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The structure of interaction and modes of reasoning can shape the evolution of conventions

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Abstract

We study the evolution of conventions in a Stag Hunt game where: (i) agents choose a location to interact locally, (ii) interactions are sometimes global and sometimes local, and (iii) agents can be either fine or coarse reasoners, i.e., agents are able or not, respectively, to distinguish between global and local interactions. We show that the structure of interaction and the mode of reasoning affect the selection of social conventions. Further, we find that the coexistence of coarse and fine reasoning may favor or hinder the adoption of the payoff dominant convention—playing Stag—depending on the structure of interaction. In particular, if interactions are mostly local, then fine reasoning increases the diffusion of Stag. Instead, if interactions are sufficiently global, then fine reasoners are never more collaborative than coarse reasoners and they may even disrupt the emergence of payoff dominant conventions.

Keywords Risk dominance · Payoff dominance · Stag Hunt game · Coarse reasoning · Fine reasoning · Location choice · Stochastic stability

JEL Classification C72 · C73 · D91

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1 Introduction

We investigate whether and how social conventions in the Stag Hunt game are influenced by heterogeneous modes of reasoning within a population, specifically when reasoning can differentiate between local and global interactions. Our approach involves a model where social interactions are either local or global, agents are mobile across locations where local interactions take place (as in Oechssler 1997; Ely 2002; Bhaskar and Vega-Redondo 2004), and the population consists of two types of myopic best responders: coarse reasoners, who do not distinguish between local and global interactions, and fine reasoners, who condition their actions based on whether the interaction is local or global. The agents who end up interacting play a Stag Hunt game in which they must simultaneously decide whether to cooperate with the other agent (hunt a Stag) or work on their own (hunt a Hare). Within this framework, we analyze the long run evolution of social conventions in the Stag Hunt game, examining how these conventions are shaped by both the prevalence of local interactions (relative to global ones) and the proportion of fine reasoners in the population.

This framework is versatile enough to capture various real-life phenomena of interest. For instance, if we interpret ‘locations’ as actual geographical places, local and global interactions can correspond to offline and online interactions, respectively. In this context, the model explores what happens as online (global) interactions become more frequent and when certain individuals behave differently in online versus offline environments. Alternatively, if ‘locations’ are understood as groups or clubs, the model examines the implications of a shift away from group-based interactions—such as the decline in the significance of religious or political affiliations over the last century—when some individuals may adopt different behaviors within group settings compared to more general social contexts. In both interpretations, the model offers insight into how social conventions evolve as the balance between local and global interactions shifts, and as the proportion of individuals who can condition their behavior on the interaction context changes.

We find that different conventions can be selected in the long run depending on the probability of local interactions and the fraction of fine reasoners in the population. More precisely, if the probability of local interactions is sufficiently low, then selection favors a convention in which (i) coarse reasoners play the action characterizing the risk dominant convention (Hare), (ii) fine reasoners play Hare in global interactions but choose Stag in local interactions, and (iii) agents separate in distinct locations according to type. Instead, if local interactions are frequent enough and the fraction of fine reasoners in the population is sufficiently high, then selection favors a convention in which (i) coarse reasoners play Stag, (ii) fine reasoners play Stag locally but Hare globally, and (iii) all agents stay in the same location. Finally, if local interactions are frequent enough and the fraction of fine reasoners is sufficiently small, then selection favors a convention in which all agents play Stag and stay in the same location.

The main contribution of this paper is to uncover the non-trivial interplay between the structure of interaction and the heterogeneity in the modes of reasoning for the evolution of conventions in the Stag Hunt game. In this regard, we find that more fine reasoners in the population do not necessarily lead to a greater efficiency: in equilibrium coordination on the payoff dominant convention may either increase or decrease, and this holds for miscoordination too.

The current paper brings together multiple streams of literature, providing contributions to each of them. First, it contributes to the literature on the evolution of conventions by showing that the heterogeneity in the mode of reasoning may matter in the selection of the ruling convention. Second, it contributes to the evolutionary literature on location-choice models by showing that heterogeneity in the mode of reasoning may give rise to the co-existence of conventions as well as to the full separation of agents' mode of reasoning even in the absence of frictions to mobility.

The remaining part of the paper is structured as follows: Sect. 2 highlights the main connections with the related literature, Sect. 3 illustrates the model, Sects. 4 and 5 present the main results, and Sect. 6 concludes. All proofs and intermediate results are collected in the Online Materials.

2 Related literature

The Stag Hunt game is often viewed as a paradigmatic representation of the obstacles to social cooperation (Skyrms 2004). In this game, cooperation can yield the highest possible payoff when both players choose to cooperate, but it also carries the risk of producing the lowest payoff if the other agent does not cooperate. By contrast, acting individually offers a safer strategy, as it ensures a relatively higher expected payoff when the behavior of the opponent is uncertain. Within this setting, it is of particular interest to understand under which conditions the payoff dominant convention is selected, since successful coordination on this outcome enables social cooperation and is socially optimal.

The literature has extensively explored the impact of the interaction structure on the evolution of conventions. A central distinction is between exogenous and endogenous interaction structures.

In the case of exogenous interaction structures, the configuration of who interacts with whom is fixed and externally determined. When interactions occur within the whole population, the risk dominant convention typically prevails due to its robustness against deviations (Kandori et al. 1993; Kandori and Rob 1995; Young 1993). Under local interactions with fixed neighbors, as in Ellison (1993), the same tendency holds when agents follow individualistic revision protocols such as myopic best reply (Newton 2021, Corollary 1). A broader framework for analyzing such fixed structures is developed in Peski (2010), and a comprehensive survey is provided by Weidenholzer (2010).

Other work has investigated endogenous interaction structures, where agents can choose their partners. Under certain conditions—such as costly interactions or constraints on the number of connections—payoff dominant conventions may emerge, particularly when frictions in changing partners are low and agents do not systematically avoid others with different strategies (Jackson and Watts 2002; Goyal and Vega-Redondo 2005; Staudigl and Weidenholzer 2014; Bilancini and Boncinelli 2018; Cui and Weidenholzer 2021; Cui and Shi 2022; Cui 2023; Cui and Jiang 2023; Cui and Liu 2024).

A particularly important case of endogenous structure involves location choice models. Here, agents interact only with those in the same location. If agents can freely optimize both their action and their location, payoff dominant conventions can emerge (Ellison 1993; Oechssler 1997; Bhaskar and Vega-Redondo 2004; Pin et al. 2017). However, when re-optimization is limited or locations have capacity constraints, both payoff dominant and risk dominant conventions may coexist in the long run (Anwar 2002).

