Signaling to Analogical Reasoners Who Can Acquire Costly Information

Ennio Bilancini* Leonardo Boncinelli†

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Abstract

We show that separation in signaling games can be obtained without the single crossing condition, in a model where the receiver reasons analogically across a pair of states and can acquire costly information on the sender's type. Beyond ordinary separation (high type sends high signal, low type sends low signal) we find that also reverse separation is sustainable in equilibrium (high type sends low signal, low type sends high signal). Further, reverse separation in one state is obtained only if ordinary separation occurs in the other state. Pooling is possible and can go along with ordinary separation in one state.

JEL classification code: D01, D82, D83.

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*Dipartimento di Economia "Marco Biagi", Università degli Studi di Modena e Reggio Emilia, Viale Berengario 51, 43 ovest, 41121 Modena, Italia. Tel.: +39 059 205 6843, fax: +39 059 205 6947, email: ennio.bilancini@unimore.it.

†Dipartimento di Scienze per l'Economia e l'Impresa, Università degli Studi di Firenze, Via delle Pandette 9, 50127 Firenze, Italia. Tel.: +39 055 2759578, fax: +39 055 2759910, email: leonardo.boncinelli@unifi.it.

1 Introduction

Separating equilibria have a prominent role in signaling games, both in theory and in applications (Riley, 2001). Typically, the existence of a separating equilibrium crucially relies on the *single crossing condition*: in a two-type two-signal setting, the condition means that the additional cost of a high signal over a low signal is smaller for high types relatively to low types.

In this paper we provide a novel set of assumptions under which separation is obtained in the absence of the single crossing condition. In particular, we show that *analogical reasoning* and *costly acquisition of information* by the receiver allows ordinary separation to arise in equilibrium. Moreover, under the same assumptions we obtain that also *reverse separation* can occur in equilibrium: the high type chooses the low signal, and the low type chooses the high signal.

Analogical reasoning is a reasonable feature of belief revision whenever a decision-maker is faced with a large variety of possible alternatives, each of which differs from the others under many respects. In such cases it may be unfeasible to form specific beliefs conditional on every informational detail of every possible alternative. Rather, the decision-maker can feasibly focus on a few dimensions that are relevant for the decision to be made and then form analogy classes on the basis of such dimensions. As an example, consider purchase decisions: a consumer may well form a belief on the quality of a specific product by averaging over all products with the same or similar packaging.

Also, individuals can often exert effort and acquire the information that is relevant for the decision to be made. Consider again purchase decisions: a consumer can invest time and cognitive resources to carefully read product information reported on packaging, in the attempt to acquire a more precise knowledge of the product quality.

Our result that separation can be sustained by analogical reasoning and costly acquisition of information is important, we think, because the single crossing condition is likely to fail in a non-negligible number of cases. Carrying on the example about purchase decisions, we see no compelling reason why different packaging options should cost relatively more for low quality sellers than for high quality sellers. Further, the possibility of reverse separation might help to rationalize situations where low types engage more than high types in signaling activities, as it may happen when low quality producers make use of fancy packaging to overcome a careful scrutiny by consumers.

In our model, the receiver can face signals in two different states, and exhibits analogical reasoning across them. This means that he is able to condition his action on the state he is actually in, but at the same time he is unable to exploit this information when updating his belief on the sender's type; so, the receiver's decisions are based on the average type that is believed to send the observed signal. In addition, the receiver can acquire information at a cost: after observing the signal and prior to taking an action, he can incur a cost to learn the actual sender's type.

The main intuition for our results is that separation becomes possible since the receiver's act of acquiring information has different consequences for the two types of sender: the receiver does purchase the good if quality is high, and does not purchase the good if quality is low. Also, analogical reasoning over distinct states helps prevent that all the information is revealed in case of separation: acquiring information can be optimal for the receiver even when separation occurs in one state, provided that a different outcome occurs in the other state.

Importantly, both analogical reasoning and costly acquisition of information are crucial for our results. If we remove either of the two assumptions, we lose the possibility of any kind of separation (ordinary and reverse). To our knowledge, we are the first to explore the relevance of jointly assuming analogical reasoning and costly acquisition of information.

The rest of the paper is organized as follows. In Section 2 we relate our contribution to the relevant literature. In Section 3 we present the model and we provide a few preliminary definitions. In Section 4 we provide a motivating application. In Section 5 we define separation outcomes and we state all our results. Finally, in Section 6 we briefly discuss pooling outcomes, we summarize our findings and we comment on the crucial role of our main assumptions.

