



Characterization of quasi-periodic and chaotic dynamics in a memristive MEMS: parameter-induced transitions and thermo-sensitive modulation

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Abstract

This work presents the mathematical modeling and numerical characterization of a Micro-Electro-Mechanical System (MEMS) integrating an HP memristor coupled to electrical and mechanical resonators. Based on the linear-drift HP memristor model, the nonlinear electromechanical dynamics are investigated through Lyapunov exponents, bifurcation diagrams, Poincaré sections, recurrence plots, phase portraits, and Fourier spectra. The numerical results demonstrate that the intrinsic memristor parameters, namely the low-resistance state R_{on} , the high-resistance state R_{off} , the oxide thickness D , and the ionic mobility μ_v , directly control the amplitude, frequency content, and dynamical regime of the electromechanical oscillations, inducing transitions between quasi-periodic and chaotic behaviors. Temperature-dependent variations of these memristive parameters are shown to modulate both the oscillation amplitudes and the distribution of quasi-periodic and chaotic regimes, highlighting the thermo-sensitive character of the proposed architecture. A qualitative comparison with Josephson-junction-based MEMS is provided, emphasizing that the HP memristor exhibits similar nonlinear dynamical features while remaining fully operational at room temperature. These results suggest that memristive thermo-sensitive MEMS constitute a promising platform for adaptive nonlinear oscillators, thermo-sensitive sensing devices, and chaos-assisted electromechanical applications.

Keywords Nonlinear oscillations · Chaotic dynamics · Thermo sensitive · Hp memristor · MEMS

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1 Introduction

Since its prediction by L. Chua in “*Memristor — the missing circuit element.*” [1], the memristor has been the subject of extensive scientific investigations, and its memory and nonlinear properties have been demonstrated theoretically, numerically, and experimentally [2–9]. Owing to their micro- and nanometric dimensions as well as their nonlinear resistive switching properties, memristive devices constitute promising candidates for Micro-Electro-Mechanical Systems (MEMS) [10, 11]. MEMS themselves are particularly attractive for high-precision sensing and actuation applications because of their ultra-small dimensions and strong sensitivity to environmental perturbations [12–14].

Several of our previous studies have employed Josephson junctions in the development of passive and active thermo-sensitive MEMS actuators and sensors [15, 16]. However, the main limitation of Josephson-junction-based architectures lies in the stringent superconducting conditions required for their operation, especially the need for

cryogenic temperatures, which significantly restrict their practical implementation under ambient conditions. Nevertheless, the remarkable nonlinear and dynamical properties of Josephson junctions remain well established and widely documented [15–22]. Through their AC and DC Josephson effects, Josephson junctions have been extensively employed in nonlinear and chaotic dynamical systems [15, 16, 18–20, 23]. However, the cryogenic constraints associated with their use may considerably limit the practical scope of applications of such nonlinear electrodynamic systems.

Motivated by these limitations, the present work investigates the integration of an HP (Hewlett-Packard) memristor into a thermo-sensitive MEMS architecture as an alternative nonlinear memory element operating under standard ambient conditions. The objective of this work is to analyze how memristive nonlinearities influence the coupled thermo-electro-mechanical dynamics of the MEMS and to assess the role of the memristor parameters on the oscillatory regimes of the system.

In this context, the HP memristor model was selected because it provides a simple, physically motivated and widely adopted framework for describing nonlinear memory effects while operating under standard room-temperature conditions [24, 25]. Unlike Josephson junction-based devices, which generally require cryogenic environments to maintain superconductivity, the HP memristor remains fully operational at room temperature, making it suitable for thermo-sensitive MEMS applications [26, 27]. Moreover, the HP model captures the essential memristive features, including state-dependent resistance and pinched hysteresis behavior, while keeping the mathematical complexity tractable [28]. Its compact formulation facilitates coupling with thermo-electro-mechanical equations and allows efficient bifurcation, stability, and Lyapunov analyses [29, 30]. Furthermore, the HP memristor has become a widely employed benchmark model in nonlinear dynamics and chaos studies, allowing comparison with previous theoretical and experimental investigations [25, 27].

The electromechanical system investigated in this work consists of an HP memristor coupled to a microbeam resonator and an electrical oscillator $r_0 L_0 C_0$. In this configuration, the memristor behaves as a central nonlinear element influencing the oscillation amplitudes, resonance frequencies, and dynamical transitions. The oscillatory regimes are characterized through bifurcation diagrams, Lyapunov exponents, recurrence plots, phase portraits, Fourier spectra, and temporal evolution analyses of the principal currents flowing through the MEMS architecture.

The main contribution of this work lies in the development of a thermo-sensitive MEMS architecture capable of exhibiting complex nonlinear dynamics in the electromechanical response of the coupled system. By coupling an HP memristor with electrical and mechanical resonators, we

show that the intrinsic memristor parameters (R_{on} , R_{off} , D , μ_v) may significantly influence oscillation amplitudes, resonance frequencies, and transitions between quasi-periodic and chaotic regimes. The proposed system exhibits thermo-sensitive dynamical behavior through temperature-dependent variations of the memristive parameters, potentially enabling modulation of the electromechanical response. Beyond the generation of chaotic oscillations, the present work aims to investigate how memristive nonlinearities interact with coupled electrical and mechanical resonators within a thermo-sensitive MEMS environment. The proposed architecture enables the analysis of nonlinear energy redistribution mechanisms and thermo-sensitive electromechanical bifurcation dynamics. It also provides a framework for exploring transitions between quasi-periodic and chaotic regimes induced by physically meaningful memristor parameters and for assessing the impact of these parameters on the oscillatory behavior of the mechanical resonator and electrically induced noise dynamics. In contrast to simplified memristor circuits commonly employed solely for chaos generation, the present system aims to model a physically coupled thermo-electro-mechanical platform potentially relevant for adaptive sensing, nonlinear actuation, and chaos-assisted MEMS applications. In addition, unlike Josephson-junction-based architectures requiring cryogenic operating conditions, the proposed HP memristor-based MEMS remains compatible with room-temperature operation, thereby offering greater practical flexibility for adaptive sensors, nonlinear oscillators, and MEMS applications involving nonlinear and chaotic dynamics.

The remainder of this paper is organized as follows. Section 2 presents the mathematical modelling of the thermo-sensitive MEMS and derives the governing electromechanical equations. Section 3 is devoted to the numerical investigation of the nonlinear dynamics, including bifurcation structures, Lyapunov analyses, recurrence plots, phase portraits, and thermo-sensitive dynamical transitions induced by the memristive parameters. Section 4 presents a qualitative comparative analysis of the dynamical behaviors and thermo-sensitive responses obtained with the proposed thermo-memristive architecture. Finally, the main conclusions and perspectives of this work are summarized in Section 5.

2 Mathematical modelling of MEMS

In this section (Subsections 2.1–2.3), we establish the mathematical model used to investigate the MEMS depicted in Fig. 1. The circuit consists of an alternating current source with amplitude I_0 and excitation frequency ω_0 , an HP memristor (Fig. 2), a mechanical resonator (microbeam), and an electrical resonator composed of a resistive inductor (L_0 , r_0)

Fig. 1 Schematic of the memristive MEMS circuit investigated

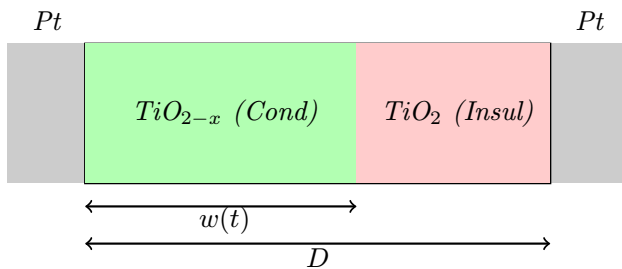
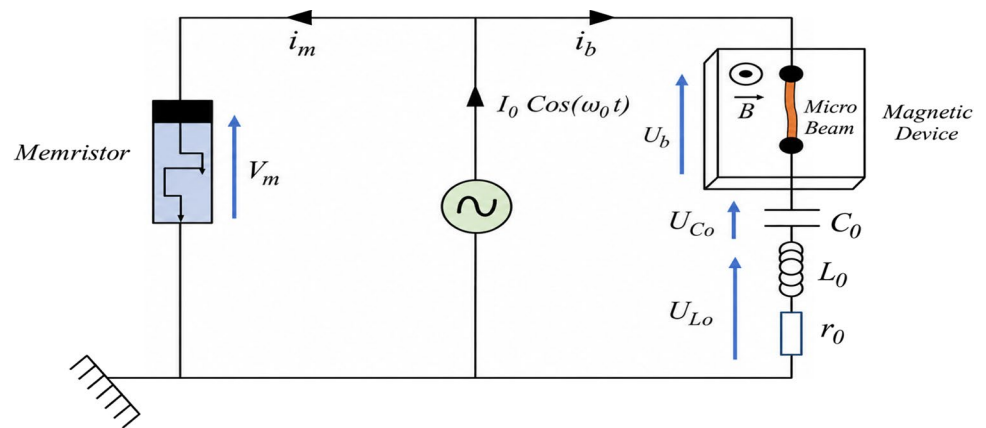


Fig. 2 Physical structure of the HP memristor illustrating the conductive TiO_{2-x} region, the insulating TiO_2 layer, and the evolution of the internal state variable $w(t)$ within the active layer thickness D

and a capacitor C_0 . The AC source provides energy to the system, while the electrical resonator (L_0, C_0) ensures sustained energy exchange through periodic charging and discharging. The memristor acts as the core control element, regulating the global dynamical response of the MEMS.

The microbeam is used to capture certain physical phenomena, such as magnetic fields and temperature, because of its hypersensitivity to environmental conditions. It may also operate as an ultra-precise micro/nanometric actuator depending on the activation parameters and oscillatory dynamics, while constituting the mechanical element responsible for the activation and transduction of the coupled electromechanical processes within the MEMS architecture. The set of differential equations governing the electrodynamic behavior of the system is derived from Kirchhoff's laws, the electrical network relations shown in the diagram illustrated in Ref. [2, 24, 31], and the classical Euler–Bernoulli beam theory, which is introduced in the following subsection.

