can provide benefits to be shared only between themselves. In contrast, Fulker et al., 2021 shows that as long as the interaction is structured in a dynamic network, spite behavior can be contagious, which is not allowed in our model since strategies that favored red color players will no longer be effective once modality is reached and the labels change.

The second crucial ingredient in our model is community enforcement, as pioneered by Kandori, 1992, which proves that, even in situations where players only have information about their own past, a community can sustain cooperation. These original ideas were extended in Okuno-Fujiwara and Postlewaite, 1995, which introduced the concept of *norm equilibrium* in the context of random matching games, and in Jindani, 2020, which shows that the strong information requirements of community enforcement can be relaxed by considering modal actions and multiple random matching. Finally, Acemoglu and Wolitzky, 2020 propose a mechanism in which enforcement is carried on by a sub- population of specialized enforcers, an idea that closely resembles that of Lee, Cleveland, and Szolnoki, 2022. Parallely, an early application of community enforcement to the evolutionary literature was developed by Johnson, Levine, and Pesendorfer, 2001, in which they use a color system of labels in an overlapping generations model in which different iterations of individuals play a Gift-Exchange Game. However, the most recent contribution that put the focus on antisocial punishment is Szolnoki and Perc, 2017, in which cooperation is proven to be restored when a community enforcement mechanism based on the second-order free-riding on antisocial punishers is introduced.

It is fair to recognize that our model is intimately related to both the literatures on reputation and social norms, provided they relate to the coevolution of cooperation and punishment. After the already mentioned seminal paper of Boyd and Richerson, 1992, their original ideas are extended in Boyd, Gintis, et al., 2003 using a simulation-based approach. Although these contributions are based on a random matching protocol, an earlier reputation mechanism based on reducing anonymity by using a lattice structure to enhance the evolution of cooperation is conceived in Nakamaru, Matsuda, and Iwasa, 1997. The crucial interactions between social norms, reputation and punishment would later be extended in Ohtsuki and Iwasa, 2004, Nakamaru and Iwasa, 2005, Ohtsuki and Iwasa, 2006, and Nakamaru and Iwasa, 2006, in which many mechanisms are explored, but all with specific focus in prosocial punishment, with the exception of the later reference that starts to identify punishment as a type of spiteful behavior in selfish agents. Finally, the reputational spillovers for the evolution of cooperation have also being explored experimentally, as in Barclay, 2006.

4.3 Model

Time is discrete, $t = 1, 2, ...$ On every period, a finite but large population $N = (1, 2, ...n)$ of agents is matched in uniform random pairwise interactions, with the probability of any agent i being matched with agent $j \neq i \in N$ being $\frac{1}{n-1}$. In every period, agents choose a strategy $s \in S$, and we denote the payoff function $\pi\colon S \times S \mapsto \mathbb{R}$ that specifies the payoff of an agent playing strategy s against an agent playing strategy s' as $\pi(s, s')$. The game a pair plays is developed in two simultaneous stages. In the first stage, each agent i takes a binary action $a_i \in \{0, 1\}$, where

1 stands for "cooperate" and 0 stands for "defect". Cooperation provides the opponent a benefit b at a cost c, with $b > c$, while defection has no payoff effects. At the same time, each agent takes a binary action $p_i \in \{0, 1\}$, where 1 stands for "punish" and 0 stands for "not punish". Punishment creates a cost γ on the opponent, and its exertion costs α, while the absence of punishment does not have any payoff effects.

The strategies of agents are transmitted using an information system that assigns a color to a strategy. Formally, an information system ι takes the vector S of possible strategies in a population and returns a vector C that assigns a color c to each strategy, $\iota\colon S \mapsto C$. An information system $\iota \in I$ defines a flag function f that assigns a color to each player. In our setting, players who adopt the modal (or modals) strategy receive the color green, and players who adopt any non-modal strategy receive the color red. Denoting $\hat{s}$ as a vector that assigns a value of 1 to a modal strategy and a value of 0 to a non-modal strategy, our flag function for individual i is a mapping $f_i\colon s_i \times \hat{s} \mapsto \{g, r\}$. This allows us to redefine payoffs as $\pi_{c,c'}(s, s')$, where c is the color of strategy s and c' is the color of strategy s'.

A strategy $s_i \in S$ is a vector that comprises four binary actions: if the agent cooperates with green, $a^G \in \{0, 1\}$, if the agent cooperates with red, $a^R \in \{0, 1\}$, if the agent punishes green, $p^G \in \{0, 1\}$ and if the agent punishes red $p^R \in \{0, 1\}$. Therefore, a strategy is a vector $s_i = (a^G, a^R, p^G, p^R)$, with the set S comprising 16 strategies.

Define a state at time t as the matrix $z_t = (s_t, f_t) = (a_t^G, a_t^R, p_t^G, p_t^R, f_t)$, where $a_t^G = (a_{1,t}^G, ..., a_{n,t}^G)$, $a_t^R = (a_{1,t}^R, ..., a_{n,t}^R)$, $p_t^R = (p_{1,t}^R, ..., p_{n,t}^R)$, $p_t^R = (p_{1,t}^R, ..., p_{n,t}^R)$ and $f_t = (f_{1,t}, ..., f_{n,t})$.

Since we adopt an evolutionary perspective, we consider an agent's strategy as the phenotype of such an agent. After every period, each agent has a fixed probability for a revision opportunity, $\rho \in (0, 1)$, and we assume that the agents revise their strategies by maximizing their payoff according to the current state z_t. In addition, this myopic best reply rule is implemented with a probability $1 - \varepsilon$, while with a small probability $\epsilon > 0$ the agent mutates and chooses a strategy uniformly at random from the strategy space.

4.4 Results

When considering the evolutionary stability of strategies, we must notice that any incumbent strategy will have the color green, while any mutant strategy will have the color red. The system has three parameter-independent evolutionary stable strategies, $(1, 0, 0, 0)$, $(1, 0, 0, 1)$ and $(0, 0, 0, 1)$.

For $(1, 0, 0, 0)$, notice that,

$$\pi_{g,g}[(1, 0, 0, 0), (1, 0, 0, 0)] = b - c,$$

$$\pi_{r,g}[s \in S \setminus (1, 0, 0, 0), (1, 0, 0, 0)] \in [-c - \alpha, 0]$$

For $(1, 0, 0, 1)$,

$$\pi_{g,g}[(1, 0, 0, 1), (1, 0, 0, 1)] = b - c,$$

$$\pi_{r,g}[s \in S \setminus (1, 0, 0, 1), (1, 0, 0, 1)] \in [-c - \alpha - \gamma, -\gamma]$$

Finally, for $(0, 0, 0, 1)$,

$$\pi_{g,g}[(0, 0, 0, 1), (0, 0, 0, 1)] = 0,$$

$$\pi_{r,g}[s \in S \setminus (0, 0, 0, 1), (0, 0, 0, 1)] \in [-c - \alpha - \gamma, -\gamma]$$

Other four strategies are evolutionary stable depending on parameter values.

Specifically, for $(1,0,1,0)$,

$$\pi_{g,g}[(1,0,1,0),(1,0,1,0)] = b - c - \alpha - \gamma,$$

$$\pi_{r,g}[s \in S \setminus (1,0,1,0),(1,0,1,0)] \in [-c-\alpha, 0]$$

And hence $(1,0,1,0)$ is evolutionary stable if $b > c + \alpha + \gamma$.

For $(1,1,0,1)$,

$$\pi_{g,g}[(1,1,0,1),(1,1,0,1)] = b - c,$$

$$\pi_{r,g}[s \in S \setminus (1,1,0,1),(1,1,0,1)] \in [b-c-\alpha-\gamma, b-\gamma]$$

And hence $(1,1,0,1)$ is evolutionary stable if $\gamma > c$.

For $(0,1,0,1)$,

$$\pi_{g,g}[(0,1,0,1),(0,1,0,1)] = 0,$$

$$\pi_{r,g}[s \in S \setminus (0,1,0,1),(0,1,0,1)] \in [b-c-\alpha-\gamma, b-\gamma]$$

And hence $(0,1,0,1)$ is evolutionary stable if $\gamma > b$.

Finally, for $(1,0,1,1)$,

$$\pi_{g,g}[(1,0,1,1),(1,0,1,1)] = b - c - \alpha - \gamma,$$

$$\pi_{r,g}[s \in S \setminus (1,0,1,1),(1,0,1,1)] \in [-c-\alpha-\gamma, -\gamma]$$

And hence $(1,0,1,1)$ is evolutionary stable if $b > c + \alpha$.

There are no other evolutionary stable strategies. To see this, it is enough to realize that $(1,1,0,0)$, $(0,1,0,0)$, $(0,1,1,0)$, $(0,0,1,0)$, $(0,1,1,1)$ and $(0,0,1,1)$ can be invaded at least by $(0,0,0,0)$, $(1,1,1,0)$ and $(1,1,1,1)$ can be invaded at least by $(1,1,0,0)$, and $(0,0,0,0)$ can be invaded at least by $(0,1,0,0)$.

These evolutionary stable strategies, always depending on the relevant parameters, coincide with the absorbing states of the system in which each strategy is played by all the agents in the population, creating a situation in which only monomorphic absorbing states emerge. To see this, it is enough to observe than whenever the system is in one of these states, an agent that receives a revision opportunity myopic best replies by not changing strategy, and therefore in the absence of mistakes the system has zero probability of changing state. We denote the set of absorbing states under parameter values $\sigma = \{b, c, \alpha, \gamma\}$ as $\mathcal{A}_\sigma$.

It is worth noticing that the fact that a red strategy can never be part of a monomorphic equilibrium, since eventually it would acquire modality, transitioning into color green and changing payoffs, implies that there is a theoretical upper bound for the proportion of mistakes that an absorbing state can support, which coincides with the moment in which a strategy loses modality, and hence, it is crucial to evaluate the resilience of absorbing states whenever the proportion of mistakes approaches $\frac{n}{2}$.

We need to establish a few concepts before starting to unravel the results. To establish the possible stochastic stability of our candidate absorbing states, we need to evaluate how vulnerable they are to transition to any other absorbing state after a string of mistakes or mutations. Denote the proportion in which an incumbent strategy s^I is played in state z as p. Incumbency implies that the strategy is modal and therefore any agent of this type has flag $f^I = g$. The average fitness attained by strategy s^I in state z is:

$$\Pi(s^I, s^{-I}, z) = p\pi(s^I, s^I) + \sum_{s^{-I} \neq s^I} \Psi(z)\pi(s^I, s^{-I})^T,$$

where incumbency requires $p \geq \frac{1}{2}$, $\Psi(z)$ is the vector of population weights in state z for strategies other than s^I, and $\pi(s^I, s^{-I})$ is the vector of payoffs of the incumbent strategy against any other strategy. Suppose that there is a single mutant strategy s^M, whose agents have flag $f^M = r$. Define a monomorphic transition path (MTP) from strategy s^I to the mutant strategy s^M as a path in which every mistake changes the type of mutant agents toward strategy s^M. The incumbent strategy cannot survive the monomorphic transition path in state z if $\Pi(s^I, s^M, z) > \Pi(s^M, s^I, z)$, meaning that the mutant strategy becomes at least a better reply before the incumbent loses modality, which requires:

$$p\pi(s^I, s^I) + (1-p)\pi(s^I, s^M) > p\pi(s^M, s^I) + (1-p)\pi(s^M, s^M), \quad p \geq \frac{1}{2}$$

Analogously, define a polymorphic transition path (PTP) from strategy s^I to the mutant strategy s^M as a path in which we evaluate the transition between these two strategies while allowing for any type of mistakes. The incumbent strategy survives the polymorphic transition path in state z if $\Pi(s^I, s^M, z) > \Pi(s^M, s^I, z)$, defined as:

$$\Pi(s^I, s^M, z) = p\pi(s^I, s^I) + \psi_0(z)\pi(s^I, s^M) + \sum_{s^k \neq s^I, s^M} \psi(z)\pi(s^I, s^k)^T$$

$$\Pi(s^M, s^I, z) = p\pi(s^M, s^I) + \psi_0(z)\pi(s^M, s^M) + \sum_{s^k \neq s^I, s^M} \psi(z)\pi(s^I, s^k)^T$$

Where $\psi_0(z)$ is the proportion of mutants that adopt strategy s^M, $\psi(z)$ is the vector of population weights for any strategy other than s^I and s^M and $\pi(\bullet, s')$ is the vector of payoffs against any strategy other than s^I and s^M. The fact that incumbency implies $p \geq \frac{1}{2}$ allows us to define $\Psi(z) = [\psi_0(z), \psi(z)] = [\psi_0(z), \psi_1(z), \psi_2(z)...]$ with $\sum \Psi(z) \leq \frac{1}{2}$.

In the general case of PTPs, we can see that $p\pi(s^I, s^I)$ and $\psi_0(z)\pi(s^I, s^M)$, the first two terms of $\Pi(s^I, s^M, z)$, and $\psi_0(z)\pi(s^I, s^M)$ and $p\pi(s^M, s^I)$, the first two terms of $\Pi(s^M, s^I, z)$ only depend on s^I and s^M.
In order to evaluate the stability of an incumbent strategy, we must minimize $\sum_{s^k \neq s^I, s^M} \psi(z)\pi(s^I, s^k)^T$, which happens with any combination of mistakes of the form $(0, 1, \cdot, \cdot)$, and maximize $\sum_{s^k \neq s^I, s^M} \psi(z)\pi(s^I, s^k)^T$, which happens with any combination of mistakes of the form $(\cdot, \cdot, 1, 0)$.
This reduces to evaluate combinations of mistakes comprising the mutant strategy s^M and strategy $(0, 1, 1, 0)$, which minimizes
$\sum_{s' \neq s^I, s^M} \psi(z)\pi(s^I, s^k)^T$, by not cooperating with green and punishing

green, while maximizes $\sum_{s' \neq s^I, s^M} \psi(z)\pi(s^I, s^k)^T$, by cooperating with red and not punishing red. Any other combination of mistakes will at most be payoff equivalent since any other type of mistake will necessarily include red strategies. Finally, we must stress the fact that any strategy that does not survive up to losing modality when we consider MTP will not survive either when we consider PTPs, since by allowing different types of mistakes we could make a transition easier, but the easiest PTP is at most as difficult as the easiest MTP. Combining all this, we get the fitness lower bound for PTP of an incumbent strategy to be payoff equivalent to a MTP towards strategy $(0, 1, 1, 0)$:

$$\underline{\Pi}[s^I, s^M] = \Pi[s^I, (0,1,1,0), z : p \to \frac{1}{2}^+] =$$
$$= \lim_{p \to \frac{1}{2}^+} p\pi(s^I, s^I) + (1-p)\pi[s^I, (0,1,1,0)]$$

While the upper bound for a mutant strategy requires evaluation of the payoff of the mutant in a PTP in which strategy $(0, 1, 1, 0)$ approaches the threshold of modality:

$$\overline{\Pi}[s^M, s^I] = \lim_{p \to \frac{1}{2}^+, \psi_0 \to 0^+} p\pi(s^M, s^I) + \psi_0\pi(s^M, s^M) + (1-p)\pi[s^M, (0,1,1,0)]$$

Using these bounds, we get that if $\underline{\Pi}[s^I, s^M] \geq \max_{s^M \in S \setminus s^I} \overline{\Pi}[s^I, s^M]$, where the non-strict inequality comes from the use of the limit, then s^I survives any PTP up to the point of losing modality. Moreover, if $\underline{\Pi}[s^I, s^M] < \max_{s^M \in S \setminus s^I} \overline{\Pi}[s^I, s^M]$, then s^M becomes a myopic best-reply before s^I loses modality.

With all the previous considerations into account, we can start by enunciating some auxiliary results that help us characterize the resiliency of the different absorbing states, by checking their radius (Ellison, 2000). Since mistakes are uniformly random, we can define the resistance of moving from state z to state z', $r(z, z')$ as the number of mistakes that are necessary to perform the transition, which in our case is instrumentalized as the proportion of mistakes given the importance of the modal threshold. Then, we define the radius of state z as $\mathcal{R}(z) = \min_{z' \in Z \setminus z} r(z, z')$. Given that all our absorbing states are monomorphic, we can also express it as $\mathcal{R}(s \in \mathcal{A}_\sigma) = \min_{s' \in \mathcal{A}_\sigma \setminus s} r(s, s')$. Also, given our flag system, the threshold of modality implies that $\max \mathcal{R}(s \in \mathcal{A}_\sigma) = \frac{n}{2}$, which in turn implies that if $\underline{\Pi}[s, s'] < \max_{s' \in S \setminus s} \overline{\Pi}[s, s]$ and $\max_{s' \in S \setminus s} \overline{\Pi}[s, s] \in \mathcal{A}_\sigma$, then s cannot have maximum radius. Finally, if $\underline{\Pi}[s, s'] < \max_{s' \in S \setminus s} \overline{\Pi}[s, s]$ and $\max_{s' \in S \setminus s} \overline{\Pi}[s, s] \notin \mathcal{A}_\sigma$, then s has maximum radius even when there exists at least a transition path such that myopic best-reply is triggered before losing modality.

Another thing is important to notice here. It is not only that $(0, 1, 1, 0)$ is the unique best way to articulate a polymorphic transition path, but also $(0, 0, 0, 0)$, which corresponds to a total free rider strategy that never cooperates and never punishes, obtains always the maximum payoffs in polymorphic transition paths, and hence will be the strategy that first triggers best reply on each possible PTM (see Appendix B.6). With that in mind, we should establish a couple of remarks that help us understand the dynamics of transition whenever we consider a best reply in a direction different from an absorbing state, given that $(0, 0, 0, 0)$ is not absorbing.

DEFINITION **1.** *A strategy s is resilient towards strategy $s^{'}$ whenever it survives a MTP towards strategy $s^{'}$ up to the point of losing modality. A strategy s is weakly resilient towards strategy $s^{'}$ whenever it survives one mistake towards strategy $s^{'}$.*

Note that week resilience of s towards any other strategy means that, as N increases, more mistakes are needed to trigger a transition from a state z in which s is played by all population towards any monomorphic state z' in which $s^{'} \neq s$ is played.

REMARK **1.** *Suppose that in a transition path between states z, where strategy s is played by all the population and z', where strategy s' is played by all the* *population, best reply is triggered in towards state z'', where strategy s'' is played by all the population. Suppose also that z is an absorbing state, while z' and z'' are not. Then, when best reply triggers and the transition protocol makes a fraction ρ of the population to receive a revision opportunity, and assuming that ρ is big enough, one out of four things can happen:*

1. *Enough individuals playing strategy s' revise their strategy towards s'' such that we enter again the basin of attraction of s, keeping s as the unique modal strategy, and the transition does not complete.*

2. *Enough individuals playing strategy s revise their strategy towards s'' such that s' becomes modal, but being s' not absorbing, the transition does not complete.*

3. *The combination of individuals playing strategies s and s' that revise their strategy is such that s'' becomes modal, and s'' is weakly resilient towards any other absorbing state. Then a transition from z has been completed, but without entering any other absorbing state, and hence we cannot say anything about the radius of z.*

4. *The combination of individuals playing strategies s and s' that revise their strategy is such that s'' becomes green, but s'' is not weakly resilient towards at least one absorbing state. Then, z cannot have maximum radius.*

REMARK **2.** *Suppose that in a transition path between states z, where strategy s is played by all the population and z', where strategy s' is played by all the population, best reply is triggered in towards state z'', where strategy s'' is played by all the population. Suppose also that z and z' are absorbing states, while z'' is not. Then, when best reply triggers and the transition protocol makes a fraction ρ of the population to receive a revision opportunity, and assuming that ρ is big enough, one out of four things can happen:*

1. *Enough individuals playing strategy s' revise their strategy towards s'' such that we enter again the basin of attraction of s, keeping s as the unique green strategy, and the transition does not complete.*

2. *Enough individuals playing strategy s revise their strategy towards s'' such that s' becomes modal, and being s' absorbing when modal, there is a positive probability that the transition completes, and z cannot have maximum radius.*

3. *The combination of individuals playing strategies s and s' that revise their strategy is such that s'' becomes modal, and s'' is weakly resilient towards*

any other absorbing state. Then a transition from z has been completed, but without entering any other absorbing state, and hence we cannot say anything about the radius of z.

