Automatica 187 (2026) 112870

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Stabilization of strictly pre-dissipative nonlinear receding horizon control by terminal costs[☆]

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ARTICLE INFO

Article history:
Received 13 January 2025
Received in revised form 14 November 2025
Accepted 12 December 2025

Keywords:
Receding horizon control
Model predictive control
Dissipativity
Practical asymptotic stability

ABSTRACT

It is known that receding horizon control with a strictly pre-dissipative optimal control problem yields a practically asymptotically stable closed loop when suitable state constraints are imposed. In this note we show that alternatively suitably bounded terminal costs can be used for stabilizing the closed loop.

1. Introduction

Receding horizon control (often used synonymously with model predictive control) is a control technique in which a finite horizon optimal control problem is solved in each time step and the first element of the resulting optimal control sequence is used in the next time step (Grüne & Pannek, 2017; Rawlings et al., 2017). Under suitable stabilizability and regularity conditions, this scheme yields a practically asymptotically stable closed loop if the system is strictly dissipative with supply function defined via the stage cost of the finite horizon optimal control problem (Grüne & Pannek, 2017, Chapter 8). In this case, we call the optimal control problem *strictly dissipative*. Here, the size of the "practical" neighborhood of the equilibrium to which the closed-loop solution converges is determined by the length of the finite optimization horizon. True (as opposed to practical) asymptotic stability can be achieved by using suitable terminal constraints and costs, see Amrit et al. (2011) and Diehl et al. (2011a) or Theorem 8.13 in Grüne and Pannek (2017). In these approaches the terminal cost is typically a local control Lyapunov function for the system and the terminal constraints are needed because the design of a global control Lyapunov function is usually a very difficult task. As a simpler alternative, it was shown in Zanon and Faulwasser (2018) that linear terminal costs can also be used to obtain true asymptotic stability.

The strict dissipativity property that is at the heart of all these results requires the existence of a so-called storage function λ mapping the state space into the reals. It is a strengthened version of the system theoretic dissipativity property introduced by Willems in his seminal papers (Willems, 1972a, 1972b) and also featured in his slightly earlier paper (Willems, 1971) on linear quadratic optimal control and the algebraic Riccati equation. Readers familiar with Lyapunov's stability theory can see the storage function λ as a generalization of a Lyapunov function. However, unlike Lyapunov functions, λ need not attain nonnegative values. However, it must be bounded from below, and this property is crucial for deriving the (practical) stability properties for receding horizon control cited above.

For generalized linear quadratic problems, i.e., problems with linear dynamics and a cost function containing quadratic and linear terms, with state space $\mathbb{R}^{n_x}$, a standard construction for a storage function results in a function of the form $\lambda(x) = x^T Px + \nu^T x$, for $P \in \mathbb{R}^{n_x \times n_x}$ and $\nu \in \mathbb{R}^{n_x}$, see Damm et al. (2014, Proposition 4.5). Clearly, such a function λ is in general *not* bounded from below and Example 2.3 in Damm et al. (2014), which we also present as Example 2.2, below, shows that storage functions unbounded from below may occur even for very simple scalar problems. While the potential unboundedness of λ has been handled somewhat informally in Damm et al. (2014), later in Grüne and Guglielmi (2018) the variant of strict dissipativity with storage function not bounded from below has been termed *strict pre-dissipativity*. For strictly pre-dissipative problems, one way to obtain strict dissipativity and thus (practical) asymptotic stability is to suitably restrict the state space by means of state

☆ The research for this paper was supported by DFG, Germany Grant No. 499435839 The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Martin Monnigmann under the direction of Editor Florian Dorfler.
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https://doi.org/10.1016/j.automatica.2026.112870

constraints, e.g., to a compact set, on which λ is bounded from below.

Since such a restriction of the state space may not always be desirable, in this note we will look at an alternative way to regain (practical) asymptotic stability. More precisely, we want to answer the following question: Given a receding horizon control scheme with strictly pre-dissipative optimal control problem, can we add a simple terminal cost that guarantees (practical) asymptotic stability? Here "simple" means that we do not want to design control Lyapunov function terminal costs but terms that are easier to compute. We will see that for obtaining practical asymptotic stability is sufficient that the terminal cost is larger than the negative storage function, while for obtaining asymptotic stability we need that the sum of the storage function and the terminal cost is positive semidefinite at the optimal equilibrium. It is worth to be noting that this implies the necessary condition from Zanon and Faulwasser (2018), cf. the discussion after Assumption 3.3, below. We emphasize that no terminal constraints are needed in our approach. This paper focuses on general nonlinear optimal control problems. In the companion paper (Zanon & Grüne, 2025) we discuss the linear-quadratic case, for which further results can be obtained.

The remainder of this note is organized as follows. In Section 2 we define the problem and the concepts we use. Section 3 contains the main results and proofs, in which we heavily rely on Grimm et al. (2005) and Grüne and Pannek (2017, Chapter 8). Section 4 contains an illustrative example and Section 5 concludes the note.

2. Problem statement

2.1. Receding horizon control

We consider discrete-time systems of the form

$$x_{k+1} = f(x_k, u_k), \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ and $u \in \mathbb{R}^{n_u}$ denote the states and controls respectively.

Receding horizon or model predictive control consists in minimizing a given stage cost $\ell : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}$ over a fixed finite prediction horizon N, possibly subject to constraints and with the addition of a terminal cost. The receding horizon optimal control problem (RH-OCP) reads

$$\min_{x_0,u_0,\ldots,x_N} \sum_{k=0}^{N-1} \ell(x_k, u_k) + V^{\mathrm{f}}(x_N) \tag{2a}$$

$$\text{s.t.} \quad x_0 = \hat{x}_j, \tag{2b}$$

$$x_{k+1} = f(x_k, u_k), \qquad k \in \mathbb{I}_0^{N-1}, \tag{2c}$$

$$h(x_k, u_k) \le 0, \qquad k \in \mathbb{I}_0^{N-1}, \tag{2d}$$

where $h : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}^l$ defines the state and input constraints and inequality (2d) is to be understood componentwise. We assume that all involved functions $f, \ell, V^{\mathrm{f}}, h$ are continuous. It is possible to introduce additional constraints on the terminal state x_N, but we will not consider that option in this note. The solution to this problem, which we assume to exist and to be unique for all $N \in \mathbb{N}$ and all $x_0 \in \mathbb{R}^n$ satisfying the constraints (2d) for some $u_0 \in \mathbb{R}^{n_u}$, is denoted by $x_k^\star, u_k^\star$. We refer to this optimal solution as the MPC prediction. For all such x_0 we define the optimal value function of the problem

$$V_N(x_0) := \inf_{u_0,x_1,\ldots,x_N} \sum_{k=0}^{N-1} \ell(x_k, u_k) + V^{\mathrm{f}}(x_N) \quad \text{s.t. (2c), (2d).}$$