In light of the critical role that interaction structure plays in the selection of conventions (see Newton 2018, and references therein), we adopt a model that allows for varying degrees of locality of interaction. Specifically, we assume that interactions are sometimes local—restricted to agents within the same location—and sometimes global—uniform across the entire population. This setup integrates endogenous and exogenous interactions, with a single parameter capturing the degree of locality, thereby enabling us to study the interplay between interaction structure and modes of reasoning, a relationship that has not yet been systematically explored.

We introduce heterogeneity in agents' mode of reasoning by distinguishing between coarse and fine reasoners. Both follow myopic best reply, but fine reasoners condition their behavior on the type of interaction (local vs. global), whereas coarse reasoners do not. This approach is reminiscent of Rubinstein (2007). In contrast to Nax and Newton (2022), we find that the greater strategic sophistication of fine reasoners can matter for the selection of long run equilibria, even without invoking higher levels of strategic depth. In particular, we observe non-trivial implications when a subpopulation of agents has a greater cognitive sophistication, along what has been shown in the parallel work on the language game (Neary 2012), where the presence of agents who face a low cost of adopting bilingualism can lead to multiple conventions surviving in the long run among different groups of interacting agents (Naono 2022).

We assume that agents adopt myopic best reply, a widely used behavioral rule in evolutionary dynamics (Newton 2018). Alternative revision protocols—particularly imitation—can lead to different outcomes. Under imitative revision protocols, the payoff dominant convention can emerge if interactions are neither global nor limited to the immediate neighbors (Alós-Ferrer and Weidenholzer 2006, 2008) or information transmission about average earned payoff is costly and all agents have many neighbors (Cui 2014). Imitation often supports the emergence of the payoff dominant convention in local interactions (Robson and Vega-Redondo 1996; Alós-Ferrer and Weidenholzer 2008), although this is not guaranteed when imitation is pairwise and based on the opponents' payoff (Bilancini et al. 2021; Neary et al. 2025). The long run outcome depends on both the specific form of imitation and the structure of information transmission (Chen et al. 2013; Cui 2014). The role of diverse revision protocols is further explored in Newton (2021), building on the strategy updating process asymmetry property from Peski (2010).

Our analysis relies on stochastic stability analysis with uniform mistakes, a method introduced by Foster and Young (1990) and developed by Young (1993) and Kandori et al. (1993) using the technical results provided by Freidlin and Wentzell (1984). In this approach, agents occasionally make mistakes—selecting strategies that deviate from myopic best responses—with a small probability. Stochastically stable outcomes are those that are most robust to such perturbations, in the sense that they are relatively easy to reach and hard to leave through mistakes. To identify these long run equilibria, we apply the radius-coradius theorems (Evans 1993; Ellison 2000), which provide tools to identify the most likely long run equilibria.

3 The model

Consider a population of agents $\mathcal{N} = \{1, \ldots, N\}$ indexed by *n* and a set of locations $\mathcal{L} = \{1, \ldots, L\}$ with $L \geq 2$ indexed by ℓ. Time is discrete and denoted by $t \in \mathbb{N}$.

At the beginning of each period of time *t*, the type of interactions agents experience is randomly determined. More precisely, interactions can be either local or global in the following sense: with probability $p \in (0, 1)$ each agent interacts with every other agent staying in its current location (local interactions) while with probability $1 - p$ each agent interacts with every other agent in the entire population (global interactions).

Under global interactions, no agent can influence its pool of potential partners like in exogenous interactions models. In contrast, under local interactions, agents can partially influence their interaction partners by choosing one location over another. Therefore, this setting combines characteristics of models with exogenous interaction structures and those with endogenous interactions.

Independently of the type of interactions, agents play a Stag Hunt game as depicted in Fig. 1. In the game, each player chooses an action between *A* (Hare, work individually) and *B* (Stag, cooperate). We assume that $b > a > c > d > 0$ and $a + c > b + d$ so that (*A*, *A*) is the risk dominant convention while (*B*, *B*) is the payoff dominant one.

In the following, we will denote with $\alpha = (a - d)/(a - d + b - c) > 1/2$ the probability with which action *B* is played in the mixed strategy equilibrium of the Stag Hunt game, so that if a proportion α of agents plays action *B* then both *A* and *B* are best replies. We can also interpret α as the size of the basin of attraction of the risk dominant convention.

When interactions are local and an agent is alone in its current location, it has no other players to interact with. In this case, the agent receives a reservation payoff denoted by u. For simplicity, we set $u = 0$, although our results remain valid for any $u < d$ - that is, for any reservation payoff strictly lower than the minimum possible in the Stag Hunt game.

Fig. 1 Normal-form representation of the Stag Hunt game

		Player n'	
		A	B
Player n	A	a, a	c, d
	B	d, c	b, b

There are two types of agents in the population, distinguished by their mode of reasoning. A fraction $q \in (0, 1)$ of the population is made of fine reasoners who can condition the action they play in the Stag Hunt game on the type of interaction (i.e., local or global) they are facing, while a fraction $1 - q$ of the population is made of coarse reasoners who play the same action in both types of interaction. Basically, coarse reasoners cannot distinguish local from global interactions, although they are aware of the likelihood that it is one or the other.

We denote with $\mathcal{C}$ and $\mathcal{F}$, respectively, the set of coarse reasoners and the set of fine reasoners, so that $\mathcal{C} \cup \mathcal{F} = \mathcal{N}$ and $\mathcal{C} \cap \mathcal{F} = \emptyset$. Moreover, we indicate with $i \in \mathcal{C}$ a generic coarse reasoner and with $j \in \mathcal{F}$ a generic fine reasoner.

Given the strategy set $\Sigma \equiv \mathcal{L} \times \{A, B\}^2$, we denote the strategy of agent $n \in \mathcal{N}$ at time *t* by means of the vector $\sigma_{nt} = (\ell_{nt}, l_{nt}, g_{nt}) \in \Sigma$ where $\ell_{nt} \in \mathcal{L}$ is the location chosen by agent *n* at time *t*, $l_{nt} \in \{A, B\}$ is the action agent *n* plays if interactions are local at time *t*, and $g_{nt} \in \{A, B\}$ is the action agent *n* plays if interactions are global at time *t*. By assumption it must be $l_{it} = g_{it}$ for every coarse reasoner $i \in \mathcal{C}$ as coarse reasoners cannot condition their action on the type of interaction they are facing. Further, we denote with $\mathcal{S} \equiv \mathcal{L}^N \times \{A, B\}^N \times \{A, B\}^N$ the state space of the system and we indicate the state of the system at time *t* via a matrix $S_t = (\sigma_{nt})_{n \in \mathcal{N}} \in \mathcal{S}$. Finally, we denote the current state of the system excluding agent n' via the matrix $S_{-n't} = (\sigma_{nt})_{n \in \mathcal{N}: n \neq n'}$.