4. *The combination of individuals playing strategies s and s' that revise their strategy is such that s'' becomes green, but s'' is not weakly resilient towards at least one absorbing state. Then, z cannot have maximum radius.*

LEMMA **3.** *If $b > c + \alpha$, $b > \gamma$, $b < c + \alpha + \gamma$ and $c > \gamma$, then polymorphic transition paths do not provide us with a suitable equilibrium selection tool.*

Proof. Under these parameters' values, we have four absorbing states,

$$\mathcal{A}_{L1} = \Big((1,0,0,0),(1,0,0,1),(0,0,0,1),(1,0,1,1)\Big)$$

We need to check if there is any absorbing state that becomes a best reply whenever we consider a polymorphic transition path in the limit in which the proportion of strategy $(0,1,1,0)$ tends to $\frac{1}{2}^{-}$. We have (see Appendix B.2),

$$\underline{\Pi}_{g,r}[(1,0,0,0),s^M] = \frac{1}{2}(b-c-\gamma),$$

$$,\quad \overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L1},(1,0,0,0)] \in \left[\frac{1}{2}(b-c)-\alpha, \frac{1}{2}(b-\alpha)\right)$$

, where, with a slight abuse of notation, we use the half-open interval to denote the fact that the upper bound is a limit. Although we can see that there are some parameters values such that other strategy belonging to the set of absorbing states when green becomes a better reply before $(1,0,0,0)$ loses modality, it is not the case that any of them becomes a best reply, given that:

$$\max \overline{\Pi}_{r,g}[s^M,(1,0,0,0)] = \overline{\Pi}_{r,g}[(0,0,0,0),(1,0,0,0)] = \frac{1}{2}b$$

Monotonicity of payoffs as mistakes accumulates, as long as the color of the different strategies do not change, ensures that this maximum applies all along the transition path. Moreover, although $(0,1,1,0)$ is not resilient towards any absorbing state when green, see Appendix B.2, the best reply to $(0,1,1,0)$ whenever is green is precisely $(0,0,0,0)$, and $(0,0,0,0)$ is itself resilient to all absorbing states, see Appendix B.2. Hence, by Remark 1, checking PTPs is not a useful equilibrium selection technique for the absorbing state of strategy $(1, 0, 0, 0)$. For the same arguments, mutatis mutandis, PTPs do not solve the selection problems for the other absorbing states, which can be checked by computing:

$$\underline{\Pi}_{g,r}[(1,0,0,1), s^M] = \frac{1}{2}(b - c - \alpha - \gamma),$$

$$, \quad \overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L1}, (1,0,0,1)] \in \Big[\frac{1}{2}(b - c - \gamma) - \alpha, \frac{1}{2}(b - c - \gamma)\Big)$$

$$\max \overline{\Pi}_{r,g}[s^M, (1,0,0,1)] = \overline{\Pi}_{r,g}[(0,0,0,0), (1,0,0,1)] = \frac{1}{2}(b - \gamma)$$

$$\underline{\Pi}_{g,r}[(0,0,0,1), s^M] = \frac{1}{2}(-\alpha - \gamma)$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L1}, (0,0,0,1)] \in \left[\frac{1}{2}(b - c - \alpha - \gamma), \begin{cases} \frac{1}{2}(b - c - \gamma) & \text{if } c < \alpha \\ \frac{1}{2}(b - \alpha - \gamma) & \text{if } c > \alpha \end{cases}\right)$$

$$\max \overline{\Pi}_{r,g}[s^M, (0,0,0,1)] = \overline{\Pi}_{r,g}[(0,0,0,0), (0,0,0,1)] = \frac{1}{2}(b - \gamma)$$

$$\underline{\Pi}_{g,r}[(1,0,1,1), s^M] = \frac{1}{2}(b - c) - \alpha - \gamma$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L1}, (1,0,1,1)] \in \left[\frac{1}{2}(b - c - \gamma) - \alpha, \begin{cases} \frac{1}{2}(b - c - \gamma) & \text{if } c < \alpha \\ \frac{1}{2}(b - \alpha - \gamma) & \text{if } c > \alpha \end{cases}\right)$$

$$\max \overline{\Pi}_{r,g}[s^M, (1,0,1,1)] = \overline{\Pi}_{r,g}[(0,0,0,0), (1,0,1,1)] = \frac{1}{2}(b - \gamma)$$

And these bounds along with Remark 1 complete the proof. □

LEMMA **4.** *If $b > c + \alpha$, $b > \gamma$, $b < c + \alpha + \gamma$ and $c < \gamma$, then polymorphic transition paths do not provide us with a suitable equilibrium selection tool.*

Proof. Under these parameters' values, we have six absorbing states,

$$\mathcal{A}_{L2} = \Big((1,0,0,0),(1,1,0,1),(1,0,0,1),(0,1,0,1),(0,0,0,1),(1,0,1,1)\Big)$$

The proof unfolds following the same arguments as Lemma 3. Computing the bounds as before:

$$\underline{\Pi}_{g,r}[(1,0,0,0),s^M] = \frac{1}{2}(b-c-\gamma)$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L2},(1,0,0,0)] \in \left[\begin{cases}\frac{1}{2}(b-\alpha)-c & \text{if } n<m\\ \frac{1}{2}(b-c)-\alpha & \text{if } n>m\end{cases}, \frac{1}{2}(b-\alpha)\right)$$

$$\max \overline{\Pi}_{r,g}[s^M,(1,0,0,0)] = \overline{\Pi}_{r,g}[(0,0,0,0),(1,0,0,0)] = \frac{1}{2}b$$

$$\underline{\Pi}[_{g,r}(1,1,0,1),s^M] = \frac{1}{2}(b-\alpha-\gamma)-c$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L2},(1,1,0,1)] \in \left[b-\alpha-\frac{1}{2}c+\gamma), \begin{cases}b-\frac{1}{2}(c+\gamma) & \text{if } c<\alpha\\ b-\frac{1}{2}(\alpha+\gamma) & \text{if } c>\alpha\end{cases}\right)$$

$$\max \overline{\Pi}_{r,g}[s^M,(1,1,0,1)] = \overline{\Pi}_{r,g}[(0,0,0,0),(1,1,0,1)] = b-\frac{1}{2}\gamma$$

$$\underline{\Pi}_{g,r}[(1,0,0,1),s^M] = \frac{1}{2}(b-c-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L2},(1,0,0,1)] \in \left[\begin{cases}\frac{1}{2}(b-\alpha-\gamma)-c & \text{if } n<m\\ \frac{1}{2}(b-c-\gamma)-\alpha & \text{if } n>m\end{cases},\right.$$

$$\left., \begin{cases}\frac{1}{2}(b-c-\gamma) & \text{if } c<\alpha\\ \frac{1}{2}(b-\alpha-\gamma) & \text{if } c>\alpha\end{cases}\right)$$

$$\max \overline{\Pi}_{r,g}[s^M,(1,0,0,1)] = \overline{\Pi}_{r,g}[(0,0,0,0),(1,0,0,1)] = \frac{1}{2}(b-\gamma)$$

$$\underline{\Pi}_{g,r}[(0,1,0,1),s^M] = -c-\alpha-\gamma$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L2},(0,1,0,1)] \in \left[\begin{cases} (b-c)-\frac{1}{2}(\alpha+\gamma) & \text{if } n<m \\ (b-\alpha)-\frac{1}{2}(c+\gamma) & \text{if } n>m \end{cases}, \right.$$

$$\left. , \begin{cases} b-\frac{1}{2}(c+\gamma) & \text{if } c<\alpha \\ b-\frac{1}{2}(\alpha+\gamma) & \text{if } c>\alpha \end{cases} \right)$$

$$\max \overline{\Pi}_{r,g}[s^M,(0,1,0,1)] = \overline{\Pi}_{r,g}[(0,0,0,0),(0,1,0,1)] = b-\frac{1}{2}\gamma$$

$$\underline{\Pi}_{g,r}[(0,0,0,1),s^M] = \frac{1}{2}(-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L2},(0,0,0,1)] \in \left[\begin{cases} \frac{1}{2}(b-\alpha-\gamma)-c & \text{if } n<m \\ \frac{1}{2}(b-c-\gamma)-\alpha & \text{if } n>m \end{cases}, \frac{1}{2}(b-c-\gamma) \right)$$

$$\max \overline{\Pi}_{r,g}[s^M,(0,0,0,1)] = \overline{\Pi}_{r,g}[(0,0,0,0),(0,0,0,1)] = \frac{1}{2}(b-\gamma)$$

$$\underline{\Pi}_{g,r}[(1,0,1,1),s^M] = \frac{1}{2}(b-c)-\alpha-\gamma$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L2},(1,0,1,1)] \in \left[\frac{1}{2}(b-\alpha-\gamma)-c, \begin{cases} \frac{1}{2}(b-c-\gamma) & \text{if } c<\alpha \\ \frac{1}{2}(b-\alpha-\gamma) & \text{if } c>\alpha \end{cases} \right)$$

$$\max \overline{\Pi}_{r,g}[s^M,(1,0,1,1)] = \overline{\Pi}_{r,g}[(0,0,0,0),(1,0,1,1)] = \frac{1}{2}(b-\gamma)$$

And these bounds along with Remark 1 complete the proof. □

LEMMA **5.** *If $b<c+\alpha$ and $b<\gamma \Rightarrow c<\gamma$, then polymorphic transition paths do not provide us with a suitable equilibrium selection tool.*

Proof. Under these parameters' values, we have five absorbing states,

$$\mathcal{A}_{L3} = \Big((1,0,0,0),(1,1,0,1),(1,0,0,1),(0,1,0,1)(0,0,0,1)\Big)$$

We need to check if there is any absorbing state that becomes the best reply whenever we consider a polymorphic transition path in the limit in which the proportion of strategy $(0,1,1,0)$ tends to $\frac{1}{2}^{-}$. We have (see Appendix B.3),

$$\underline{\Pi}_{g,r}[(1,0,0,0),s^M] = \frac{1}{2}(b-c-\gamma), \quad \overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L3},$$
$$,(1,0,0,0)] \in \Big[\frac{1}{2}(b-\alpha)-c, \frac{1}{2}(b-\alpha)\Big)$$

$$\max \overline{\Pi}_{r,g}[s^M,(1,0,0,0)] = \overline{\Pi}_{r,g}[(0,0,0,0),(1,0,0,0)] = \frac{1}{2}b$$

$$\underline{\Pi}_{g,r}[(1,1,0,1),s^M] = \frac{1}{2}(b-\alpha-\gamma)-c$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L3},(1,1,0,1)] \in \left[b-\frac{1}{2}(c+\alpha+\gamma), \begin{cases} b-\frac{1}{2}(c+\gamma) & \text{if } c<\alpha \\ b-\frac{1}{2}(\alpha+\gamma) & \text{if } c>\alpha \end{cases}\right)$$

$$\max \overline{\Pi}_{r,g}[s^M,(1,1,0,1)] = \overline{\Pi}_{r,g}[(0,0,0,0),(1,1,0,1)] = b-\frac{1}{2}\gamma$$

$$\underline{\Pi}_{g,r}[(1,0,0,1),s^M] = \frac{1}{2}(b-c-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L3},(1,0,0,1)]\left[\frac{1}{2}(b-\alpha-\gamma)-c, \begin{cases} \frac{1}{2}(b-c-\gamma) & \text{if } n<m \\ \frac{1}{2}(b-\alpha-\gamma) & \text{if } n>m \end{cases}\right)$$

$$\max \overline{\Pi}_{r,g}[s^M,(1,0,0,1)] = \overline{\Pi}_{r,g}[(0,0,0,0),(1,0,0,1)] = \frac{1}{2}(b-\gamma)$$

$$\underline{\Pi}_{g,r}[(0,1,0,1),s^M] = -c-\alpha-\gamma$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L3}, (0,1,0,1)] \left[b - c - \frac{1}{2}(\alpha+\gamma), \begin{cases} b - \frac{1}{2}(c+\gamma) & \text{if } c < \alpha \\ b - \frac{1}{2}(\alpha+\gamma) & \text{if } c > \alpha \end{cases} \right)$$

$$\max \overline{\Pi}_{r,g}[s^M, (0,1,0,1)] = \overline{\Pi}_{r,g}[(0,0,0,0),(0,1,0,1)] = b - \frac{1}{2}\gamma$$

$$\underline{\Pi}_{g,r}[(0,0,0,1), s^M] = \frac{1}{2}(-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L3}, (0,0,0,1)] \in \left[\frac{1}{2}(b-\alpha-\gamma) - c, \frac{1}{2}(b-c-\gamma) \right)$$

$$\max \overline{\Pi}_{r,g}[s^M, (0,0,0,1)] = \overline{\Pi}_{r,g}[(0,0,0,0),(0,0,0,1)] = \frac{1}{2}(b-\gamma)$$

And these bounds along with Remark 1 complete the proof. □

LEMMA **6.** *If $b > c + \alpha + \gamma$ and $c > \gamma$, then polymorphic transition paths do not provide us with a suitable equilibrium selection tool.*

Proof. Under these parameters' values, we have five absorbing states,

$$\mathcal{A}_{L4} = \Big((1,0,0,0), (1,0,1,0), (1,0,0,1), (0,0,0,1), (1,0,1,1) \Big)$$

We need to check if there is any absorbing state that becomes the best reply whenever we consider a polymorphic transition path in the limit in which the proportion of strategy $(0,1,1,0)$ tends to $\frac{1}{2}^-$. We have (see Appendix B.3),

$$\underline{\Pi}_{g,r}[(1,0,0,0), s^M] = \frac{1}{2}(b-c-\gamma),$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L4}, (1,0,0,0)] \in \left[\frac{1}{2}(b-c) - \alpha, \frac{1}{2}(b-\alpha) \right)$$

,

$$\max \overline{\Pi}_{r,g}[s^M, (1,0,0,0)] = \overline{\Pi}_{r,g}[(0,0,0,0),(1,0,0,0)] = \frac{1}{2}b$$

$$\underline{\Pi}_{g,r}[(1,0,1,0),s^M] = \frac{1}{2}(b-c-\alpha)-\gamma$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L4},(1,0,1,0)]\left[\frac{1}{2}(b-c)-\alpha, \begin{cases} \frac{1}{2}(b-c) & \text{if } c<\alpha \\ \frac{1}{2}(b-\alpha) & \text{if } c>\alpha \end{cases}\right)$$

$$\max \overline{\Pi}_{r,g}[s^M,(1,0,1,0)] = \overline{\Pi}_{r,g}[(0,0,0,0),(1,0,1,0)] = \frac{1}{2}b$$

$$\underline{\Pi}_{g,r}[(1,0,0,1),s^M] = \frac{1}{2}(b-c-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L4},(1,0,0,1)]\left[\frac{1}{2}(b-c-\gamma)-\alpha, \begin{cases} \frac{1}{2}(b-c-\gamma) & \text{if } c<\alpha \\ \frac{1}{2}(b-\alpha-\gamma) & \text{if } c>\alpha \end{cases}\right)$$

$$\max \overline{\Pi}_{r,g}[s^M,(1,0,0,1)] = \overline{\Pi}_{r,g}[(0,0,0,0),(1,0,0,1)] = \frac{1}{2}(b-\gamma)$$

$$\underline{\Pi}_{g,r}[(0,0,0,1),s^M] = \frac{1}{2}(-\alpha-\gamma),$$

$$, \quad \overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L4},(0,0,0,1)] \in \left[\frac{1}{2}(b-c-\gamma)-\alpha, \frac{1}{2}(b-c-\gamma)\right.$$

$$\max \overline{\Pi}_{r,g}[s^M,(0,0,0,1)] = \overline{\Pi}_{r,g}[(0,0,0,0),(0,0,0,1)] = \frac{1}{2}(b-\gamma)$$

$$\underline{\Pi}_{g,r}[(1,0,1,1),s^M] = \frac{1}{2}(b-c)-\alpha-\gamma$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L4},(1,0,1,1)]\left[\frac{1}{2}(b-c-\alpha-\gamma), \begin{cases} \frac{1}{2}(b-c-\gamma) & \text{if } c<\alpha \\ \frac{1}{2}(b-\alpha-\gamma) & \text{if } c>\alpha \end{cases}\right)$$

$$\max \overline{\Pi}_{r,g}[s^M,(1,0,1,1)] = \overline{\Pi}_{r,g}[(0,0,0,0),(1,0,1,1)] = \frac{1}{2}(b-\gamma)$$

And these bounds along with Remark 1 complete the proof. □

LEMMA 7. *If $b > c+\alpha+\gamma$ and $c < \gamma$, then polymorphic transition paths do not provide us with a suitable equilibrium selection tool.*

Proof. Under these parameters' values, we have six absorbing states,

$$\mathcal{A}_{L5} = \Big((1,0,0,0),(1,0,1,0),(1,1,0,1),(1,0,0,1),(0,0,0,1),(1,0,1,1)\Big)$$

We need to check if there is any absorbing state that becomes the best reply whenever we consider a polymorphic transition path in the limit in which the proportion of strategy $(0,1,1,0)$ tends to $\frac{1}{2}^{-}$. We have (see Appendix B.3),

$$\underline{\Pi}_{g,r}[(1,0,0,0),s^M] = \frac{1}{2}(b-c-\gamma)$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L5},(1,0,0,0)] \in \left[\begin{cases}\frac{1}{2}(b-\alpha)-c & \text{if } n<m\\ \frac{1}{2}(b-c)-\alpha & \text{if } n>m\end{cases},\frac{1}{2}(b-\alpha)\right)$$

$$\max\overline{\Pi}_{r,g}[s^M,(1,0,0,0)] = \overline{\Pi}_{r,g}[(0,0,0,0),(1,0,0,0)] = \frac{1}{2}b$$

$$\underline{\Pi}_{g,r}[(1,0,1,0),s^M] = \frac{1}{2}(b-c-\alpha)-\gamma,$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L5},(1,0,1,0)] \in \left[\begin{cases}\frac{1}{2}(b-c)-\alpha & \text{if } n<m\\ \frac{1}{2}(b-\alpha)-c & \text{if } n>m\end{cases},\right.$$
$$\left.,\begin{cases}\frac{1}{2}(b-c) & \text{if } c<\alpha\\ \frac{1}{2}(b-\alpha) & \text{if } c>\alpha\end{cases}\right)$$

$$\max\overline{\Pi}_{r,g}[s^M,(1,0,1,0)] = \overline{\Pi}_{r,g}[(0,0,0,0),(1,0,1,0)] = \frac{1}{2}b$$

$$\underline{\Pi}_{g,r}[(1,1,0,1),s^M] = \frac{1}{2}(b-\alpha-\gamma)-c$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L5},(1,1,0,1)] \in \left[b-\alpha-\frac{1}{2}(c+\gamma),\begin{cases}b-\frac{1}{2}(c+\gamma) & \text{if } c<\alpha\\ b-\frac{1}{2}(\alpha+\gamma) & \text{if } c>\alpha\end{cases}\right]$$

$$\max \overline{\Pi}_{r,g}[s^M, (1,1,0,1)] = \overline{\Pi}_{r,g}[(0,0,0,0),(1,1,0,1)] = b - \frac{1}{2}\gamma$$

$$\underline{\Pi}_{g,r}[(1,0,0,1), s^M] = \frac{1}{2}(b - c - \alpha - \gamma)$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L5}, (1,0,0,1)] \left[\begin{cases} \frac{1}{2}(b-\alpha-\gamma) - c & \text{if } n < m \\ \frac{1}{2}(b-c-\gamma) - \alpha & \text{if } n > m \end{cases}, \right.$$

$$\left. , \begin{cases} \frac{1}{2}(b-c-\gamma) & \text{if } c < \alpha \\ \frac{1}{2}(b-\alpha-\gamma) & \text{if } c > \alpha \end{cases} \right)$$

$$\max \overline{\Pi}_{r,g}[s^M, (1,0,0,1)] = \overline{\Pi}_{r,g}[(0,0,0,0),(1,0,0,1)] = \frac{1}{2}(b - \gamma)$$

$$\underline{\Pi}_{g,r}[(0,0,0,1), s^M] = \frac{1}{2}(-\alpha - \gamma)$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L5}, (0,0,0,1)] \in \left[\begin{cases} \frac{1}{2}(b-\alpha-\gamma) - c & \text{if } n < m \\ \frac{1}{2}(b-c-\gamma) - \alpha & \text{if } n > m \end{cases}, x \right)$$

$$\max \overline{\Pi}_{r,g}[s^M, (0,0,0,1)] = \overline{\Pi}_{r,g}[(0,0,0,0),(0,0,0,1)] = \frac{1}{2}(b - \gamma)$$

$$\underline{\Pi}_{g,r}[(1,0,1,1), s^M] = \frac{1}{2}(b - c) - \alpha - \gamma$$

$$\overline{\Pi}_{r,g}[s^M \in \mathcal{A}_{L5}, (1,0,1,1)] \in \left[\frac{1}{2}(b-\alpha-\gamma) - c, \begin{cases} \frac{1}{2}(b-c-\gamma) & \text{if } c < \alpha \\ \frac{1}{2}(b-\alpha-\gamma) & \text{if } c > \alpha \end{cases} \right)$$

$$\max \overline{\Pi}_{r,g}[s^M, (1,0,1,1)] = \overline{\Pi}_{r,g}[(0,0,0,0),(1,0,1,1)] = \frac{1}{2}(b - \gamma)$$

And these bounds along with Remark 1 complete the proof. $\square$

A sufficient condition for having maximum radius is to survive all the possible monomorphic transition paths while, at the same time, best reply is not triggered toward any other strategy along those trasition paths, so that we can guarantee that Remark 2 does not play an effect. In other words, to eliminate an absorbing state as candidate for maximum radius, we need to find a MTP towards another absorbing state such that the strategy does not survive the transition or the strategy survives but best reply is triggered towards any direction, moment in which Remark 2 helps us complete the transition with positive probability. If several absorbing states have maximum radius, then they are all stochastically stable. This is not obvious by looking at radius-coradius arguments, since those states would have their radius equal to their coradius, but we can support our claim in a threee surgery argument. To see this, consider a tree T' rooted at $z' \in \mathcal{A}$ and with Z' as set of nodes, which minimizes the sum of the resistances over its links among all trees rooted at z'. Starting from T' we add all links connecting consecutive states in such path from z' to z'', where $z'' \in \mathcal{A}$, and we remove all links in T' which were outgoing from any state in the path. By doing so, we obtain a tree T'' rooted at z''. We now compare the sum of the resistances over links in T' and T''. If we were to remove the branches of the tree that go from z' to z'' or vice versa, and since they both have maximum radius, the easiest way to reconnect both states is precisely through the easiest path that characterizes that maximum radius, which implies that both states have minimum stochastic potential, and hence both are stochastically stable.

PROPOSITION **4.** *If $b > c + \alpha$, $b > \gamma$, $b < c + \alpha + \gamma$ and $c > \gamma$, then:*

(a) $(1, 0, 0, 0)$ *is stochastically stable.*

(b) $(1, 0, 0, 1)$ *is stochastically stable.*

(c) $(0, 0, 0, 1)$ *is stochastically stable.*

(d) $(1, 0, 1, 1)$ *is stochastically stable.*

Proof. Under these parameters' values, we have four absorbing states,

$$\mathcal{A}_{P1} = \Big((1, 0, 0, 0), (1, 0, 0, 1), (0, 0, 0, 1), (0, 0, 0, 1)\Big)$$

First, notice that every absorbing state survives all MTPs towards every other absorbing state, which is granted by the fact that $b - c - \alpha > 0$ (see Appendix B.5). We will refer Appendix B.5 whenever we check for MTPs in the following propositions. We need to check that $(0,0,0,0)$, which attains the maximum payoff between off-transiton red strategies (see Appendix B.6), does not become a best reply in any MTP for any strategy belonging to an absorbing state. Denote the payoff of strategy $(0,0,0,0)$ in a transition between strategy s^I and strategy s^M, with $s^I, s^M \in \mathcal{A}$ as $\mu(s^I, s^M)$. For *(a)*, we have:

$$\pi_{g,r}[(1,0,0,0),(1,0,0,1)] = b{+}p(-c) > (1{-}p)(-\gamma) = \mu_{g,r}[(1,0,0,0),(1,0,0,1)]$$

$$\pi_{g,r}[(1,0,0,0),(0,0,0,1)] = p(b{-}c) > (1{-}p)(-\gamma) = \mu_{g,r}[(1,0,0,0),(0,0,0,1)]$$

$$\pi_{g,r}[(1,0,0,0),(1,0,1,1)] = b + p(-c) + (1-p)(-\gamma) > (1-p)(-\gamma) = \\ = \mu_{g,r}[(1,0,0,0),(0,0,0,1)]$$

These expressions always hold since $b{-}c > 0$. Hence $(1,0,0,0)$ is stochastically stable.