In order to distinguish the time in the MPC closed loop from the time k of the MPC prediction, we denote the former by j. Starting in the initial value $\hat{x}_0$ at time instant $j = 0$, for every time instant $j \ge 0$ the state $\hat{x}_j$ is measured, Problem (2) is solved with $x_0 = \hat{x}_j$, and the first optimal input $u_0^\star(\hat{x}_j)$ is applied to the system to obtain

$$\hat{x}_{j+1} = f(\hat{x}_j, u_0^\star(\hat{x}_j)). \tag{3}$$

This procedure is repeated iteratively for all $j \ge 0$.

Associated to the RH-OCP (2), we define the steady-state optimization problem (SOP)

$$\min_{\bar{x},\bar{u}} \ \ell(\bar{x}, \bar{u}) \qquad \text{s.t.} \ \ \bar{x} = f(\bar{x}, \bar{u}), \qquad h(\bar{x}, \bar{u}) \le 0. \tag{4}$$

We assume that a unique optimal solution of SOP (4) exists and denote it by $\bar{z}^\star = (\bar{x}^\star, \bar{u}^\star)$.

In this note, we are interested in obtaining stability properties of the closed loop system (3) at the optimal steady state $\bar{z}^\star$. While stability results are abundant for the case of suitably formulated terminal constraints and Lyapunov function terminal costs (Angeli et al., 2012; Diehl et al., 2011b; Faulwasser et al., 2018; Müller et al., 2013, 2015), we focus here on the case of no terminal constraints. This case has been analyzed, e.g., in Faulwasser and Zanon (2018), Grüne (2013), Müller and Grüne (2016) and Zanon and Faulwasser (2018), where we can further distinguish between formulations without terminal cost and formulations with simple terminal costs that need not be Lyapunov functions, which are usually difficult to design. This last approach has in particular been taken in Faulwasser and Zanon (2018) and Zanon and Faulwasser (2018) by using a linear terminal cost and the present note can be seen as a continuation of this research. As in these references, our analysis is based on dissipativity concepts and the turnpike property.

2.2. Strict dissipativity

The stability theory of receding horizon control and economic MPC is often based on strict dissipativity (Angeli et al., 2012; Diehl et al., 2011b; Faulwasser et al., 2018; Grüne, 2013; Müller et al., 2013, 2015). Next we define the weaker notion of *strict pre-dissipativity*, which we will use throughout this note.

Definition 2.1. We say that the RH-OCP (2) is *strictly pre-dissipative* if there exists a continuous *storage function* $\lambda : \mathbb{R}^{n_x} \to \mathbb{R}$ such that for all $(x, u) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_u}$ with $h(x, u) \le 0$ the *rotated cost*

$$L(x, u) := \ell(x, u) - \ell(\bar{x}^\star, \bar{u}^\star) + \lambda(x) - \lambda(f(x, u)) \tag{5}$$

satisfies

$$L(x, u) \ge \alpha(\|x - \bar{x}^\star\|), \tag{6}$$

where α is a class $\mathcal{K}$ function.

In contrast to strict dissipativity, strict pre-dissipativity, introduced under this name in Grüne and Guglielmi (2018), does not require the storage function λ to be bounded from below. This implies that one cannot use arguments as, e.g., in Grüne (2013) and Grüne and Stieler (2014) in order to conclude (practical) stability properties of the closed loop (3), and in fact stability may fail to hold, as we will show by means of the following example.

Example 2.2. Consider the optimal control problem with dynamics, stage cost, and terminal cost

$$x_{k+1} = 2x_k + u_k, \qquad \ell(x, u) = u^2, \qquad V^{\mathrm{f}}(x) = 0.$$

One easily sees that for any initial condition x_0 and any horizon N the optimal control sequence is $u_k^\star \equiv 0$, as this is the only control

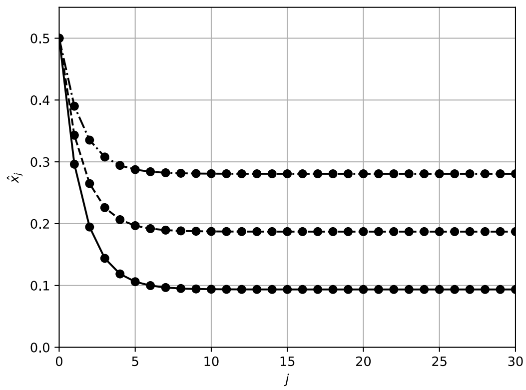

Fig. 1. Closed-loop solutions $\hat{x}_j$ for compact state constraint sets $[-1, 1]$ (solid, bottom), $[-2, 2]$ (dashed, middle), $[-3, 3]$ (dash-dotted, top), for optimization horizon $N = 5$.

that produces 0 cost, while all other control sequences produce positive costs. This implies that system (3) becomes

$$\hat{x}_{j+1} = 2\hat{x}_j,$$

for which the origin is obviously exponentially unstable. Yet, one checks that this problem is strictly pre-dissipative at the optimal equilibrium $\bar{z}^\star = (0, 0)$ with storage function $\lambda(x) = -cx^2$ for each $c \in (0, 1]$. This shows that strict pre-dissipativity does not imply asymptotic stability of the optimal equilibrium.

As already mentioned in the introduction and as also seen in this example, storage functions that are not bounded from below appear naturally already for linear quadratic problems. In order to achieve closed-loop stability, often a compact state constraint set is imposed, as compactness implies boundedness of the storage function provided it is continuous (which is often the case). For Example 2.2, it was shown in Damm et al. (2014, Example 2.3) that this indeed renders the origin practically asymptotically stable for the closed loop. Yet, imposing compact state constraints just for the sake of achieving stability may not always be desirable. This is on the one hand because one may not always know beforehand the operating region on which the controller is supposed to work. On the other hand, and more importantly, the performance of the closed loop controller depends on the size of the compact set. Fig. 1 illustrates this for Example 2.2.