2 Related Literature

Analogical reasoning has been formally introduced by Jehiel (2005) with the equilibrium notion called *analogy-based expectation equilibrium*, and then extended by Jehiel and Koessler (2008) to games of incomplete information. The analogy-based expectation equilibrium captures a form of bounded rationality that concerns expectation formation by agents, rather than best-response selection. This solution concept has been fruitfully applied to explain a number of phenomena (Ettinger and Jehiel, 2010; Jehiel and Samuelson, 2012; Hagenbach and Koessler, 2017). Analogical reasoning is related to, but different from, the so-called *coarse reasoning*, where agents interpret messages by means of a limited number of categories, and are unable to distinguish objects falling in the same category (Mullainathan, 2002; Mullainathan et al., 2008).[1]

Among the many contributions that consider the acquisition of information as a costly strategic choice, Dewatripont and Tirole (2005) is particularly relevant since they focus on a sender-receiver setup, developing a theory of costly communication.[2] The cost to acquire information can be cognitive in nature, stemming from the limited amount of cognitive resources (Simon, 1955),[3] or it can measure the search effort to acquire information on

[1]A different bound to belief revision is implied by the notion of *cursed equilibrium* (Eyster and Rabin, 2005). Interestingly, Eyster and Rabin (2005) observe that in a classical signaling game a partially cursed equilibrium might allow for separation when Nash equilibrium does not.

[2]Other recent contributions considering the costly acquisition of information are Dewatripont (2006), Caillaud and Tirole (2007), Tirole (2009) and Butler et al. (2013).

[3]See also Bilancini and Boncinelli (2018) for a model of signaling where the costly processing of information

products' characteristics, as more typical in models of advertising (see, e.g., Gardete and Guo, 2014).

The reverse separation that emerges in our model can be related to *counter-signaling* (Feltovich et al., 2002): a situation where a sender has a quality that can be mistaken only for close qualities, and this allows the emergence of a signaling outcome where medium-quality senders choose high signals to separate from low-quality senders, while high-quality senders choose low signals to separate from medium-quality senders, thus yielding an inverted U-shaped relationship between types and signals.[4] Reverse separation, instead, produces a negative monotonic relationship between types and signals.

The only other paper, to our knowledge, where separation does not rely on the single crossing condition is Daley and Green (2014). Even if they assume single crossing in their model, their results hold also when the cost of signals is independent of the sender's type, as they note in Remark 3.5. Indeed, in their model, in addition to choosing a signal, senders undergo a test that provides a noisy grade about their actual type. Such a grading mechanism has different effects on low types and high types, indirectly making the benefits of signaling type-dependent, and hence allowing separation outcomes.

Finally, while we just assume that the single crossing condition does not hold, we point out to a recent stream of literature where violations of the single crossing condition are derived from further sources of agents' heterogeneity (see Boone and Schottmüller, 2017, and references therein).

is related to dual process theories in psychology, and Bilancini and Boncinelli (2016) for an application to persuasion games with labelling. We stress that agents in both these contributions are coarse reasoners, and not analogical reasoners.

[4] Harbaugh et al. (2017) and Harbaugh and Rasmusen (2018) explore a similar idea in the setup of certifiable quality disclosure. Mayzlin and Shin (2011) obtain a counter-signaling equilibrium where medium quality firms choose to make informative advertising, while high and low quality firms opt for uninformative advertising, which works as an invitation to search for consumers.

3 The Model

Consider a Sender-Receiver game with two states, ω_1 and ω_2. We denote the generic state with $\omega \in \{\omega_1, \omega_2\}$. The common prior of being in ω_1 is p_{ω_1}.

The Sender, denoted by S, has two types: a high type, denoted by H, and a low type, denoted by L. We use $t \in \{H, L\}$ to indicate the generic S's type. The common prior of S being of type H is potentially different in ω_1 and ω_2, and is denoted by $p_{H|\omega}$. In each ω, S sends one of two different signals, denoted by x and y, with generic signal $z \in \{x, y\}$.

The Receiver, denoted by R, makes decisions after observing the signal sent by S. In particular, R has to decide on two issues: whether to acquire S's type t at cost c, and which action to take. We denote the generic decision to acquire information with $i \in I = \{0, 1\}$, with 1 indicating acquisition. Also, R chooses between a high action $\overline{a}$ and a low action $\underline{a}$, where the generic action is denoted by $a \in \{\overline{a}, \underline{a}\}$.