For *(b)*, we have:

$$\pi_{g,r}[(1,0,0,1),(1,0,0,0)] = p(b-c) + (1-p)(b-\alpha) > p(-\gamma)t = \\ = \mu_{g,r}[(1,0,0,1),(1,0,0,0)]$$

$$\pi_{g,r}[(1,0,0,1),(0,0,0,1)] = p(b-c) + (1-p)(-\alpha) > (-\gamma) = \\ = \mu_{g,r}[(1,0,0,1),(0,0,0,1)]$$

$$\pi_{g,r}[(1,0,0,1),(1,0,1,1)] = p(b-c) + (1-p)(b-\alpha-\gamma) > (-\gamma) = \\ = \mu_{g,r}[(1,0,0,1),(1,0,1,1)]$$

These expressions always hold since $b > c + \alpha$. Hence $(1,0,0,1)$ is stochastically stable.

For *(c)*, we have:

$$\pi_{g,r}[(0,0,0,1),(1,0,0,0)] = (1{-}p)(b{-}\alpha) > p(-\gamma) = \mu_{g,r}[(0,0,0,1),(1,0,0,0)]$$

$$\pi_{g,r}[(0,0,0,1),(1,0,0,1)] = (1{-}p)(b{-}\alpha) > (-\gamma) = \mu_{g,r}[(0,0,0,1),(1,0,0,1)]$$

$$\pi_{g,r}[(0,0,0,1),(1,0,1,1)] = (1{-}p)(b{-}\alpha{-}\gamma) > p(-\gamma) = \mu_{g,r}[(0,0,0,1),(1,0,1,1))]$$

These expressions always hold since $b > \alpha$. Hence $(0,0,0,1)$ is stochastically stable.

Finally, for *(d)*, we have:

$$\pi_{g,r}[(1,0,1,1),(1,0,0,0)] = p(b-c-\alpha-\gamma) + (1-p)(b-\alpha) > p(-\gamma) =$$
$$= \mu_{g,r}[(1,0,1,1),(1,0,0,0)]$$
$$\pi_{g,r}[(1,0,1,1),(1,0,0,1)] = p(b-c-\alpha-\gamma) + (1-p)(b-\alpha) > (-\gamma) =$$
$$= \mu_{g,r}[(1,0,1,1),(1,0,0,1)]$$
$$\pi_{g,r}[(1,0,1,1),(1,0,1,1)] = p(b-c-\alpha-\gamma) + (1-p)(-\alpha) > (-\gamma) =$$
$$= \mu_{g,r}[(1,0,1,1),(1,0,1,1)]$$

These expressions always hold since $b > c + \alpha$. Hence $(1,0,1,1)$ is stochastically stable, which completes the proof. □

PROPOSITION **5.** *If $b > c+\alpha$, $b > \gamma$, $b < c+\alpha+\gamma$ and $c < \gamma$, then:*

- (a) $(1,0,0,0)$ *is stochastically stable.*
- (b) $(1,1,0,1)$ *is stochastically stable as long as $b - c > 2(b-\gamma)$ and $2\gamma > 2c+\alpha$*
- (c) $(1,0,0,1)$ *is stochastically stable as long as $2\gamma > c+\alpha$.*
- (d) $(0,1,0,1)$ *is not stochastically stable.*
- (e) $(0,0,0,1)$ *is stochastically stable as long as $2\gamma > b-c$ and $2\gamma - b > \alpha$.*
- (f) $(1,0,1,1)$ *is stochastically stable as long as $\gamma > 2\alpha + c$.*

Proof. Under these parameters' values, we have six absorbing states,

$$\mathcal{A}_{P2} = \Big((1,0,0,0),(1,1,0,1),(1,0,0,1),(0,1,0,1),(0,0,0,1),(1,0,1,1)\Big)$$

For *(a)*, $(1,0,0,0)$ survives all MTPs towards every other absorbing state. The expressions for $\pi_{g,r}[(1,0,0,0),(1,0,0,1)]$, $\mu_{g,r}[(1,0,0,0),(1,0,0,1)]$, $\pi_{g,r}[(1,0,0,0),(0,0,0,1)]$, $\mu_{g,r}[(1,0,0,0),(0,0,0,1)]$, $\pi_{g,r}[(1,0,0,0),(0,0,0,1)]$ and $\mu_{g,r}[(1,0,0,0),(0,0,0,1)]$ hold in the same way as in Proposition 4. In addition, we have:

$$\pi_{g,r}[(1,0,0,0),(1,1,0,1)] = p(b-c) + (1-p)b > (1-p)(b-\gamma) =$$

$$= \mu_{g,r}[(1,0,0,0),(1,1,0,1)]$$

$$\pi_{g,r}[(1,0,0,0),(0,1,0,1)] = p(b-c) > (1-p)(b-\gamma) = \mu_{g,r}[(1,0,0,0),(0,1,0,1)]$$

Those expressions always hold since the $c < \gamma$. Hence, $(1,0,0,0)$ is stochastically stable.

For *(b)*, notice that $(1,1,0,1)$ might not survive a MTP towards $(0,1,0,1)$ in the limit when $p \to \frac{1}{2}^+$ if $b-c < 2(b-\gamma)$. In addition, we have that:

$$\pi_{g,r}[(1,1,0,1),(1,0,0,0)] = p(b-c) + (1-p)(b-c-\alpha) > p(b-\gamma) =$$

$$= \mu_{g,r}[(1,1,0,1),(1,0,0,0)]$$

$$\pi_{g,r}[(1,1,0,1),(1,0,0,1)] = p(b-c)+(1-p)(b-c-\alpha) > p(b-\gamma)+(1-p)(-\gamma) =$$

$$= \mu_{g,r}[(1,1,0,1),(1,0,0,1)]$$

These expressions hold since $c < \gamma$.

Also, we have:

$$\pi_{g,r}[(1,1,0,1),(0,1,0,1)] = p(b-c)+(1-p)(b-c-\alpha) > p(b-\gamma)+(1-p)(b-\gamma) =$$

$$= \mu_{g,r}[(1,1,0,1),(0,1,0,1)] \iff$$

$$\lim p \to \frac{1}{2}^+ b-c(1-p)(-\alpha) > \lim p \to \frac{1}{2}^+ b-\gamma \iff 2(b-c)-\alpha > 2(b-\gamma)$$

If $2(b-c)-\alpha < 2(b-\gamma)$, then Remark 2 tells us that a transition from $(1,1,0,1)$ is possible towards $(0,1,0,1)$ before losing modality, and $(1,1,0,1)$ cannot be stochastically stable. This expression, however, is less stringent than the previous one for MTP.

Continuing our checking:

$$\pi_{g,r}[(1,1,0,1),(0,0,0,1)] = p(b-c)+(1-p)(-c-\alpha) > p(b-\gamma)+(1-p)(-\gamma) =$$

$$= \mu_{g,r}[(1,1,0,1),(0,0,0,1)] \iff$$

$$\lim p \to \frac{1}{2}^+ p(b)+(1-p)(-\alpha)-c > \lim p \to \frac{1}{2}^+ p(b)+-\gamma \iff 2\gamma > 2c+\alpha$$

If $2\gamma < 2c+\alpha$, then Remark 2 tells us that a transition from $(1,1,0,1)$ is possible towards $(0,0,0,1)$ before losing modality, and $(1,1,0,1)$ cannot be stochastically stable.

Finally, we have:

$$\pi_{g,r}[(1,1,0,1),(1,0,1,1)] = p(b-c) + (1-p)(b-c-\alpha-\gamma) >$$

$$> p(b-c-\alpha-\gamma) + (1-p)(-\alpha-\gamma) = \mu_{g,r}[(1,1,0,1),(1,0,1,1)]$$

And this always hold since $b-c>0$.

For *(c)*, $(1,0,0,1)$ survives all MTPs towards every other absorbing state. The expressions for $\pi_{g,r}[(1,0,0,1),(1,0,0,0)]$, $\mu_{g,r}[(1,0,0,1),(1,0,0,0)]$, $\pi_{g,r}[(1,0,0,1),(0,0,0,1)]$, ${}_{g,r}\mu[(1,0,0,1),(0,0,0,1)]$, $\pi_{g,r}[(1,0,0,1),(1,0,1,1)]$ and $\mu_{g,r}[(1,0,0,1),(1,0,1,1)]$ are as in Proposition 4.

In addition, we have:

$$\pi_{g,r}[(1,0,0,1),(1,1,0,1)] = p(b-c)+(1-p)(b-\alpha) > p(-\gamma)+(1-p)(b-\gamma) =$$

$$= \mu_{g,r}[(1,0,0,1),(1,1,0,1)] \iff$$

$$\iff \lim p \to \frac{1}{2}^{+} p(b-c)+(1-p)(-\alpha) > p(-\gamma)+(1-p)(b-\gamma) \iff 2\gamma > c+\alpha$$

If $2\gamma < c+\alpha$, then Remark 2 tells us that a transition from $(1,0,0,1)$ is possible towards $(1,1,0,1)$ before losing modality, and $(1,0,0,1)$ cannot be stochastically stable.

Finally, we have

$$\pi_{g,r}[(1,0,0,1),(0,1,0,1)] = p(b-c) + (1-p)(b-\alpha) >$$

$$> p(-\gamma) + (1-p)(b-\gamma) = \mu_{g,r}[(1,0,0,1),(0,1,0,1)],$$

which always holds given that $b > c+\alpha$.

For *(d)*, notice that $(0,1,0,1)$ might not survive a MTP towards $(1,0,0,0)$ in the limit when $p \to \frac{1}{2}^{+}$ if $\alpha > \gamma$. Notice that, at the beggining of the transition, $(0,1,0,1)$ might not survive a MTP towards $(1,0,0,0)$ if $b-c-\gamma > 0$, which also holds if $\alpha > \gamma$. For the same arguments, $(0,1,0,1)$ might not survive a MTP towards $(1,1,0,1)$ if $b-c>\gamma$, and $(0,1,0,1)$ might not survive a MTP towards $(0,0,0,1)$ if $b+c>2\gamma$. In addition, we have that, whenever a small mutation begin a transition towards $(1,0,0,0)$, we have that:

$$\pi_{g,r}[(0,1,0,1),(1,0,0,0)] = (1-p)(b-c-\alpha) < p(b-\gamma) =$$

$$= \mu_{g,r}[(0,1,0,1),(1,0,0,0)],$$

which is due to the fact that p is very close to one at the beginning of the transitions. At this point, the system starts moving towards $(0,0,0,0)$ without further mistakes, and can arbitrary enter another absorbing state. Hence $(0,1,0,1)$ cannot be stochastically stable.

For *(e)*, notice that $(0,0,0,1)$ might not survive a MTP towards $(0,1,0,1)$ in the limit when $p \to \frac{1}{2}^+$ if $2\gamma < b - c$.

The expressions for $\pi_{g,r}[(0,0,0,1),(1,0,0,0)]$, $\mu_{g,r}[(0,0,0,1),(1,0,0,0)]$, $\pi_{g,r}[(0,0,0,1),(1,0,0,1)]$, $\mu_{g,r}[(0,0,0,1),(1,0,0,1)]$, $\pi_{g,r}[(0,0,0,1),(1,0,1,1)]$ and $\mu_{g,r}[(0,0,0,1),(1,0,1,1)]$ hold in the same way as in Proposition 4.

In addition, we have that:

$$\pi_{g,r}[(0,0,0,1),(1,1,0,1)] = (1-p)(b-\alpha) > p(-\gamma) + (1-p)(b-\gamma) =$$

$$= \mu_{g,r}[(0,0,0,1),(1,1,0,1)] \iff$$

$$\iff \lim p \to \frac{1}{2}^+ (1-p)(-\alpha) > -\gamma \iff 2\gamma > \alpha$$

If $2\gamma < \alpha$, then Remark 2 tells us that a transition from $(0,0,0,1)$ is possible towards $(1,1,0,1)$ before losing modality, and $(0,0,0,1)$ cannot be stochastically stable.

Finally, we have

$$\pi_{g,r}[(0,0,0,1),(0,1,0,1)] = (1-p)(-\alpha) > p(-\gamma) + (1-p)(b-\gamma) =$$

$$= \mu_{g,r}[(0,0,0,1),(0,1,0,1)] \iff$$

$$\iff \lim p \to \frac{1}{2}^+ (1-p)(-\alpha) > \lim p \to \frac{1}{2}^+ (1-p)b - \gamma \iff 2\gamma - b > \alpha$$

If $2\gamma - b < \alpha$, then Remark 2 tells us that a transition from $(0,0,0,1)$ is possible towards $(0,1,0,1)$ before losing modality, and $(0,0,0,1)$ cannot be stochastically stable. This condition is more stringent than the previous one for a transition from $(0,0,0,1)$ towards $(1,1,0,1)$.

For *(f)*, notice that $(1,0,1,1)$ might not survive a MTP towards $(0,1,0,1)$ in the limit when $p \to \frac{1}{2}^+$ if $\alpha > \gamma$. The expressions for $\pi_{g,r}[(1,0,1,1),(1,0,0,0)]$,

$\mu_{g,r}[(1,0,1,1),(1,0,0,0)]$, $\pi_{g,r}[(1,0,1,1),(1,0,0,1)]$, $\mu_{g,r}[(1,0,1,1),(1,0,0,1)]$, $\pi_{g,r}[(1,0,1,1),(0,0,0,1)]$ and $\mu_{g,r}[(1,0,1,1),(0,0,0,1)]$ hold in the same way as in Proposition 4.

In addition, we have that:

$$\pi_{g,r}[(1,0,1,1),(1,1,0,1)] = p(b-c-\alpha-\gamma)+(1-p)(b-\alpha) > p(-\gamma)+(1-p)(b-\gamma) =$$

$$= \mu_{g,r}[(1,0,1,1),(0,1,0,1)] \iff$$

$$\iff \lim p \to \frac{1}{2}^{+} p(b-c)-\alpha > \lim p \to \frac{1}{2}^{+} (1-p)(-\gamma) \iff b+\gamma > c+\alpha$$

If $b+\gamma < c+\alpha$, then Remark2 tells us that a transition from $(1,0,1,1)$ is possible towards $(1,1,0,1)$ before losing modality, and $(1,0,1,1)$ cannot be stochastically stable.

$$\pi_{g,r}[(1,0,1,1),(0,1,0,1)] = p(b-c-\alpha-\gamma)+(1-p)(-\alpha) > p(-\gamma)+(1-p)(b-\gamma) =$$

$$= \mu_{g,r}[(1,0,1,1),(0,1,0,1)] \iff$$

$$\iff \lim p \to \frac{1}{2}^{+} p(b-c)-\alpha > \lim p \to \frac{1}{2}^{+} (1-p)(b-\gamma) \iff \gamma > c+2\alpha$$

If $\gamma < c+2\alpha$, then Remark 2 tells us that a transition from $(1,0,1,1)$ is possible towards $(0,1,0,1)$ before losing modality, and $(1,0,1,1)$ cannot be stochastically stable, which completes the proof. This condition is more stringent than the previous one for the survival of MTPs and the previous one for a transition from $(1,0,1,1)$ towards $(1,1,0,1)$ □

PROPOSITION **6.** *If $b < c+\alpha$ and $b < \gamma \Rightarrow c < \gamma$, then:*

(a) $(1,0,0,0)$ *is stochastically stable.*

(b) $(1,1,0,1)$ *is stochastically stable as long as* $b-c-\alpha > -\gamma$ *and* $2\gamma > c+\alpha$.

(c) $(1,0,0,1)$ *is stochastically stable as long as* $2b+\gamma > \alpha$ *and* $2\gamma > c+\alpha$.

(d) $(0,1,0,1)$ *is stochastically stable as long as* $\gamma > c+\alpha$.

(e) $(0,0,0,1)$ *is stochastically stable as long as* $c+\gamma > b+\alpha$.

Proof. Under these parameters' values, we have five absorbing states,

$$\mathcal{A}_{P3} = \Big((1,0,0,0),(1,1,0,1),(1,0,0,1),(0,1,0,1)(0,0,0,1)\Big)$$

For *(a)*, $(1,0,0,0)$ survives all MTPs towards every other absorbing state. The expressions for $\pi_{g,r}[(1,0,0,0),(1,0,0,1)]$, $\mu_{g,r}[(1,0,0,0),(1,0,0,1)]$, $\pi_{g,r}[(1,0,0,0),(1,1,0,1)]$, $\mu_{g,r}[(1,0,0,0),(1,1,0,1)]$, $\pi_{g,r}[(1,0,0,0),(0,0,0,1)]$, $\mu_{g,r}[(1,0,0,0),(0,0,0,1)]$, $\pi_{g,r}[(1,0,0,0),(0,0,0,1)]$ and $\mu_{g,r}[(1,0,0,0),(0,0,0,1)]$ hold in the same way as in Proposition 4 and Proposition 5.
Hence, $(1,0,0,0)$ is stochastically stable.

For *(b)*, notice that $(1,1,0,1)$ might not survive a MTP towards $(1,0,0,0)$ in the limit when $p \to \frac{1}{2}^+$ if $b-c-\alpha < -\gamma$. The expressions for $\pi_{g,r}[(1,1,0,1),(1,0,0,0)]$, $\mu_{g,r}[(1,1,0,1),(1,0,0,0)]$, $\pi_{g,r}[(1,1,0,1),(1,0,0,1)]$, $\mu_{g,r}[(1,1,0,1),(1,0,0,1)]$, $\pi_{g,r}[(1,1,0,1),(0,0,0,1)]$, $mu_{g,r}[(1,1,0,1),(0,0,0,1)]$, $\pi_{g,r}[(1,1,0,1),(1,0,1,1)]$ and $\mu_{g,r}[(1,1,0,1),(1,0,1,1)]$ hold in the same way as in Proposition 5.

For *(c)*, notice that $(1,0,0,1)$ might not survive a MTP towards $(1,0,0,0)$ in the limit when $p \to \frac{1}{2}^+$ if $2b+\gamma < \alpha$.
In addition, we have that:

$$\pi_{g,r}[(1,0,0,1),(1,0,0,0)] > \mu_{g,r}[(1,0,0,1),(1,0,0,0)] \iff 2b+\gamma > c+\alpha$$
$$\pi_{g,r}[(1,0,0,1),(1,1,0,1)] > \mu_{g,r}[(1,0,0,1),(1,1,0,1)] \iff b+2\gamma > c+\alpha$$
$$\pi_{g,r}[(1,0,0,1),(0,1,0,1)] > \mu_{g,r}[(1,0,0,1),(0,1,0,1)] \iff 2\gamma > c+\alpha$$
$$\pi_{g,r}[(1,0,0,1),(0,0,0,1)] > \mu_{g,r}[(1,0,0,1),(0,0,0,1)] \iff b+2\gamma > c+\alpha$$

And these expressions hold as long as $2b+\gamma > \alpha$ and $2\gamma > c+\alpha$.

For *(d)*, notice that $(0,1,0,1)$ might not survive a MTP towards $(1,0,0,0)$ at the beginning of the transition and in the limit when $p \to \frac{1}{2}^+$ if $\alpha > \gamma$. In addition, we have that:

$$\pi_{g,r}[(0,1,0,1),(1,0,0,0)] = (1-p)(b-c-\alpha) > p(b-\gamma) =$$
$$= \mu_{g,r}[(0,1,0,1),(1,0,0,0)] \iff \lim p \to \frac{1}{2}^+ (1-p)(b-c-\alpha) >$$
$$> \lim p \to \frac{1}{2}^+ p(b-\gamma) \iff \gamma > c+\alpha$$

$$\pi_{g,r}[(0,1,0,1),(1,1,0,1)] = (1-p)(b-c-\alpha) > p(b-\gamma)+(1-p)(b-\gamma) =$$
$$= \mu_{g,r}[(0,1,0,1),(1,1,0,1)] \iff \lim p \to \frac{1}{2}^{+} (1-p)(-c-\alpha) >$$
$$> pb-\gamma \iff 2\gamma > b+c+\alpha$$
$$\pi_{g,r}[(0,1,0,1),(1,0,0,1)] = (1-p)(b-c-\alpha) > p(b-\gamma)+(1-p)(-\gamma) =$$
$$= \mu_{g,r}[(0,1,0,1),(1,0,0,1)] \iff \lim p \to \frac{1}{2}^{+} (1-p)(b-c-\alpha) >$$
$$> \lim p \to \frac{1}{2}^{+} pb-\gamma \iff 2\gamma > c+\alpha$$
$$\pi_{g,r}[(0,1,0,1),(0,0,0,1)] = (1-p)(-c-\alpha) > p(b-\gamma)+(1-p)(-\gamma) =$$
$$= \mu_{g,r}[(0,1,0,1),(0,0,0,1)] \iff \lim p \to \frac{1}{2}^{+} (1-p)(-c-\alpha) >$$
$$> \lim pb-\gamma \to \frac{1}{2}^{+} 2\gamma > b+c+\alpha \iff x > y$$

And these expressions hold as long as $\gamma > c+\alpha$.

Finally, for *(e)*, notice that notice that $(0,0,0,1)$ might not survive a MTP towards $(1,0,0,0)$ in the limit when $p \to \frac{1}{2}^{+}$ if $c+\gamma < b+\alpha$. In addition, we have that:

$$\pi_{g,r}[(0,0,0,1),(1,0,0,0)] > \mu_{g,r}[(0,0,0,1),(1,0,0,0)] \iff \gamma > b-\alpha$$
$$\pi_{g,r}[(0,0,0,1),(1,1,0,1)] > \mu_{g,r}[(0,0,0,1),(1,1,0,1)] \iff 2\gamma > \alpha$$
$$\pi_{g,r}[(0,0,0,1),(1,0,0,1)] > \mu_{g,r}[(0,0,0,1),(1,0,0,1)] \iff b+2\gamma > \alpha$$
$$\pi_{g,r}[(0,0,0,1),(0,1,0,1)] > \mu_{g,r}[(0,0,0,1),(0,1,0,1)] \iff 2\gamma-b > \alpha$$

And these expressions hold as long as $2\gamma - b > \alpha$. This condition is less stringent than the previous one for the MTP.

□

PROPOSITION 7. *If $b > c+\alpha+\gamma$ and $c > \gamma$, then:*

(a) $(1,0,0,0)$ *is stochastically stable.*

(b) $(1,0,1,0)$ *is stochastically stable.*

(c) $(1,0,0,1)$ *is stochastically stable.*

(d) $(0,0,0,1)$ *is stochastically stable as long as* $x<y$.

(e) $(1,0,1,1)$ *is stochastically stable as long as* $x<y$.