As the simulations[1] in Fig. 1 show, the neighborhood of the optimal equilibrium $\bar{x}^\star = 0$ to which the closed-loop solutions converge grows with the size of the compact state constraint set, meaning that a larger operating region causes a larger distance from the optimal equilibrium. This could be counteracted by increasing the optimization horizon N, but this would, in turn, increase the computational effort. Hence, an alternative method for obtaining (practical) asymptotic stability of the optimal equilibrium $\bar{x}^\star$ may be preferred. As we will prove in this note, stability can be alternatively achieved by a suitably defined terminal cost, cf. the end of Section 3.5, below.

3. Main results

We start with the following lemma, which shows the relation between the optimal control problems for the original and the rotated stage cost, respectively.

[1] All simulations in this paper were performed with the nMPyC Package for Python, https://pypi.org/project/nmpyc/.

Lemma 3.1. *Consider the RH-OCP (2) for an arbitrary finite horizon N. Assume strict pre-dissipativity with storage function λ. Then the problem with stage cost ℓ and terminal cost V^{f} has the same optimal trajectories $x_k^\star$ and control sequences $u_k^\star$ as the RH-OCP problem with rotated stage cost L from (5) and adapted terminal cost $V^{\mathrm{f}} + \lambda$. Moreover, if we denote the optimal value function of the problem with rotated stage cost L and adapted terminal cost $V^{\mathrm{f}} + \lambda$ by $V_{\lambda,N}$, then the identity $V_{\lambda,N} = V_N + \lambda - N\ell(\bar{x}^\star, \bar{u}^\star)$ holds.*

Proof. The proof follows from the observation that

$$\begin{aligned}
&\sum_{k=0}^{N-1} L(x_k, u_k) + V^{\mathrm{f}}(x_N) + \lambda(x_N)\\
&= \sum_{k=0}^{N-1} \big[\ell(x_k, u_k) - \ell(\bar{x}^\star, \bar{u}^\star) + \lambda(x_k) - \lambda(x_{k+1})\big]\\
&\quad + V^{\mathrm{f}}(x_N) + \lambda(x_N)\\
&= \lambda(x_0) - N\ell(\bar{x}^\star, \bar{u}^\star) + \sum_{k=0}^{N-1} \ell(x_k, u_k) + V^{\mathrm{f}}(x_N)
\end{aligned}$$

implies that the optimization objectives of the two problems differ only by the constant $\lambda(x_0) - N\ell(\bar{x}^\star, \bar{u}^\star)$. Note that all values in the above sums are finite, as $\lambda(x) \in \mathbb{R}$ for all $x \in \mathbb{R}^{n_x}$. From this, both the statements about the optimal solutions, i.e., the minimizers as well as the statement about the optimal value functions follow. ■

Lemma 3.1 now allows us to use existing results on stability of MPC for either stabilizing or strictly dissipative optimal control problems and carry them over to the strictly pre-dissipative case. For doing this, we distinguish between two cases.

3.1. Results for positive semidefinite $V^{\mathrm{f}} + \lambda$

In this subsection we make the assumption that $V^{\mathrm{f}} + \lambda$ is positive semidefinite in the following sense. This will allow us to use stability results from Grimm et al. (2005).

Definition 3.2. A function $\Phi : \mathbb{R}^{n_x} \to \mathbb{R}$ is called positive semidefinite at a point $x^\star \in \mathbb{R}^{n_x}$, if $\Phi(x^\star) = 0$ and $\Phi(x) \geq 0$ for all $x \in \mathbb{R}^{n_x}$.

The main structural assumption we make in this subsection is the following, where we use the state and control constraint sets

$$\mathcal{X} := \{x \in \mathbb{R}^{n_x} \mid \text{there is } u \in \mathbb{R}^{n_u} \text{ with } h(x, u) \leq 0\},$$

and

$$\mathcal{U} := \{u \in \mathbb{R}^{n_u} \mid \text{there is } x \in \mathbb{R}^{n_x} \text{ with } h(x, u) \leq 0\}.$$

Assumption 3.3. The optimal control problem (2) is strictly pre-dissipative at an equilibrium $x^\star \in \operatorname{int}\mathcal{X}$ with continuous storage function λ and class $\mathcal{K}$ function α and the function $x \mapsto V^{\mathrm{f}}(x) + \lambda(x)$ is positive semidefinite at $x^\star$.

We note that this assumption implies that $V^{\mathrm{f}} + \lambda$ has a global minimum at $x^\star$, which satisfies the necessary optimality conditions of a local minimum since it is in the interior of $\mathcal{X}$. If both functions are differentiable, this implies that $\nabla V^{\mathrm{f}}(x^\star) = -\nabla\lambda(x^\star)$ must hold. Hence, the linear term in the terminal cost V^{f} provides a gradient correction in the sense of Faulwasser and Zanon (2018) and Zanon and Faulwasser (2018), which is known to be necessary for obtaining asymptotic stability in the absence of terminal constraints.

In addition, for invoking the results from Grimm et al. (2005) we need the following technical assumption on the rotated stage cost.

Assumption 3.4. Either $\mathcal{U}$ is compact or for each compact set $C \subset \mathcal{X}$, each $N \geq 1$, and each $\mu > 0$ there is $\eta > 0$ such that

$$\sum_{k=0}^{N-1} L(x_k, u_k) + V^f(x_N) + \lambda(x_N) \leq \eta$$

implies $\|u_k\| \leq \mu$ for all $k = 0, \dots, N-1$.

We note that the second alternative of the assumption follows, e.g., if $\ell(x, u) = \ell_1(x) + \ell_2(u)$ and $\ell_2(u) \geq \gamma(\|u\|)$ for some $\gamma \in \mathcal{K}$ and V^f is bounded from below. It can thus be seen as a coercivity condition of the stage cost in u, which prevents the optimal control values from becoming arbitrarily large.