We assume that agents aim to maximize the average payoff from their interactions. Since an agent may engage in multiple interactions within a given period, we interpret this as follows: during each time period—from *t* to $t + 1$—an agent is endowed with a unit of time, which it allocates equally across all interactions it participates in. That is, if an agent engages in *k* interactions at time *t*, it spends 1/*k* units of time on each. Consequently, the agent evaluates its overall payoff by weighting each interaction payoff by the proportion of time spent on it, which is equivalent to maximizing the average payoff.

We assume that agents are myopic best responders. Coarse reasoners select a single best response based on the overall distribution of behaviors across both local and global interactions, whereas fine reasoners determine distinct best responses for each context. Despite this difference, in Online Appendix A we show that both types of agents ultimately choose the location that maximizes coordination on the action employed in local interactions.

The system evolves according to synchronous myopic best reply with inertia and uniform random mistakes. More precisely, at the end of each period of time *t* every agent $n \in \mathcal{N}$ has a fixed probability $\rho \in (0, 1)$ to revise its strategy. If given a revision opportunity, with probability $1 - \varepsilon \in (0, 1]$ the agent myopically best replies to the current state of the system S_t by choosing a strategy providing the highest average payoff and randomizing over best replies if the best reply is not unique; instead, with probability ε the agent makes a mistake and adopts a strategy uniformly at random. Formally, let $\pi_{nt}(\sigma) = \pi(\sigma; S_{-nt})$ be the average payoff of agent *n* associated to strategy σ given that the other agents adopt strategies S_{-nt}. With probability $1 - \varepsilon$ the agent best replies to the current state of the system, selecting with positive probability a strategy σ^* if and only if

$$\sigma^* \in \underset{\sigma}{\operatorname{argmax}}\ \pi_{nt}(\sigma)$$

Instead, with probability ε the agent makes a mistake and, so, it chooses a strategy uniformly at random.

The system described induces a Markov chain over the state space $\mathcal{S}$. We say that the system evolves according to an unperturbed dynamics if $\varepsilon = 0$, while we say that the system evolves according to a perturbed dynamics if $\varepsilon > 0$. We study the two cases in turn.

4 Unperturbed dynamics

In this section, we examine the system dynamics in the absence of mistakes - that is, agents myopically best respond to the current state of the system with probability one. Under this dynamic, every strict Nash equilibrium is an absorbing state: if the system reaches a configuration where each agent plays their unique best response to the prevailing state at time t, it will remain in that state with probability one for all subsequent times $t + k$, for any $k > 0$. We proceed by characterizing all such absorbing states and demonstrating that no other absorbing sets exist.

We begin with some needed additional notation. We use an *ad hoc* labeling to refer to the states of the system where every coarse reasoner $i \in \mathcal{C}$ adopts strategy $\sigma_i = (\ell^*, X, X)$, $X \in \{A, B\}$, and where every fine reasoner $j \in \mathcal{F}$ adopts strategy $\sigma_j = (\ell^{**}, Y, Z)$, $Y, Z \in \{A, B\}$, made of the following three components:

- The action played by coarse reasoners: *X*;
- The spatial distribution of types: if coarse reasoners and fine reasoners stay in the same location, i.e., if $\ell^* = \ell^{**}$, then *X* will be followed by the symbol "-"; if, instead, coarse and fine reasoners are segregated according to type, i.e. if $\ell^* \neq \ell^{**}$, then *X* will be followed by the symbol "/";
- The actions played by fine reasoners in local (*Y*) and global (*Z*) interactions: *YZ*.

For example, the label A-BA refers to the states of the system in which $\sigma_i = (\ell^*, A, A)$ for every $i \in \mathcal{C}$ and $\sigma_j = (\ell^*, B, A)$ for every $j \in \mathcal{F}$ with everybody choosing the same location $\ell^* \in \mathcal{L}$. Note that there are L distinct states like this, one for each location in $\mathcal{L}$.

By adopting this notation we aim at stressing that the specific location chosen by agents is not of particular interest as all locations are identical: what really matters is whether coarse and fine reasoners stay in the same or in a different location.

With this notation at hand, we can state our first result.

Theorem 1 *If the population is large enough, then all the states of the following types are absorbing:*

(1.1) *A-AA, if* $p, q \in (0, 1)$*;*
(1.2) *A-AB, if* $p \in (0, 1)$ *and* $q \in \left(\alpha, \min\left\{\frac{\alpha}{1-p}, 1\right\}\right)$*;*

(1.3) *A-BA, if* $p \in (0,1)$ *and* $q \in \left(\alpha, \min\left\{\frac{\alpha}{p}, 1\right\}\right)$*;*

(1.4) *A/BA, if* $p \in \left(0, \frac{a-d}{b-d}\right)$ *and* $q \in (0,1)$*;*

(1.5) *B-AB, if* $p \in (0,1)$ *and* $q \in \left(1-\alpha, \min\left\{\frac{1-\alpha}{p}, 1\right\}\right)$*;*

(1.6) *B-BA, if* $p \in (0,1)$ *and* $q \in \left(1-\alpha, \min\left\{\frac{1-\alpha}{1-p}, 1\right\}\right)$*;*

(1.7) *B-BB, if* $p \in (0,1)$*;Further:*

(1.8) *there are no absorbing sets other than the states of types A-AA, A-AB, A-BA, A/BA, B-AB, B-BA, and B-BB.*

As established in Theorem 1—with the proof provided in Online Appendix A—the system admits multiple types of absorbing states, most of which arise only within specific sub-regions of the (*p*, *q*) parameter space, as illustrated in Fig. 2. This result becomes clearer when considered in light of the following observations.

First, since agents of a given type are identical and there are no location capacity constraints nor costs associated to changing location, agents of the same type must adopt the same strategy in any absorbing state of the system. Consequently, only configurations that can be represented in the form X-YZ or X/YZ, with X, Y, Z ∈ {A, B}, can result in absorbing states—yielding a total of 16 distinct types of such states.

Second, an absorbing state requires some degree of coordination between coarse reasoners and fine reasoners—either in local or global interactions. But then, configurations such as A-BB, A/BB, B-AA, and B/AA cannot constitute absorbing states, as they necessarily involve one group (either coarse or fine reasoners) playing a suboptimal strategy.

Third, if coarse and fine reasoners choose the same action in local interactions, then agents cannot be segregated by type in equilibrium. As a result, states such as A/AA, A/AB, B/BA, and B/BB are not absorbing. In these configurations, segregation is unsustainable because both types of agents are indifferent between staying in their current location and moving to the one chosen by the other type. Consequently, there is a positive probability that all agents will eventually converge to the same location. Once this occurs, segregation becomes unattainable.