2.1 Equations of mechanical oscillations

According to [32–34], the dynamics of the microbeams can be described by the partial differential equation Eq. (1), derived from the classical Euler–Bernoulli beam theory.

$$EI_y \frac{\partial^4 u(z, t)}{\partial z^4} + \rho A \frac{\partial^2 u(z, t)}{\partial t^2} + \lambda \frac{\partial u(z, t)}{\partial t} + NLT = f(t), \quad (1)$$

where E denotes the Young's modulus of the beam material, I_y the second moment of inertia of the beam, ρ the volume density, A the cross-section of the beam, $f(t)$ the beam activating force, $u(z, t)$ the transverse displacement function, and λ the damping coefficient. The nonlinearity term is assumed to be negligible, i.e., $NLT = 0$. The actuating force, $f(t)$ is a Lorentz force due to the current i_b through the beam and is expressed as:

$$f(t) = BL i_b = BL \left(\frac{dq}{dt} + \frac{e_b}{r_b} \right) \quad (2)$$

with $\vec{B} \perp \vec{L}$ and $\frac{e_b}{r_b} = \frac{-2BA}{\mu L} \int_0^L \frac{\partial u(z, t)}{\partial t} dz$. L is the length of the micro-beam; $\vec{B}$ the magnetic field; Here, e_b/r_b denotes the self-induced current resulting from the beam motion. According to Lenz's law, this induced current acts in opposition to the cause that generate it, which accounts for the negative sign. The term e_b represents the electromotive force induced in the beam, while the resistance of the beam is expressed as $r_b = \frac{\mu L}{A}$, with A the cross-sectional area, and μ the resistivity of the micro-beam material. The capacitor C_0 has a time-dependent charge $q(t)$, and therefore the total current in the beam is the sum of the capacitor charging current $C_0 (dq/dt)$ and the self-induced current e_b/r_b .

By substituting Eq. (2) into Eq. (1), we obtain the general model expressed in Eq. (3).

$$EI_y \frac{\partial^4 u(z, t)}{\partial z^4} + \rho A \frac{\partial^2 u(z, t)}{\partial t^2} + \lambda \frac{\partial u(z, t)}{\partial t} = BL \left[\frac{dq}{dt} - \frac{2BA}{\mu L} \int_0^L \frac{\partial u(z, t)}{\partial t} dz \right] \quad (3)$$

The shape of the modes must satisfy the differential equation and boundary conditions [32] (Chapter 3 Eq.(3.29)). The deflection $u(z, t)$ of the beam can then be written as

follows: $u(z, t) = \sum_{n=1}^{\infty} Z_n(z) T_n(t)$ where n indicates the mode of vibration; $T_n(t)$ represents the generalized coordinate of the amplitudes, and $Z_n(z)$ the set of eigenfunction of the equation:

$$\frac{\partial^4 u(z, t)}{\partial z^4} + \frac{\rho A}{E I_y} \frac{\partial^2 u(z, t)}{\partial t^2} = 0. \quad (4)$$

In our case, we have $u(0, t) = u(L, t) = 0$ (boundary conditions) and the set eigen functions $Z_n(z)$ are written:

$$Z_n(z) = a_n(\cos \xi_n z - \cosh \xi_n z) + b_n(\sin \xi_n z - \sinh \xi_n z) \quad (5)$$

with ξ_n the solution of the transcendental equation:

$$\cos \xi_n L \cosh \xi_n L - 1 = 0. \quad (6)$$

As we focus on the study of the fundamental state, we shall take it for the rest $n = 1$.

In this study, we will be interested in the dynamics of the middle of the beam ($z = \frac{L}{2}$) in the fundamental mode ($n = 1$). In addition, for this study we will use the normalization made by N. Lobontiu [35, 36]. Indeed, $Z(z) = \eta_1 Z^*(z^*)$, with $z^* = \frac{z}{L}$; $\eta_1 = 16$ and $Z^*(z^*) = z^{*2}(z^* - 1)^2$

Using this normalization and the conditions defined above, we obtain: $Z_1(z) = Z_1(\frac{L}{2}) = 1$ and $\int_0^L Z_n(z) dz = 0.5333 L = \frac{8}{15} L$

Integrating all these transformations into the equation (3), we have:

$$\ddot{T}(t) = -\left(\frac{\lambda}{\rho A} + \frac{16 B^2 L}{15 \mu \rho}\right) \dot{T}(t) - \frac{E I_y \xi_n^4}{\rho A} T(t) + \frac{B L}{\rho A} \dot{q}(t). \quad (7)$$

2.2 Equation modelling the variation of the charge $q(t)$ of capacitor C_0

The voltage V_m at the terminals of a memristor supplied with a current i_m is written as follows [2]:

$$\begin{cases} V_m(t) = M[q_m(t)] i_m(t), \\ M[q_m(t)] = \frac{d\phi_m(t)}{dq_m(t)} = \frac{\dot{\phi}_m(t)}{i_m(t)}. \end{cases} \quad (8)$$

Then we can write: (See [31, 37])

$$\begin{aligned} V_m(t) = \dot{\phi}_m(t) &\Rightarrow V_m \\ &= R_{on} \left[\frac{R_{off}}{R_{on}} - \mu_v \frac{(R_{off} - R_{on})}{D^2} q_m(t) \right] i_m(t) \end{aligned} \quad (9)$$

where ϕ_m is the flux linkage across the memristor; D is the memristor's length; μ_v the memristor ionic mobility; R_{off} the memristor resistance in the Off state and R_{on} the memristor resistance in the On state.

Applying Kirchhoff's voltage law (KVL) $V_m - U_{beam} - U_{C_0} - U_{L_0} = 0$, we have:

$$\ddot{q}(t) = \frac{1}{L_0} \dot{\phi}_m(t) - \frac{r_p + r_0}{L_0} \dot{q}(t) + \frac{16 B L}{15 L_0} \dot{T}(t) - \frac{1}{L_0 C_0} q(t) \quad (10)$$

where r_0 is the resistance of the Coil; C_0 the self-capacitance of the capacitor, L_0 the self-inductance of the coil, I_0 the amplitude of the excitation current, and

$$\begin{aligned} \dot{\phi}_m(t) &= R_{on} \left\{ \frac{R_{off}}{R_{on}} - \mu_v \frac{(R_{off} - R_{on})}{D^2} \right. \\ &\quad \left. \left\{ \frac{I_0}{\omega_0} \sin(\omega_0 t) - q(t) + \frac{16 B A}{15 \mu} T(t) \right\} \right\} \\ &\times \left[I_0 \cos(\omega_0 t) - \left(\dot{q}(t) - \frac{16 B A}{15 \mu} \dot{T}(t) \right) \right] \end{aligned} \quad (11)$$

2.3 Memristor flux equation and governing dynamical system.

From the ϕ_m equation obtained earlier, and deriving Eq. (11) once with respect to time, we obtain the following.

$$\begin{aligned} \ddot{\phi}_m(t) &= -\mu_v \frac{(R_{off} - R_{on}) R_{on}}{D^2} \left\{ I_0 \cos(\omega_0 t) - \dot{q}(t) + \frac{16 B A}{15 \mu} \dot{T}(t) \right\}^2 \\ &+ R_{on} \left\{ \frac{R_{off}}{R_{on}} - \mu_v \frac{(R_{off} - R_{on})}{D^2} \left\{ \frac{I_0}{\omega_0} \sin(\omega_0 t) - q(t) + \frac{16 B A}{15 \mu} T(t) \right\} \right\} \\ &\left\{ -I_0 \omega_0 \sin(\omega_0 t) - \frac{1}{L_0} \dot{\phi}_m(t) + \left\{ \frac{r_p + r_0}{L_0} + \frac{16 B^2 L}{15 \mu \rho} \right\} \dot{q}(t) - \frac{16 B L}{15} \right. \\ &\left. \left\{ \frac{1}{L_0} + \frac{\lambda}{\mu \rho L} + \frac{16 B^2 A}{15 \mu^2 \rho} \right\} \dot{T}(t) + \frac{1}{L_0 C_0} q(t) - \frac{16 B E I_y \xi_n^4}{15 \mu \rho} T(t) \right\} \end{aligned} \quad (12)$$

The flow governing the dynamical system is written as follows:

$$\begin{cases} x = \dot{\alpha}_m(\tau) \\ \dot{x} = -\Delta_{R_m} (I_0 \cos(\omega_0^* \tau) - \omega_1 Q_0 z + \sigma_0 y)^2 \\ \quad + \sigma_1 \{ R_0^* - \Delta_{R_{on}} [I_0^* \sin(\omega_0^* \tau) - Q_0 \gamma(\tau) + \sigma_2 \theta(\tau)] \} \\ \quad \times \{ \{ -I_0^* \sin(\omega_0^* \tau) - \vartheta_2 x + \delta_{r_2} z - \sigma_3 y + \sigma_4 \gamma(\tau) - \sigma_5 \theta(\tau) \} \\ y = \dot{\theta}(\tau) \\ \dot{y} = -\beta_1 y - \beta_2 \theta(\tau) + \beta_3 z \\ z = \dot{\gamma}(\tau) \\ \dot{z} = \vartheta_1 x - \delta_{r_1} z + \eta y - \Omega_{C_0}^2 \gamma(\tau) \end{cases} \quad (13)$$

With $\tau = \omega_1 t$; $\theta = \frac{T(t)}{T_0}$; $\gamma = \frac{q(t)}{Q_0}$ and $\alpha_m = \frac{\phi_m(t)}{\phi_0}$ where $\omega_1, T_0, Q_0, \Phi_0$ are scaling coefficients.

$$\beta_1 = \left(\frac{\lambda}{\omega_1 \rho A} + \frac{16 B^2 L}{15 \omega_1 \mu \rho} \right); \beta_3 = \frac{B L Q_0}{\rho \omega_1 A T_0};$$

$$\omega_{C_0}^2 = \frac{1}{L_0 C_0}; \vartheta_1 = \frac{\phi_0}{L_0 Q_0 \omega_1}; \Omega_{C_0}^2 = \frac{1}{\omega_1^2 L_0 C_0};$$

$$\delta_{r_1} = \frac{r_p + r_0}{L_0 \omega_1}; \delta_{r_2} = \left(\delta_{r_0} + \frac{16 B^2 L}{15 \mu \rho} \right) \omega_1 Q_0;$$

$$\Delta_{R_{on}} = \mu_v \frac{(R_{off} - R_{on})}{D^2}; \delta_{r_0} = \frac{r_p + r_0}{L_0}$$

$$\Delta_{R_m} = \mu_v \frac{(R_{off} - R_{on}) R_{on}}{D^2 \omega_1^2 \phi_0};$$

$$R_0^* = \frac{R_{off}}{R_{on}}; \sigma_2 = \frac{16 B A T_0}{15 \mu};$$

$$\vartheta_0 = \omega_1 \phi_0; \vartheta_2 = \frac{\omega_1 \phi_0}{L_0};$$

$$\sigma_3 = \frac{16 B L \omega_1 T_0}{15} \left(\frac{1}{L_0} + \frac{\lambda}{\mu \rho L} + \frac{16 B^2 A}{15 \mu^2 \rho} \right);$$

$$\beta_2 = \frac{E I_y \xi^4}{\rho \omega_1^2 A};$$

$$\sigma_1 = \frac{R_{on}}{\omega_1^2 \phi_0}; \omega_0^* = \frac{\omega_0}{\omega_1}; I_0^* = I_0 \omega_0;$$

$$\sigma_0 = \frac{16 B A \omega_1 T_0}{15 \mu}; \sigma_4 = \frac{Q_0}{L_0 C_0};$$

$$\sigma_5 = \frac{16 B E I_y \xi^4 T_0}{15 \mu \rho}.$$

The main currents of the system are given by:

$$\begin{cases} i_{C_0}(\tau) = \dot{\gamma} \sim i_b \\ i_m = \frac{\vartheta_0 x}{R_0^* - \Delta_{R_{on}} \{ I_0^* \sin(\omega_0^* \tau) - Q_0 \gamma(\tau) + \sigma_2 \theta(\tau) \}} \end{cases} \quad (14)$$