Proof. Under these parameters' values, we have five absorbing states,

$$\mathcal{A}_{L4} = \Big((1,0,0,0),(1,0,1,0),(1,0,0,1),(0,0,0,1),(1,0,1,1)\Big)$$

First, notice that every absorbing state survives all MTPs towards every other absorbing state, which is granted by the fact that $b>c+\alpha+\gamma$. For *(a)*, we have that:

$$\pi_{g,r}[(1,0,0,0),(1,0,1,0)] = p(b-c)+(1-p)(b-\gamma) > 0 = \mu_{g,r}[(1,0,0,0),(1,0,1,0)]$$

, which holds since $b>c+\alpha+\gamma$.

The expressions for $\pi_{g,r}[(1,0,0,0),(1,0,0,1)]$, $\mu_{g,r}[(1,0,0,0),(1,0,0,1)]$, $\pi_{g,r}[(1,0,0,0),(0,0,0,1)]$, $\mu_{g,r}[(1,0,0,0),(0,0,0,1)]$, ${}_{g,r}\pi[(1,0,0,0),(0,0,0,1)]$ and $\mu_{g,r}[(1,0,0,0),(0,0,0,1)]$ hold in the same way as in Proposition 4. Hence, $(1,0,0,0)$ is stochastically stable.

For *(b)*, we have that:

$$\pi_{g,r}[(1,0,1,0),(1,0,0,0)] = p(b-c-\alpha-\gamma)+(1-p)b > 0 = \mu_{g,r}[(1,0,1,0),(1,0,0,0)]$$

$$\pi_{g,r}[(1,0,1,0),(1,0,0,1)] = p(b-c-\alpha-\gamma)+(1-p)b > (1-p)(-\gamma) =$$
$$= \mu_{g,r}[(1,0,1,0),(1,0,0,1)]$$
$$\pi_{g,r}[(1,0,1,0),(0,0,0,1)] = p(b-c-\alpha-\gamma) > (1-p)(-\gamma) =$$
$$= \mu_{g,r}[(1,0,1,0),(0,0,0,1)]$$
$$\pi_{g,r}[(1,0,1,0),(1,0,1,1)] = p(b-c-\alpha-\gamma)+(1-p)(b-\gamma) > (1-p)(-\gamma) =$$
$$= \mu_{g,r}[(1,0,1,0),(1,0,1,1)]$$

These expressions hold since $b>c+\alpha+\gamma$. Hence, $(1,0,1,0)$ is stochastically stable.

For *(c)*, we have that:

$$\pi_{g,r}[(1,0,0,1),(1,0,1,0)] = p(b-c)+(1-p)(b-\alpha-\gamma) > p(-\gamma) =$$
$$= \mu_{g,r}[(1,0,0,1),(1,0,1,0)]$$

, which holds since $b > c + \alpha + \gamma$.

The expressions for $\pi_{g,r}[(1,0,0,1),(1,0,0,0)]$, $\mu_{g,r}[(1,0,0,1),(1,0,0,0)]$, $\pi_{g,r}[(1,0,0,1),(0,0,0,1)]$, $\mu_{g,r}[(1,0,0,1),(0,0,0,1)]$, $\pi_{g,r}[(1,0,0,1),(1,0,1,1)]$ and $\mu_{g,r}[(1,0,0,1),(1,0,1,1)]$ hold in the same way as in Proposition 4. Hence, $(1,0,0,1)$ is stochastically stable.

For *(d)*, we have that:

$$\pi_{g,r}[(0,0,0,1),(1,0,1,0)] = (1-p)(b-\alpha-\gamma) > p(-c-\alpha-\gamma) =$$
$$= \mu[(0,0,0,1),(1,0,1,0)]$$

, which holds since $b > c + \alpha + \gamma$.

The expressions for $\pi_{g,r}[(0,0,0,1),(1,0,0,0)]$, $\mu_{g,r}[(0,0,0,1),(1,0,0,0)]$, $\pi_{g,r}[(0,0,0,1),(1,0,0,1)]$, $\mu_{g,r}[(0,0,0,1),(1,0,0,1)]$, $\pi_{g,r}[(0,0,0,1),(1,0,1,1)]$ and $\mu_{g,r}[(0,0,0,1),(1,0,1,1)]$ hold in the same way as in Proposition 4. Hence, $(0,0,0,1)$ is stochastically stable.

Finally, for *(e)*, we have that:

$$\pi_{g,r}[(1,0,1,1),(1,0,1,0)] = p(b-c-\alpha-\gamma)+(1-p)(b-\alpha-\gamma) > p(-\gamma) =$$
$$= \mu_{g,r}[(1,0,1,1),(1,0,1,0)]$$

, which holds since $b > c + \alpha + \gamma$.

The expressions for $\pi_{g,r}[(1,0,1,1),(1,0,0,0)]$, $\mu_{g,r}[(1,0,1,1),(1,0,0,0)]$, $\pi_{g,r}[(1,0,1,1),(1,0,0,1)]$, $\mu_{g,r}[(1,0,1,1),(1,0,0,1)]$, $\pi_{g,r}[(1,0,1,1),(0,0,0,1)]$ and $\mu_{g,r}[(1,0,1,1),(0,0,0,1)]$ hold in the same way as in Proposition 4. Hence, $(1,0,1,1)$ is stochastically stable.

□

PROPOSITION **8.** *If $b > c + \alpha + \gamma$ and $c < \gamma$, then:*

(a) $(1,0,0,0)$ *is stochastically stable.*

(b) $(1,0,1,0)$ *is stochastically stable.*

(c) $(1,1,0,1)$ *is stochastically stable as long as* $2\gamma > c + \alpha$

(d) $(1,0,0,1)$ *is stochastically stable.*

(e) $(0,0,0,1)$ *is stochastically stable as long as* $2\gamma > \alpha$*.*

(f) $(1,0,1,1)$ *is stochastically stable as long as* $b - c > 2\alpha - \gamma$*.*

Proof. Under these parameters' values, we have six absorbing states,

$$\mathcal{A}_{L5} = \Big((1,0,0,0),(1,0,1,0),(1,1,0,1),(1,0,0,1),(0,0,0,1),(1,0,1,1)\Big)$$

First, notice that every absorbing state survives all MTPs towards every other absorbing state, which is granted by the fact that $b > c + \alpha + \gamma$.
For *(a)*, the expressions for $\pi_{g,r}[(1,0,0,0),(1,0,1,0)]$, $\mu_{g,r}[(1,0,0,0),(1,0,1,0)]$, $\pi_{g,r}[(1,0,0,0),(1,1,0,1)]$, $\mu_{g,r}[(1,0,0,0),(1,1,0,1)]$, $\pi[_{g,r}(1,0,0,0),(1,0,0,1)]$, $\mu_{g,r}[(1,0,0,0),(1,0,0,1)]$, $\pi_{g,r}[(1,0,0,0),(0,0,0,1)]$, $\mu_{g,r}[(1,0,0,0),(0,0,0,1)]$, $\pi_{g,r}[(1,0,0,0),(1,0,1,1)]$ and $\mu_{g,r}[(1,0,0,0),(1,0,1,1)]$ hold in the same way as in Proposition 4, Proposition 5 and Proposition 7. Hence, $(1,0,0,0)$ is stochastically stable.

For *(b)*, we have that:

$$\pi_{g,r}[(1,0,1,0),(1,1,0,1)] = p(b-\alpha-\gamma) > (1-p)(-c-\alpha-\gamma) =$$
$$= \mu_{g,r}[(1,0,1,0),(1,1,0,1)]$$

This expression holds since $b > c+\alpha+\gamma$. In addition, the expressions for $\pi_{g,r}[(1,0,1,0),(1,0,0,0)]$, $\mu_{g,r}[(1,0,1,0),(1,0,0,0)]$, $\pi_{g,r}[(1,0,1,0),(1,0,0,1)]$, $\mu_{g,r}[(1,0,1,0),(1,0,0,1)]$, $\pi_{g,r}[(1,0,1,0),(0,0,0,1)]$, $\mu_{g,r}[(1,0,1,0),(0,0,0,1)]$, $\pi_{g,r}[(1,0,1,0),(1,0,1,1)]$ and $\mu_{g,r}[(1,0,1,0),(1,0,1,1)]$ hold in the same way as in Proposition 7. Hence, $(1,0,1,0)$ is stochastically stable.

For *(c)*, we have that $(1,1,0,1)$ survives all MTPs towards every other absorbing state. The expressions for $\pi_{g,r}[(1,1,0,1),(1,0,0,0)]$, $\mu_{g,r}[(1,1,0,1),(1,0,0,0)]$, $\pi_{g,r}[(1,1,0,1),(1,0,0,1)]$, $\mu_{g,r}[(1,1,0,1),(1,0,0,1)]$, $\pi_{g,r}[(1,1,0,1),(0,0,0,1)]$, $mu_{g,r}[(1,1,0,1),(0,0,0,1)]$, $\pi_{g,r}[(1,1,0,1),(1,0,1,1)]$ and $\mu_{g,r}[(1,1,0,1),(1,0,1,1)]$ hold in the same way as in Proposition 5 and Proposition 6.

Finally we have that:

$$\pi_{g,r}[(1,1,0,1),(1,0,1,0)] = p(b-c) + (1-p)(b-c-\alpha-\gamma) > p(b-\gamma) =$$
$$= \mu_{g,r}[(1,1,0,1),(1,0,1,0)]$$

And this expression hold since $c < \gamma$

For *(d)*, we have that:

$$\pi_{g,r}[(1,0,0,1),(1,1,0,1)] = p(b-c)+(1-p)(b-\alpha) > p(-\gamma)+(1-p)(b-\gamma) =$$
$$= \mu_{g,r}[(1,0,0,1),(1,1,0,1)]$$

, which always hold since $b > c+\alpha+\gamma$

The expressions for $\pi_{g,r}[(1,0,0,1),(1,0,0,0)]$, $\mu_{g,r}[(1,0,0,1),(1,0,0,0)]$, $\pi_{g,r}[(1,1,0,1),(1,0,1,0)]$, $\mu_{g,r}[(1,1,0,1),(1,0,1,0)]$, $\pi_{g,r}[(1,0,0,1),(0,0,0,1)]$, $\mu_{g,r}[(1,0,0,1),(0,0,0,1)]$, $\pi_{g,r}[(1,0,0,1),(1,0,1,1)]$ and $\mu_{g,r}[(1,0,0,1),(1,0,1,1)]$ hold in the same way as in Proposition 4 and Proposition 7. Hence, $(1,0,0,1)$ is stochastically stable.

For *(e)*, we have that:

$$\pi_{g,r}[(0,0,0,1),(1,1,0,1)] > \mu_{g,r}[(0,0,0,1),(1,1,0,1)] \iff 2\gamma > \alpha$$

If $2\gamma < \alpha$, then Remark 2 tells us that a transition from $(0,0,0,1)$ is possible towards $(1,1,0,1)$ before losing modality, and $(0,0,0,1)$ cannot be stochastically stable.

In addition, the expressions for $\pi_{g,r}[(0,0,0,1),(1,0,0,0)]$, $\mu_{g,r}[(0,0,0,1),(1,0,0,0)]$, $\pi_{g,r}[(0,0,0,1),(1,0,1,0)]$, $\mu_{g,r}[(0,0,0,1),(1,0,1,0)]$, $\pi_{g,r}[(0,0,0,1),(1,0,0,1)]$, $\mu_{g,r}[(0,0,0,1),(1,0,0,1)]$, $\pi_{g,r}[(0,0,0,1),(1,0,1,1)]$ and $\mu_{g,r}[(0,0,0,1),(1,0,1,1)]$ hold in the same way as in Proposition 4 and Proposition 7.

Finally, for *(f)*, we have that:

$$\pi_{g,r}[(1,0,1,1),(1,1,0,1)] > \mu_{g,r}[(1,0,1,1),(1,1,0,1)] \iff$$
$$p(b-c-\alpha-\gamma)+(1-p)(b-\alpha) > p(-\gamma)+(1-p)(b-\gamma) \iff$$
$$\iff \lim p \to \frac{1}{2}^{+} p(b-c)-\alpha > \lim p \to \frac{1}{2}^{+} (1-p)(-\gamma) \iff b-c > 2\alpha-\gamma$$

If $b-c < 2\alpha-\gamma$, then Remark 2 tells us that a transition from $(1,0,1,1)$ is possible towards $(1,1,0,1)$ before losing modality, and $(1,0,1,1)$ cannot be stochastically stable.

In addition, the expressions for $\pi_{g,r}[(1,0,1,1),(1,0,0,0)]$, $\mu_{g,r}[(1,0,1,1),(1,0,0,0)]$, $\pi_{g,r}[(1,0,1,1),(1,0,1,0)]$, $\mu_{g,r}[(1,0,1,1),(1,0,1,0)]$, $\pi_{g,r}[(1,0,1,1),(1,0,0,1)]$, $\mu_{g,r}[(1,0,1,1),(1,0,0,1)]$, $\pi_{g,r}[(1,0,1,1),(0,0,0,1)]$ and $\mu_{g,r}[(1,0,1,1),(0,0,0,1)]$ hold in the same way as in Proposition 4 and Proposition 7.

$\square$

4.5 Discussion and conclusions

A summary of Propositions 4 to 8 can be found in Table 1. As we can see, strategy $(1,0,0,0)$, which represents conditional norm cooperators, is always stochastically stable, which is not surprising given that our flag system makes that strategy incredibly resistant given that saving both the costs of punishing and cooperating with any other strategy apart from itself. In contrast, strategy $(1,0,1,0)$, which also comprises conditional norm cooperators that likewise are conditional norm punishers, only manages to get stochastically stable whenever the benefit of cooperation b is big enough to compensate for the fact that the strategy punishes itself. The obvious way to solve this problem is to be a pro-norm punisher, which is the modality of punishment that accounts for more strategies being stochastically stable, with a total of four. Strategy $(1,1,0,1)$, unconditional cooperators that are pro-norm punishers requires not only that the costs of being punished γ are high enough but also that the difference between the benefits of cooperation and the cost of cooperation c is substantial so that to avoid excessive costs of unconditional cooperation. The other three strategies, $(1,0,0,1)$, which represents conditional norm cooperators that are pro-norm punishers, $(0,1,0,1)$, anti-norm cooperators that are pro-norm punishers, and $(0,0,0,1)$, free riders that are pro-norm punishers, only require conditions in which the damage γ that they inflict to the red strategies is large enough. Finally, we have only an instance of spiteful punishment, with strategy $(1,0,1,1)$ representing conditional norm cooperators that punish everybody. Similar to the anti-norm punishers of before, they need the benefits of cooperation to pay back for both the costs of punishment and the costs of being punished.

The conclusions to be taken from the model comprise a mixture of in-

Table 1: Summary of conditions for stochastic stability

	Model's parameters				
	$b > c + \alpha$ $b > \gamma$ $b < c + \alpha + \gamma$ $c > \gamma$	$b > c + \alpha$ $b > \gamma$ $b < c + \alpha + \gamma$ $c > \gamma$	$b < c + \alpha$ $b < \gamma \Rightarrow c < \gamma$	$b > c + \alpha + \gamma$ $c > \gamma$	$b > c + \alpha + \gamma$ $c < \gamma$
Strategies					
$(1, 0, 0, 0)$	ALWAYS	ALWAYS	ALWAYS	ALWAYS	ALWAYS
$(1, 0, 1, 0)$	NEVER	NEVER	NEVER	ALWAYS	ALWAYS
$(1, 1, 0, 1)$	NEVER	if $b - c > 2(b - \gamma)$ and if $2\gamma > 2c + \alpha$	if $b - c - \alpha > -\gamma$ and if $2\gamma > c + \alpha$	NEVER	if $2\gamma > c + \alpha$
$(1, 0, 0, 1)$	ALWAYS	if $2\gamma > c + \alpha$	if $2b + \gamma > \alpha$ and if $2\gamma > c + \alpha$	ALWAYS	ALWAYS
$(0, 1, 0, 1)$	NEVER	NEVER	if $\gamma > c + \alpha$	NEVER	NEVER
$(0, 0, 0, 1)$	ALWAYS	if $2\gamma > b - c$ and if $2\gamma - b > \alpha$	if $c + \gamma > b + \alpha$	ALWAYS	if $2\gamma > \alpha$
$(1, 0, 1, 1)$	ALWAYS	if $\gamma > 2\alpha + c$	NEVER	ALWAYS	if $b - c > 2\alpha - \gamma$

tuitive and counterintuitive facts that, however, are able to explain pretty well what we see in the experimental literature about punishment. On one side, prosocial punishment, instrumented as pro-norm punishment, is the most common widespread modality of punishment and can effectively sustain cooperation in a population, as in Fehr and Gächter, 2002. On the other side, we will observe small amounts of antisocial and spiteful punishment, both because some cooperators following non modal strategies are punished by pro-norm punishment and because under some parameter configuration, anti-norm and spiteful punishment can be stochastically stable. Altogether, our results also explain why we observe extensive heterogeneity in punishment behavior across societies (Herrmann, Thoni, and Gachter, 2008), probably due to differences in parameter values in different groups evolved under different environments.

Another interesting implication of our models regards the resiliency of stochastically stable states and the coexistence of monomorphic stochastically stable states. Community enforcement is a tool that usually expands the support for equilibrium, and applying it to evolutionary models is both relatively weak in assumptions, as long as we accept that agents in a population can myopicly best reply, and helpful in finding new insights about the evolution of different traits. Coexistence is also less and less common as our toolbox for stochastic stability analysis is increased, with many techniques allowing us to grant uniqueness in the recent models developed. In our particular case, the existence of a particularly harmful strategy such as $(0, 1, 1, 0)$, anti-norm cooperators that are anti-norm punishers, prevent us from being able to apply techniques such as p-dominance (Morris, Rob, and Shin, 1995) or generalized risk dominance (Maruta, 1997; Peski, 2010). Paradoxically, the existence of a fully antisocial strategy is what allows us to get the coexistence of different long-run equilibria, a road that could be explored with other settings and structures. Overall, we believe that our results help us shed light on the survival of cooperation in the presence of antisocial modalities of punishment, the existing heterogeneity of modalities of punishment in different social groups, and the possibility of coexistence of stochastically stable states.

Chapter 5

Conclusion

In this doctoral thesis, I aimed to investigate how conflict, aggressivity, and punishment shape the way humans interact in social situations. To do so, I used the expertise of evolutionary and theoretical games theory, behavioral and experimental economics and neuroscience. The three papers I presented approached the issue from different aspects, both theoretical and experimental, and from different levels, from group to individual conflicts.

The emerging picture posits the pivotal role of conflict in human interactions in different ways. From a multi-group point of view, in Chapter 2, we prove that the determinants of conflict initiation are crucial to understanding which distribution of power will prevail more often in the long run. Specifically, hegemony will be observed more often when the relatively stronger groups attack more often. In comparison, balance of power will be observed more often whenever the relatively weaker groups attack more often.

At the individual level, conflict amounts to behaving aggressively or hostilely, and the roots of this behavior were investigated in Chapter 3, where, with two highly controlled experiments, one with electroencephalographic recordings (N=44) and one without (N=403), we investigated the role of frontal fatigue into influencing social interaction. With an ego depletion paradigm, we demonstrate that extended practice with tasks that require the use of self-control influences decisions in a Hawk and Dove game in which subjects can behave either peacefully or hostilely. At the same time, the distribution of punishment modalities shifts with respect to the control group, with a reduction in prosocial punishment and an increase in spiteful punishment.

Given our interesting results in punishment, Chapter 4 looks at the societal level to explore, from a theoretical perspective, how it is possible that cooperation is so pervasive in groups given the existence of small, but not negligible, amounts of antisocial and spiteful modalities of punishment. Merging techniques of the stochastic stability and community enforcement literature, we show that cooperation can survive in the presence of antisocial punishment whenever agents can conditional their cooperative and punishment behaviors to the modality of the other players' strategy. Interestingly, antisocial strategies allow us to characterize the coexistence of different stochastically stable social norms regarding punishment and cooperation and shed some light on the empirically heterogeneous distribution of punishment norms across societies.

Appendix A

Appendix: Prolonged Impulse Control Prompts Sleep-like Brain Activity and Aggressivity

Appendix of the Chapter *Prolonged Impulse Control Prompts Sleep-like Brain Activity and Aggressivity* will comprehend additional tables and figures with results that did not enter in the main manuscript, but nonetheless provide useful pieces of information regarding the study.

A.1 Matched sample characteristic

Table 2: Matching of participants (Study 1)

	Means			St. Deviation		
	No Fatigue	Fatigue	p t-test	No Fatigue	Fatigue	p F-test
Age	30.0455	30.5	0.8484	7.5497	8.1167	0.7431
Gender	0.5	0.5	1	0.5118	0.5118	1
Education	16.0455	15.3182	0.3669	2.4972	2.7841	0.6227
Pos. Reciprocity	-0.0657363	0.0718753	0.5776	0.8779334	0.7426719	0.4497
Neg. Reciprocity	0.0075612	-0.0066722	0.9516	0.6652603	0.8680993	0.2308
Altruism	-0.0402158	0.0564778	0.7095	0.8181046	0.8906829	0.7005
Patience	-0.2343495	0.2343489	0.0535	0.8666674	0.6882082	0.2985
Risk	-0.0145818	0.0145822	0.8989	0.7588576	0.7545047	0.9792
Trust	0.2137509	-0.2137513	0.1586	1.020427	0.9542618	0.7615

The table shows the matched characteristics of the two groups, which included biological sex, age, level of education, and the indices derived from the Global Preferences Survey. For these latter indices, standardized (z-scores) values are reported instead of raw values. Results about means are robust to the Wilcoxon rank-sum tests and results about standard deviations are robust to Levene's robust test statistic, and Brown and Forsythe robust test statistics.

Table 3: Matching of participants (Study 2)

	Means			St. Deviation		
	No Fatigue	Fatigue	p t-test	No Fatigue	Fatigue	p F-test
Age	23.3168	23.2985	0.9565	3.3899	3.3512	0.8712
Gender	0.6287	0.6169	0.8076	0.4843	0.4874	0.9302
Education	13.7574	13.7761	0.8959	1.3625	1.5015	0.1695
Pos. Reciprocity	0.0241005	-0.0247033	0.5136	0.7387023	0.7596906	0.6919
Neg. Reciprocity	-0.001979	-0.0031	0.9892	0.8524511	0.7999507	0.3701
Altruism	-0.0288995	0.0320133	0.436	0.8009949	0.7650098	0.5163
Patience	0.0227037	-0.0120349	0.6652	0.8414756	0.7666625	0.1883
Risk	-0.0083538	0.008395	0.8274	0.7437159	0.7966453	0.331
Trust	-0.0272027	0.0274749	0.5842	1.034398	0.9658329	0.3331

The table shows the matched characteristics of the two groups, which included biological sex, age, level of education, and the indices derived from the Global Preferences Survey. For these latter indices, standardized (z-scores) values are reported instead of raw values. Results about means are robust to the Wilcoxon rank-sum tests and results about standard deviations are robust to Levene's robust test statistic, and Brown and Forsythe robust test statistics.