Theorem 3.5. *Consider the MPC closed loop (3) with optimal control problem (2) satisfying Assumptions 3.3 and 3.4. Assume there is $\rho \in \mathcal{K}$ such that the optimal value function of (2) satisfies*

$$V_N(x) + \lambda(x) - N\ell(\bar{x}^\star, \bar{u}^\star) \leq \inf_{u \in \mathcal{U}} \rho(L(x, u)) \tag{7}$$

for all $x \in \mathcal{X}$ and all $N \geq 1$. Then there are $\beta \in \mathcal{KL}$, and $\Delta(N) > \delta(N) > 0$ with $\Delta(N) \to \infty$ and $\delta(N) \to 0$ as $N \to \infty$, such that for all sufficiently large N the solutions $\hat{x}_j$ of the closed loop (3) with $\hat{x}_0 \in \mathcal{X}$ and $\|\hat{x}_0 - \bar{x}^\star\| \leq \Delta(N)$ satisfy

$$\|\hat{x}_j\| \leq \max\{\beta(\|\hat{x}_0 - \bar{x}^\star\|, j),\ \delta(N)\}. \tag{8}$$

In words, $x^\star$ is a semiglobally practically asymptotically stable equilibrium of the closed loop (3).

If the inequality for V_N holds with a linear function $\rho(s) = cs$ with $c \geq 0$, then $x^\star$ is an asymptotically stable equilibrium of the closed loop (3), i.e., inequality (8) holds with $\delta(N) = 0$ for all initial conditions $\hat{x}_0$.

Proof. In order to prove the theorem, we apply results from Grimm et al. (2005), which can be found as Corollaries A.1 and A.2 in the Appendix , to the problem with rotated stage cost L and modified terminal cost $V^f + \lambda$. In order to apply these corollaries, we need to check whether the standing assumptions SA1–SA4 in Grimm et al. (2005), which can also be found in the Appendix, are satisfied. SA1 follows since f, ℓ and λ and thus also L and $V^f + \lambda$ are continuous. SA2 and SA4 are part of the assumptions of this theorem. For checking SA3, we note that since strict pre-dissipativity implies $L(x, u) \geq \alpha(\|x - \bar{x}^\star\|)$, cf. (6), SA3 holds with $W \equiv 0$, $\sigma(x) = \alpha(\|x - \bar{x}^\star\|)$, and $\gamma_W(r) = \bar{\alpha}_W(r) = \alpha_W(r) = r$.

Thus, we can apply Corollary A.1 and, in case that ρ is linear, also Corollary A.2 to the problem with rotated stage cost and modified terminal cost. Since by Lemma 3.1 the optimal controls for the original problem coincide with the optimal controls for the rotated problem, the resulting closed loop systems are identical. Thus, fixing $\Delta_\sigma, \delta_\sigma > 0$, using the definition of σ from above and the monotonicity of α, the inequality from Corollary A.1 implies

$$\|\hat{x}_j - \bar{x}^\star\| \leq \max\{\alpha^{-1}(\beta_\sigma(\alpha(\|\hat{x}_0 - \bar{x}^\star\|), j)),\ \alpha^{-1}(\delta_\sigma)\}$$

for all $N \geq N^\star$ and all $\hat{x}_0 \in \mathcal{X}$ with $\|\hat{x}_j - \bar{x}^\star\| \leq \alpha^{-1}(\Delta_\sigma)$.

In order to construct $\Delta(N)$ and $\delta(N)$, consider monotone sequences $\Delta_n \to \infty$, $\delta_n \to 0$, $\Delta_n > \delta_n$ for $n = 0, 1, 2, \dots$, and let N_n^* be the $N^\star$ from Corollary A.1 for $\Delta_\sigma = \alpha(\Delta_n)$ and $\delta_\sigma = \alpha(\delta_n)$. Then, $\widehat{N}_n := \max\{n, N_n^\star\}$ is an unbounded increasing sequence and we set $\Delta(N) := \max\{\Delta_n \mid \widehat{N}_n \leq N\}$ and $\delta(N) := \min\{\delta_n \mid \widehat{N}_n \leq N\}$.

We claim that for this choice of $\Delta(N)$ and $\delta(N)$ the inequality (8) follows for any $N \geq \widehat{N}_0$. To prove this claim, fix $N \geq \widehat{N}_0$ and let $n(N)$ be maximal with $\widehat{N}_{n(N)} \leq N$. Then $\Delta(N) = \Delta_{n(N)}$ and the $N^\star$ corresponding to $\Delta_\sigma = \alpha(\Delta(N))$ satisfies $N^\star = N^\star_{n(N)} \leq \widehat{N}_{n(N)} \leq N$. Likewise, the $N^\star$ corresponding to $\delta_\sigma = \alpha(\delta(N))$ satisfies $N^\star \leq N$. This implies the desired inequality (8) with $\beta(r, t) = \alpha^{-1}(\beta_\sigma(\alpha(r), t))$.

If ρ is linear, the second statement follows analogously from Corollary A.2, with $\beta(r, t) = \alpha^{-1}(Me^{-\nu t}\alpha(r))$. ■

Remark 3.6. Even though the storage function λ is usually unknown, it is not difficult to find candidates for V^f rendering $\lambda + V^f$ positive definite. This is based on the observation that any function V^f that satisfies $\nabla\lambda(\bar{x}^\star) + \nabla V^f(\bar{x}^\star) = 0$ has a critical point in $\bar{x}^\star$. As the gradient $\nabla\lambda(\bar{x}^\star)$ in the optimal equilibrium can be computed by solving a linear system of equations (actually, it is a by-product when computing $\bar{x}^\star$ via necessary optimality conditions, see Zanon and Faulwasser (2018, Theorem 4 and eq. (7))), we can make the ansatz

$$V^f(x) = \nabla\lambda(\bar{x}^\star)(\bar{x}^\star - x) + \gamma(\|\bar{x}^\star - x\|).$$

This function has a critical point at $\bar{x}^\star$ if $\gamma : \mathbb{R} \to \mathbb{R}$ is a C^1 function with $\gamma'(0) = 0$. Hence, in order to obtain positive definiteness of $\lambda + V^f$ it remains to choose γ sufficiently steep and positive. For instance, in a linear-quadratic problem it is known that λ grows at most quadratically, hence $\gamma(s) = cs^2$ for sufficiently large $c > 0$ will always work. A suitable c can, e.g., be determined based on simulations of the MPC closed loop.