The Jacobian matrix of the system (13) is written as:

$$J_x = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & A_x & B_y & C_y & D_z & E_z \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\beta_2 & -\beta_1 & 0 & \beta_3 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & v_1 & 0 & \eta & -\Omega_c^2 & -\delta_{r_1} \end{pmatrix}$$

where

$$A_x = -\sigma_1 \{ R_0^* - \Delta_{R_{on}} I_0^* \sin(\omega_0^* \tau) - Q_0 \gamma(\tau) + \sigma_2 \theta(\tau) \} \vartheta_2 \}$$

$$B_y = -\sigma_1 (\Delta_{R_{on}} \sigma_2) \{ -I_0^* \sin(\omega_0^* \tau) - \vartheta_2 x + \delta_{r_2} z - \sigma_3 y + \sigma_4 \gamma(\tau) - \sigma_5 \theta(\tau) \} \\ - \sigma_5 \sigma_1 \{ R_0^* - \Delta_{R_{on}} [I_0^* \sin(\omega_0^* \tau) - Q_0 \gamma(\tau) + \sigma_2 \theta(\tau)] \}$$

$$C_y = -2 \Delta_{R_m} \sigma_0 \{ I_0 \cos(\omega_0^* \tau) - \omega_1 Q_0 z + \sigma_0 y \} \\ - \sigma_1 \sigma_3 \{ R_0^* - \Delta_{R_{on}} [I_0^* \sin(\omega_0^* \tau) - Q_0 \gamma(\tau) + \sigma_2 \theta(\tau)] \}$$

$$D_z = \Delta_{R_{on}} Q_0 \sigma_1 \{ -I_0^* \sin(\omega_0^* \tau) - \vartheta_2 x + \delta_{r_2} z - \sigma_3 y + \sigma_4 \gamma(\tau) - \sigma_5 \theta(\tau) \} \\ + \sigma_4 \sigma_1 \{ R_0^* - \Delta_{R_{on}} [I_0^* \sin(\omega_0^* \tau) - Q_0 \gamma(\tau) + \sigma_2 \theta(\tau)] \}$$

$$E_z = \omega_1 Q_0 \Delta_{R_m} \{ I_0 \cos(\omega_0^* \tau) - \omega_1 Q_0 z + \sigma_0 y \} \\ + \delta_{r_2} \sigma_1 \{ R_0^* - \Delta_{R_{on}} [I_0^* \sin(\omega_0^* \tau) - Q_0 \gamma(\tau) + \sigma_2 \theta(\tau)] \}$$

For the autonomous system ($I_0 = 0$), the fixed point is simply the origin $P(x^*, \dot{x}^*, y^*, \dot{y}^*, z^*, \dot{z}^*) = (0, 0, 0, 0, 0, 0)$.

In the case of sinusoidal excitation, for an applied current $i(t) = I_0 \cos(\omega_0 t)$, the dynamics become nonlinear and the instantaneous thickness of the conductive region is given by:

$$i_m(t) = \frac{V_m(t)}{M[q_m(t)]} \quad \text{and} \quad \frac{dw(t)}{dt} = \mu_v \frac{R_{on}}{D} \frac{V_m(t)}{M[q_m(t)]} \quad (15)$$

In this case, no simple closed-form analytical solution exists, and the solution must be obtained numerically.

3 Numerical study and analysis

3.1 Physical description of the HP memristor key parameters employed in numerical analyses

The HP memristor model as presented in Fig. 2 is characterized by several parameters, three of which are particularly important: the current flowing through the memristor, the voltage across its terminals, and the flux $\phi[q_m(t)]$. These quantities are related through the memristance $M[q_m(t)]$, which depends on R_{on} , R_{off} , and μ_v . The parameter R_{on} represents the minimum resistance of the memristor when it is in its conducting state, while R_{off} corresponds to the maximum resistance when it is in its non-conducting state. These parameters play a crucial role in characterizing the behavior of the HP memristor, particularly in applications related to memory devices and neuromorphic computing [2, 38–40].

In the HP memristor model [2], the total resistance results from the series connection of two distinct material regions (See Fig. 2):

- a TiO_{2-x} conductive doped region of thickness $w(t)$ as previously defined in Eq. (15)

- a insulating non-doped TiO_2 region of thickness $D - w(t)$, where D denote the total thickness of the active film.

The values of R_{on} and R_{off} are determined exclusively by:

- the intrinsic electronic resistivity ρ_{on} , ρ_{off} ,
- the geometry of the device (thickness D , surface area S),
- the density of electron carriers,
- the conduction mechanisms (bands, hopping, and polarons).

$$R_{on} = \rho_{on} \frac{D}{S}, \quad R_{off} = \rho_{off} \frac{D}{S}. \quad (16)$$

Ionic mobility μ_v describes the drift velocity of oxygen vacancies under an electric field E_m : $v_{ion} = \mu_v E_m$. Ionic mobility μ_v is only involved in the dynamical Eq. (15), that is, the time rate of change of the boundary position $w(t)$, and not in instantaneous electronic conduction. Thus, in the ideal linear HP model, R_{on} and R_{off} do not depend on μ_v , as they are related to electron transport, while μ_v governs only the kinetics of ion migration.

Accordingly, Figs. 3–11 were generated to investigate the various electromechanical oscillatory regimes exhibited by the proposed memristive MEMS within parameter domains identified from the Lyapunov exponent diagrams. In most of these analyses, the ionic mobility μ_v is assumed constant (without intrinsic or extrinsic thermal activation) in order to isolate the influence of the intrinsic memristor parameters R_{on} and R_{off} on the nonlinear electrodynamic behavior of the system. Variations of these parameters significantly modify the internal resistive state of the memristor, given by the following expression [2, 41, 42]:

$$R(t) = R_{on}w(t) + R_{off}(1 - w(t)), \quad (17)$$

with $R_{on} \leq R(t) \leq R_{off}$.

They also alter the accumulated memristive charge $q_m(t)$. Consequently, variations in $R(t)$ and $q_m(t)$ modify the nonlinear energy transfer mechanisms governing the coupled thermo-electro-mechanical system, leading to changes in the observed dynamical regimes (Figs. 3–8, 10–11). As a result, the system may undergo transitions between distinct oscillatory regimes ranging from quasi-periodic to chaotic dynamics. Nevertheless, a dedicated Lyapunov analysis involving variations of μ_v was also performed to evaluate the influence of ionic migration kinetics on the emergence and modulation of chaotic dynamics (Fig. 9). In this framework, the memristor does not merely behave as a passive memory element, but could rather be viewed as an active nonlinear regulator capable of dynamically redistributing energy between the electrical and mechanical resonators. Through the modulation of the memristive parameters, the device directly influences resonance amplification, oscillatory stability,

and the emergence of nonlinear dynamical states within the MEMS architecture (Figs. 3–11). It should be emphasized that the ionic mobility μ_v in oxide-based memristive devices strongly depends on several physical factors, including oxygen vacancy concentration, initial doping conditions, defect density, fabrication process, local electric field, and temperature [43–45]. Consequently, the effective mobility may vary over several orders of magnitude depending on the material composition and operating conditions. In the present work, μ_v was varied within the interval 10^{-12} to $5 \times 10^{-10} \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$, consistent with the order of magnitude considered in the original HP memristor model proposed by Strukov et al. [2]. Furthermore, slightly extended values of μ_v were considered (Figs. 12 and 13) in order to explore a broader thermo-electro-mechanical nonlinear dynamical domain and to evaluate the sensitivity of the oscillatory regimes to enhanced ionic transport effects. Such a parametric approach remains physically plausible owing to the strong variability of ionic mobility reported in oxide-based memristive materials. The present work mainly highlights the indirect influence of temperature on energy redistribution and nonlinear dynamical mode reconfiguration. It should be noted that the present study does not provide a quantitative determination of the thermal operating limits of the proposed MEMS architecture. Nevertheless, recent investigations on TiO_x -2-based memristive devices generally report operating temperatures between approximately 200 and 450 K. The temperature-dependent evolutions and variation ranges of μ_v , R_{on} , and R_{off} , however, depend on the material composition, fabrication process, and experimental conditions considered in each study. [43, 45], [46].

The numerical study begins with an analysis of the distribution of the principal currents of the system as a function of the HP memristor parameters R_{on} and R_{off} . Subsequently, a comprehensive nonlinear dynamics investigation is carried out using bifurcation diagrams, Lyapunov exponents, Poincaré sections, recurrence plots, Fourier amplitude spectra, phase portraits, and time evolution responses. This analysis reveals the emergence of chaotic and quasi-periodic regimes induced by variations in the memristor parameters. Furthermore, a regulation strategy is proposed to control these dynamical behaviors. Finally, a brief examination is conducted to assess the plausible electromechanical responses of the system under moderate thermal variations affecting the HP memristor.