Table 4: Matching of participants (pooled data)

	Means			St. Deviation		
	No Fatigue	Fatigue	p t-test	No Fatigue	Fatigue	p F-test
Age	23.9777	24.0090	0.9416	4.4444	4.5806	0.6529
Gender	0.6161	0.6054	0.8172	0.4874	0.4899	0.9407
Education	13.9821	13.9283	0.7360	1.6511	1.7253	0.5122
Pos. Reciprocity	0.0163753	-0.0166556	0.643	0.7482413	0.7572825	0.8579
Neg. Reciprocity	0.0002061	0.0004495	0.9975	0.8367353	0.8085697	0.6108
Altruism	-0.0307196	0.0328269	0.404	0.8155425	0.7910178	0.6496
Patience	-0.002936	0.0027184	0.9408	0.8469781	0.7597402	0.1057
Risk	-0.0091699	0.009211	0.7997	0.7422482	0.7875032	0.3779
Trust	-0.0035312	0.0035631	0.8964	0.5940963	0.5547887	0.3086

The table shows matched characteristics of the two groups, which included biological sex, age, level of education, and the indexes derived from the Global Preferences Survey (pooled data). For these latter indices, standardized values are reported instead of raw values. Results about means are robust to the Wilcoxon rank-sum tests and results about standard deviations are robust to Levene's robust test statistic and Brown's and Forsythe robust test statistics.

A.2 Likert Scales/Moods Questionnaires

Table 5: Likert Scales (Study 1)

	(1) Sleepiness	(2) Mood	(3) Fatigue	(4) Motivation	(5) Pos. affects	(6) Neg. affects
Condition	-0.0606	0.801	-0.784	0.621	-0.0358	0.362
	[0.779]	[0.516]	[0.598]	[0.432]	[0.179]	[0.190]
Time	0.545	0.591	1.955***	-0.273	-0.205	-0.0682
	[0.716]	[0.499]	[0.519]	[0.483]	[0.201]	[0.0866]
Interaction	0.318	-0.909	0.909	0.136	0.00909	-0.241
	[1.107]	[0.697]	[0.823]	[0.615]	[0.274]	[0.176]
Demographics	YES	YES	YES	YES	YES	YES
Preferences	YES	YES	YES	YES	YES	YES
Questionnaires	YES	YES	YES	YES	YES	YES
Constant	3.186	5.054	-2.288	8.548**	6.089***	0.188
	[5.011]	[3.923]	[3.497]	[3.134]	[1.276]	[0.934]
Observations	88	88	88	88	88	88

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The table displays the estimates for the Likert Scales compiled by participants of Study 1 during the experiment. We used linear regression with robust errors, since the tobit maximum likelihood was not achieving convergence in a couple of cases. The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale. Results are not corrected for multiple comparisons.

Table 6: Likert Scales - tobit estimates (Study 2)

	(1) Sleepiness	(2) Mood	(3) Fatigue	(4) Motivation	(5) Pos. affects	(6) Neg. affects
Condition	-0.0459	-0.0409	-0.154	-1.525*	-0.127	0.0571
	[0.288]	[0.646]	[0.244]	[0.616]	[0.0707]	[0.0425]
Time	1.056***	-0.701	1.994***	-1.771**	-0.423***	-0.100*
	[0.302]	[0.617]	[0.267]	[0.608]	[0.0705]	[0.0424]
Interaction	-0.437	-0.166	0.690	1.833*	0.121	0.0214
	[0.420]	[0.856]	[0.379]	[0.793]	[0.0999]	[0.0600]
Demographics	YES	YES	YES	YES	YES	YES
Preferences	YES	YES	YES	YES	YES	YES
Questionnaires	YES	YES	YES	YES	YES	YES
Constant	0.134	13.61***	1.890	4.351	3.655***	-0.0557
	[1.797]	[3.802]	[1.571]	[3.242]	[0.422]	[0.254]
Observations	798	798	798	798	798	798

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The table displays diff-in-diff tobit estimates for the Likert Scales compiled by participants of Study 2 during the experiment The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale. Results are not corrected for multiple comparisons.

Table 7: Likert Scales - tobit estimates (pooled data)

	(1) Sleepiness	(2) Mood	(3) Fatigue	(4) Motivation	(5) Pos. affects	(6) Neg. affects
Condition	-0.127	-0.137	-0.189	-1.454*	-0.122	0.0827*
	[0.273]	[0.632]	[0.232]	[0.614]	[0.0672]	[0.0405]
Time	1.003***	-0.531	2.020***	-1.915**	-0.401***	-0.0968*
	[0.287]	[0.616]	[0.254]	[0.605]	[0.0670]	[0.0404]
Interaction	-0.349	-0.262	0.663	1.948*	0.110	-0.00474
	[0.399]	[0.847]	[0.359]	[0.791]	[0.0948]	[0.0571]
Demographics	YES	YES	YES	YES	YES	YES
Preferences	YES	YES	YES	YES	YES	YES
Questionnaires	YES	YES	YES	YES	YES	YES
Constant	0.233	13.00***	0.959	4.862	4.055***	0.0144
	[1.643]	[3.612]	[1.439]	[3.080]	[0.386]	[0.233]
Observations	886	886	886	886	886	886

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets.

The table displays diff-in-diff tobit estimates for the Likert Scales compiled by participants during the experiment (pooled data). The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale. Results are not corrected for multiple comparisons.

A.3 Go-Nogo Task

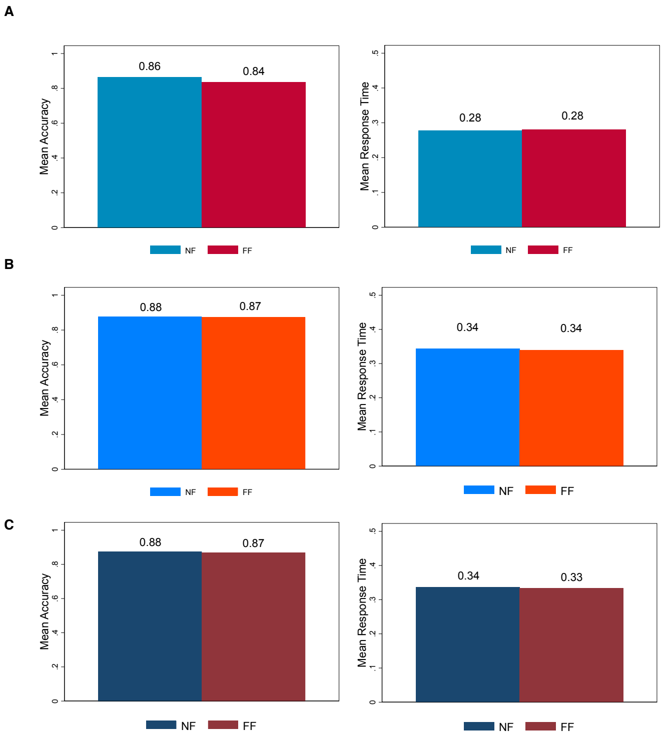

Figure 6: Results of mean accuracy and mean reaction times for the Go-Nogo Task in the two experiments. Panel A shows the results of Study 1, panel B the ones of Study 2 and finally, panel C shows the results of the two studies merged.

A.4 Beliefs

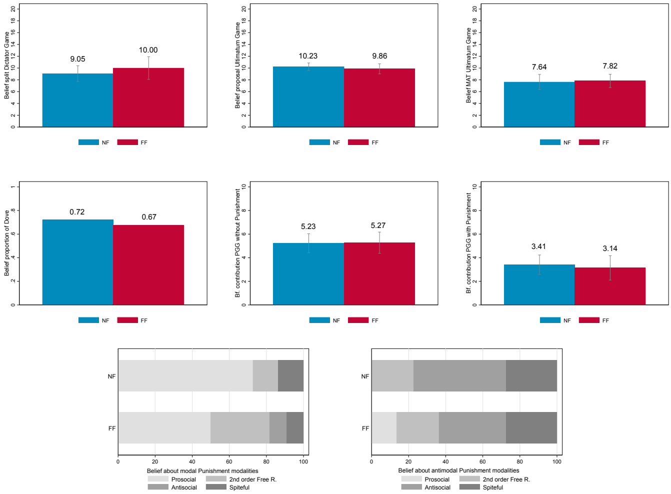

Figure 7: Results of the beliefs stated in Study 1. The order of the panel is the following: Dictator Game (average split), Ultimatum Game (average split and minimum acceptance threshold - MAT), Hawk and Dove Game (proportion of people that played "dove"), Public Goods Game without punishment (average contribution), Public Goods Game with Punishment (average contribution, most frequent and infrequent punishing behaviors). No significant results were found.

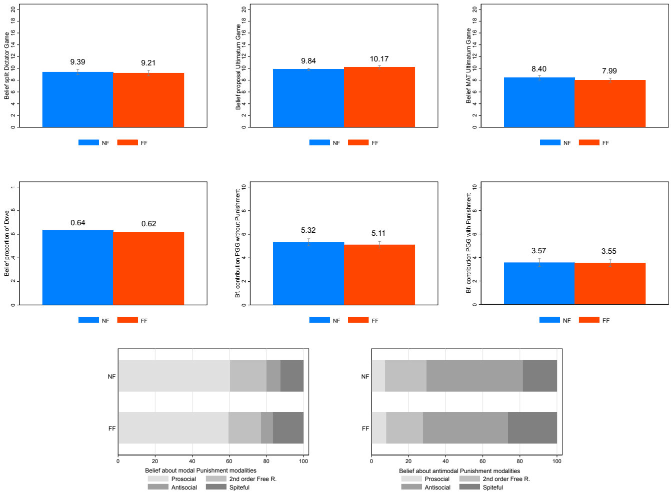

Figure 8: Results of the beliefs stated in Study 2. The order of the panel is the same as for Fig 7. A marginally significant result was found in the subjects' beliefs about the minimal proposal in the Ultimatum Game (MAT, $p = 0.0444$) but did not survive when pooling the data.

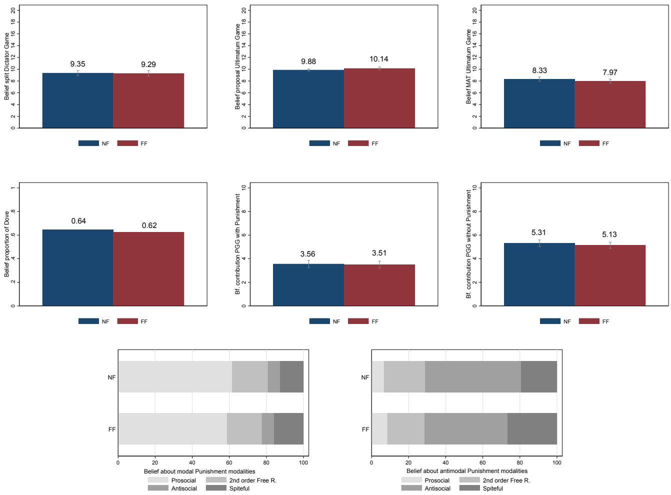

Figure 9: Results of the beliefs stated pooling the data of the two experiments. The order of the panel is the same as for Fig 7. No significant results were found.

A.5 Robustness of behavioral results

Other analysis on EEG data. Here is the graph containing the topographic plot of the analysis conducted using as a reference the period T2 as well as the source modeling results for the theta frequency band in the same period.

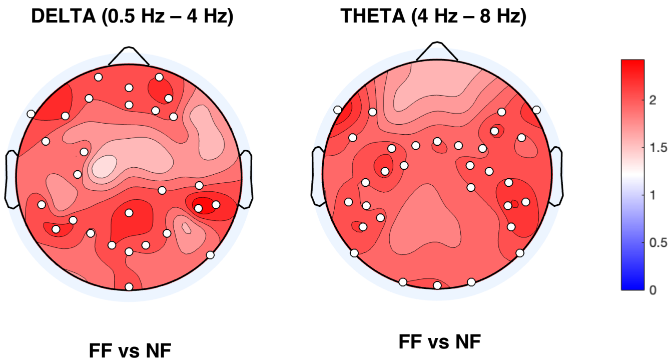

Figure 10: Differences in low-frequency activity variations after T2. The topographic plots show the statistical comparison between experimental conditions (FF vs NF) for delta and theta frequency bands at time T2 after subjects completed the economic games block. Colormap indicates T-values between +3 and -3, while white dots mark electrodes significant at $p < 0.05$ (cluster-based correction).

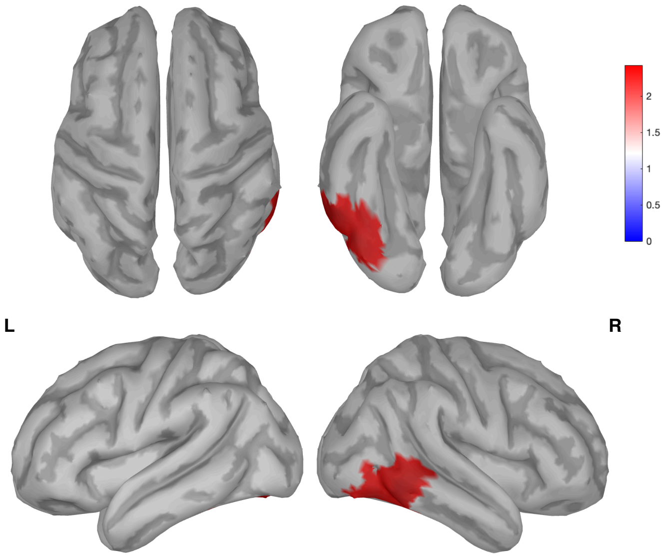

Figure 11: Source analysis of the theta EEG activity associated with the FF condition in T2. Significant differences ($p < 0.01$, cluster-based correction) in T2/T0 theta ratios between FF and NF. The difference peak was found in the right temporal lobe (64, -54, -10).

Results of the other games. All the additional results regarding the other games played that were not mentioned in the results section of the paper are reported below. Tables with regression analysis on all the results are also displayed.

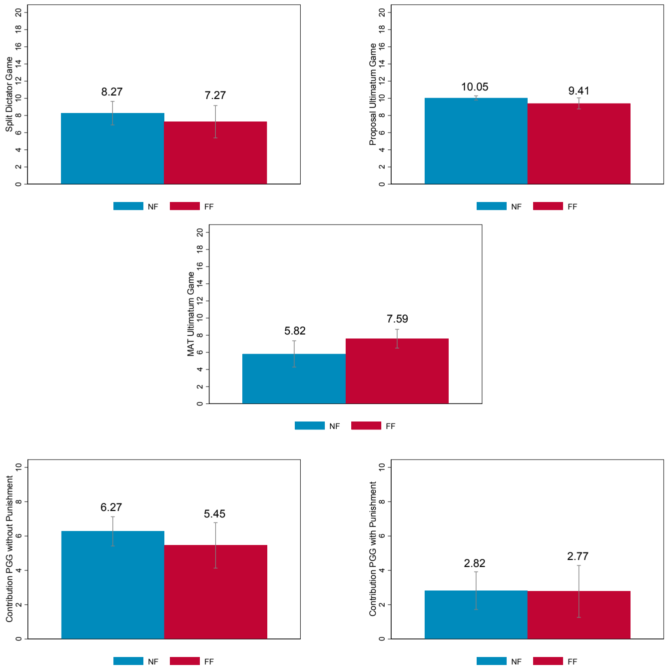

Figure 12: Results of the games of Study 1 not shown in the Results section. In order: split of the Dictator Game, proposition in the Ultimatum Game, minimum acceptance threshold for the Receiver of the Ultimatum Game, contribution in the PGG without and with punishment.

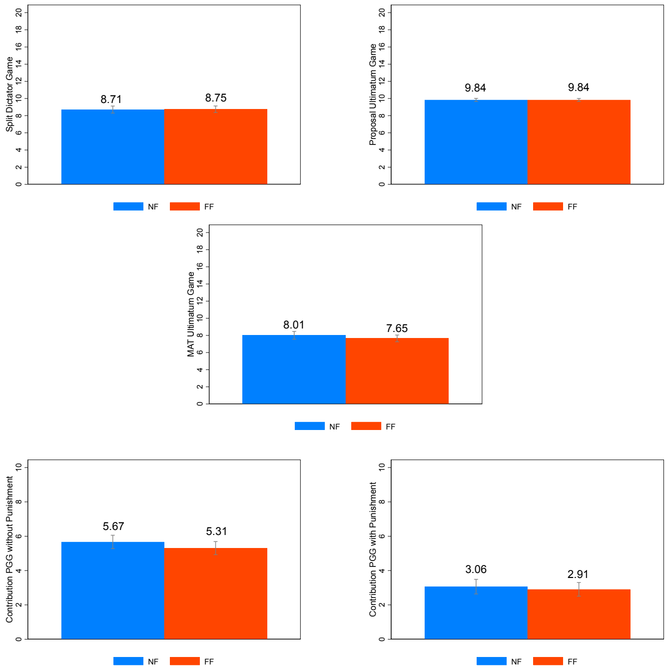

Figure 13: Results of the games of Study 2. Order of the results is the same as in Fig. 12.

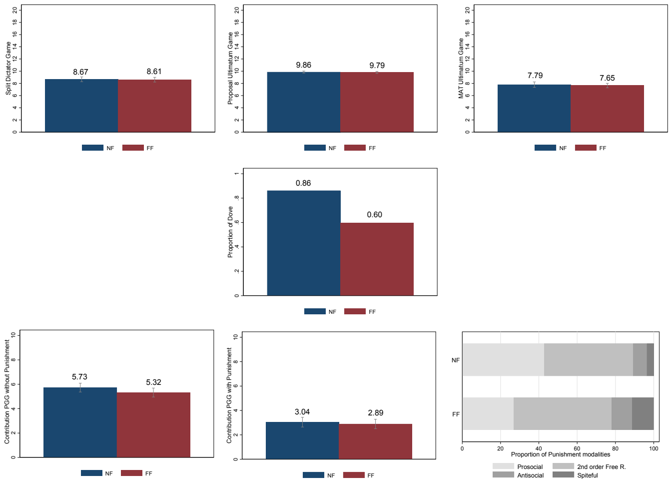

Figure 14: Results of the games (pooled data). Order of the results is the same as in Fig. 12, with the addition of the proportion of punishment modalities.

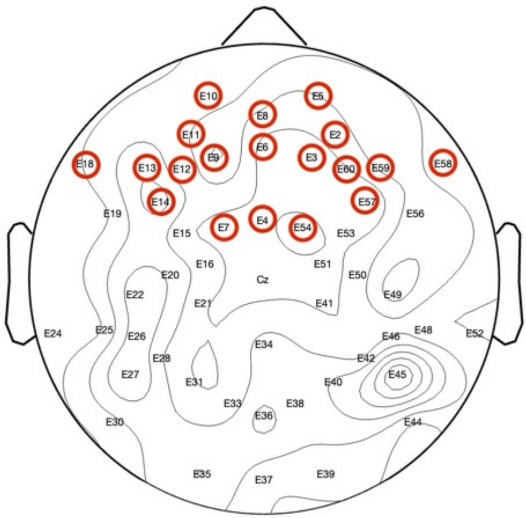

Figure 15: Cortical map of the electrodes used (red circles) to compute the additional analysis.

Table 8: Dictator Game splits - tobit estimates (Study 1)

	(1) Split DG	(2) Split DG	(3) Split DG	(4) Split DG
Condition	-1.178	-1.390	-0.0326	0.234
	[1.303]	[1.288]	[1.359]	[1.323]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	8.123***	13.09*	11.84*	17.75
	[0.918]	[4.856]	[4.587]	[10.89]
Observations	44	44	44	44

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets. The table displays tobit estimates for the dictator's split in the Dictator Game for Study 1. The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 9: Dictator Game splits - tobit estimates (Study 2)

	(1) Split DG	(2) Split DG	(3) Split DG	(4) Split DG
Condition	0.0561	0.0674	0.00247	0.0152
	[0.303]	[0.301]	[0.299]	[0.298]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	8.647***	8.123***	9.197***	9.032***
	[0.214]	[1.551]	[1.563]	[2.469]
Observations	403	403	399	399

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets. The table displays tobit estimates for the dictator's split in the Dictator Game for Study 2. The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 10: Dictator Game splits - tobit estimates (pooled data)

	(1) Split DG	(2) Split DG	(3) Split DG	(4) Split DG
Condition	-0.0562	-0.0540	-0.0887	-0.0707
	[0.302]	[0.299]	[0.297]	[0.296]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	8.596***	9.908***	10.63***	9.567***
	[0.213]	[1.326]	[1.327]	[2.355]
Observations	447	447	443	443

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Standard errors in brackets.

The table displays tobit estimates for the dictator's split in the Dictator Game (pooled data). The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 11: Ultimatum Game proposal - tobit estimates (Study 1)

	(1) Proposal UG	(2) Proposal UG	(3) Proposal UG	(4) Proposal UG
Condition	-0.636	-0.694*	-0.820*	-0.859*
	[0.335]	[0.333]	[0.350]	[0.348]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	10.05***	10.88***	11.39***	10.35**
	[0.237]	[1.254]	[1.180]	[2.853]
Observations	44	44	44	44

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets. The table displays tobit estimates for the sender's proposal in the Ultimatum Game for Study 1. The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 12: Ultimatum Game proposal - tobit estimates (Study 2)

	(1) Proposal UG	(2) Proposal UG	(3) Proposal UG	(4) Proposal UG
Condition	-0.000789	0.00185	-0.0293	-0.0306
	[0.126]	[0.125]	[0.123]	[0.123]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	9.837***	8.795***	9.084***	7.893***
	[0.0893]	[0.644]	[0.642]	[1.014]
Observations	403	403	399	399

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets. The table displays tobit estimates for the sender's proposal in the Ultimatum Game for Study 2. The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 13: Ultimatum Game proposal - tobit estimates (pooled data)

	(1) Proposal UG	(2) Proposal UG	(3) Proposal UG	(4) Proposal UG
Condition	-0.0634	-0.0588	-0.0787	-0.0757
	[0.119]	[0.118]	[0.116]	[0.116]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	9.857***	9.301***	9.485***	8.339***
	[0.0841]	[0.523]	[0.519]	[0.923]
Observations	447	447	443	443

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Standard errors in brackets.