Remark 3.7. (i) We note that Grimm et al. (2005, Corollary 3) also provides an estimate for the horizon length N for which asymptotic stability holds in case of a linear ρ, but this bound is very conservative. Tighter bounds were provided, e.g., in Tuna et al. (2006) or L. Grüne et al. (2010), see also Grüne and Pannek (2017, Chapter 6).

(ii) The condition on V_N in Theorem 3.5 is effectively a condition on the optimal value function $V_{\lambda,N}$ of the problem with rotated cost and adapted terminal cost, cf. Lemma 3.1. In essence, it requires that the system can be controlled asymptotically to $\bar{x}^\star$ with sufficiently low cost. In particular, if L and the terminal cost terms are polynomial and the system can be controlled to $\bar{x}^\star$ exponentially fast, then ρ can be chosen as a linear function. We refer to Grüne and Pannek (2017, Chapter 6) for an extensive discussion and examples.

3.2. Results for $V^f + \lambda$ bounded from below

The design of a terminal cost V^f for which $V^f + \lambda$ is positive semidefinite requires rather accurate knowledge of the storage function λ. This may either not be available or difficult to obtain. To this end, we now present a semiglobal practical asymptotic stability result under the following significantly weaker assumption.

Assumption 3.8. The optimal control problem (2) is strictly pre-dissipative at an equilibrium $x^\star \in \text{int}\mathcal{X}$ with continuous storage function λ and class $\mathcal{K}$ function α and the function $x \mapsto V^f(x) + \lambda(x)$ is bounded from below on $\mathcal{X}$.

In contrast to requiring positive semidefiniteness, this lower boundedness assumption only requires V^f to be sufficiently large for large x, as on compact sets $V^f + \lambda$ is bounded from below by continuity, regardless of how V^f is chosen.

Again, besides this structural assumption we need a couple of technical assumptions. Here we use the assumptions from Grüne and Pannek (2017, Chapter 8), which are somewhat more streamlined than the assumptions in the original paper (Grüne & Stieler, 2014), where the result we use appeared for the first time.

Assumption 3.9. There are class $\mathcal{K}$ functions γ_V and γ_{V_λ}, such that the inequalities

$$|V_N(x) - V_N(\bar{x}^\star)| \leq \gamma_V(\|x - \bar{x}^\star\|)$$

and

$$|V_{\lambda,N}(x) - V_{\lambda,N}(\bar{x}^\star)| \leq \gamma_{V_\lambda}(\|x - \bar{x}^\star\|)$$

hold for all $N \geq 1$ and all $x \in \mathcal{X}$.

Theorem 3.10. *Consider the MPC closed loop (3) with optimal control problem (2) satisfying Assumptions 3.8 and 3.9. Assume there is $\rho \in \mathcal{K}$ such that the optimal value function of (2) satisfies*

$$V_N(x) + \lambda(x) - N\ell(\bar{x}^\star, \bar{u}^\star) \leq \inf_{u \in \mathcal{U}} \rho(L(x, u))$$

for all $x \in \mathcal{X}$ and all $N \geq 1$. Then there are $\beta \in \mathcal{KL}$, and $\Delta(N) > \delta(N) > 0$ with $\Delta(N) \to \infty$ and $\delta(N) \to 0$ as $N \to \infty$, such that for all sufficiently large N the solutions $\hat{x}_j$ of the closed loop (3) with $\hat{x}_0 \in \mathcal{X}$ and $\|\hat{x}_0 - \bar{x}^\star\| \leq \Delta(N)$ satisfy

$$\|\hat{x}_j\| \leq \max\{\beta(\|\hat{x}_0 - \bar{x}^\star\|, j), \delta(N)\}. \tag{9}$$

In words, $x^\star$ is a semiglobally practically asymptotically stable equilibrium of the closed loop (3).

Proof. The proof proceeds almost identical to the proof of Grüne and Pannek (2017, Theorem 8.33), with the following two changes:

- In the proof of Grüne and Pannek (2017, Theorem 8.33) practical asymptotic stability is obtained by showing that the optimal value function for the problem with stage cost L and without terminal cost is a practical Lyapunov function for the closed loop generated by the problem with stage cost L and terminal cost λ. As no particular storage function properties of λ are exploited in this proof, we can apply the same reasoning to the closed loop generated by the problem with stage cost L and terminal cost $V^{\mathrm{f}}+\lambda$ instead of λ, which by Lemma 3.1 yields the same closed loop system as (2); see also Faulwasser and Zanon (2018) for a similar reasoning.
- In Grüne and Pannek (2017, Theorem 8.33), boundedness of λ (which, as just mentioned, needs to be replaced here by $V^{\mathrm{f}} + \lambda$) is required. However, an inspection of the proof shows that the upper bound on this function is only used when the function is evaluated in the initial condition. Hence, for any fixed $\Delta > 0$ we can apply the proof for all solutions with initial conditions $\|\hat{x}_0\| \leq \Delta$. Thus we obtain a lower bound on N, depending on Δ, for which practical asymptotic stability holds for these solutions. As we can find such bounds on N for arbitrary $\Delta > 0$, this implies the claimed semiglobal practical stability. ■

3.3. Relation to the required supply

One of the standard constructions of a storage function for a dissipative system with a given supply rate is the so-called required supply. Here we follow the definition in Lopezlena and Scherpen (2006). In order to adapt this definition to our strict setting, we assume that the optimal control problem (2) satisfies Definition 2.1 and define the supply rate via $s(x, u) := \ell(x, u) - \ell(\bar{x}^\star, \bar{u}^\star) - \alpha(\|x - \bar{x}^\star\|)$, with α from Definition 2.1. For the definition of the required supply we need to make the assumption that every $x \in \mathbb{R}^{n_x}$ can be reached from $\bar{x}^\star$. This is a rather strong condition for nonlinear systems, but it is only required in this subsection for defining the required supply, such that we can relate it to our approach. Under this condition, the function defined by

$$\lambda_{rs}(x) := \inf_{\substack{u_0,\ldots,u_k,N_x \\ x_0=\bar{x}^\star, x_{N_x}=x}} \sum_{k=0}^{N_x-1} s(x_k, u_k)$$

is finite and it is a storage function for the strict dissipativity property in Definition 2.1, cf. Lopezlena and Scherpen (2006, Theorem 3.2).