Since the system proposed in this work is purely excited by a sinusoidal current and to assess the influence of memristor on the system's oscillation modes, it is therefore relevant to map the behavior of the system's main currents as a function of R_{off} and R_{on} . The behavior of the memristor current i_m was numerically evaluated in Fig. 3 by simultaneously varying R_{on} and R_{off} . This allowed us not only to confirm the expected nonlinear nature of the device [3, 5], but

also to verify the consistency of the mathematical model through the conformity of the current dynamics with Kirchhoff's laws. The numerical analysis is performed in Python using Runge–Kutta RK45 (Dormand–Prince), as implemented in *scipy.integrate.solve_ivp*. The maximum Lyapunov exponent was computed using the classical Benettin renormalization algorithm [47, 48]. All axes representing dimensionless variables are shown without units during the numerical analysis. For brevity, certain parameter symbols whose unit are lengthy, such as μ_v , are defined in Table 1 and recalled in the figure captions. The various curves shown in this section are obtained with the following parameter values: $L = 200 \mu m$; $l = h = 1 \mu m$; $\mu = 0.2582 \Omega.m$; $E = 150.10^9 Pa$; $\rho = 2330 kg.m^{-3}$; $D = 5 nm$; $\mu_v = 10^{-10} (cm^2 s^{-1} V^{-1})$; $L_0 = 50 mH$; $r_0 = 5 \Omega$; $C_0 = 50 pF$; $\omega_0 = \omega_b/35$; $\omega_1 = \omega_b = 2\pi \times 1.05 \times \sqrt{\frac{E}{\rho}} \frac{h}{L^2}$ with ω_b the natural frequency of the beam (see [2, 12, 38, 49–51]). The intensity of the magnetic field $\vec{B}$ is $B = 1 T$ (see [12, 51]). The initial conditions are chosen as $\alpha_0 = 0.2$; $x_0 = 0.05$; $\theta_0 = 0.05$; $y_0 = 0.35$; $\gamma_0 = -1$; $z_0 = 0.25$.

A brief analysis of the first diagram in Fig. 3 highlights the nonlinear properties of the memristor. For very low values of R_{on} , a maximum current flows through the memristor (Fig. 3a), while a minimum current i_b flows through the capacitor C_0 (Fig. 3b). When R_{on} is very small, current saturation occurs in the memristor. As R_{on} increases, the current i_b flowing through the beam and charging the capacitor C_0 also increases. The approximately nonlinear variation of $\max(\dot{\gamma})$ with the ratio $\frac{R_{off}}{R_{on}}$ suggests a variable dynamics of the current i_b flowing through the beam, leading to increased

mechanical amplitudes. This behavior occurs as the memristor current i_m reaches a peak for very small values of R_{on} before reaching an apparent saturation plateau as R_{on} increases (see Fig. 3 and Fig. 6a and b). It should be noted that for values of R_{off} between $10 k\Omega$ and $15 k\Omega$, which correspond to the preferred operating range of the HP memristor for periodic oscillations (see [41]), the current i_b flowing through the beam exhibits an almost linear dependence on the ratio $\frac{R_{off}}{R_{on}}$.

In order to evaluate the general dynamics of the system and the actual effects of the other memristor parameters on the electromechanical dynamics, we use R_{off} in the largest range of linearity of $\dot{\gamma}$ ($10 k\Omega \leq R_{off} \leq 15 k\Omega$, see Fig. 3 b). This allows the dynamics to remain consistent throughout this range of R_{off} . These two diagrams of Fig. 3 confirm that the mathematical model is consistent with the physical laws and properties of the memristor used.

3.2 Bifurcation analysis and nonlinear dynamics of the memristive MEMS without thermal effects

Given the strong sensitivity of the system currents to variations in R_{on} (Fig. 3), the bifurcation structure of the memristor current i_m , together with the corresponding maximum Lyapunov exponent (LE) distributions, was investigated as functions of R_{on} and the excitation current amplitude I_0 . Figure 4 reveals two main dynamical regions. The first corresponds to a central domain characterized by alternating chaotic and quasi-periodic oscillations, whose

Table 1 Nature and units of the physical parameters of the system used in the numerical study

Designation	Nature & unit	Designation	Nature & unit
R_{on}	Scalar parameter (Ω)	$(\alpha_m; \dot{\alpha}_m)$	Dimensionless memristor variables
R_{off}	Scalar parameter (Ω)	$(\theta; \dot{\theta})$	Dimensionless beam variables
μ_v	Scalar parameter ($cm^2 s^{-1} V^{-1}$)	$(\gamma; \dot{\gamma})$	Dimensionless C_0 charge variables
D	Scalar parameter (nm)	i_m	Dimensionless memristor current
I_0	Parameter with algebraic value (mA)	τ	Dimensionless time
μ	Scalar parameter ($\Omega.m$)	$i_{C_0} = \dot{\gamma} \sim i_b$	Dimensionless capacitor C_0 current

Fig. 3 System currents as a function of R_{on} and R_{off} : **a** The memristor current i_m and **b** The current i_b through the beam and capacitor C_0 . **b** is rotated with respect to **a**, to better visualize the current distribution. $D = 5 nm$; $\mu_v = 10^{-10} cm^2 s^{-1} V^{-1}$, $I_0 = 2 mA$

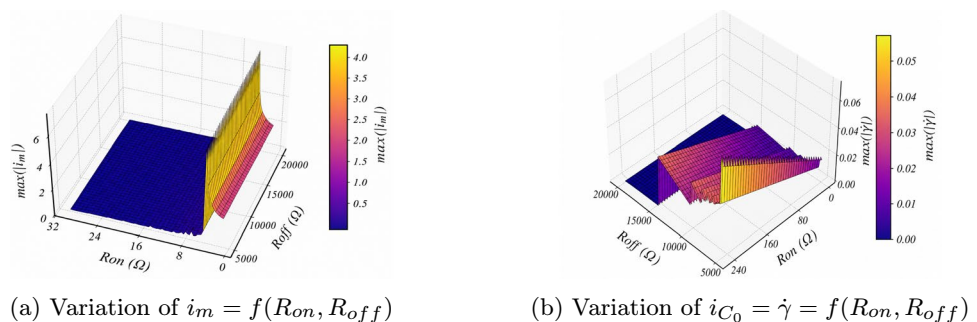
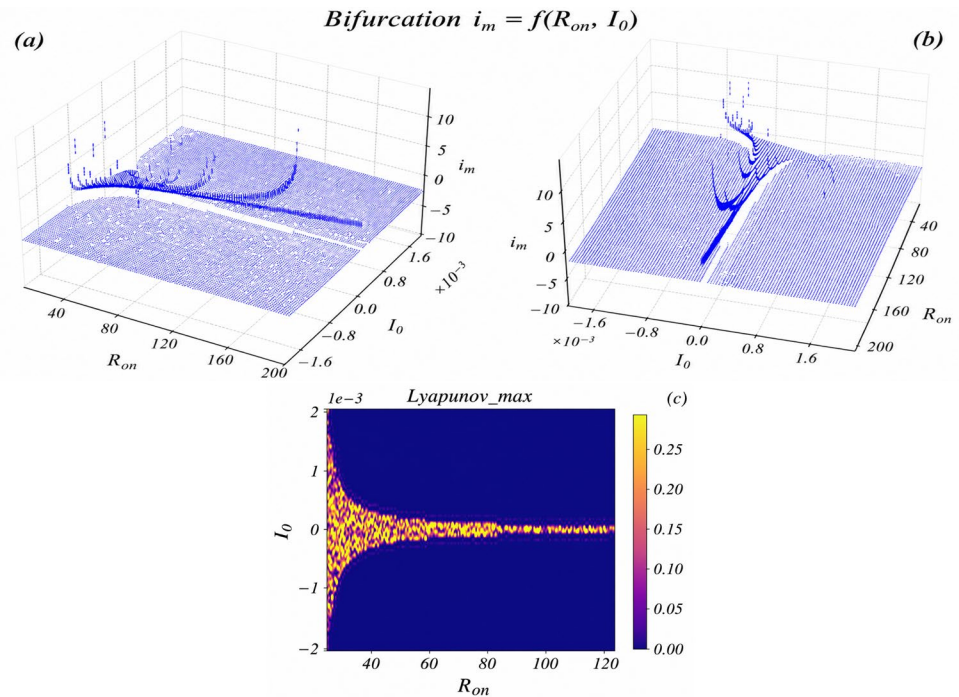


Fig. 4 Bifurcation diagrams and maximum Lyapunov exponent map of the memristive MEMS as functions of R_{on} (Ω) and I_0 (A) for $\mu_v = 10^{-10} \text{ cm}^2 \text{ s}^{-1} \text{ V}^{-1}$, $R_{off} = 12 \text{ k}\Omega$, and $D = 5 \text{ nm}$. Panels **a** and **b** correspond to two viewing orientations of the bifurcation structure of the memristor current i_m , whereas panel **c** shows the associated Lyapunov map identifying regular and chaotic dynamical regions



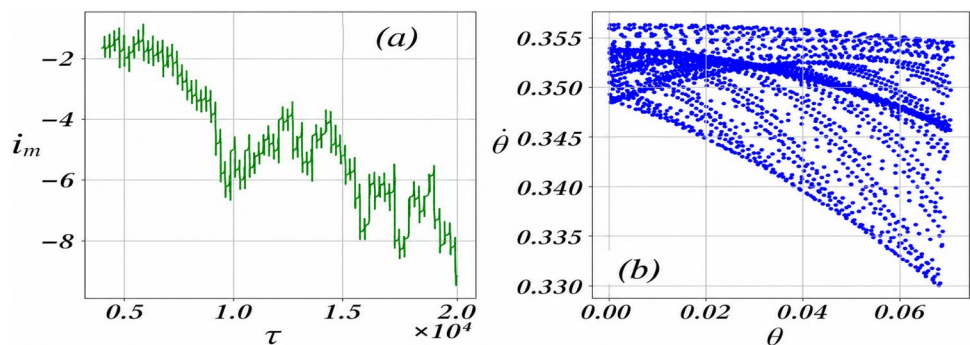
width progressively decreases as R_{on} increases. The second region, approximately symmetric with respect to the central axis, is predominantly associated with quasi-periodic electromechanical oscillations. In addition, for $|I_0| \geq 2 \text{ mA}$, the oscillatory dynamics tend to become more regular. The maximum Lyapunov exponent remains close to zero in these regions, suggesting the predominance of quasi-periodic rather than purely periodic oscillatory regimes. The progressive reduction of the chaotic region with increasing R_{on} suggests that larger ON-state resistances tend to attenuate the nonlinear electromechanical interactions responsible for complex oscillatory responses. This behavior may be related to a reduction in the effective energy transfer between the electrical and mechanical subsystems, thereby promoting more regular dynamical responses. Furthermore, the approximate symmetry of the bifurcation structures with respect to $I_0 = 0 \text{ mA}$ indicates that the system exhibits comparable dynamical responses

for opposite excitation polarities within the investigated parameter range.