Fourth, since the set of strategies available to coarse reasoners is a strict subset of those available to fine reasoners, coarse reasoners cannot earn a strictly higher average payoff than fine reasoners in equilibrium. This has two important implications. On one side, states of the type B/AB cannot be absorbing, because in these states fine reasoners would have an incentive to emulate coarse reasoners in order to improve their payoff—contradicting the equilibrium condition. On the other side, this constraint limits the parameter space in which states like A-AB, A-BA, B-AB, and B-BA can be absorbing. Specifically, for these states to persist, the fraction of fine reasoners must be sufficiently high to ensure they earn a higher payoff than coarse reasoners. Graphically, in Fig. 2, this requirement translates into horizontal lines defined by the conditions $q > 1-\alpha$ for states B-AB and B-BA, and $q > \alpha$ for states A-AB and A-BA. Notably, the condition becomes stricter when fine reasoners choose Stag in cases of miscoordination with coarse reasoners.

Finally, when miscoordination arises in a given state, coarse reasoners must have no incentive to change their strategy. In Fig. 2, this condition is represented by the curved boundaries for states of the type A-AB, A-BA, B-AB, and B-BA, and by a

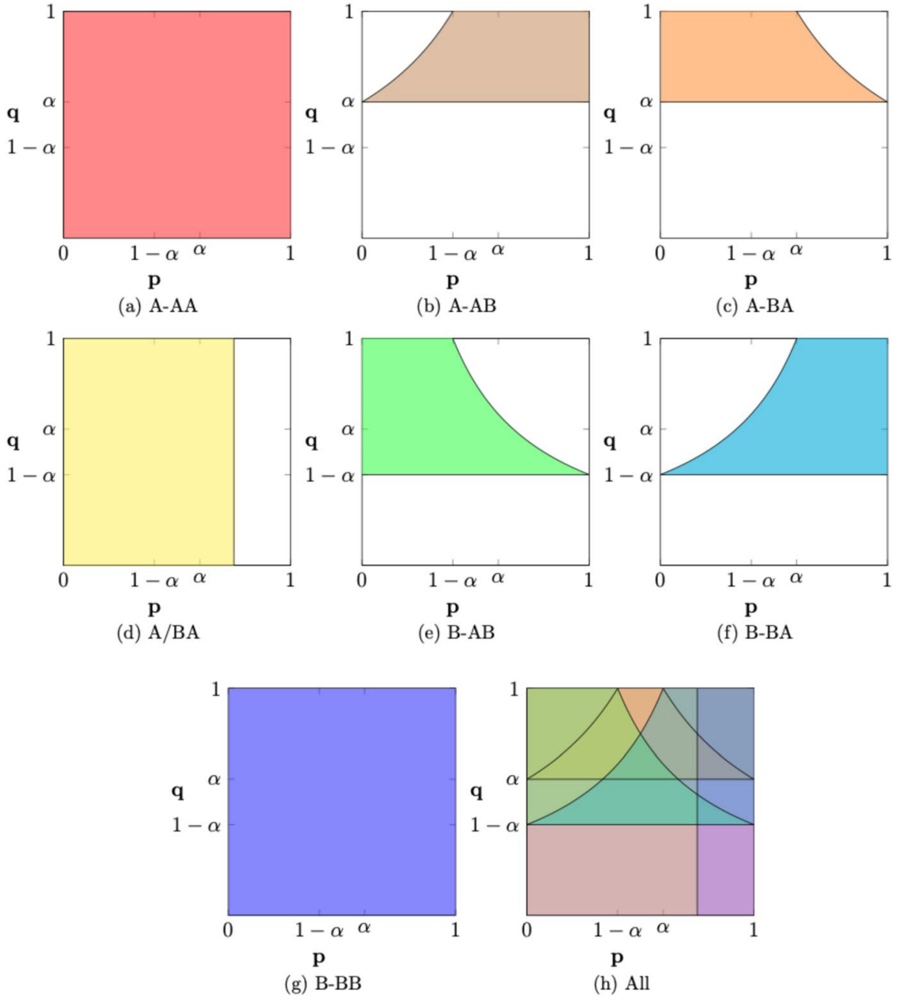

Fig. 2 Parameter regions in which each state type is absorbing. Case $a = 4$, $b = 5$, $c = 3$, $d = 1$ and, consequently, $\alpha = 0.6$ and $(a - d)/(b - d) = 0.75$

vertical line for states of the type A/BA.[1] For instance, states of the type B-BA fail to be absorbing when the fraction of fine reasoners is sufficiently high (q is large) and local interactions are relatively infrequent (p is small). Under these conditions, coarse reasoners prefer to switch to the strategy (ℓ^*, A, A), which allows them to better coordinate with the more numerous fine reasoners during global interactions - which are, in this scenario, the most common form of interaction.

[1] The line is vertical rather than curved because agents are segregated by type.

5 Perturbed dynamics

In this section, we examine the dynamics of the system in the presence of mistakes: with probability $1 - \varepsilon$ an agent myopically best responds to the current state of the system, while with probability $\varepsilon > 0$, the agent adopts a strategy at random. Our focus is on understanding the system long-run behavior under such perturbed dynamics.

To analyze the long-run behavior of this system, we adopt two complementary approaches. In Sect. 5.1, we use stochastic stability to identify which absorbing states of the unperturbed dynamics are likely to be selected in the long run—i.e., which are stochastically stable for some values of p and q. We also derive sufficient conditions on p and q for the selection of specific absorbing states.

Then, in Sect. 5.2, we conduct simulations to validate and extend the analytical findings. These simulations show that the conclusions drawn from the stochastic stability analysis remain robust across the full parameter space. Additionally, they suggest that the insights of the model hold even when small but persistent mistakes are present, and over extended, though finite, time horizons.

5.1 Stochastic stability analysis

Since, under perturbed dynamics, mistakes allow every state - and every strict subset of the state space $\mathcal{S}$—to be left with positive probability, the system no longer has absorbing states or absorbing sets. Furthermore, starting from any state $S \in \mathcal{S}$, the system can reach any other state $S' \in \mathcal{S}$ with positive probability. As a result, the system forms an ergodic Markov chain.

Since the system is an ergodic Markov chain, it admits a unique invariant distribution, denoted by μ^ε. This distribution can be interpreted as a probability measure over the state space $\mathcal{S}$ where $\mu^\varepsilon(S)$ approximates the fraction of time the system spends in state S in the long run. This probability distribution exists and is unique for any given $\varepsilon > 0$. However, following standard practice in the literature, we focus on the limit distribution μ^*, obtained as $\varepsilon \to 0$.