As shown in Willems (1972a, Theorem 2(ii)), the required supply is the largest possible storage function satisfying $\lambda_{rs}(\bar{x}^\star) = 0$. This means that by choosing $\lambda = \lambda_{rs}$ in Theorems 3.5 and 3.10, we obtain the least demanding condition on V^{f}.

Moreover, the choice $\lambda = \lambda_{rs}$ also gives an intuitive explanation for the requirements in Assumptions 3.3 and 3.8. First note that Assumption 3.3 implies $V^{\mathrm{f}} \geq -\lambda_{rs}$ while under Assumption 3.8 we can assume $V^{\mathrm{f}} \geq -\lambda_{rs}$ without loss of generality, because adding a constant to V^{f} does not change the solutions to the optimal control problem. Moreover, even if one of the assumptions (or both) are violated, we can always without loss of generality assume that $V^{\mathrm{f}}(\bar{x}^\star) \geq -\lambda_{rs}(\bar{x}^\star) = 0$, again because adding a constant to V^{f} does not change the optimal solutions.

We provide the intuitive explanation of the two assumptions for such a choice of V^{f}. To this end, consider an arbitrary solution x_k with control u_k starting in $x_0 = \bar{x}^\star$ and reaching $x_N \neq \bar{x}^\star$ in $N \geq 1$ steps. Then, by nonnegativity of $\alpha(r)$, the definition of λ_{rs} yields that

$$\begin{aligned}\lambda_{rs}(x) &\leq \sum_{k=0}^{N-1} s(x_k, u_k) \\ &= \sum_{k=0}^{N-1} \Big(\ell(x_k, u_k) - \ell(\bar{x}^\star, \bar{u}^\star) - \alpha(\|x_k - \bar{x}^\star\|)\Big) \\ &\leq \sum_{k=0}^{N-1} \ell(x_k, u_k) - N\ell(\bar{x}^\star, \bar{u}^\star).\end{aligned}$$

Now, if we assume that $V^{\mathrm{f}} \geq -\lambda_{rs}$ is violated in x_N, i.e., if $V^{\mathrm{f}}(x_N) < -\lambda_{rs}(x_N)$, then the overall cost of the trajectory x_k satisfies

$$\begin{aligned}\sum_{k=0}^{N-1} \ell(x_k, u_k) + V^{\mathrm{f}}(x_N) &< \sum_{k=0}^{N-1} \ell(x_k, u_k) - \lambda_{rs}(x_N) \\ &\leq N\ell(\bar{x}^\star, \bar{u}^\star) \\ &\leq N\ell(\bar{x}^\star, \bar{u}^\star) + V^{\mathrm{f}}(\bar{x}^\star).\end{aligned}$$

Observing that the last expression is precisely the cost to stay in $\bar{x}^\star$ for N steps, this means that the finite horizon optimal solutions will not stay in $\bar{x}^\star$ because it is cheaper to move to x_N. While this does not exactly contradict stability of $\bar{x}^\star$ for the RH closed loop (the solutions could still stay in or near $\bar{x}^\star$ for a couple of steps before moving towards x_N), it is a clear indication that the optimal control problem is not well designed for obtaining asymptotic stability in or near $\bar{x}^\star$.

From this observation, one may conjecture that $V^{\mathrm{f}} \geq -\lambda_{rs}$ is a necessary condition for obtaining (practical) asymptotic stability of the RH closed loop, but a formal proof appears technically involved and is beyond the scope of this note. However, in the companion paper (Zanon & Grüne, 2025) we present a tight lower bound for linear-quadratic problems.

3.4. Comparison with other terminal cost and constraints

The MPC literature describes various ways for the definition of terminal costs and/or terminal constraints that guarantee asymptotic stability of the closed loop. In this section we compare the two most well known approaches with our proposed method. We refer to Grüne and Pannek (2017, Chapter 8) for more information on these approaches.

Probably the simplest way is to impose the so-called equilibrium terminal constraint

$$x_N = \bar{x}^\star \tag{10}$$

as an additional constraint in (2). Fig. 2 compares the closed-loop solution based on our approach with the equilibrium terminal constraint for the example from Section 4.

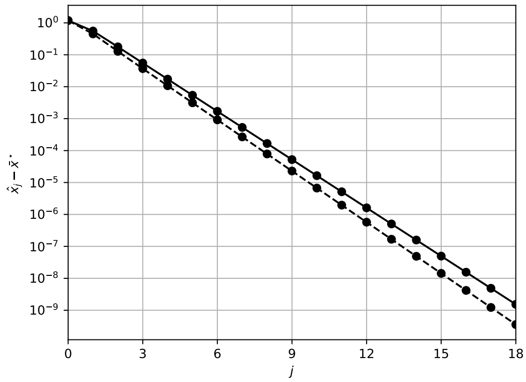

Fig. 2. Difference between closed-loop solution $\hat{x}_j$ and $\bar{x}^\star$ with terminal cost $V^{\mathrm{f}}(x) = x^2 - \nu x$ from Section 4 and no terminal constraint (solid) and without terminal cost but with equilibrium terminal constraint (10) (dashed), both with optimization horizon $N = 3$.

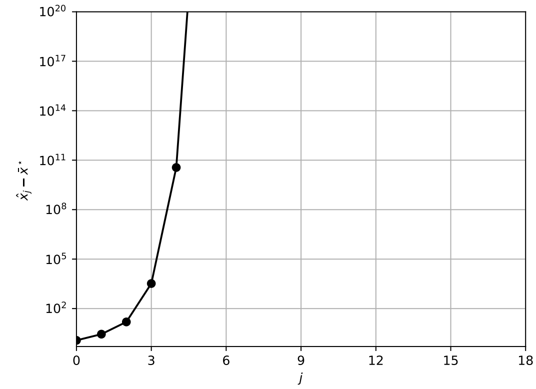

Fig. 3. Difference between closed-loop solution $\hat{x}_j$ and $\bar{x}^\star$ with $V^{\mathrm{f}} \equiv 0$.