3.2.1 Chaotic and quasi-periodic behavior in memristor-based MEMS

3.2.1.1 Alternating electromechanical dynamical regimes in the positive Lyapunov exponent region The dynamical responses illustrated in Fig. 5 were obtained by selecting a parameter point located within the region of positive Lyapunov exponents. The time series were computed over 20 000 time units (τ), after discarding the initial 5 000 time units to eliminate the transient effects. The memristor current exhibits irregular and aperiodic temporal oscillations, indicative of a sensitive dependence on initial conditions. Furthermore, the Poincaré section of the beam oscillations reveals a highly scattered and unstructured point distribution, characteristic of a chaotic attractor. These observations confirm the

Fig. 5 Temporal memristive response and associated beam Poincaré section highlighting alternating nonlinear electromechanical dynamics. **a**: Memristor current time evolution. **b**: Beam Poincaré section for $D = 5 \text{ nm}$, $\mu_v = 10^{-10} \text{ cm}^2 \text{ s}^{-1} \text{ V}^{-1}$, $I_0 = -0.5 \text{ mA}$, $R_{on} = 20 \Omega$, and $R_{off} = 12 \text{ k}\Omega$



presence of chaotic electromechanical oscillations, in agreement with the bifurcation and Lyapunov analyses reported in Fig. 4.

The alternating dynamical behavior is supported by both the temporal evolution of the memristor current and the beam Poincaré section. In Fig. 5a, the memristor current exhibits successive intervals of relatively organized oscillations separated by irregular amplitude modulations. This behavior is further confirmed by the Poincaré map in Fig. 5b, where locally concentrated trajectories are interspersed with strongly dispersed point distributions. The alternation of organized and scattered regions suggests alternating transitions between quasi-periodic and more complex chaotic electromechanical dynamics. Such alternation may be associated with the temporal evolution of the effective memristive resistance $R(t)$ (Eq. (17)), whose dynamics continuously modulate the nonlinear electromechanical coupling.

3.2.1.2 Quasi-periodic dynamics in the uniform LE domain Numerical analysis of the dynamics of the HP memristor current i_m Figure 6 illustrates the oscillatory regimes of the memristor current i_m through its phase portraits, time evolution, Fourier amplitude spectrum, and recurrence plots for different values of R_{on} . The values of R_{on} and I_0 were selected within the quasi-periodic region

identified in Fig. 4, thus ensuring that the system operates in a regular oscillatory domain. Recurrence plots (Fig. 6e) exhibit a dominant main diagonal in all cases, which confirms deterministic behavior. However, a clear evolution in the degree of periodicity is observed as R_{on} increases. For $R_{on} = 10 \Omega$, the recurrence plot lacks pronounced secondary diagonals, indicating a weak periodic structure despite the apparent regularity of the time-domain signal. This behavior is consistent with the Fourier amplitude spectrum of the memristor current i_m , shown in Fig. 6d, which reveals a broadened and noisy frequency distribution, suggesting multi-frequency oscillations. As R_{on} increases, the oscillations become progressively more structured. Recurrence plots display well-defined secondary diagonals and finer line patterns (e.g., for $R_{on} = 200 \Omega$), indicating enhanced periodic recurrence. The Fourier amplitude spectrum of the memristor current i_m becomes more concentrated around dominant frequencies, reflecting a transition towards more coherent and quasi-monochromatic oscillations. These observations demonstrate that increasing R_{on} promotes greater dynamical regularity, reducing spectral dispersion and reinforcing the quasi-periodic character of the memristor current.

Numerical analysis of the current flowing through mechanical and electrical resonators The phase-space portraits shown in Fig. 7 exhibit bi-periodic and highly regular

Fig. 6 a–c and e: Illustration of quasi-periodic electrical dynamics of the memristor oscillations. (a)–(c) and (e): Highlighting the electrical noise for different values of R_{on} and I_0 selected within the uniform quasi-periodic region of the maximum LE map shown in Fig. 4. Parameters: $R_{off} = 12 \text{ k}\Omega$, $R_{on} = 200, 100$ and 10Ω , $D = 5 \text{ nm}$, $\mu_v = 10^{-10} \text{ cm}^2 \text{ s}^{-1} \text{ V}^{-1}$, and $I_0 = 2 \text{ mA}$ (color figure online)

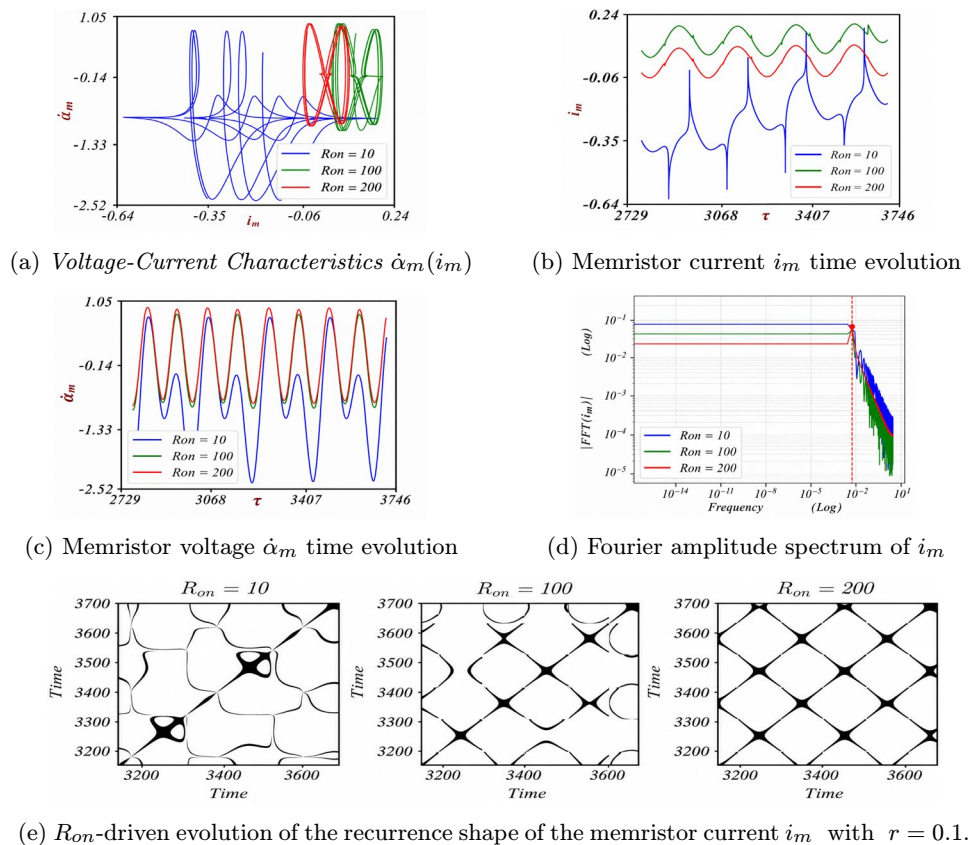
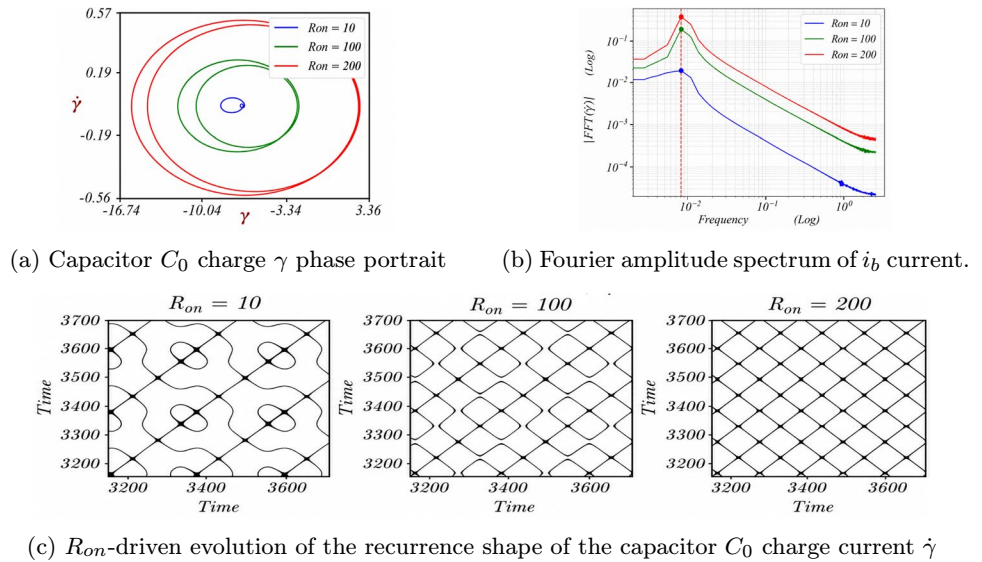


Fig. 7 **a, c** Illustration of the quasi-periodic dynamics of the capacitor C_0 charge current $\dot{\gamma} \sim i_b$. **(b)**: Fourier amplitude spectrum of i_b , showing amplitude power gain as R_{on} increases. R_{on} and I_0 chosen within the uniform quasi-periodic oscillations region of the maximum LE map in Fig. 4. Parameters: $R_{off} = 12 \text{ k}\Omega$, $R_{on} = 200, 100$ and $10 \text{ }\Omega$, $D = 5 \text{ nm}$, $\mu_v = 10^{-10} \text{ cm}^2 \text{ s}^{-1} \text{ V}^{-1}$ and $I_0 = 2 \text{ mA}$ (color figure online)