The table displays tobit estimates for the sender's proposal in the Ultimatum Game (pooled data). The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 14: Ultimatum Game MAT - tobit estimates (Study 1)

	(1) MAT UG	(2) MAT UG	(3) MAT UG	(4) MAT UG
Condition	1.773	1.718	1.428	1.583
	[0.914]	[0.913]	[0.932]	[0.856]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	5.818***	5.210	5.693	10.30
	[0.647]	[3.442]	[3.145]	[7.019]
Observations	44	44	44	44

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets. The table displays tobit estimates for the receiver's minimum acceptance threshold (MAT) in the Ultimatum Game for Study 1. The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 15: Ultimatum Game MAT - tobit estimates (Study 2)

	(1) MAT UG	(2) MAT UG	(3) MAT UG	(4) MAT UG
Condition	-0.356	-0.352	-0.372	-0.385
	[0.315]	[0.311]	[0.313]	[0.312]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	7.982***	7.553***	8.047***	6.591*
	[0.222]	[1.601]	[1.632]	[2.580]
Observations	403	403	399	399

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets. The table displays tobit estimates for the receiver's minimum acceptance threshold (MAT) in the Ultimatum Game for Study 2. The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 16: Ultimatum Game MAT - tobit estimates (pooled data)

	(1) MAT UG	(2) MAT UG	(3) MAT UG	(4) MAT UG
Condition	-0.147	-0.138	-0.138	-0.153
	[0.301]	[0.297]	[0.299]	[0.298]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	7.769***	8.382***	8.676***	8.218***
	[0.213]	[1.316]	[1.334]	[2.369]
Observations	447	447	443	443

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Standard errors in brackets.

The table displays tobit estimates for the receiver's minimum acceptance threshold (MAT) in the Ultimatum Game (pooled data). The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 17: Hawk and Dove Game - probit estimates (Study 1)

	(1) Play Dove	(2) Play Dove	(3) Play Dove	(4) Play Dove
Condition	-1.327**	-1.331**	-1.831*	-1.881*
	[0.430]	[0.439]	[0.762]	[0.784]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	1.097**	0.710	1.796	-5.221
	[0.335]	[1.611]	[2.418]	[6.773]
Observations	44	44	44	44

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets. The table displays probit estimates for the "dove" action of the Hawk and Dove Game for Study 1. The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 18: Hawk and Dove Game - probit estimates (Study 2)

	(1) Play Dove	(2) Play Dove	(3) Play Dove	(4) Play Dove
Condition	-0.789***	-0.797***	-0.823***	-0.831***
	[0.142]	[0.143]	[0.146]	[0.147]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	1.087***	0.634	0.913	2.123
	[0.110]	[0.712]	[0.731]	[1.193]
Observations	403	403	399	399

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets. The table displays probit estimates for the "dove" action of the Hawk and Dove Game for Study 2. The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 19: Hawk and Dove Game - probit estimates (pooled data)

	(1) Play Dove	(2) Play Dove	(3) Play Dove	(4) Play Dove
Condition	-0.843***	-0.849***	-0.877***	-0.889***
	[0.135]	[0.136]	[0.139]	[0.139]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	1.088***	0.860	1.090	1.945
	[0.104]	[0.579]	[0.596]	[1.091]
Observations	447	447	443	443

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets.

The table displays probit estimates for the "dove" action of the Hawk and Dove Game (pooled data). The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 20: Public Goods Game without punishment contribution - tobit estimates (Study 1)

	(1) Contribution	(2) Contribution	(3) Contribution	(4) Contribution
Condition	-0.776	-0.842	0.183	0.307
	[0.901]	[0.896]	[0.881]	[0.862]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	6.469***	9.520**	8.264**	15.05*
	[0.637]	[3.346]	[2.934]	[7.021]
Observations	44	44	44	44

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets. The table displays tobit estimates for contribution to the Public Goods Game without punishment for Study 1. The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 21: Public Goods Game without punishment contribution - tobit estimates (Study 2)

	(1) Contribution	(2) Contribution	(3) Contribution	(4) Contribution
Condition	-0.397	-0.398	-0.438	-0.421
	[0.344]	[0.344]	[0.342]	[0.340]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	5.854***	4.537*	5.337**	5.022
	[0.244]	[1.777]	[1.789]	[2.820]
Observations	403	403	399	399

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets. The table displays tobit estimates for contribution to the Public Goods Game without punishment for Study 2. The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 22: Public Goods Game without punishment contribution - tobit estimates (pooled data)

(4)		(1)	(2)	(3)
	Contribution	Contribution	Contribution	Contribution
Condition	-0.433	-0.434	-0.443	-0.432
	[0.323]	[0.323]	[0.320]	[0.319]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	5.916***	5.084***	5.508***	5.190*
	[0.229]	[1.427]	[1.426]	[0.862]
Observations	447	447	443	443

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets.

The table displays tobit estimates for contribution to the Public Goods Game without punishment (pooled data). The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 23: Public Goods Game with punishment contribution - tobit estimates (Study 1)

	(1)	(2)	(3)	(4)
	Contribution	Contribution	Contribution	Contribution
Condition	-0.366	-0.0111	2.305	2.171
	[1.556]	[1.469]	[1.313]	[1.251]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	1.776	-4.863	-6.712	8.515
	[1.102]	[6.005]	[4.752]	[10.00]
Observations	44	44	44	44

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets. The table displays tobit estimates for the contributions to the Public Goods Game with a punishment option for Study 1. The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 24: Public Goods Game with punishment contribution - tobit estimates (Study 2)

	(1) Contribution	(2) Contribution	(3) Contribution	(4) Contribution
Condition	-0.290	-0.283	-0.307	-0.303
	[0.454]	[0.451]	[0.454]	[0.452]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	2.383***	3.252	3.511	-0.194
	[0.323]	[2.348]	[2.389]	[3.747]
Observations	403	403	399	399

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets. The table displays tobit estimates for the contributions to the Public Goods Game with a punishment option for Study 2. The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 25: Public Goods Game with punishment contribution - tobit estimates (pooled data)

	(1) Contribution	(2) Contribution	(3) Contribution	(4) Contribution
Condition	-0.293	-0.275	-0.260	-0.250
	[0.436]	[0.432]	[0.433]	[0.432]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	2.327***	1.853	2.056	-1.048
	[0.310]	[1.928]	[1.951]	[3.437]
Observations	447	447	443	443

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Standard errors in brackets.

The table displays tobit estimates for the contributions to the Public Goods Game with a punishment option (pooled data). The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 26: Prosocial punishment - probit estimates (Study 2)

	(1) ProPun	(2) ProPun	(3) ProPun	(4) ProPun
Prosocial Punishment				
Condition	-0.457***	-0.457***	-0.491***	-0.492***
	[0.130]	[0.130]	[0.133]	[0.134]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	-0.175*	-0.596	-0.612	-2.566*
	[0.0887]	[0.663]	[0.687]	[1.104]
Observations	403	403	399	399

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets. The table displays pobit estimates of the prosocial punishment option of the Public Goods Game for Study 2. The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 27: Antisocial punishment - probit estimates (Study 2)

	(1) AntiPun	(2) AntiPun	(3) AntiPun	(4) AntiPun
Antisocial Punishment				
Condition	0.181	0.182	0.177	0.192
	[0.174]	[0.175]	[0.177]	[0.180]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	-1.410***	-2.091*	-2.066*	0.178
	[0.129]	[0.848]	[0.880]	[1.500]
Observations	403	403	399	399

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets. The table displays pobit estimates of the antisocial punishment option of the Public Goods Game for Study 2. The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 28: Spiteful punishment - probit estimates (Study 2)

	(1) SpitePun	(2) SpitePun	(3) SpitePun	(4) SpitePun
Spiteful Punishment Condition	0.602**	0.613**	0.651**	0.646**
	[0.197]	[0.198]	[0.205]	[0.207]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	-1.755***	-1.620	-1.566	-0.638
	[0.161]	[0.962]	[1.008]	[1.590]
Observations	403	403	399	399

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets. The table displays pobit estimates of the spiteful punishment option of the Public Goods Game for Study 2. The demographics group variable includes age, education, and biological sex. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 29: Punishment modalities - multinomial logit estimates (Study 2)

	Free Rider	Prosocial	Antisocial	Spiteful
Free Rider				
(base outcome)				
Prosocial punisher				
Condition	-0.600**	-0.601**	-0.659**	-0.665**
	[0.226]	[0.227]	[0.233]	[0.235]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	-0.0450	-1.055	-0.991	-3.639
	[0.150]	[1.168]	[1.214]	[1.936]
Antisocial punisher				
Condition	0.214	0.214	0.185	0.225
	[0.359]	[0.360]	[0.363]	[0.367]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	-1.738***	-3.469*	-3.398	0.0491
	[0.271]	[1.718]	[1.759]	[2.945]
Spiteful punisher				
Condition	1.035*	1.040*	1.063*	1.044*
	[0.431]	[0.432]	[0.440]	[0.442]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	-2.431***	-2.752	-2.700	-1.526
	[0.369]	[1.972]	[2.043]	[3.260]
Observations	403	403	399	399

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Standard errors in brackets

The table displays multinomial logit estimates for each individual modality of punishment of Study 2 (prosocial punishment, second-order free-riding, antisocial punishment, and spiteful punishment), as compared with the reference category of second-order free-riding (i.e., not taking a punishment action). Individual probit estimates for each modality of punishment separately obtained qualitative equivalent results. The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Table 30: Punishment modalities - multinomial logit estimates (pooled data)

	Free Rider	Prosocial	Antisocial	Spiteful
Free Rider (base outcome)				
Prosocial punisher				
Condition	-0.562**	-0.559**	-0.579**	-0.577**
	[0.213]	[0.214]	[0.219]	[0.220]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	-0.0800	-1.295	-1.304	-2.815
	[0.142]	[0.933]	[0.964]	[1.734]
Antisocial punisher				
Condition	0.314	0.315	0.314	0.345
	[0.350]	[0.350]	[0.353]	[0.356]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	-1.872***	-2.425	-2.413	1.136
	[0.269]	[1.516]	[1.534]	[2.789]
Spiteful punisher				
Condition	1.048*	1.047*	1.097*	1.085*
	[0.428]	[0.428]	[0.437]	[0.438]
Demographics	NO	YES	YES	YES
Preferences	NO	NO	YES	YES
Questionnaires	NO	NO	NO	YES
Constant	-2.565***	-1.525	-1.460	-0.0677
	[0.367]	[1.791]	[1.865]	[3.183]
Observations	447	447	443	443

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Standard errors in brackets

The table displays multinomial logit estimates for each individual modality of punishment (prosocial punishment, second-order free-riding, antisocial punishment, and spiteful punishment), as compared with the reference category of second-order free-riding (i.e., not taking a punishment action). Individual probit estimates for each modality of punishment separately obtained qualitative equivalent results (pooled data). The preferences group variable includes Falk's et al. (2018) Global Preferences survey proxies for altruism, positive reciprocity, negative reciprocity, trust, patience, and risk aversion. Finally, the questionnaire group variable includes the Buss-Perry Aggression Questionnaire, the Interpersonal Reactivity Index, and Barratt's Impulsiveness Scale.

Appendix B

Appendix: Community enforcement can help sustain cooperation even in the presence of antisocial punishment

B.1 Payoffs for Evolutionary Stable Strategies

For $(1,0,0,0)$, we have that $\pi_{g,g}[(1,0,0,0),(1,0,0,0)] = b - c$ when incumbent, while when mutant, the other strategies get:

$$\pi_{r,g}[(1,1,0,0),(1,0,0,0)] = \pi_{r,g}[(1,1,0,1),(1,0,0,0)] =$$

$$= \pi_{r,g}[(1,0,0,1),(1,0,0,0)] = -c$$

$$\pi_{r,g}[(0,1,0,0),(1,0,0,0)] = \pi_{r,g}[(0,0,0,0),(1,0,0,0)] =$$

$$= \pi_{r,g}[(0,1,0,1),(1,0,0,0)] = \pi_{r,g}[(0,0,0,1),(1,0,0,0)] = 0$$

$$\pi_{r,g}[(1,1,1,0),(1,0,0,0)] = \pi_{r,g}[(1,0,1,0),(1,0,0,0)] =$$

$$= \pi_{r,g}[(1,1,1,1),(1,0,0,0)] = \pi_{r,g}[(1,0,1,1),(1,0,0,0)] = -c - \alpha$$

$$\pi_{r,g}[(0,1,1,0),(1,0,0,0)] = \pi_{r,g}[(0,0,1,0),(1,0,0,0)] =$$

$$= \pi_{r,g}[(0,1,1,1),(1,0,0,0)] = \pi_{r,g}[(0,0,1,1),(1,0,0,0)] = -\alpha$$

And hence, $(1,0,0,0)$ is Evolutionary Stable.

For $(1,0,1,0)$, we have that $\pi_{g,g}[(1,0,1,0,),(1,0,1,0)] = b-c-\alpha-\gamma$ when incumbent, while when mutant, the other strategies get:

$$\pi_{r,g}[(1,1,0,0),(1,0,1,0)] = \pi_{r,g}[(1,0,0,0),(1,0,1,0)] =$$

$$= \pi_{r,g}[(1,1,0,1),(1,0,1,0)] = \pi_{r,g}[(1,0,0,1),(1,0,1,0)] = -c$$

$$\pi_{r,g}[(0,1,0,0),(1,0,1,0)] = \pi_{r,g}[(0,0,0,0),(1,0,1,0)] =$$

$$= \pi_{r,g}[(0,1,0,1),(1,0,1,0)] = \pi_{r,g}[(0,0,0,1),(1,0,1,0)] = 0$$

$$\pi_{r,g}[(1,1,1,0),(1,0,1,0)] = \pi_{r,g}[(1,1,1,1),(1,0,1,0)] =$$

$$= \pi_{r,g}[(1,0,1,1),(1,0,1,0)] = -c-\alpha$$

$$\pi_{r,g}[(0,1,1,0),(1,0,1,0)] = \pi_{r,g}[(0,0,1,0),(1,0,1,0)] =$$

$$= \pi_{r,g}[(0,1,1,1),(1,0,1,0)] = \pi_{r,g}[(0,0,1,1),(1,0,1,0)] = -\alpha$$

And hence, $(1,0,1,0)$ is Evolutionary Stable if $b > c+\alpha+\gamma$.

For $(1,1,0,1)$, we have that $\pi_{g,g}[(1,1,0,1),(1,1,0,1)] = b-c$ when incumbent, while when mutant, the other strategies get:

$$\pi_{r,g}[(1,1,0,0),(1,1,0,1)] = \pi_{r,g}[(1,0,0,0),(1,1,0,1)] =$$

$$= \pi_{r,g}[(1,0,0,1),(1,1,0,1)] = b-c-\alpha$$

$$\pi_{r,g}[(0,1,0,0),(1,1,0,1)] = \pi_{r,g}[(0,0,0,0),(1,1,0,1)] =$$

$$= \pi_{r,g}[(0,1,0,1),(1,1,0,1)] = \pi_{r,g}[(0,0,0,1),(1,1,0,1)] = b-\gamma$$

$$\pi_{r,g}[(1,1,1,0),(1,1,0,1)] = \pi_{r,g}[(1,0,1,0),(1,1,0,1)] =$$

$$= \pi_{r,g}[(1,1,1,1),(1,1,0,1)] = \pi_{r,g}[(1,0,1,1),(1,1,0,1)] = b-c-\alpha-\gamma$$

$$\pi_{r,g}[(0,1,1,0),(1,1,0,1)] = \pi_{r,g}[(0,0,1,0),(1,1,0,1)] =$$

$$= \pi_{r,g}[(0,1,1,1),(1,1,0,1)] = \pi_{r,g}[(0,0,1,1),(1,1,0,1)] = b - \alpha - \gamma$$

And hence, $(1,1,0,1)$ is Evolutionary Stable if $\gamma > c$.

For $(1,0,0,1)$, we have that $\pi_{g,g}[(1,0,0,1),(1,0,0,1)] = b - c$ when incumbent, while when mutant, the other strategies get:

$$\pi_{r,g}[(1,1,0,0),(1,0,0,1)] = \pi_{r,g}[(1,0,0,0),(1,0,0,1)] =$$

$$= \pi_{r,g}[(1,1,0,1),(1,0,0,1)] = -c - \gamma$$

$$\pi_{r,g}[(0,1,0,0),(1,0,0,1)] = \pi_{r,g}[(0,0,0,0),(1,0,0,1)] =$$

$$= \pi_{r,g}[(0,1,0,1),(1,0,0,1)] = \pi_{r,g}[(0,0,0,1),(1,0,0,1)] = -\gamma$$

$$\pi_{r,g}[(1,1,1,0),(1,0,0,1)] = \pi_{r,g}[(1,0,1,0),(1,0,0,1)] =$$

$$= \pi_{r,g}[(1,1,1,1),(1,0,0,1)] = \pi_{r,g}[(1,0,1,1),(1,0,0,1)] = -c - \alpha - \gamma$$

$$\pi_{r,g}[(0,1,1,0),(1,0,0,1)] = \pi_{r,g}[(0,0,1,0),(1,0,0,1)] =$$

$$= \pi_{r,g}[(0,1,1,1),(1,0,0,1)] = \pi_{r,g}[(0,0,1,1),(1,0,0,1)] = -\alpha - \gamma$$

And hence, $(1,0,0,1)$ is Evolutionary Stable.

For $(0,1,0,1)$, we have that $\pi_{g,g}[(0,1,0,1),(0,1,0,1)] = 0$ when incumbent, while when mutant, the other strategies get:

$$\pi_{r,g}[(1,1,0,0),(0,1,0,1)] = \pi_{r,g}[(1,0,0,0),(0,1,0,1)] =$$

$$= \pi_{r,g}[(1,1,0,1),(0,1,0,1)] = \pi_{r,g}[(1,0,0,1),(0,1,0,1)] = b - c - \gamma$$

$$\pi_{r,g}[(0,1,0,0),(0,1,0,1)] = \pi_{r,g}[(0,0,0,0),(0,1,0,1)] =$$

$$= \pi_{r,g}[(0,0,0,1),(0,1,0,1)] = b - \gamma$$

$$\pi_{r,g}[(1,1,1,0),(0,1,0,1)] = \pi_{r,g}[(1,0,1,0),(0,1,0,1)] =$$

$$= \pi_{r,g}[(1,1,1,1),(0,1,0,1)] = \pi_{r,g}[(1,0,1,1),(0,1,0,1)] = b - c - \alpha - \gamma$$

$$\pi_{r,g}[(0,1,1,0),(0,1,0,1)] = \pi_{r,g}[(0,0,1,0),(0,1,0,1)] =$$

$$= \pi_{r,g}[(0,1,1,1),(0,1,0,1)] = \pi_{r,g}[(0,0,1,1),(0,1,0,1)] = b - \alpha - \gamma$$

And hence, $(0,1,0,1)$ is Evolutionary Stable if $\gamma > b$.

For $(0,0,0,1)$, we have that $\pi_{g,g}[(0,0,0,1),(0,0,0,1)] = 0$ when incumbent, while when mutant, the other strategies get:

$$\pi_{r,g}[(1,1,0,0),(0,0,0,1)] = \pi_{r,g}[(1,0,0,0),(0,0,0,1)] =$$

$$= \pi_{r,g}[(1,1,0,1),(0,0,0,1)] = \pi_{r,g}[(1,0,0,1),(0,0,0,1)] = -c-\gamma$$

$$\pi_{r,g}[(0,1,0,0),(0,0,0,1)] = \pi_{r,g}[(0,0,0,0),(0,0,0,1)] =$$

$$= \pi_{r,g}[(0,1,0,1),(0,0,0,1)] = -\gamma$$

$$\pi_{r,g}[(1,1,1,0),(0,0,0,1)] = \pi_{r,g}[(1,0,1,0),(0,0,0,1)] =$$

$$= \pi_{r,g}[(1,1,1,1),(0,0,0,1)] = \pi_{r,g}[(1,0,1,1),(0,0,0,1)] = -c-\alpha-\gamma$$

$$\pi_{r,g}[(0,1,1,0),(0,0,0,1)] = \pi_{r,g}[(0,0,1,0),(0,0,0,1)] =$$

$$= \pi_{r,g}[(0,1,1,1),(0,0,0,1)] = \pi_{r,g}[(0,0,1,1),(0,0,0,1)] = -\alpha-\gamma$$

And hence, $(0,0,0,1)$ is Evolutionary Stable.

For $(1,0,1,1)$, we have that $\pi_{g,g}[(1,0,1,1),(1,0,1,1)] = b-c-\alpha-\gamma$ when incumbent, while when mutant, the other strategies get:

$$\pi_{r,g}[(1,1,0,0),(1,0,1,1)] = \pi_{r,g}[(1,0,0,0),(1,0,1,1)] =$$

$$= \pi_{r,g}[(1,1,0,1),(1,0,1,1)] = \pi_{r,g}[(1,0,0,1),(1,0,1,1)] = -c-\gamma$$

$$\pi_{r,g}[(0,1,0,0),(1,0,1,1)] = \pi_{r,g}[(0,0,0,0),(1,0,1,1)] =$$

$$= \pi_{r,g}[(0,1,0,1),(1,0,1,1)] = \pi_{r,g}[(0,0,0,1),(1,0,1,1)] = -\gamma$$

$$\pi_{r,g}[(1,1,1,0),(1,0,1,1)] = \pi_{r,g}[(1,0,1,0),(1,0,1,1)] =$$

$$= \pi_{r,g}[(1,1,1,1),(1,0,1,1)] = -c-\alpha-\gamma$$

$$\pi_{r,g}[(0,1,1,0),(1,0,1,1)] = \pi_{r,g}[(0,0,1,0),(1,0,1,1)] =$$

$$= \pi_{r,g}[(0,1,1,1),(1,0,1,1)] = \pi_{r,g}[(0,0,1,1),(1,0,1,1)] = -\alpha-\gamma$$

And hence, $(1,0,1,1)$ is Evolutionary Stable if $b > c+\alpha$.