It is clearly visible that the (dashed) equilibrium terminal constrained closed-loop solution approaches the equilibrium somewhat faster than the solid solution without terminal constraints, which is a common phenomenon. Otherwise, however, the solutions are qualitatively very similar. A drawback of the equilibrium terminal constraint (10) is that it requires exact controllability to $\bar{x}^\star$, which is a strong condition for nonlinear systems. Moreover, when the dynamics is slow (e.g., in the presence of input constraints) the set of initial values x_0 for which the constraint (10) is feasible may be rather small for moderate N.

The first drawback is removed when using terminal costs V^{f} forming a local Lyapunov function that is compatible with the stage cost ℓ. By this we mean that there exists a region $\mathbb{X}^{\mathrm{f}} \subset \mathcal{X}$ containing $\bar{x}^\star$ in its interior, such that for each $x \in \mathbb{X}^{\mathrm{f}}$ there is $u \in \mathcal{U}$ satisfying $h(x, u) \le 0$, $f(x, u) \in \mathbb{X}^{\mathrm{f}}$, and

$$V^{\mathrm{f}}(f(x, u)) \le V^{\mathrm{f}}(x) - \ell(x, u) + \ell(\bar{x}^\star, \bar{u}^\star).$$

A practical way to find such a V^{f} is to obtain a linear-quadratic approximation (without state or input constraints) to the optimal control problem near $(\bar{x}^\star, \bar{u}^\star)$ and solve a Lyapunov equation and a system of linear equations, see Amrit et al. (2011, Section 4, particularly eq. (22)). The size of the set $\mathbb{X}^{\mathrm{f}}$ then depends on the accuracy of the linear-quadratic approximation to the nonlinear control problem. This approach also works for merely stabilizable systems, but may still lead to rather small feasible sets for moderate values of N if the terminal constraint set $\mathbb{X}^{\mathrm{f}}$ is small. By appropriate scaling of V^{f} it may be possible to remove this constraint, but we are not aware of a general result that would justify this approach for general nonlinear systems. Such a general result can be obtained if in addition to scaling V^{f} a multiple of the storage function λ is added, see Amrit et al. (2011, Section 5), but this is only feasible if λ is known.

Compared to these constructions of V^{f}, our construction of V^{f} from Remark 3.6 uses different calculations and does not require the knowledge of λ. Hence, for some use cases it might provide a useful alternative to the established approaches.

3.5. Consequences for the linear quadratic case

In the particular case of strictly pre-dissipative generalized linear quadratic optimal control problems, it was shown in Grüne and Guglielmi (2018) that the storage function can always be chosen to be of the form $\lambda(x) = x^\top P x + \upsilon^\top x$. Moreover, the technical Assumption 3.4 is always satisfied if $R > 0$ and the technical Assumption 3.9 as well as the bound on V_N assumed in Theorem 3.5 are satisfied if (A, B) are stabilizable. Hence, for applying Theorems 3.5 and 3.10, it suffices to check the Assumptions 3.3 and 3.8, respectively.

If we restrict ourselves to terminal cost functions that contain only quadratic and linear terms, i.e., $V^{\mathrm{f}}(x) = x^\top P^{\mathrm{f}} x + \nu^\top x$, it is easy to see that for symmetric P and P^{f} Assumption 3.3 is equivalent to the conditions

$$P^{\mathrm{f}} \succeq -P \quad \text{and} \quad \nu = -\upsilon - (2P + 2P^{\mathrm{f}})\bar{x}^\star.$$

Assumption 3.8, in turn, is implied by the condition

$$P^{\mathrm{f}} \succ -P$$

in case $\upsilon \neq 0$ and is equivalent to $P^{\mathrm{f}} \succeq P$ in the special case that $\upsilon = 0$. Further conditions for linear quadratic case can be found in the companion paper (Zanon & Grüne, 2025).

Given that the optimal control problem from Example 2.2 is strictly pre-dissipative with storage function $\lambda(x) = -cx^2$ for any $c \in (0, 1]$, we conclude that any terminal cost of the form $V^{\mathrm{f}}(x) = ax^2$ for any $a > 0$ stabilizes the RH closed loop. As in this example we obtain an unstable closed loop without terminal cost, we see that the condition on a is tight here. The next section illustrates our results with a somewhat more complicated example.

4. Illustrative example

We illustrate our findings by an example that can be seen as a more involved nonlinear version of Example 2.2. It is given by

$$x_{k+1} = 2x_k + x_k^3 + 1 + u_k, \quad \ell(x, u) = u^2$$

with scalar x and u. With a little bit of computation one checks that the optimal control problem is strictly pre-dissipative at the equilibrium

$$\bar{x}^\star = -\frac{\alpha^2 - 12}{6\alpha} \approx -0.6823278, \quad \bar{u}^\star = 0,$$

where $\alpha = (108 + 12\sqrt{93})^{1/3}$, with storage function

$$\lambda(x) = -x^2 + \nu x, \tag{11}$$

where

$$\nu = \frac{\alpha(-8\sqrt{93} - 72) + \alpha^2(-6\sqrt{93} - 54) + 144\sqrt{93} + 1392}{\alpha(18\sqrt{93} + 174) - (\alpha^2 - 12)(9 + \sqrt{93})} \approx -1.3646556.$$

Fig. 3 shows that the receding horizon closed loop is unstable for terminal cost $V^{\mathrm{f}} \equiv 0$. The computation was done for $N = 3$ but the results are similar for other optimization horizons.