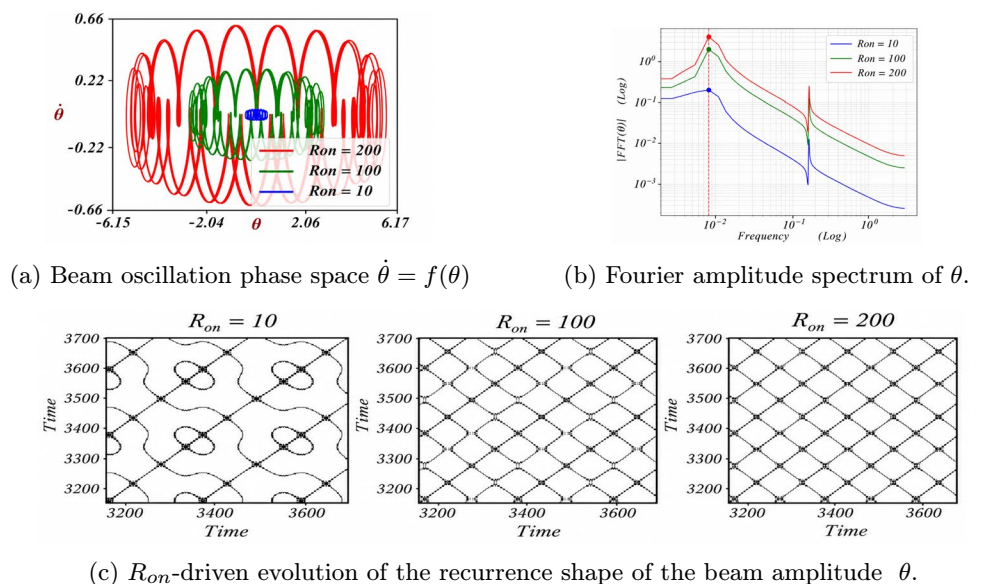


oscillatory trajectories for all values considered of R_{on} . This regularity is further confirmed by the recurrence plots, which display well-defined primary and secondary diagonals, indicating strong periodic structure and deterministic behavior. Both the charge γ and the charging current $\dot{\gamma} \sim i_b$ increase proportionally with R_{on} . This scaling behavior explains the predictable and stable response of the MEMS, which remains consistent with Kirchhoff's current and voltage laws. As R_{on} increases, the memristor becomes more resistive and consequently draws less current (see Fig. 6). This redistribution of current results in a larger fraction of the electrical energy being transferred to the electromechanical resonators (see Fig. 7). The Fourier amplitude spectrum of the $\dot{\gamma} \sim i_b$ current reveals a single dominant resonance frequency that remains essentially unchanged with R_{on} ,

confirming that the system operates in a stable resonant regime. Moreover, the spectral content becomes cleaner at higher R_{on} values, with reduced high-frequency noise components. This behavior reflects an internal redistribution of electrical power within the coupled system: as R_{on} increases, the memristor dissipates less electrical power (see Fig. 6d), while the electromechanical resonators experience a corresponding increase in electrical power (see Figs. 7 and 8). This energy transfer between subsystems remains fully consistent with Kirchhoff's laws and the global conservation of current and energy.

Numerical analysis of mechanical oscillation of the beam The recurrence plots of the mechanical oscillations of the beam, represented by the transverse displacement θ of the center of the beam (see Fig. 8, c), exhibit a dynamical

Fig. 8 **a,c** Illustration of the quasi-periodic dynamics of the beam amplitude θ . **(b)**: Highlighting the mechanical double resonance frequencies and power gain as R_{on} increases. The parameters R_{on} and I_0 are selected within the uniform quasi-periodic region of the maximum LE map in Fig. 4. Parameters: $R_{off} = 12 \text{ k}\Omega$, $R_{on} = 200, 100$ and $10 \text{ }\Omega$, $D = 5 \text{ nm}$, $\mu_v = 10^{-10} \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$, and $I_0 = 2 \text{ mA}$ (color figure online)



structure that closely resembles that of the charging current i_b . The dotted recurrence lines reflect slight modulation effects while preserving deterministic quasi-periodic behavior. The corresponding phase-space portraits in Fig. 8a reveal quasi-periodic trajectories for all values of R_{on} . Here, θ denotes the transverse deflection amplitude of the center of the beam with respect to its equilibrium position, thus directly characterizing the mechanical vibration state of the resonator. The maximum deflection amplitude increases proportionally with R_{on} , indicating improved electromechanical coupling and greater mechanical energy transfer from the electrical subsystem to the beam. The Fourier amplitude spectrum of the beam amplitude θ displays two dominant resonance peaks, confirming a bi-frequency response consistent with quasi-periodic motion. Moreover, the mechanical spectral amplitude power increases with R_{on} , while no broadband components are observed. The absence of spectral spreading indicates that the mechanical oscillations remain coherent and dynamically stable within the explored parameter range. These results demonstrate that increasing R_{on} strengthens the mechanical response of the beam without inducing chaotic frequency components, thus preserving a structured quasi-periodic regime.

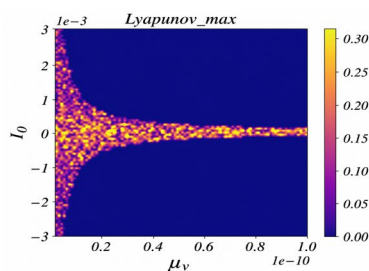
The first numerical investigations reveal two distinct electromechanical dynamical behaviors depending on the regions identified in the maximum Lyapunov exponent maps. When the parameters (R_{on}, I_0) are selected within the uniform quasi-periodic regions, the coupled electro-mechanical system with the memristor tends to exhibit organized and dynamically stable oscillations (see Figs. 6, 7, and 8). In these regimes, the effective memristive resistance $R(t)$ appears to evolve more regularly, which may maintain a nearly uniform electromechanical coupling and promote coherent quasi-periodic responses in both the electrical and mechanical subsystems. In contrast, parameters chosen within the transitional regions of the Lyapunov maps tend to produce alternating dynamical behavior characterized

by successive quasi-periodic and chaotic oscillations (see Figs. 4 and 5). Stronger fluctuations of the internal state variable $w(t)$ may induce more irregular variations of the effective resistance $R(t)$, which could continuously modulate the nonlinear electromechanical coupling and contribute to complex energy redistribution between the electrical and mechanical resonators.

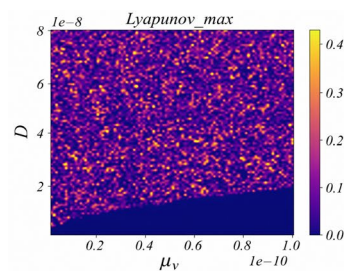
3.2.2 Regulation of chaotic and quasi-periodic regimes through memristor parameters

To highlight the nonlinear dynamics with the memristor parameters, we plot in Fig. 9 the maximum values of the Lyapunov exponents (LE) with another parameter of the memristor. As the ion mobility μ_v governs the charge transport dynamics of the memristor, we choose to evaluate its influence on the oscillatory regimes of the system when the amplitude I_0 of the excitation current varies. Figures 9a and 9b illustrate an amplification of the strictly positive LE domain as μ_v decreases. For $\mu_v = 8 \times 10^{-12} \text{ cm}^2 \text{ s}^{-1} \text{ V}^{-1}$, Fig. 10, shows an expansion of the chaotic domain of $LE = f(R_{on}, I_0)$ obtained in Fig. 4. While the chaotic zone was almost zero for $R_{on} \geq 50 \Omega$ in Fig. 4, the diagram in Fig. 10 predicts chaotic regimes for the values of $R_{on} \geq 100 \Omega$ and $|I_0| \geq 1 \text{ mA}$. The bifurcation diagram of Fig. 10a and 10b confirms a sharp reduction in regions of pure quasi-periodicity in favor of an extended positive Lyapunov exponent domain.

The more accentuated chaotic behavior can be observed for $R_{on} = 60 \Omega$ and $I_0 = -1.5 \text{ mA}$ in Fig. 11. Despite a value of R_{on} three times larger than that considered in Fig. 5, the memristor current exhibits a markedly stronger irregular behavior, while the corresponding beam Poincaré section becomes significantly more dispersed. In comparison with Fig. 5a, the relatively organized intervals observed in the temporal evolution of the memristor current (Fig. 11a) become noticeably shorter, indicating a reduction of the



(a) Max LE $I_0 = f(\mu_v)$ for $D = 5 \text{ nm}$



(b) Max LE $D = f(\mu_v)$ for $I_0 = -1 \text{ mA}$

Fig. 9 Influence of the memristor ionic mobility μ_v and geometric length D on the regulation of chaotic and quasi-periodic regime boundaries through the maximum Lyapunov exponent. Panels **a** and **b** show the dependence of the max LE on (μ_v, I_0) and (μ_v, D) ,

respectively. Here, D denotes the active layer length, while μ_v represents the ionic mobility. Parameters: $R_{off} = 12 \text{ k}\Omega$, and $R_{on} = 100 \Omega$, $\mu_v (\text{cm}^2 \text{ V}^{-1} \text{ s}^{-1})$

Fig. 10 Bifurcation structures and maximum Lyapunov exponent distributions of the memristive MEMS as functions of R_{on} and I_0 for $\mu_v = 8 \times 10^{-12} \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$, $R_{off} = 12 \text{ k}\Omega$, and $D = 5 \text{ nm}$. Panels **a** and **b** show two viewing orientations of the bifurcation diagram of the memristor current i_m , while panel **c** presents the corresponding Lyapunov map highlighting the extension of alternating chaotic and quasi-periodic dynamical regions

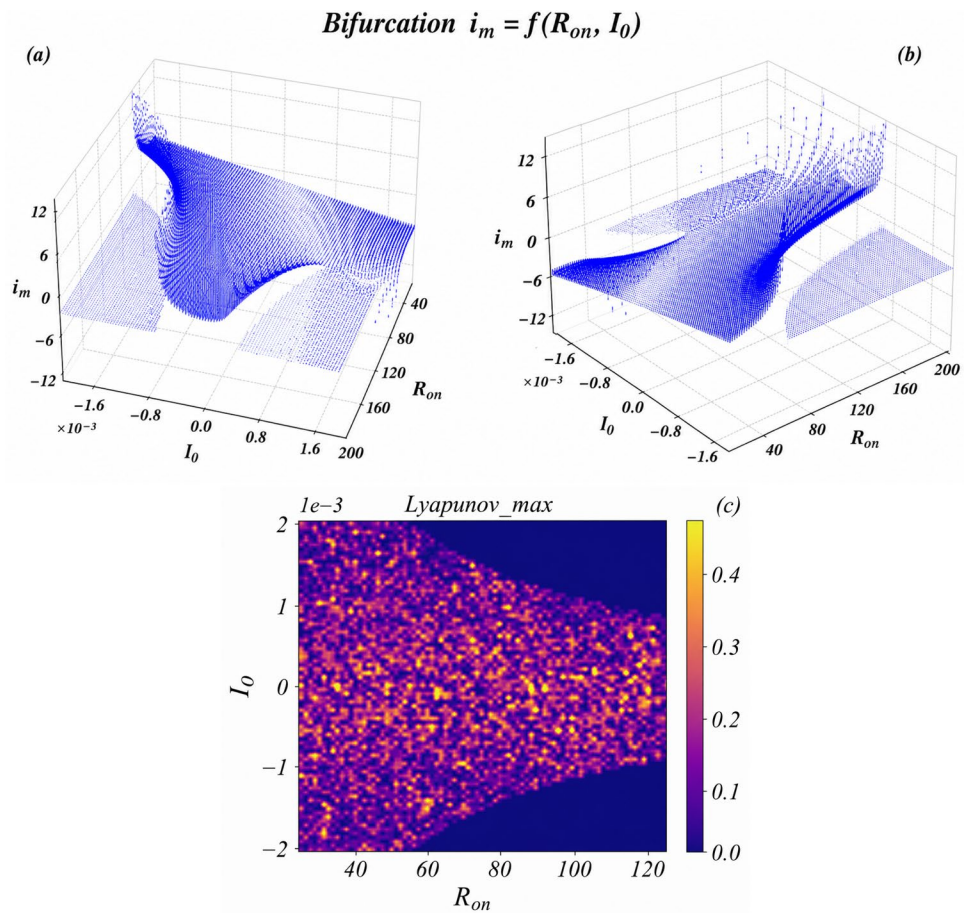
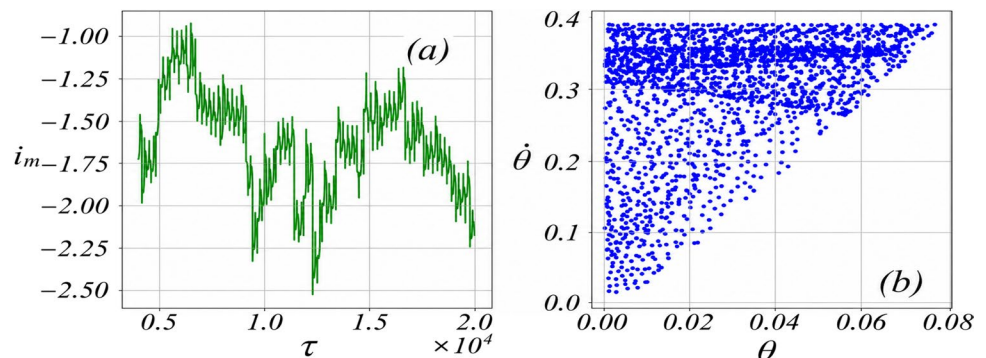