B.2 $(0,0,0,0)$ always gets the maximum payoff within the red strategies in a transition path

The proof for this claim is quite straightforward since it suffices to compare the payoffs when red of $(0,0,0,0)$ with the payoffs of any other strategy when red. In any transition path, monomorphic or polymorphic, any red strategy, no matter its form, gains the benefits and suffers the costs imposed by the green strategies. At the same time, first, any strategy of the form $(1,\cdot,\cdot,\cdot)$ suffers the cost of cooperating with the green strategies, while $(0,0,0,0)$ does not. Second, any strategy of the form $(\cdot,1,\cdot,\cdot)$ suffers the cost of cooperating with red strategies and receives the benefits of cooperating with itself, while $(0,0,0,0)$ does not suffer the cost while receiving the benefits of cooperation from $(\cdot,1,\cdot,\cdot)$ strategies. Third, any strategy of the form $(\cdot,\cdot,1,\cdot)$ suffers the costs of punishing the green strategies, while $(0,0,0,0)$ does not. Fourth and final, any strategy of the form $(\cdot,\cdot,\cdot,1)$ suffers both the costs of punishing the red strategies and punishing itself, while $(0,0,0,0)$ suffers the costs of punishment from $(\cdot,\cdot,\cdot,1)$ while never receiving the costs of punishing. Hence, in any transition path, $(0,0,0,0)$ will receive the maximum payoff and its best reply is triggered towards any red strategy; it will be precisely $(0,0,0,0)$.

B.3 Bounds for polymorphic transition paths for Lemmas 3, 4, 5, 6, and 7

For $(1,0,0,0)$

$$\underline{\Pi}_{g,r}[(1,0,0,0),s^M] = \lim_{p\to\frac{1}{2}} p\pi_{g,g}[(1,0,0,0),(1,0,0,0)]+$$

$$+(1-p)\pi_{g,r}[(1,0,0,0),(0,1,1,0)] = \lim_{p\to\frac{1}{2}} p(b-c)+(1-p)(-\gamma) = \frac{1}{2}(b-c-\gamma)$$

$$\overline{\Pi}_{r,g}[(1,0,1,0),(1,0,0,0)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(1,0,1,0),(1,0,0,0)]+$$

$$+\psi_0\pi_{r,r}[(1,0,1,0),(1,0,1,0)]+(1-p)\pi_{r,r}[(1,0,1,0),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}} p(-c-\alpha)+(1-p)b=\frac{1}{2}(b-c-\alpha)$$

$$\overline{\Pi}_{r,g}[(1,1,0,1),(1,0,0,0)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to 0+} p\pi_{r,g}[(1,1,0,1),(1,0,0,0)]+$$

$$+\psi_0\pi_{r,r}[(1,1,0,1),(1,1,0,1)]+(1-p)\pi_{r,r}[(1,1,0,1),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}} p(-c)+(1-p)(b-c-\alpha)=\frac{1}{2}(b-\alpha)-c$$

$$\overline{\Pi}_{r,g}[(1,0,0,1),(1,0,0,0)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to 0+} p\pi_{r,g}[(1,0,0,1),(1,0,0,0)]+$$

$$+\psi_0\pi_{r,r}[(1,0,0,1),(1,0,0,1)]+(1-p)\pi_{r,r}[(1,0,0,1),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}} p(-c)+(1-p)(b-\alpha)=\frac{1}{2}(b-c-\alpha)$$

$$\overline{\Pi}_{r,g}[(0,1,0,1),(1,0,0,0)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to 0+} p\pi_{r,g}[(0,1,0,1),(1,0,0,0)]+$$

$$+\psi_0\pi_{r,r}[(0,1,0,1),(0,1,0,1)]+(1-p)\pi_{r,r}[(0,1,0,1),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}}(1-p)(b-c-\alpha)=\frac{1}{2}(b-c-\alpha)$$

$$\overline{\Pi}_{r,g}[(0,0,0,1),(1,0,0,0)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to 0+} p\pi_{r,g}[(0,0,0,1),(1,0,0,0)]+$$

$$+\psi_0\pi_{r,r}[(0,0,0,1),(0,0,0,1)]+(1-p)\pi_{r,r}[(0,0,0,1),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}}(1-p)(b-\alpha)=\frac{1}{2}(b-\alpha)$$

$$\overline{\Pi}_{r,g}[(1,0,1,1),(1,0,0,0)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to 0+} p\pi_{r,g}[(1,0,1,1),(1,0,0,0)]+$$

$$+\psi_0\pi_{r,r}[(1,0,1,1),(1,0,1,1)]+(1-p)\pi_{r,r}[(1,0,1,1),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}} p(-c-\alpha)+(1-p)(b-\alpha)=\frac{1}{2}(b-c)-\alpha$$

For $(1,0,1,0)$

$$\underline{\Pi}_{g,r}[(1,0,1,0),s^M]=\lim_{p\to\frac{1}{2}} p\pi_{g,g}[(1,0,1,0),(1,0,1,0)]+$$

$$+(1-p)\pi_{g,r}[(1,0,1,0),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}} p(b-c-\alpha-\gamma)+(1-p)(-\gamma)=\frac{1}{2}(b-c-\alpha)-\gamma$$

$$\overline{\Pi}_{r,g}[(1,0,0,0),(1,0,1,0)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to0^+} p\pi_{r,g}[(1,0,0,0),(1,0,1,0)]+$$

$$+\psi_0\pi_{r,r}[(1,0,0,0),(1,0,0,0)]+(1-p)\pi_{r,r}[(1,0,0,0),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}} p(-c)+(1-p)b=\frac{1}{2}(b-c)$$

$$\overline{\Pi}_{r,g}[(1,1,0,1),(1,0,1,0)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to0^+} p\pi_{r,g}[(1,1,0,1),(1,0,1,0)]+$$

$$+\psi_0\pi_{r,r}[(1,1,0,1),(1,1,0,1)]+(1-p)\pi_{r,r}[(1,1,0,1),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}} p(-c)+(1-p)(b-c-\alpha)=\frac{1}{2}(b-\alpha)-c$$

$$\overline{\Pi}_{r,g}[(1,0,0,1),(1,0,1,0)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to0^+} p\pi_{r,g}[(1,0,0,1),(1,0,1,0)]+$$

$$+\psi_0\pi_{r,r}[(1,0,0,1),(1,0,0,1)]+(1-p)\pi_{r,r}[(1,0,0,1),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}} p(-c)+(1-p)(b-\alpha)=\frac{1}{2}(b-c-\alpha)$$

$$\overline{\Pi}_{r,g}[(0,1,0,1),(1,0,1,0)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to0^+} p\pi_{r,g}[(0,1,0,1),(1,0,1,0)]+$$

$$+\psi_0\pi_{r,r}[(0,1,0,1),(0,1,0,1)]+(1-p)\pi_{r,r}[(0,1,0,1),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}}(1-p)(b-c-\alpha)=\frac{1}{2}(b-c-\alpha)$$

$$\overline{\Pi}_{r,g}[(0,0,0,1),(1,0,1,0)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+}p\pi_{r,g}[(0,0,0,1),(1,0,1,0)]+$$

$$+\psi_0\pi_{r,r}[(0,0,0,1),(0,0,0,1)]+(1-p)\pi_{r,r}[(0,0,0,1),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}}(1-p)(b-\alpha)=\frac{1}{2}(b-\alpha)$$

$$\overline{\Pi}_{r,g}[(1,0,1,1),(1,0,1,0)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+}p\pi_{r,g}[(1,0,1,1),(1,0,1,0)]+$$

$$+\psi_0\pi_{r,r}[(1,0,1,1),(1,0,1,1)]+(1-p)\pi_{r,r}[(1,0,1,1),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}}p(-c-\alpha)+(1-p)(b-\alpha)=\frac{1}{2}(b-c)-\alpha$$

For $(1,1,0,1)$

$$\underline{\Pi}_{g,r}[(1,1,0,1),s^M]=\lim_{p\to\frac{1}{2}}p\pi_{g,g}[(1,1,0,1),(1,1,0,1)]+$$

$$+(1-p)\pi_{g,r}[(1,1,0,1),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}}p(b-c)+(1-p)(-c-\alpha-\gamma)=\frac{1}{2}(b-\alpha-\gamma)-c$$

$$\overline{\Pi}_{r,g}[(1,0,0,0),(1,1,0,1)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+}p\pi_{r,g}[(1,0,0,0),(1,1,0,1)]+$$

$$+\psi_0\pi_{r,r}[(1,0,0,0),(1,0,0,0)]+(1-p)\pi_{r,r}[(1,0,0,0),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}}p(b-c-\gamma)+(1-p)b=b+\frac{1}{2}(-c-\gamma)$$

$$\overline{\Pi}_{r,g}[(1,0,1,0),(1,1,0,1)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+}p\pi_{r,g}[(1,0,1,0),(1,1,0,1)]+$$

$$+\psi_0\pi_{r,r}[(1,0,1,0),(1,0,1,0)]+(1-p)\pi_{r,r}[(1,0,1,0),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}} p(b-c-\alpha-\gamma)+(1-p)b=b+\frac{1}{2}(-c-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[(1,0,0,1),(1,1,0,1)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(1,0,0,1),(1,1,0,1)]+$$

$$+\psi_0\pi_{r,r}[(1,0,0,1),(1,0,0,1)]+(1-p)\pi_{r,r}[(1,0,0,1),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}} p(b-c-\gamma)+(1-p)(b-\alpha)=b+\frac{1}{2}(-c-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[(0,1,0,1),(1,1,0,1)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(0,1,0,1),(1,1,0,1)]+$$

$$+\psi_0\pi_{r,r}[(0,1,0,1),(0,1,0,1)]+(1-p)\pi_{r,r}[(0,1,0,1),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}} p(b-\gamma)+(1-p)(b-c-\alpha)=b+\frac{1}{2}(-c-\alpha-\gamma)+\frac{1}{2}(b-c-\alpha)$$

=

$$\overline{\Pi}_{r,g}[(0,0,0,1),(1,1,0,1)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(0,0,0,1),(1,1,0,1)]+$$

$$+\psi_0\pi_{r,r}[(0,0,0,1),(0,0,0,1)]+(1-p)\pi_{r,r}[(0,0,0,1),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}} p(b-\gamma)+(1-p)(b-\alpha)=b+\frac{1}{2}(-\alpha-\gamma)$$

=

$$\overline{\Pi}_{r,g}[(1,0,1,1),(1,1,0,1)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(1,0,1,1),(1,1,0,1)]+$$

$$+\psi_0\pi_{r,r}[(1,0,1,1),(1,0,1,1)]+(1-p)\pi_{r,r}[(1,0,1,1),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}} p(b-c-\alpha-\gamma)+(1-p)(b-\alpha)=b-\alpha\frac{1}{2}(-c-\gamma)+\frac{1}{2}(b-\alpha)$$

For $(1,0,0,1)$

$$\underline{\Pi}_{g,r}[(1,0,0,1),s^M] = \lim_{p\to\frac{1}{2}} p\pi_{g,g}[(1,0,0,1),(1,0,0,1)]+$$

$$+(1-p)\pi_{g,r}[(1,0,0,1),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(b-c)+(1-p)(-\alpha-\gamma) = \frac{1}{2}(b-c-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[(1,0,0,0),(1,0,0,1)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(1,0,0,0),(1,0,0,1)]+$$

$$+\psi_0\pi_{r,r}[(1,0,0,0),(1,0,0,0)] + (1-p)\pi_{r,r}[(1,0,0,0),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(-c-\gamma)+(1-p)b = \frac{1}{2}(b-c-\gamma)$$

$$\overline{\Pi}_{r,g}[(1,0,1,0),(1,0,0,1)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(1,0,1,0),(1,0,0,1)]+$$

$$+\psi_0\pi_{r,r}[(1,0,1,0),(1,0,1,0)] + (1-p)\pi_{r,r}[(1,0,1,0),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(-c-\alpha-\gamma)+(1-p)b = \frac{1}{2}(b-c-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[(1,1,0,1),(1,0,0,1)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(1,1,0,1),(1,0,0,1)]+$$

$$+\psi_0\pi_{r,r}[(1,1,0,1),(1,1,0,1)] + (1-p)\pi_{r,r}[(1,1,0,1),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(-c-\gamma)+(1-p)(b-\alpha) = \frac{1}{2}(b-\alpha-\gamma)-c$$

$$\overline{\Pi}_{r,g}[(0,1,0,1),(1,0,0,1)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(0,1,0,1),(1,0,0,1)]+$$

$$+\psi_0\pi_{r,r}[(0,1,0,1),(0,1,0,1)] + (1-p)\pi_{r,r}[(0,1,0,1),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(-\gamma)+(1-p)(b-c-\alpha) = \frac{1}{2}(b-c-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[(0,0,0,1),(1,0,0,1)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(0,0,0,1),(1,0,0,1)]+$$

$$+\psi_0\pi_{r,r}[(0,0,0,1),(0,0,0,1)] + (1-p)\pi_{r,r}[(0,0,0,1),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(-\gamma) + (1-p)(b-\alpha) = \frac{1}{2}(b-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[(1,0,1,1),(1,0,0,1)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(1,0,1,1),(1,0,0,1)]+$$

$$+\psi_0\pi_{r,r}[(1,0,1,1),(1,0,1,1)] + (1-p)\pi_{r,r}[(1,0,1,1),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(-c-\alpha-\gamma) + (1-p)(b-\alpha) = \frac{1}{2}(b-c-\gamma)-\alpha$$

For $(0,1,0,1)$

$$\underline{\Pi}_{g,r}[(0,1,0,1),s^M] = \lim_{p\to\frac{1}{2}} p\pi_{g,g}[(0,1,0,1),(0,1,0,1)]+$$

$$+(1-p)\pi_{g,r}[(0,1,0,1),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} (1-p)(-c-\alpha-\gamma) = \frac{1}{2}(-c-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[(1,0,0,0),(0,1,0,1)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(1,0,0,0),(0,1,0,1)]+$$

$$+\psi_0\pi_{r,r}[(1,0,0,0),(1,0,0,0)] + (1-p)\pi_{r,r}[(1,0,0,0),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(b-c-\gamma) + (1-p)b = b + \frac{1}{2}(-c-\gamma)$$

$$\overline{\Pi}_{r,g}[(1,0,1,0),(0,1,0,1)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(1,0,1,0),(0,1,0,1)]+$$

$$+\psi_0\pi_{r,r}[(1,0,1,0),(1,0,1,0)] + (1-p)\pi_{r,r}[(1,0,1,0),(0,1,1,0)] =$$

$$= \lim_{p \to \frac{1}{2}} p(b-c-\alpha-\gamma) + (1-p)b = b + \frac{1}{2}(-c-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[(1,1,0,1),(0,1,0,1)] = \lim_{p \to \frac{1}{2}^+, \psi_0 \to 0^+} p\pi_{r,g}[(1,1,0,1),(0,1,0,1)] +$$

$$+\psi_0 \pi_{r,r}[(1,1,0,1),(1,1,0,1)] + (1-p)\pi_{r,r}[(1,1,0,1),(0,1,1,0)] =$$

$$= \lim_{p \to \frac{1}{2}} p(b-c-\gamma) + (1-p)(b-c-\alpha) = b - c + \frac{1}{2}(-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[(1,0,0,1),(0,1,0,1)] = \lim_{p \to \frac{1}{2}^+, \psi_0 \to 0^+} p\pi_{r,g}[(1,0,0,1),(0,1,0,1)] +$$

$$+\psi_0 \pi_{r,r}[(1,0,0,1),(1,0,0,1)] + (1-p)\pi_{r,r}[(1,0,0,1),(0,1,1,0)] =$$

$$= \lim_{p \to \frac{1}{2}} p(b-c-\gamma) + (1-p)(b-\alpha) = b + \frac{1}{2}(-c-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[(0,0,0,1),(0,1,0,1)] = \lim_{p \to \frac{1}{2}^+, \psi_0 \to 0^+} p\pi_{r,g}[(0,0,0,1),(0,1,0,1)] +$$

$$+\psi_0 \pi_{r,r}[(0,0,0,1),(0,0,0,1)] + (1-p)\pi_{r,r}[(0,0,0,1),(0,1,1,0)] =$$

$$= \lim_{p \to \frac{1}{2}} p(b-\gamma) + (1-p)(b-\alpha) = b + \frac{1}{2}(-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[(1,0,1,1),(0,1,0,1)] = \lim_{p \to \frac{1}{2}^+, \psi_0 \to 0^+} p\pi_{r,g}[(1,0,1,1),(0,1,0,1)] +$$

$$+\psi_0 \pi_{r,r}[(1,0,1,1),(1,0,1,1)] + (1-p)\pi_{r,r}[(1,0,1,1),(0,1,1,0)] =$$

$$= \lim_{p \to \frac{1}{2}} p(b-c-\alpha-\gamma) + (1-p)(b-\alpha) = b - \alpha + \frac{1}{2}(-c-\gamma)$$

For $(0,0,0,1)$

$$\underline{\Pi}_{g,r}[(0,0,0,1),s^M] = \lim_{p \to \frac{1}{2}} p\pi_{g,g}[(0,0,0,1),(0,0,0,1)] +$$

$$+(1-p)\pi_{g,r}[(0,0,0,1),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}}(1-p)(-\alpha-\gamma) = \frac{1}{2}(-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[(1,0,0,0),(0,0,0,1)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(1,0,0,0),(0,0,0,1)]+$$

$$+\psi_0\pi_{r,r}[(1,0,0,0),(1,0,0,0)] + (1-p)\pi_{r,r}[(1,0,0,0),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(-c-\gamma) + (1-p)b = \frac{1}{2}(b-c-\gamma)$$

$$\overline{\Pi}_{r,g}[(1,0,1,0),(0,0,0,1)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(1,0,1,0),(0,0,0,1)]+$$

$$+\psi_0\pi_{r,r}[(1,0,1,0),(1,0,1,0)] + (1-p)\pi_{r,r}[(1,0,1,0),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(-c-\alpha-\gamma) + (1-p)b = \frac{1}{2}(b-c-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[(1,1,0,1),(0,0,0,1)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(1,1,0,1),(0,0,0,1)]+$$

$$+\psi_0\pi_{r,r}[(1,1,0,1),(1,1,0,1)] + (1-p)\pi_{r,r}[(1,1,0,1),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(-c-\gamma) + (1-p)(b-c-\alpha) = \frac{1}{2}(b-\alpha-\gamma) - c$$

$$\overline{\Pi}_{r,g}[(1,0,0,1),(0,0,0,1)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(1,0,0,1),(0,0,0,1)]+$$

$$+\psi_0\pi_{r,r}[(1,0,0,1),(1,0,0,1)] + (1-p)\pi_{r,r}[(1,0,0,1),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(-c-\gamma) + (1-p)(b-\alpha) = \frac{1}{2}(b-c-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[(0,1,0,1),(0,0,0,1)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(0,1,0,1),(0,0,0,1)]+$$

$$+\psi_0\pi_{r,r}[(0,1,0,1),(0,1,0,1)]+(1-p)\pi_{r,r}[(0,1,0,1),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}} p(-\gamma)+(1-p)(b-c-\alpha)=\frac{1}{2}(b-c-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[(1,0,1,1),(0,0,0,1)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(1,0,1,1),(0,0,0,1)]+$$

$$*\psi_0\pi_{r,r}[(1,0,1,1),(1,0,1,1)]+(1-p)\pi_{r,r}[(1,0,1,1),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}} p(-c-\alpha-\gamma)+(1-p)(b-\alpha)=\frac{1}{2}(b-c-\gamma)-\alpha$$

For $(1,0,1,1)$

$$\underline{\Pi}_{g,r}[(1,0,1,1),s^M]=\lim_{p\to\frac{1}{2}} p\pi_{g,g}[(1,0,1,1),(1,0,1,1)]+$$

$$+(1-p)\pi_{g,r}[(1,0,1,1),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}} p(b-c-\alpha-\gamma)+(1-p)(-\alpha-\gamma)=\frac{1}{2}(b-c)-\alpha-\gamma$$

$$\overline{\Pi}_{r,g}[(1,0,0,0),(1,0,1,1)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(1,0,0,0),(1,0,1,1)]+$$

$$+\psi_0\pi_{r,r}[(1,0,0,0),(1,0,0,0)]+(1-p)\pi_{r,r}[(1,0,0,0),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}} p(-c-\gamma)+(1-p)b=\frac{1}{2}(b-c-\gamma)$$

$$\overline{\Pi}_{r,g}[(1,0,1,0),(1,0,1,1)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(1,0,1,0),(1,0,1,1)]+$$

$$+\psi_0\pi_{r,r}[(1,0,1,0),(1,0,1,0)]+(1-p)\pi_{r,r}[(1,0,1,0),(0,1,1,0)]=$$

$$=\lim_{p\to\frac{1}{2}} p(-c-\alpha-\gamma)+(1-p)b=\frac{1}{2}(b-c-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[(1,1,0,1),(1,0,1,1)]=\lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(1,1,0,1),(1,0,1,1)]+$$

$$+\psi_0\pi_{r,r}[(1,1,0,1),(1,1,0,1)] + (1-p)\pi_{r,r}[(1,1,0,1),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(-c-\gamma) + (1-p)(b-c-\alpha) = \frac{1}{2}(b-\alpha-\gamma) - c$$

$$\overline{\Pi}_{r,g}[(1,0,0,1),(1,0,1,1)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(1,0,0,1),(1,0,1,1)]+$$

$$+\psi_0\pi_{r,r}[(1,0,0,1),(1,0,0,1)] + (1-p)\pi_{r,r}[(1,0,0,1),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(-c-\gamma) + (1-p)(b-\alpha) = \frac{1}{2}(b-c-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[(0,1,0,1),(1,0,1,1)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(0,1,0,1),(1,0,1,1)]+$$

$$+\psi_0\pi_{r,r}[(0,1,0,1),(0,1,0,1)] + (1-p)\pi_{r,r}[(0,1,0,1),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(-\gamma) + (1-p)(b-c-\alpha) = \frac{1}{2}(b-c-\alpha-\gamma)$$

$$\overline{\Pi}_{r,g}[(0,0,0,1),(1,0,1,1)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(0,0,0,1),(1,0,1,1)]+$$

$$+\psi_0\pi_{r,r}[(0,0,0,1),(0,0,0,1)] + (1-p)\pi_{r,r}[(0,0,0,1),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(-\gamma) + (1-p)(b-\alpha) = \frac{1}{2}(b-\alpha-\gamma)$$

B.4 Bounds for $(0,0,0,0)$ for lemmas 3, 4, 5, 6, and 7

For $(1,0,0,0)$

$$\overline{\Pi}_{r,g}[(0,0,0,0),(1,0,0,0)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(0,0,0,0),(1,0,0,0)]+$$

$$+\psi_0\pi_{r,r}[(0,0,0,0),(0,0,0,0)] + (1-p)\pi_{r,r}[(0,0,0,0),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} (1-p)b = \frac{1}{2}b$$