The limit distribution μ^* is an approximation of the invariant distribution for sufficiently small values of the mutation rate ε. States with strictly positive probability in μ^* are referred to as stochastically stable. These states must represent absorbing sets of the system under unperturbed dynamics (Young 1993). In our framework, this means that only the absorbing states identified in Theorem 1 can be candidates for stochastic stability. To identify which of these absorbing states are stochastically stable, we apply radius-coradius arguments (Evans 1993; Ellison 2000).

In the following, we refer to stochastically stable *sets* to indicate sets containing only stochastically stable states. This is useful when we refer to the types of states listed in Fig. 2 because if any single state of a given type is stochastically stable then all states of that type are so.

In Theorem 2 we identify the types of absorbing states of the system that are never stochastically stable.

Theorem 2 *If the population is large enough, then all the absorbing states of the types A-AA, A-AB, A-BA, and B-AB are never stochastically stable.*

By stating that all absorbing states of the types A-AA, A-AB, A-BA, and B-AB are never stochastically stable (as demonstrated in Online Appendix B), Theorem 2 narrows down the set of candidate stochastically stable states to those of the types A/BA, B-BA, and B-BB. This result leads to two important implications.

Stochastically stable states in the system are characterized by perfect coordination in local interactions. This coordination arises either because fine and coarse reasoners choose the same action in local interactions—such as in the B-BA and B-BB cases—or because players segregate by type when selecting different actions, as seen in the A/BA scenario. Interpreting local interactions as group interactions, this implies that stability in our framework requires each group to exhibit a consistent, shared pattern of behavior.

In a stochastically stable state of the system, fine reasoners consistently choose action B—the one associated with the payoff dominant convention—in local interactions. This behavior highlights an evolutionary advantage of being a fine reasoner: the ability to reliably secure coordination benefits within local interactions.

In Theorem 3 we further analyze the set of stochastically stable states of the system and provide sufficient conditions for the stochastic stability of the states of the types A/BA, B-BA, and B-BB.

Theorem 3 *If the population is large enough, then all and only absorbing states of the following types are stochastically stable:*

(3.1) *A/BA, if* $p \in \left(0, \frac{2\alpha-1}{\alpha}\right)$ *and* $q \in (0,1)$*;*
(3.2) *B-BA, if* $p \in \left(\frac{a-d}{b-d}, 1\right)$ *and* $q \in \big(2(1-\alpha), 1\big)$*;*
(3.3) *B-BB, if* $p \in \left(\frac{a-d}{b-d}, 1\right)$ *and* $q \in \big(0, 1-\alpha\big)$.

We prove this result in Online Appendix C, while in Fig. 3 we provide a graphical representation of the implications of Theorem 3. In the figure, setting the payoffs to $a = 4$, $b = 5$, $c = 3$, and $d = 1$, we color the regions in the (p, q) space where states of type A/BA, B-BA, and B-BB are or can be stochastically stable.

If the frequency of local interactions is sufficiently small, then the set of states of the type A/BA is stochastically stable. These are states in which coarse reasoners always play Hare; instead, fine reasoners do so only in global interactions while they coordinate on Stag if interactions are local. This configuration emerges because—given the low frequency of local interactions—coarse reasoners aim at coordinating with fine reasoners in global interactions, while fine reasoners end up playing A in global interactions as it is the action characterizing the risk dominant convention. Moreover, this configuration is stable independently of the fraction of fine reasoners q in the population because agents are segregated according to their type and, consequently, they achieve perfect coordination in both local and global interactions.

On the contrary, if the frequency of local interactions is sufficiently high, then two different scenarios are possible and, more precisely, the actual long run prediction depends on the fraction of fine reasoners in the society.

If the fraction of fine reasoners is sufficiently high, then the set of states of the type B-BA is stochastically stable. In these states, coarse reasoners consistently choose Stag, while fine reasoners play Stag in local interactions but switch to Hare in global

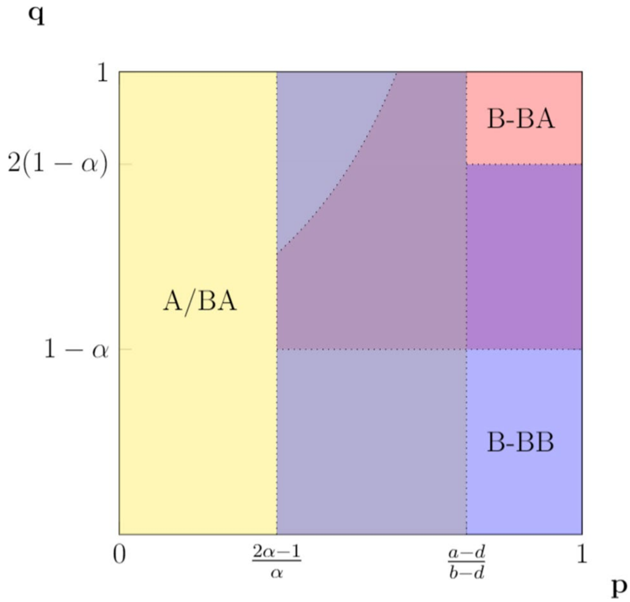

Fig. 3 Types of stochastically stable states in the parameter space (p, q). Case $a = 4$, $b = 5$, $c = 3$, $d = 1$

interactions. This configuration arises because—given the high frequency of local interactions—coarse reasoners seek to coordinate with fine reasoners locally. Meanwhile, fine reasoners opt for action A (Hare) in global interactions, as it represents the risk-dominant convention and the presence of enough fine reasoners ensures that doing so yields them the highest expected payoff. However, since all agents remain in the same location, persistent miscoordination occurs—specifically in global interactions between fine and coarse reasoners. Consequently, the frequency of miscoordination decreases with both the probability of local interactions p and the fraction of coarse reasoners in the population $1 - q$.

Instead, if the fraction of fine reasoners is sufficiently small, then the set of states of the type B-BB is stochastically stable. These are states in which all agents play Stag independently of the type of interaction they face. This configuration emerges because— given the high frequency of local interactions—coarse reasoners seek to coordinate with fine reasoners locally and, thus, play B. Moreover, fine reasoners are so few that they are better off by coordinating on B with coarse reasoners—who are relatively many— rather than choosing A in global interactions. In states of the type B-BB there is perfect coordination on Stag for all levels of p and q that do not compromise stochastic stability.