Fig. 11 Accentuation of the chaotic regime illustrated in Fig. 4 for $\mu_v = 8 \times 10^{-12} \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$. **a** Time evolution of the memristor current. **b** Beam Poincaré section for $D = 5 \text{ nm}$, $I_0 = -1.5 \text{ mA}$, $R_{on} = 60 \Omega$, and $R_{off} = 12 \text{ k}\Omega$



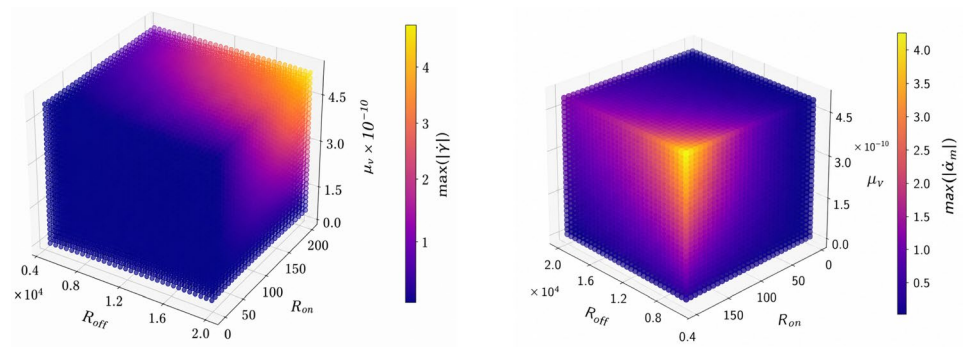
alternating times associated with quasi-organized dynamical episodes. The alternation between organized and irregular phases therefore occurs more frequently, reflecting an intensification of the chaotic dynamics. This interpretation is supported by the broader distribution of points in the Poincaré section shown in Fig. 11b, which reveals a stronger loss of dynamical regularity in the mechanical response. As the beam deflection amplitude decreases while the memristor current increases (see Figs. 6 and 7), the mechanical trajectories become progressively more irregular. The enhancement of chaotic oscillations in the memristor current is thus

transferred to the beam through the nonlinear electromechanical coupling, leading to a pronounced degradation of the regularity of the mechanical motion.

3.3 Thermo-induced bifurcation dynamics of the memristive MEMS

Temperature plays a crucial role in the electrical behavior of memristive devices. In the HP memristor model, the total resistance depends on the relative position of the boundary between the conductive doped region TiO_{2-x} and the insulating

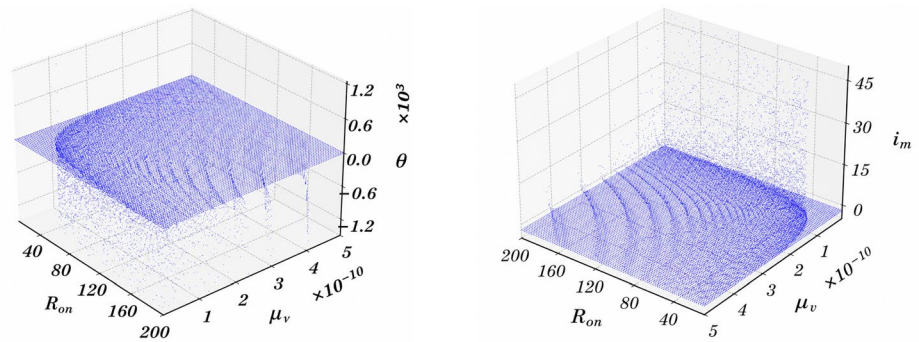
Fig. 12 4D diagram illustrating: **a** the sensitivity of the current $i_b \sim \dot{\gamma}$ flowing through the mechanical (micro-beam) and electrical (r_0, L_0, C_0) resonators and **b** the sensitivity of the voltage $V_m \sim \dot{\alpha}_m$ across the memristor, as a function of the three thermo-sensitive parameters (R_{on}, R_{off}, μ_v) of the HP memristor. Parameters: μ_v ($\text{cm}^2 \text{s}^{-1} \text{V}^{-1}$), R_{on} (Ω), R_{off} (Ω), $I_0 = 2 \text{ mA}$, and $D = 5 \text{ nm}$



(a) $\dot{\gamma} = f(R_{on}, R_{off}, \mu_v)$: Response in the current $\dot{\gamma} \sim i_b$ flowing through the microbeam as a function of the thermo-sensitive parameters of the HP memristor.

(b) $\dot{\alpha}_m = f(R_{on}, R_{off}, \mu_v)$: Voltage $\dot{\alpha}_m \sim V_m$ across the memristor as a function of the thermo-sensitive parameters of the HP memristor.

Fig. 13 Bifurcation transitions between quasi-periodic and chaotic electromechanical regimes driven by thermo-sensitive parameters R_{on} and μ_v . The parameter ranges are $\mu_v \in [10^{-12}, 5 \cdot 10^{-10}] \text{ cm}^2 \text{V}^{-1} \text{s}^{-1}$ and $R_{on} \in [5, 200] \Omega$, $I_0 = -1.5 \text{ mA}$, and $D = 5 \text{ nm}$



(a) Bifurcation diagram of the micro-beam amplitude $\theta = f(R_{on}, \mu_v)$.

(b) Bifurcation diagram of the memristor current $i_m = f(R_{on}, \mu_v)$.

region TiO_2 [2]. In realistic experimental memristive devices, indirect coupling between ionic mobility, conductivity, and resistive states may arise through self-heating effects, local defect redistribution, or thermally activated vacancy migration mechanisms [52]. The present study evaluates this local thermally induced modulation through the temperature-dependence of the memristive parameters R_{on} , R_{off} , and μ_v . Consequently, the observed dynamical transitions can be interpreted as the electromechanical manifestation of local temperature-induced modifications of the memristive state. Several experimental and theoretical studies have demonstrated that the principal memristive parameters R_{on} , R_{off} , and μ_v exhibit significant temperature sensitivity [42–45]. Consequently, temperature-dependent formulations reported in the literature are considered in the present work to qualitatively interpret the thermo-sensitive nonlinear dynamics of the proposed MEMS architecture. Based on experimentally supported conductivity relations, the ionic mobility of oxygen vacancies and the resistive states of the HP memristor may be expressed as [45]:

$$\mu_v(T_p) = \frac{\sigma(T_p) M(\text{TiO}_2)}{N_A e n \rho(\text{TiO}_2)} \quad (18)$$

$$R_{on}(T_p) = \frac{w}{\sigma(T_p) A_m} \quad \text{and} \quad R_{off}(T_p) = \frac{D - w}{\sigma(T_p) A_m}. \quad (19)$$

The electrical conductivity $\sigma(T_p)$ follows the temperature-dependent relation

$$\sigma(T_p) = \sigma_{0,Mott} T_p^{-2s} \exp \left[- \left(\frac{T_{0,Mott}}{T_p} \right)^s \right] + \sigma_{0,ES} T_p^{-2s} \exp \left[- \left(\frac{T_{0,ES}}{T_p} \right)^s \right]. \quad (20)$$

which combines both Mott variable-range hopping and Efros–Shklovskii hopping conduction mechanisms frequently reported in oxide-based memristive materials [43, 45]. T_p is the temperature, and the other parameters are fully described in [45]. These thermo-activated relations indicate

that moderate temperature variations may substantially modify the effective ionic transport and the internal resistive state of the memristor, thereby affecting the nonlinear electrodynamic response of the coupled thermo-electro-mechanical system. For completeness, the temperature-dependent evolutions of μ_v , R_{on} , and R_{off} can be found in the work of Singh and Raj [45], where these dependencies are presented and analyzed in detail. In the present work, temperature effects are assumed to act globally only on the memristive parameters in order to avoid additional perturbations associated with the strong thermo-mechanical hypersensitivity of the microbeam. Nevertheless, local self-heating effects in the HP TiO₂ memristor may induce non-uniform thermal gradients capable of locally modifying the ionic mobility μ_v , the ON-state resistance R_{on} , and the OFF-state resistance R_{off} . As the temperature increases, resistances R_{on} and R_{off} tend to decrease due to the increase in electronic conductivity, while ionic mobility μ_v increases as a result of the thermally activated diffusion mechanisms. Figure 12 was generated by simultaneously varying these three (R_{on} , R_{off} , μ_v) HP memristor parameters. It clearly shows in Fig. 12a, the current response $i_b \sim \dot{\gamma}$, represented here by the fourth color dimension (color bar), which propagates through both the mechanical (micro-beam) and electrical (r_0 , L_0 , C_0) resonators.