For $(1,0,1,0)$

$$\overline{\Pi}_{r,g}[(0,0,0,0),(1,0,1,0)] = \lim_{p\to\frac{1}{2}^{+},\psi_0\to 0^{+}} p\pi_{r,g}[(0,0,0,0),(1,0,1,0)]+$$

$$+\psi_0\pi_{r,r}[(0,0,0,0),(0,0,0,0)] + (1-p)\pi_{r,r}[(0,0,0,0),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} (1-p)b = \frac{1}{2}b$$

For $(1,1,0,1)$

$$\overline{\Pi}_{r,g}[(0,0,0,0),(1,1,0,1)] = \lim_{p\to\frac{1}{2}^{+},\psi_0\to 0^{+}} p\pi_{r,g}[(0,0,0,0),(1,1,0,1)]+$$

$$+\psi_0\pi_{r,r}[(0,0,0,0),(0,0,0,0)] + (1-p)\pi_{r,r}[(0,0,0,0),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(b-\gamma) + (1-p)b = b - \frac{1}{2}\gamma$$

For $(1,0,0,1)$

$$\overline{\Pi}_{r,g}[(0,0,0,0),(1,0,0,1)] = \lim_{p\to\frac{1}{2}^{+},\psi_0\to 0^{+}} p\pi_{r,g}[(0,0,0,0),(1,0,0,1)]+$$

$$+\psi_0\pi_{r,r}[(0,0,0,0),(0,0,0,0)] + (1-p)\pi_{r,r}[(0,0,0,0),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(-\gamma) + (1-p)b = \frac{1}{2}(b-\gamma)$$

For $(0,1,0,1)$

$$\overline{\Pi}_{r,g}[(0,0,0,0),(0,1,0,1)] = \lim_{p\to\frac{1}{2}^{+},\psi_0\to 0^{+}} p\pi_{r,g}[(0,0,0,0),(0,1,0,1)]+$$

$$+\psi_0\pi_{r,r}[(0,0,0,0),(0,0,0,0)] + (1-p)\pi_{r,r}[(0,0,0,0),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(b-\gamma) + (1-p)b = \frac{1}{2}(b-\gamma)$$

For $(0,0,0,1)$

$$\overline{\Pi}_{r,g}[(0,0,0,0),(0,0,0,1)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(0,0,0,0),(0,0,0,1)]+$$

$$+\psi_0\pi_{r,r}[(0,0,0,0),(0,0,0,0)] + (1-p)\pi_{r,r}[(0,0,0,0),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(-\gamma) + (1-p)b = \frac{1}{2}(b-\gamma)$$

For $(1,0,1,1)$

$$\overline{\Pi}_{r,g}[(0,0,0,0),(1,0,1,1)] = \lim_{p\to\frac{1}{2}^+,\psi_0\to 0^+} p\pi_{r,g}[(0,0,0,0),(1,0,1,1)]+$$

$$+\psi_0\pi_{r,r}[(0,0,0,0),(0,0,0,0)] + (1-p)\pi_{r,r}[(1,0,0,0),(0,1,1,0)] =$$

$$= \lim_{p\to\frac{1}{2}} p(-\gamma) + (1-p)b = \frac{1}{2}(b-\gamma)$$

B.5 Resiliency of $(0,0,0,0)$ 3, 4, 5, 6, and 7

For $(1,0,0,0)$:

$$\pi_{g,r}[(0,0,0,0),(1,0,0,0)] = (1-p)b >$$

$$> p(-c) = \pi_{r,g}[(1,0,0,0),(0,0,0,0)]$$

For $(1,0,1,0)$:

$$\lim_{p\to 1^-} \pi_{g,r}[(0,0,0,0),(1,0,1,0)] = (1-p)(b-\gamma) = 0 >$$

$$> p(-c-\alpha) = -c-\alpha = \lim_{p\to 1^-} \pi_{r,g}[(1,0,1,0),(0,0,0,0)]$$

For $(1,1,0,1)$:

$$\pi_{g,r}[(0,0,0,0),(1,1,0,1)] = (1-p)(b) >$$

$$> p(-c) + (1-p)(b-c-\alpha-\gamma) = \pi_{r,g}[(1,1,0,1),(0,0,0,0)]$$

For $(1,0,0,1)$:

$$\pi_{g,r}[(0,0,0,0),(1,0,0,1)] = (1-p)(b) >$$

$$> p(-c) + (1-p)(-\alpha-\gamma) = \lim_{p\to 1^-} \pi_{r,g}[(1,0,0,1),(0,0,0,0)]$$

For $(0,1,0,1)$:

$$\lim_{p\to 1^-} \pi_{g,r}[(0,0,0,0),(0,1,0,1)] = 0 >$$

$$> (1-p)(b-c-\alpha-\gamma) = \lim_{p\to 1^-} \pi_{r,g}[(0,1,0,1),(0,0,0,0)]$$

For clarifying this case, note that if $b > c+\alpha+\gamma$, then $(0,1,0,1)$ is not absorbing.

For $(0,0,0,1)$:

$$\pi_{g,r}[(0,0,0,0),(0,0,0,1)] = 0 >$$

$$> (1-p)(-\alpha-\gamma) = \pi_{r,g}[(0,0,0,1),(0,0,0,0)]$$

For $(1,0,1,1)$:

$$\pi_{g,r}[(0,0,0,0),(1,0,1,1)] = (1-p)(b-\gamma) >$$

$$> p(-c-\alpha) + (1-p)(-c-\gamma) = \pi_{r,g}[(1,0,1,1),(0,0,0,0)]$$

B.6 Monomorphic transition paths between absorbing states for Propositions 4, 5, 6, 7, and 8

B.6.1 For $(1,0,0,0)$

Transition to $(1,0,1,0)$

$$\pi_{g,r}[(1,0,0,0),(1,0,1,0)] > \pi_{r,g}[(1,0,1,0),(1,0,0,0)] \iff$$

$$\iff p(b-c)+(1-p)(b-\gamma) > p(-c-\alpha) \Rightarrow b > (1-p)\gamma - p\alpha$$

Transition to $(1,1,0,1)$

$$\pi_{g,r}[(1,0,0,0),(1,1,0,1)] > \pi_{r,g}[(1,1,0,1),(1,0,0,0)] \iff$$

$$\iff p(b-c)+(1-p)b > p(-c)+(1-p)(b-c-\alpha-\gamma) \Rightarrow pb > (1-p)(-c-\alpha-\gamma),$$

, which is always true

Transition to $(1,0,0,1)$

$$\pi_{g,r}[(1,0,0,0),(1,0,0,1)] > \pi_{r,g}[(1,0,0,1),(1,0,0,0)] \iff$$

$$\iff p(b-c)+(1-p)(b) > p(-c)+(1-p)(-\alpha-\gamma) \Rightarrow b > (1-p)(-\alpha-\gamma),$$

, which is always true

Transition to $(0,1,0,1)$

$$\pi_{g,r}[(1,0,0,0),(0,1,0,1)] > \pi_{r,g}[(0,1,0,1),(1,0,0,0)] \iff$$

$$p(b-c) > (1-p)(b-c-\alpha-\gamma),$$

, which is always true

Transition to $(0,0,0,1)$

$$\pi_{g,r}[(1,0,0,0),(0,0,0,1)] > \pi_{r,g}[(0,0,0,1),(1,0,0,0)] \iff$$

$$\iff p(b-c) > (1-p)(-\alpha-\gamma),$$

, which is always true

Transition to $(1,0,1,1)$

$$\pi_{g,r}[(1,0,0,0),(1,0,1,1)] > \pi_{r,g}[(1,0,1,1),(1,0,0,0)] \iff$$

$$\iff p(b-c)+(1-p)(b-\gamma) > p(-c-\alpha)+(1-p)(-\alpha-\gamma) \Rightarrow b > -\gamma,$$

, which is always true

B.6.2 For $(1,0,1,0)$

Transition to $(1,0,0,0)$

$$\pi_{g,r}[(1,0,1,0),(1,0,0,0)] > \pi_{r,g}[(1,0,0,0),(1,0,1,0)] \iff$$

$$\iff p(b-c-\alpha-\gamma)+(1-p)b > p(-c) \Rightarrow b+p(-\alpha-\gamma) > 0$$

Transition to $(1,1,0,1)$

$$\pi_{g,r}[(1,0,1,0),(1,1,0,1)] > \pi_{r,g}[(1,1,0,1),(1,0,1,0)] \iff$$

$$\iff p(b-c-\alpha-\gamma)+(1-p)b > p(-c)+(1-p)(b-c-\alpha-\gamma) \Rightarrow$$

$$\Rightarrow p(b-\alpha-\gamma) > (1-p)(-c-\alpha-\gamma)$$

Transition to $(1,0,0,1)$

$$\pi_{g,r}[(1,0,1,0),(1,0,0,1)] > \pi_{r,g}[(1,0,0,1),(1,0,1,0)] \iff$$

$$\iff p(b-c-\alpha-\gamma)+(1-p)b > p(-c)+(1-p)(-\alpha-\gamma) \Rightarrow$$

$$\Rightarrow b > (2p\alpha-\alpha)+(1-p)(2p\gamma-\gamma)$$

Transition to $(0,1,0,1)$

$$\pi_{g,r}[(1,0,1,0),(0,1,0,1)] > \pi_{r,g}[(0,1,0,1),(1,0,1,0)] \iff$$

$$\iff p(b-c-\alpha-\gamma) > (1-p)(b-c-\alpha-\gamma) \Rightarrow 0 > (b-2pb)+(2p\alpha-\alpha)+(1-p)(2p\gamma-\gamma)$$

Transition to $(0,0,0,1)$

$$\pi_{g,r}[(1,0,1,0),(0,0,0,1)] > \pi_{r,g}[(0,0,0,1),(1,0,1,0)] \iff$$

$$\iff p(b-c-\alpha-\gamma) > (1-p)(-\alpha-\gamma) \Rightarrow p(b-c)+(2p\alpha-\alpha)+(1-p)(2p\gamma-\gamma)$$

Transition to $(1,0,1,1)$

$$\pi_{g,r}[(1,0,1,0),(1,0,1,1)] > \pi_{r,g}[(1,0,1,1),(1,0,1,0)] \iff$$

$$\iff p(b-c-\alpha-\gamma)+(1-p)(b-\gamma) > p(-c-\alpha)+(1-p)(-\alpha-\gamma) \Rightarrow$$

$$\Rightarrow b+p(-\gamma) > (1-p)(-\alpha)$$

B.6.3 For $(1,1,0,1)$

Transition to $(1,0,0,0)$

$$\pi_{g,r}[(1,1,0,1),(1,1,0,1)] > \pi_{r,g}[(1,1,0,1),(1,1,0,1)] \iff$$

$$\iff p(b-c)+(1-p)(b-c-\alpha) > p(b-c-\gamma) \Rightarrow b-c+(1-p)(-\alpha) > p(b-c-\gamma)$$

Transition to $(1,0,1,0)$

$$\pi_{g,r}[(1,1,0,1),(1,0,1,0)] > \pi_{r,g}[(1,0,1,0),(1,1,0,1)] \iff$$

$$\iff p(b-c)+(1-p)(b-c-\alpha-\gamma) > p(b-c-\alpha-\gamma) \Rightarrow (1-p)(b-c-\alpha-\gamma) > p(-\alpha-\gamma)$$

Transition to $(1,0,0,1)$

$$\pi_{g,r}[(1,1,0,1),(1,0,0,1)] > \pi_{r,g}[(1,0,0,1),(1,1,0,1)] \iff$$

$$\iff p(b-c)+(1-p)(b-c-\alpha) > p(b-c-\gamma)+(1-p)(-\alpha-\gamma) \Rightarrow (1-p)(b-c) > -\gamma$$

Transition to $(0,1,0,1)$

$$\pi_{g,r}[(1,1,0,1),(0,1,0,1)] > \pi_{r,g}[(0,1,0,1),(1,1,0,1)] \iff$$

$$\iff p(b-c)+(1-p)(-c-\alpha) > p(b-\gamma)+(1-p)(b-c-\alpha-\gamma) \Rightarrow p(b-c)+b-\gamma$$

Transition to $(0,0,0,1)$

$$\pi_{g,r}[(1,1,0,1),(0,0,0,1)] > \pi_{r,g}[(0,0,0,1),(1,1,0,1)] \iff$$

$$p(b-c)+(1-p)(-c-\alpha) > p(b-\gamma)+(1-p)(-\alpha-\gamma) \Rightarrow -c > -\gamma$$

Transition to $(1,0,1,1)$

$$\pi_{g,r}[(1,1,0,1),(1,0,1,1)] > \pi_{r,g}[(1,0,1,1),(1,1,0,1)] \iff$$

$$\iff p(b-c)+(1-p)(b-c-\alpha-\gamma) > p(b-c-\alpha-\gamma)+(1-p)(-\alpha-\gamma) \Rightarrow$$

$$\Rightarrow (1-p)(b-c) > p(-\alpha-\gamma)$$

B.6.4 For $(1,0,0,1)$

Transition to $(1,0,0,0)$

$$\pi_{g,r}[(1,0,0,1),(1,0,0,0)] > \pi_{r,g}[(1,0,0,0),(1,0,0,1)] \iff$$

$$\iff p(b-c)+(1-p)(b-\alpha) > p(-c-\alpha) \Rightarrow b > p(-\gamma)+(1-p)(\alpha)$$

Transition to $(1,0,1,0)$

$$\pi_{g,r}[(1,0,0,1),(1,0,1,0)] > \pi_{r,g}[(1,0,1,0),(1,0,0,1)] \iff$$

$$p(b-c)+(1-p)(b-\alpha-\gamma) > p(-c-\alpha-\gamma) \Rightarrow b+(1-p)(-\alpha-\gamma) > p(-\alpha-\gamma),$$

, which is always true.

Transition to $(1,1,0,1)$

$$\pi_{g,r}[(1,0,0,1),(1,1,0,1)] > \pi_{r,g}[(1,1,0,1),(1,0,0,1)] \iff$$

$$p(b-c)+(1-p)(b-\alpha) > p(-c-\gamma)+(1-p)(b-c-\alpha-\gamma) \Rightarrow b > (1-p)(-c)-\gamma,$$

, which is always true.

Transition to $(0,1,0,1)$

$$\pi_{g,r}[(1,0,0,1),(0,1,0,1)] > \pi_{r,g}[(0,1,0,1),(1,0,0,1)] \iff$$

$$p(b-c)+(1-p)(-\alpha) > p(-\gamma)+(1-p)(b-c-\alpha-\gamma) \Rightarrow$$

$$\Rightarrow p(b-c) > (1-p)(b-c) - gamma,$$

, which is always true.

Transition to $(0,0,0,1)$

$$\pi_{g,r}[(1,0,0,1),(0,0,0,1)] > \pi_{r,g}[(0,0,0,1),(1,0,0,1)] \iff$$

$$\iff p(b-c)+(1-p)(-\alpha) > p(-\gamma)+(1-p)(-\alpha-\gamma) \Rightarrow p(b-c) > -\gamma,$$

, which is always true.

Transition to $(1,0,1,1)$

$$\pi_{g,r}[(1,0,0,1),(1,0,1,1)] > \pi_{r,g}[(1,0,1,1),(1,0,0,1)] \iff$$

$$p(b-c)+(1-p)(b-\alpha-\gamma) > p(-c-\alpha-\gamma)+(1-p)(-\alpha-\gamma) \Rightarrow b > p(-\alpha-\gamma),$$

, which is always true.

B.6.5 For $(0,1,0,1)$

Transition to $(1,0,0,0)$

$$\pi_{g,r}[(0,1,0,1),(1,0,0,0)] > \pi_{r,g}[(1,0,0,0),(0,1,0,1)] \iff$$

$$\iff (1-p)(b-c-\alpha) > p(b-c-\gamma)$$

Transition to $(1,0,1,0)$

$$\pi_{g,r}[(0,1,0,1),(1,0,1,0)] > \pi_{r,g}[(1,0,1,0),(0,1,0,1)] \iff$$

$$\iff (1-p)(b-c-\alpha-\gamma) > p(b-c-\alpha-\gamma) \Rightarrow b-2pb > (c-2pc)+(\alpha-2p\alpha)+(\gamma-2p\gamma)$$

Transition to $(1,1,0,1)$

$$\pi_{g,r}[(0,1,0,1),(1,1,0,1)] > \pi_{r,g}[(1,1,0,1),(0,1,0,1)] \iff$$

$$\iff (1-p)(b-c-\alpha) > p(b-c-\gamma)+(1-p)(b-c-\alpha-\gamma) \Rightarrow 0 > p(b-c)-\gamma$$

Transition to $(1,0,0,1)$

$$\pi_{g,r}[(0,1,0,1),(1,0,0,1)] > \pi_{r,g}[(1,0,0,1),(0,1,0,1)] \iff$$

$$\iff (1-p)(b-c-\alpha) > p(b-c-\gamma)+(1-p)(-\alpha-\gamma) \Rightarrow (1-p)(b-c) > p(b-c)-\gamma$$

Transition to $(0,0,0,1)$

$$\pi_{g,r}[(0,1,0,1),(0,0,0,1)] > \pi_{r,g}[(0,0,0,1),(0,1,0,1)] \iff$$

$$\iff (1-p)(-c-\alpha) > p(b-\gamma)+(1-p)(-\alpha-\gamma) \Rightarrow 0 > pb+(1-p)c-\gamma$$

Transition to $(1,0,1,1)$

$$\pi_{g,r}[(0,1,0,1),(1,0,1,1)] > \pi_{r,g}[(1,0,1,1),(0,1,0,1)] \iff$$

$$\iff (1-p)(b-c-\alpha-\gamma) > p(b-c-\alpha-\gamma)+(1-p)(-\alpha-\gamma) \Rightarrow$$
$$\Rightarrow (1-p)(b-c) > p(b-c-\alpha-\gamma)$$

B.6.6 For $(0,0,0,1)$

Transition to $(1,0,0,0)$

$$\pi_{g,r}[(0,0,0,1),(1,0,0,0)] > \pi_{r,g}[(1,0,0,0),(0,0,0,1)] \iff$$

$$\iff (1-p)(b-\alpha) > p(-c-\gamma)$$

Transition to $(1,0,1,0)$

$$\pi_{g,r}[(0,0,0,1),(1,0,1,0)] > \pi_{r,g}[(1,0,1,0),(0,0,0,1)] \iff$$

$$\iff (1-p)(b-\alpha-\gamma) > p(-c-\alpha-\gamma),$$

, which is always true.

Transition to $(1,1,0,1)$

$$\pi_{g,r}[(0,0,0,1),(1,1,0,1)] > \pi_{r,g}[(1,1,0,1),(0,0,0,1)] \iff$$

$$\iff (1-p)(b-\alpha) > p(-c-\gamma)+(1-p)(b-c-\alpha-\gamma) \Rightarrow 0 > -c-\gamma,$$

, which is always true.

Transition to $(1,0,0,1)$

$$\pi_{g,r}[(0,0,0,1),(1,0,0,1)] > \pi_{r,g}[(1,0,0,1),(0,0,0,1)] \iff$$

$$\iff (1-p)(b-\alpha) > p(-c-\gamma)+(1-p)(-\alpha-\gamma) \Rightarrow (1-p)b > p(-c)-\gamma,$$

, which is always true.

Transition to $(0,1,0,1)$

$$\pi_{g,r}[(0,0,0,1),(0,1,0,1)] > \pi_{r,g}[(0,1,0,1),(0,0,0,1)] \iff$$

$$\iff (1-p)(-\alpha) > p(-\gamma)+(1-p)(b-c-\alpha-\gamma) \Rightarrow 0 > (1-p)(b-c)-\gamma$$

Transition to $(1,0,1,1)$

$$\pi_{g,r}[(0,0,0,1),(1,0,1,1)] > \pi_{r,g}[(1,0,1,1),(0,0,0,1)] \iff$$

$$\iff (1-p)(b-\alpha-\gamma) > p(-c-\alpha-\gamma)+(1-p)(-\alpha-\gamma) \Rightarrow (1-p)b > p(-c-\alpha-\gamma),$$

, which is always true.

B.6.7 For $(1,0,1,1)$

Transition to $(1,0,0,0)$

$$\pi_{g,r}[(1,0,1,1),(1,0,0,0)] > \pi_{r,g}[(1,0,0,0),(1,0,1,1)] \iff$$

$$p(b-c-\alpha-\gamma)+(1-p)(b-\alpha) > p(-c-\gamma) \Rightarrow b-\alpha > 0$$

Transition to $(1,0,1,0)$

$$\pi_{g,r}[(1,0,1,1),(1,0,1,0)] > \pi_{r,g}[(1,0,1,0),(1,0,1,1)] \iff$$

$$\iff p(b-c-\alpha-\gamma)+(1-p)(b-\alpha-\gamma) > p(-c-\alpha-\gamma) \Rightarrow b+(1-p)(-\alpha-\gamma) > 0$$

Transition to $(1,1,0,1)$

$$\pi_{g,r}[(1,0,1,1),(1,1,0,1)] > \pi_{r,g}[(1,1,0,1),(1,0,1,1)] \iff$$

$$\iff p(b-c-\alpha-\gamma)+(1-p)(b-\alpha) > p(-c-\gamma)+(1-p)(b-c-\alpha-\gamma) \Rightarrow$$

$$\Rightarrow p(b-\alpha) > p(z)+(1-p)(-c-\gamma)$$

Transition to $(1,0,0,1)$

$$\pi_{g,r}[(1,0,1,1),(1,0,0,1)] > \pi_{r,g}[(1,0,0,1),(1,0,1,1)] \iff$$

$$\iff p(b-c-\alpha-\gamma)+(1-p)(b-\alpha) > p(-c-\gamma)+(1-p)(-\alpha-\gamma) \Rightarrow$$

$$\Rightarrow b-p\alpha > (1-p)(-\gamma)$$

Transition to $(0,0,0,1)$

$$\pi_{g,r}[(1,0,1,1),(1,0,1,1)] > \pi_{r,g}[(1,0,1,1),(1,0,1,1)] \iff$$

$$\iff p(b-c-\alpha-\gamma)+(1-p)(-\alpha) > p(-\gamma)+(1-p)(-\alpha-\gamma) \Rightarrow$$

$$\Rightarrow p(b-c-\alpha) > (1-p)(-\gamma)$$

Transition to $(0,1,0,1)$

$$\pi_{g,r}[(1,0,1,1),(0,1,0,1)] > \pi_{r,g}[(0,1,0,1),(1,0,1,1)] \iff$$

$$\iff p(b-c-\alpha-\gamma)+(1-p)(-\alpha) > p(-\gamma)+(1-p)(b-c-\alpha-\gamma) \Rightarrow$$

$$\Rightarrow p(b-c-\alpha) > (1-p)(b-c-\gamma)$$

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