Figure 3 also illustrates that Theorem 3 identifies the stochastically stable states only within a subset of the parameter space. Broadly speaking, two regions of uncertainty remain regarding which states are selected in the long run. First, for intermediate values of the probability of local interactions p, it is unclear whether states of type A/BA or B-BA and B-BB are selected. Second, when q takes intermediate values and

p is sufficiently high, it remains uncertain whether the stochastically stable states are of type B-BA or B-BB.

It is also important to emphasize that Theorem 3 establishes the stochastic stability of entire sets of states corresponding to each type—A/BA, B-BA, and B-BB. That is, either all states of a given type are stochastically stable, or none are. This outcome follows directly from the assumption that all locations are identical. Since no location offers a long-run advantage, agents have no incentive to favor one over another. As a result, stochastic stability analysis cannot distinguish between equilibrium states of the same type that differ only in their location choices.

Finally, We observe that the threshold $(a-d)/(b-d)$ is the same as the one found in Newton and Sercombe (2020) for agency autonomy (see Remark 2), in Naono (2022) for persistence of the bilingual strategy (see Proposition 1) and in Cui (2023) for existence of the action-heterogeneous absorbing sets (see Proposition 2) and expected convergence time to depend on linking frictions (see Theorem 2).

Overall, these findings suggest different outcomes depending on the dominant mode of interaction. When interactions are mostly global, social coordination emerges alongside segregation by reasoning type, with fine reasoners more likely to behave cooperatively in local interactions. Conversely, when interactions are mostly local, social coordination is still achieved but without segregation—potentially accompanied by lower cooperation from fine reasoners in global interactions. If we interpret global and local interactions as proxies for online and offline settings, respectively, the model predicts that a shift from offline to online interactions (i.e., a decrease in p) should be associated with declining overall cooperation and a relative increase in cooperation by fine reasoners compared to coarse reasoners.

These dynamics yield important insights. On the one hand, increasing the frequency of local interactions can promote the diffusion of the payoff-dominant convention, as it facilitates a transition from less efficient states like A/BA to more efficient states such as B-BA or B-BB. On the other hand, a higher proportion of fine reasoners in the population can have ambiguous effects. If interactions are primarily global (low p), states like A/BA become stochastically stable, and a greater presence of fine reasoners leads to more frequent use of Stag in local interactions. However, when interactions are mostly local (high p), a higher fraction of fine reasoners can have two negative effects on the spread of the payoff-dominant convention: (i) it may shift the system from B-BB to the less desirable B-BA; and (ii) if B-BA is already in place, it reduces the use of Stag in global interactions, while decreasing the frequency of miscoordination between coarse and fine reasoners in global interactions.

Interestingly, the frequency of local interactions and the proportion of fine reasoners interact in a nontrivial way. The benefits of increasing local interactions (raising p) are weakly decreasing in the fraction of fine reasoners. This is because, in a population dominated by fine reasoners, increasing p is more likely to move the system from A/BA to B-BA rather than to the more efficient B-BB, thereby limiting the potential gains from enhanced local coordination.

5.2 Simulation analysis

The stochastic stability analysis performed in Sect. 5.1 provides sharp predictions about the long run behavior of the system in the perturbed dynamics, but only for a portion of the (p, q)-space. To complete the analysis we employ agent-based simulations exploring the long run behavior of the system in the entire parameter space. An additional advantage of agent-based simulations is that they allow us to study the evolution of the system for a fixed small population size and a non-vanishing mistake probability, less demanding conditions with respect to those of Theorems 2 and 3.

We run simulations setting $N = 100$, $L = 3$, $\rho = 0.2$, and $\varepsilon = 0.1$. Moreover, we explore the (p, q)-space for $p, q \in \{0.1, 0.2, \ldots, 0.9\}$ for a total of $9^2 = 81$ combinations. For each of these combinations, we perform 10 independent simulations of the model and in each simulation we run the system for 10^5 iterations.

In each iteration, every agent has a 0.2 probability of receiving a revision opportunity. If given this opportunity, the agent updates its strategy using a myopic best response with probability 0.9. With the remaining 0.1 probability, the agent selects a strategy uniformly at random, introducing noise into the dynamics.

The simulation results presented below are based on the following payoff structure for the Stag Hunt game: $a = 4$, $b = 5$, $c = 3$, and $d = 1$. These values were chosen to avoid extreme threshold parameters, yielding $\alpha = 0.6$ and $(a - d)/(b - d) = 0.75$.

Applying Theorem 1, we can identify which types of states are absorbing across different regions of the (p, q) parameter space. However, Theorem 2 allows us to rule out the possibility that the system spends a significant amount of time in absorbing states of types A-AA, A-AB, A-BA, and B-AB in the long run.

According to Theorem 3, the system tends to spend most of its time in absorbing states of the following types, depending on the values of p and q: (i) A/BA if $p \in (0, 0.33)$, (ii) B-BA if $p \in (0.75, 1)$ and $q \in (0.8, 1)$, and (iii) B-BB if $p \in (0.75, 1)$ and $q \in (0, 0.4)$.

These findings are summarized in Table 1. It is important to note that the system is not expected to remain in a single absorbing state of a given type. Rather, it frequently transitions between absorbing states of the same type that differ only in the specific locations occupied. This reflects the symmetry of the environment, where identical locations prevent long-term preference for any particular one.

Table 1 Values of p and q for which states of a given type are absorbing (by Theorem 1) and for which they are guaranteed to be stochastically stable (by Theorems 2 and 3), if $a = 4$, $b = 5$, $c = 3$, $d = 1$

Type	Absorbing		Stochastically Stable	
	p	q	p	q
A-AA	$(0.00, 1.00)$	$(0.00, 1.00)$	$\emptyset$	$\emptyset$
A-AB	$(0.00, 1.00)$	$\left(0.60, \frac{0.60}{1-p}\right)$	$\emptyset$	$\emptyset$
A-BA	$(0.00, 1.00)$	$\left(0.60, \frac{0.60}{p}\right)$	$\emptyset$	$\emptyset$
A/BA	$(0.00, 0.75)$	$(0.00, 1.00)$	$(0.00, 0.33)$	(0.00, 1.00)
B-AB	$(0.00, 1.00)$	$\left(0.40, \frac{0.40}{p}\right)$	$\emptyset$	$\emptyset$
B-BA	$(0.00, 1.00)$	$\left(0.40, \frac{0.40}{1-p}\right)$	$(0.75, 1.00)$	(0.80, 1.00)
B-BB	$(0.00, 1.00)$	$(0.00, 1.00)$	$(0.75, 1.00)$	(0.00, 0.40)