A strong dependence of the electrical current amplitude i_b is observed on the memristor parameters R_{on} , R_{off} , and μ_v . Since the deflection of the micro-beam is proportional to the current intensity i_b (see Figs 7a and, 8a), it can be concluded that both the mechanical and electrical response amplitudes are thermally dependent on the HP memristor parameters. Indeed, a simultaneous decrease in R_{on} , R_{off} , and an increase in μ_v induces a variation of the electrical response amplitudes across the two resonators (see Fig. 12 a). Figure 12b illustrates the variation of the voltage across the memristor. The results reveal a stronger sensitivity to variations of μ_v and R_{on} than to those of R_{off} . Indeed, only slight modifications of $\dot{\alpha}_m$ and $\dot{\gamma}$ are observed when R_{off} decreases, whereas significantly larger variations occur for changes in R_{on} and μ_v . These observations suggest that the thermo-induced energy redistribution within the coupled electromechanical system remains weakly sensitive to variations of R_{off} . Furthermore, the combined analysis of Figs. 12a and 12b indicates that the thermo-electromechanical dynamics of the system may be more sensitive to variations in the ionic mobility μ_v and the ON-state resistance R_{on} than to changes in the OFF-state resistance R_{off} .

To investigate the electromechanical responses of the thermo-sensitive MEMS to the thermal sensitivity of the HP memristor, the ratio $\frac{R_{off}}{R_{on}}$ was varied through modifications of R_{on} . As previously shown in Fig. 3, a range of R_{off} values exists for which the current i_b exhibits an approximately linear evolution, together with a weak sensitivity of the

sensitivity to thermo-induced energy redistribution shown in Fig. 12. Within this parameter range, variations of R_{off} produce only slight amplitude changes without significantly modifying the electrodynamic regimes of the resonators. Consequently, tuning R_{on} provides an efficient way to control the ratio $\frac{R_{off}}{R_{on}}$ and analyze the resulting nonlinear dynamical responses. Figure 13 presents the three-dimensional bifurcation structures associated with the mechanical response of the microbeam and the electrical response of the memristor. These structures are obtained when the ratio $\frac{R_{off}}{R_{on}}$ (adjusted through variations of R_{on}) and the ionic mobility μ_v are simultaneously varied.

Figure 13 illustrates dynamical transitions between quasi-periodic and chaotic regimes as a function of the principal memristor parameters. Both the mechanical and electrical oscillations exhibit a clear tendency toward increasingly pronounced chaotic behavior as the ionic mobility μ_v decreases. The bifurcation structures shown in Fig. 13 reveal thermo-sensitive dynamical transitions generated by simultaneous variations of the ionic mobility μ_v and the ON-state resistance R_{on} . These transitions demonstrate the strong influence of the thermo-sensitive memristive parameters on the organization of the electromechanical attractors and on the distribution of quasi-periodic and chaotic regimes. Considering that the memristor temperature is expected to vary continuously in time due to Joule heating, the thermo-sensitive parameters R_{on} and μ_v may also evolve dynamically. Such thermo-induced parameter fluctuations may promote alternations between quasi-periodic and chaotic electromechanical behaviors. In this context, the bifurcation structures of Fig. 13 provide qualitative insight into the thermo-sensitive mechanisms that may govern these dynamical alternations. The combined analysis of Figs. 12 and 13 therefore suggests that temperature may indirectly influence both the amplitudes and the dynamical modes of the electromechanical responses through its effect on the thermo-sensitive memristive properties. The bifurcation structures further indicate that variations of μ_v and R_{on} substantially modify the organization of the electromechanical attractors and the relative distribution of quasi-periodic and chaotic responses. These observations suggest that thermo-sensitive modifications of the memristive parameters may alter the balance between quasi-periodic and chaotic dynamical behaviors, thereby influencing the overall nonlinear response of the coupled electromechanical system.

Nevertheless, no broad operating region completely suppresses chaotic dynamics. Quasi-periodic and chaotic regimes remain distributed throughout the investigated parameter domain. This complex distribution of dynamical regimes highlights the strong sensitivity of the

electromechanical response to variations of the memristive thermo-sensitive parameters and confirms their important role in governing the nonlinear behavior of the proposed MEMS architecture.

The numerical results based on time series, phase portraits, Fourier spectra, recurrence plots, Lyapunov exponents, and bifurcation diagrams consistently demonstrate that the memristive parameters govern not only the amplitudes and frequency content of the electromechanical oscillations but also their overall nonlinear dynamics. In particular, since the ionic mobility μ_v , R_{on} , and R_{off} are temperature dependent, moderate thermal variations in the vicinity of the memristor may modify the electromechanical behavior and promote alternating transitions between quasi-periodic and chaotic oscillations.

4 Comparative analysis: HP memristor versus Josephson junction dynamical systems

In order to place the proposed thermo-sensitive MEMS architecture into a broader context, a comparative analysis with the Josephson-junction-based electro-mechanical architecture previously investigated in Ref. [15] is presented in Table 2. The objective of this comparison is not to establish the superiority of one technology over the other, but rather to highlight their respective operating domains, nonlinear dynamical characteristics, and implementation constraints for thermo-electro-mechanical applications.

As summarized in Table 2, both systems exhibit rich nonlinear dynamics capable of supporting complex oscillatory behaviors. However, they rely on fundamentally different physical mechanisms. The Josephson-junction architecture derives its nonlinear response from superconducting phase dynamics and offers extremely fast operation, whereas the memristive architecture exploits state-dependent resistance and ionic transport processes to generate tunable nonlinear electromechanical responses.

The comparison further emphasizes the complementary nature of the two approaches. Josephson-junction-based systems are particularly attractive for ultra-fast nonlinear applications but generally require cryogenic operating conditions. In contrast, the HP memristor provides nonlinear memory effects and dynamical tunability under standard ambient conditions, making it more readily compatible with practical thermo-sensitive MEMS implementations.

Overall, the results obtained in the present work suggest that the HP memristor constitutes a promising nonlinear element for thermo-sensitive MEMS architectures, combining room-temperature operation, intrinsic memory

Table 2 Comparative analysis of the HP memristor model and the Josephson junction model for thermo-sensitive MEMS applications

Aspect	HP Memristor Model	Josephson Junction Model
Operating Temperature	Room-temperature operation; typically investigated within 200–450 K in TiO ₂ devices [45], [46]	Cryogenic operation; Nb/Al junctions typically operate around 4.2 K, while higher- T_c variants still require cryogenic environments; qubit-related JJ systems often explore mK to few K regimes [53–56]
Nonlinear Dynamics Regulation	Strong amplitude modulation and nonlinear dynamics regulated through temperature-dependent variations of R_{on} , R_{off} , and μ_v (see Figs. 4–11) [24, 27, 29]	Nonlinear RCSJ dynamics governed by the Josephson phase difference. (see [19, 53–55, 57])
Intrinsic Memory Properties	Native nonvolatile memory; pinched hysteresis loops; state-dependent resistance; dynamic resistive switching (see Fig. 6a), [2, 24]	No intrinsic nonvolatile resistive memory; hysteretic I–V characteristics possible in underdamped regimes. [53, 55, 56]
Thermo-sensitive Effects	Thermally activated ionic transport; temperature-induced dynamical reconfiguration (see Fig. 12), [43, 45], [46]	Increasing temperature reduces the critical current $I_c(T)$, progressively weakens the Josephson nonlinearity, and ultimately suppresses the Josephson effect as $T \rightarrow T_c \leq 77\text{K}$. [55, 57, 58]
Implementation Complexity	Room-temperature operation facilitates integration with standard electronic platforms and avoids the cryogenic requirements.	Requires cryogenic infrastructure and specialized measurement equipment, resulting in increased implementation complexity and operational cost [54–56]
Frequency response and switching speed	Typically operating from the Hz–MHz range (and extending to the GHz range in optimized nanoscale devices); switching times generally ranging from nanoseconds to microseconds depending on the memristive technology [2, 59]	Intrinsically ultra-fast response extending from microwave to terahertz frequencies through the AC Josephson effect; characteristic switching times typically in the picoseconds to nanoseconds range [54, 55, 60]

properties, flexible implementation, and regulation of electromechanical dynamical regimes through physically meaningful memristive parameters.

5 Conclusions

The present work has reported the mathematical modeling and numerical characterization of a thermo-sensitive Micro-Electro-Mechanical System (MEMS) architecture based on the coupling of an HP memristor with electrical and mechanical resonators. The governing electromechanical equations were derived from Kirchhoff's laws, Euler–Bernoulli beam theory, and the linear-drift HP memristor model, and the resulting nonlinear dynamics were investigated through Lyapunov exponents, bifurcation diagrams, Poincaré sections, recurrence plots, phase portraits, and Fourier spectra. The numerical results demonstrate that the intrinsic memristor parameters, R_{on} , R_{off} , D , and μ_v , strongly influence the amplitude, frequency content, and dynamical regime of the coupled electromechanical oscillations. In particular, two distinct dynamical behaviors were identified depending on the region of the Lyapunov exponent map: quasi-periodic regimes characterized by organized and stable oscillations, and transitional regions exhibiting successive quasi-periodic and chaotic behaviors. Increasing R_{on} was shown to promote greater dynamical regularity, reducing spectral dispersion and reinforcing the quasi-periodic character of both the memristor current and the beam oscillations. Conversely, decreasing the ionic mobility μ_v was found to expand the chaotic domain and intensify the irregularity of the electromechanical response. The analysis performed using temperature-dependent memristive parameters demonstrated that thermal variations can modulate both the oscillation amplitudes and the distribution of quasi-periodic and chaotic regimes, supporting the thermo-sensitive character of the proposed architecture. The thermo-electromechanical response was found to be more sensitive to variations in μ_v and R_{on} , suggesting that ionic transport mechanisms play a dominant role in thermally induced dynamical reconfigurations.

Compared with Josephson-junction-based MEMS architectures, the proposed system exhibits nonlinear dynamical features qualitatively similar to those reported for such systems, including chaotic oscillations and complex bifurcation structures, while operating at room temperature without the cryogenic infrastructure required by superconducting devices. This characteristic may constitute a practical advantage for the development of adaptive nonlinear oscillators, thermo-sensitive sensing devices, and chaos-assisted electromechanical systems.

The present study is primarily numerical, and a fully coupled thermal model remains beyond its current scope. Future work may extend the present framework by incorporating explicit heat-transfer mechanisms within the memristive layer, including Joule self-heating and thermo-ionic coupling, and by experimentally validating the predicted dynamical transitions through the fabrication and characterization of memristive microbeam prototypes. In addition, the thermo-sensitive memristive parameters identified in this work could be exploited for the development of regulation strategies enabling targeted switching between quasi-periodic and chaotic regimes in sensing and signal-processing applications.

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Data availability No datasets were generated or analysed during the current study.

Declarations

Conflict of interest The authors declare that they have no conflict of interest and no conflict of interest.

Ethics approval and consent to participate Not applicable.

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