Fig. 4 shows the practical asymptotic stability for terminal cost $V^{\mathrm{f}}(x) = 2x^2$, for which $V^{\mathrm{f}} + \lambda$ is bounded from below but

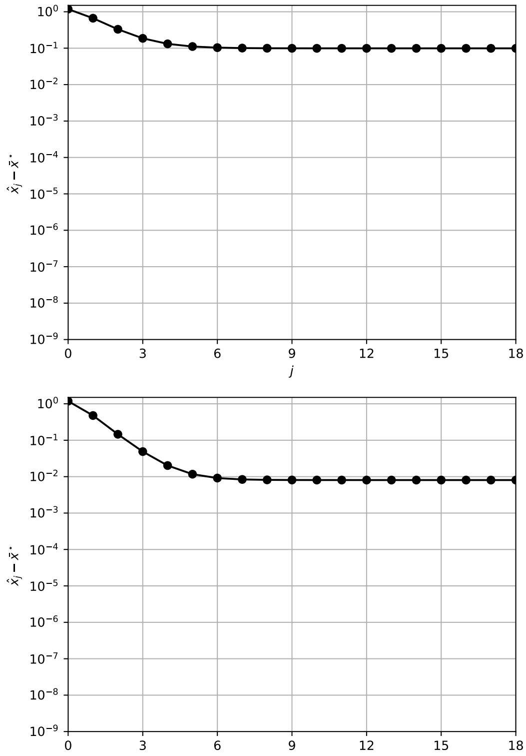

Fig. 4. Difference between closed-loop solution $\hat{x}_j$ and $\bar{x}^\star$ with $V^{\mathrm{f}}(x) = 2x^2$ and optimization horizon $N = 3$ (top) and $N = 5$ (bottom).

not positive definite in $\bar{x}^\star$. The closed-loop solution ends up near the optimal equilibrium $\bar{x}^\star$ (the figure shows the distance to the equilibrium in a logarithmic scale), and for the larger value $N = 5$ (bottom) it ends up closer to $\bar{x}^\star$ then for the smaller value $N = 3$ (top).

Finally, Fig. 5 shows that already for optimization horizon $N = 3$ "true" asymptotic stability (up to roundoff errors) holds for terminal cost $V^{\mathrm{f}}(x) = x^2 - \nu x$, for which $V^{\mathrm{f}} + \lambda$ vanishes and is thus positive semidefinite at $\bar{x}^\star$. Note that the inequality on the optimal value function in Theorem 3.5 is satisfied with linear ρ, because $V_{\lambda,N}$ grows quadratically and $\inf_u L$ grows at least quadratically[2] in $x - \bar{x}^\star$, hence the reasoning from Remark 3.7(ii) applies.

5. Conclusions

We have shown that receding horizon control with strictly pre-dissipative optimal control problem with storage function λ can be stabilized by suitable designed terminal costs V^{f}. For obtaining practical asymptotic stability it is sufficient that $V^{\mathrm{f}} + \lambda$ is bounded from below. If this sum is, in addition, positive definite, then "true", i.e., non-practical asymptotic stability can be concluded.

Acknowledgments

We thank Jonas Schiessl for his help with the numerical simulations and the anonymous reviewers for valuable suggestions that helped to improve this note significantly.

[2] More precisely, $\inf_u L$ grows quadratically near $\bar{x}^\star$ and with the order $(x - \bar{x}^\star)^4$ away from $\bar{x}^\star$.

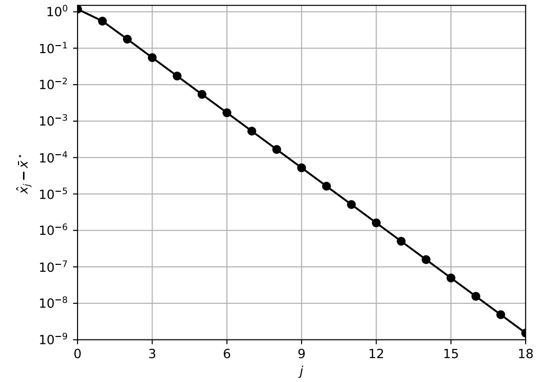

Fig. 5. Difference between closed-loop solution $\hat{x}_j$ and $\bar{x}^\star$ with $V^{\mathrm{f}}(x) = 2x^2 - 2\nu x$ and optimization horizon $N = 3$.

Appendix

The proof of Theorem 3.5 relies on two results from Grimm et al. (2005), which we repeat here as Corollaries A.1 and A.2 together with the standing assumptions from this paper for the convenience of the reader. We start with the standing assumptions SA1–SA4 from Grimm et al. (2005), which we write using expressions that already appeared in the paper.

SA1: The functions L and V^{f} are continuous.
SA2: Assumption 3.4 holds.
SA3: The following detectability condition holds: There are functions $W : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$, $\sigma : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$, two class $\mathcal{K}$ functions α_W and γ_W, and a continuous and nondecreasing function $\bar{\alpha}_W : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ with $\bar{\alpha}_W(0) = 0$ that satisfy the inequalities

$$W(x) \leq \bar{\alpha}_W(\sigma(x))$$
$$W(f(x,u)) - W(x) \leq -\alpha_W(\sigma(x)) + \gamma_W(L(x,u))$$

for all $x \in \mathcal{X}$, $u \in \mathcal{U}$ with $h(x,u) \leq 0$.
SA4: There exists $\rho \in \mathcal{K}_\infty$ such that inequality (7) holds.

We next state the corollaries from Grimm et al. (2005) we use in the proof of Theorem 3.5. We somewhat simplify the statements by leaving out details that are not relevant for our considerations (such as the precise value of N^* in Corollary A.2) and adapt the notation to the one used in this paper.

Corollary A.1 (*Corollary 1 in Grimm et al. (2005)*)**.** *Let SA1–SA4 hold. Then there exists $\beta_\sigma \in \mathcal{KL}$ and for each $\delta_\sigma, \Delta_\sigma > 0$ there exists $N^* \geq 1$ such that for all $N \geq N^*$ and $\hat{x}_0 \in \mathcal{X}$ with $\sigma(\hat{x}_0) \leq \Delta_\sigma$ the closed-loop solutions $\hat{x}_j$ satisfy*

$$\sigma(\hat{x}_j) \leq \max\{\beta_\sigma(\sigma(\hat{x}_0), j),\ \delta_\sigma\}.$$

Corollary A.2 (*Corollary 3 in Grimm et al. (2005)*)**.** *Let SA1–SA4 hold and assume that γ_W, $\bar{\alpha}_W$, α_W, and ρ from S3 and S4 are all linear functions. Then there exists $N^* \geq 1$, depending on these linear functions, as well as $M > 0$ and $\mu > 0$ such that for all $N \geq N^*$ and all $\hat{x}_0 \in \mathcal{X}$ the closed-loop solutions $\hat{x}_j$ satisfy*

$$\sigma(\hat{x}_j) \leq Me^{-\mu j}\sigma(\hat{x}_0).$$

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