Simulation results are displayed in Fig. 4, where various measures are considered to summarize key system characteristics. Panel (a) presents the assortativity of coarse reasoners, defined as:

$$\mathcal{A}_t^{\mathcal{C}} \equiv \sum_{\ell \in \mathcal{L}} \frac{n_{\ell t}^{\mathcal{C}}}{n^{\mathcal{C}}} \frac{n_{\ell t}^{\mathcal{C}} - 1}{n^{\mathcal{C}} - 1}$$

Here, $n_{\ell t}^{\mathcal{C}}$ denotes the number of coarse reasoners located in ℓ at time t. The assortativity measure ranges from 0 to 1, where 0 indicates that no coarse reasoners share the same location, and 1 indicates that all coarse reasoners stay in the same location. Panel (b) presents the corresponding assortativity measure for fine reasoners, while panel (d) shows agents' segregation by type, measured as the complement of the agents mixing measure:

$$\mathcal{M}_t = \sum_{\ell \in \mathcal{L}} \frac{n_{\ell t}^{\mathcal{C}}}{n^{\mathcal{C}}} \frac{n_{\ell t}^{\mathcal{F}}}{n^{\mathcal{F}}}$$

Panel (c) reports the fraction of coarse reasoners playing action B (Stag), while panels (e) and (f) illustrate the fraction of fine reasoners playing B under local and global interactions, respectively. Figure 4 presents average values calculated as follows: for each simulation, we computed each measure at every time step, beginning from period 1000 to eliminate initialization bias. We then averaged these values within each simulation and across all 10 independent simulations performed for a given (p, q) pair. Standard errors across simulations were also computed and reported in parentheses.

The results presented in Fig. 4 suggest that for $p \leq 0.6$ the system spends most of the time in states of the type A/BA, regardless of the fraction of fine reasoners q. This is evidenced by the spatial distribution of agents: coarse reasoners tend to cluster in a single location (panel (a)), as do fine reasoners (panel (b)), leading to segregation by type (panel (d)). In terms of Stag Hunt behavior, coarse reasoners predominantly play A (panel (c)), while fine reasoners play A only when interactions are global (as shown in panels (e) and (f)).

In contrast, when $p > 0.6$ two distinct long-run outcomes emerge depending on the value of q. If $q < 0.5$, the system spends most of the time in states of the type B-BB; whereas, if $q > 0.5$, the system spends most of the time in states of the type B-BA.

These findings are visually summarized in Fig. 5, alongside the theoretical predictions discussed in Sect. 5.1.

In Online Appendix D, we present simulation results for various payoff configurations in the Stag Hunt game. Taken together, these results indicate that the threshold value of the probability of local interactions, denoted by p^*—which marks the boundary of the region where states of type A/BA are stochastically stable—consistently falls in the interval $(\alpha, (a - d)/(b - d))$. Furthermore, the threshold value for the fraction of fine reasoners in the population, q^*—which separates the region where

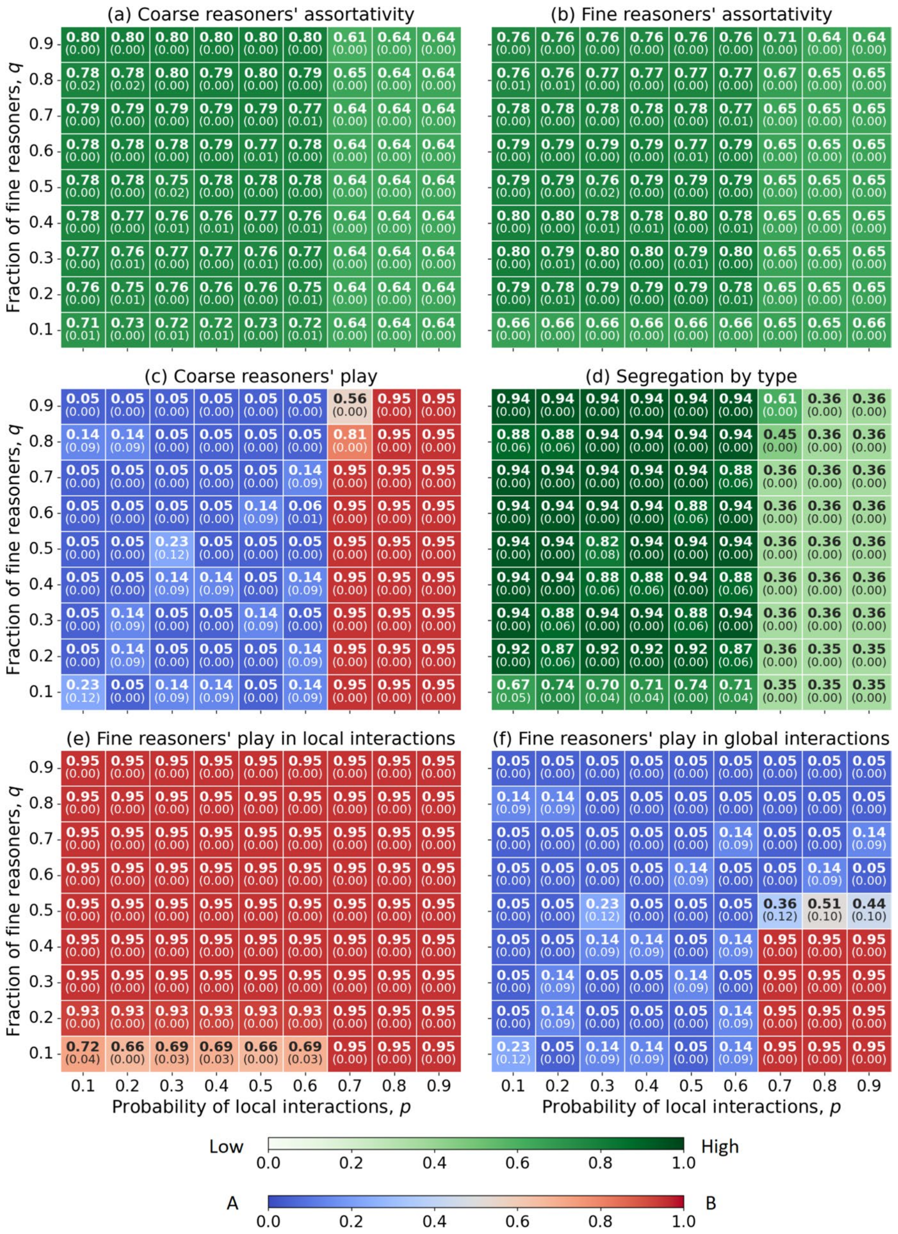

Fig. 4 Simulation results. Case $a = 4, b = 5, c = 3, d = 1$

states of type B-BB are selected from the region where B-BA states are stochastically stable—varies systematically with the value of α. Specifically, when α is high, q^* tends to approach its lower-bound $(1 - \alpha)$; conversely, if α is low, q^* is closer to its upper-bound $2(1 - \alpha)$.

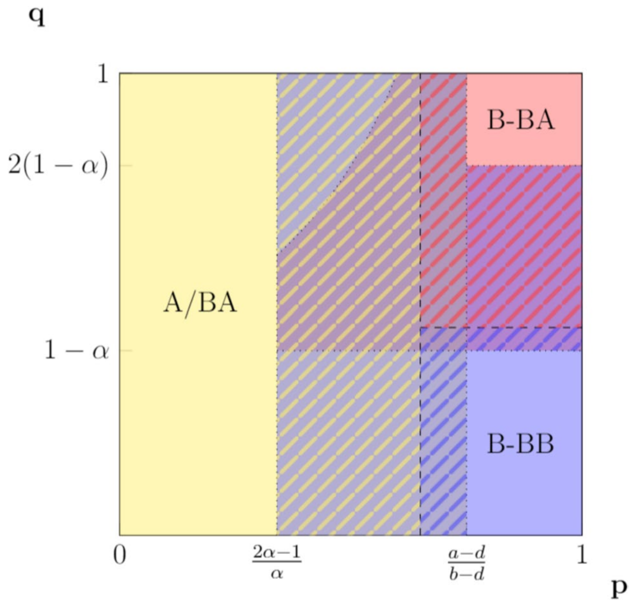

Fig. 5 Types of stochastically stable states in the entire parameter space (p, q): uniformly colored regions refer to both stochastic stability and simulation results, while striped regions refer to simulation results. Case $a = 4, b = 5, c = 3, d = 1$

Overall, these findings suggest that the qualitative results provided by Theorem 3 hold more generally: the threshold values for the sufficient conditions extend naturally to nearby regions of the parameter space. Moreover, these findings hold even in the case of non-vanishing mistakes probabilities and a relatively small population size.

6 Discussion

We have considered a finite population of agents who myopically best reply with a small probability of making a mistake, and we have studied how the mode of reasoning and the structure of interaction can affect the evolution of conventions in this setup.

We have found that if interactions are mostly global then selection favors A/BA conventions where agents are separated into different locations according to their mode of reasoning (/), with coarse reasoners playing Hare (A) in all interactions and fine reasoners playing Stag locally and Hare globally (BA). If, instead, interactions are mostly local, then all agents stay in the same location (-) and there are two scenarios. If coarse reasoners are sufficiently numerous, all agents play Stag both globally and locally (B-BB). Instead, if coarse reasoners are not numerous enough, coarse reasoners play Stag, while fine reasoners play Hare globally and Stag locally (B-BA). This implies that the co-existence of coarse and fine reasoning may favor or

Table 2 Qualitative summary of results in terms of conventions selected in the long run

Qualitative summary of results

fraction q of fine reasoners ↑ / fraction p of local interactions →	"low"	"high"
"high"	**A/BA** fine reasoners locally play Stag	**B-BA** fine reasoners globally play Hare
"low"	**A/BA** fine reasoners locally play Stag	**B-BB**

hamper the diffusion of the payoff dominant convention depending on the structure of interaction. Table 2 summarizes these qualitative features of our results.

Interestingly, despite the absence of constraints on the number of interactions, location capacities, or frictions in agents mobility, our results reveal the coexistence of both the payoff dominant and risk dominant conventions in equilibrium (A/BA and B-BA). We also observe the emergence of agents segregation based on their mode of reasoning (A/BA), as well as instances of persistent miscoordination in equilibrium (B-BA).

From the comparison of the welfare associated with the viable long run outcomes we can draw some conclusions. By looking at Table 2, if interactions are mostly global (p "low"), increasing the fraction of fine reasoners (moving vertically in the table) turns out to be welfare enhancing because the long run outcome remains the same (A/BA) and fine reasoners are more prosocial than coarse reasoners in this case. Instead, if interactions are mostly local (p "high"), increasing the fraction of fine reasoners turns out to be welfare reducing because it changes the long run outcome from full adoption of Stag (B-BB) to a partial adoption of it (B-BA). Furthermore, we notice that increasing the fraction of local interactions (moving horizontally in the table) is always welfare enhancing.

If we look at local and global interactions as representing offline and online interactions, respectively. This model suggests that an increase in online interactions (corresponding to a decrease in p) is always associated with a reduction in the adoption of Stag.

Alternatively, if we interpret locations as representing groups or clubs, the model suggests that individuals capable of distinguishing between group-salient and generic interactions (i.e., fine reasoners) tend to adopt a strategy that prescribes playing Stag in group interactions and Hare in non-group interactions. This behavior can be understood as a way of maximizing the benefits of group affiliation—cooperating within the group to reinforce cohesion, while acting more individualistically outside the group to undermine the welfare of outsiders or affiliated to different groups. This behavior by fine reasoners is constrained only when group interactions are frequent and the majority of individuals do not differentiate based on group membership.

A simplifying assumption in our model is that agents are always fine reasoners or coarse reasoners. In fact, we can expect real decision-makers to reason sometimes coarsely and sometimes finely depending on the choice problem faced, i.e., we can expect the mode of reasoning to be endogenous. Such endogeneity could disrupt the equilibria where fine reasoners and coarse reasoners co-exist, behave differently, and possibly live in different locations, as all agents might end up facing the same payoff structure and, therefore, following the same optimal behavior. However, if we allow for heterogeneity in cognitive abilities, i.e., different costs to reason finely, we may obtain that in equilibrium agents have heterogeneous probabilities to resort to fine reasoning, so that an equilibrium may arise where individuals are segregated in different locations and behave differently according to their mode of reasoning. Further research is needed to establish the precise conditions for this to happen, but we can already note that only for states where everybody plays the Stag both locally and globally (i.e., of the B-BB type) we can be confident that all agents will reason coarsely (since coordination is obtained without conditioning actions to the type of interaction), while for all other states which turned out to be stochastically stable in our model (i.e., of the A/BA and B-BA types) there are good reasons to expect that those with small costs of fine reasoning will reason finely (since conditioning one's action to the type of interaction allows better coordination) while those with high costs will not.

A last remark stems from the observation that fine reasoners and coarse reasoners may end up in the long run living in different locations, which is what happens in states of type A/BA. This outcome is reminiscent of real-world phenomena characterized by globalization and polarization, which is a widespread phenomenon at least for online interactions. We stress that when this cognitive segregation happens, the frequency of interaction with a fine reasoner is larger for a fine reasoner than for a coarse reasoner. This creates a novel form of assortativity – namely, assortativity in cognition – which deserves further investigation both theoretically, along the lines of Bilancini et al. (2023), and empirically.

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