The utility of S is given by $V : \{H, L\} \times \{\omega_1, \omega_2\} \times \{x, y\} \times \{\overline{a}, \underline{a}\} \to \mathbb{R}$. We assume that signal x always costs more than signal y, i.e., $V(t, \omega, x, a) < V(t, \omega, y, a)$, for all t, ω, and a. Also, we assume that x is always worth to send if it gets a reply of $\overline{a}$ instead of $\underline{a}$, i.e., $V(t, \omega, x, \overline{a}) - V(t, \omega, y, \underline{a}) > 0$, for all t and ω. Finally, we assume that no single crossing condition holds in both ω_1 and ω_2, i.e., $V(H, \omega, y, a) - V(H, \omega, x, a) = V(L, \omega, y, a) - V(L, \omega, x, a)$, for all ω and a.

The utility of R, gross of acquisition costs, is given by $U : \{H, L\} \times \{\omega_1, \omega_2\} \times \{x, y\} \times \{\overline{a}, \underline{a}\} \to \mathbb{R}$. The cost of acquiring t is denoted by c and is the same in ω_1 and ω_2. We assume that, other things being equal, $\overline{a}$ is the optimal choice when $t = H$ while $\underline{a}$ is the optimal choice when $t = L$. Formally, $U(H, \omega, z, \overline{a}) > U(H, \omega, z, \underline{a})$ and $U(L, \omega, z, \overline{a}) < U(L, \omega, z, \underline{a})$ for all ω and z.

Further, we crucially assume increasing differences in actions and states: $U(t, \omega_2, z, \overline{a}) - U(t, \omega_2, z, \underline{a}) > U(t, \omega_1, z, \overline{a}) - U(t, \omega_1, z, \underline{a})$ for all t and z. This property amounts to impose that choosing $\overline{a}$ over $\underline{a}$ pays more in ω_2 than in ω_1.

A strategy for S is described by a function $\sigma : \{H, L\} \times \{\omega_1, \omega_2\} \to \{x, y\}$. To simplify the exposition, we exploit the fact that, whenever R chooses $i = 1$, he will then optimally take $a = \underline{a}$ if S's type is L, and $a = \overline{a}$ if S's type is H. Hence, we can describe a strategy for R as a function $\rho : \{x, y\} \times \{\omega_1, \omega_2\} \to \{(0, \underline{a}), 1, (0, \overline{a})\}$, where $(0, \underline{a})$ and $(0, \overline{a})$ mean that information is not acquired (i.e., $i = 0$) and either action $\underline{a}$ or action $\overline{a}$ is taken, respectively, while 1 means that information is acquired (i.e., $i = 1$) and then the optimal action is taken (i.e., $a = \underline{a}$ if S's type is L, and $a = \overline{a}$ if S's type is H). We restrict attention to pure strategies, as this turns out to be sufficient for our primary goal of showing what separation patterns can arise.[5]

As equilibrium concept, we rely on Jehiel and Koessler (2008), where we find a definition of analogy-based expectation equilibrium for games of incomplete information. The distinguishing feature of such equilibrium concept is that players have analogy classes (i.e., collections of states of the world possibly different from information sets)[6] and they consider the average behavior of the opponent, and the average prior as well, over states belonging to the same class.

In our setting, the analogy classes for S are $\{(H, \omega_1)\}, \{(L, \omega_1)\}, \{(H, \omega_2)\}, \{(L, \omega_2)\}$, while for R the only analogy class is $\{(H, \omega_1), (L, \omega_1), (H, \omega_2), (L, \omega_2)\}$. We stress that the receiver maintains the possibility to condition his action on ω. Here, the states of the world correspond to the four pairs (H, ω_1), (L, ω_1), (H, ω_2), (L, ω_2), but for the ease of exposition we keep on referring to ω_1 and ω_2 simply as states.

To simplify the analysis, we define $\beta : \{\omega_1, \omega_2\} \times \{x, y\} \to [0, 1]$ as the function describing the posterior beliefs held by R after observing a signal and prior to deciding whether to acquire t. Given the analogy classes of R, we note that $\beta(\omega_1, z) = \beta(\omega_2, z)$ for all z.

[5]Considering the mixed extension of the game would add neither to the quality of our findings nor to the intuition behind them, while it would make proofs and statements substantially longer and less intuitive.

[6]Information sets for S are $\{(H, \omega_1)\}, \{(L, \omega_1)\}, \{(H, \omega_2)\}, \{(L, \omega_2)\}$, while for R are $\{(H, \omega_1), (L, \omega_1)\}, \{(H, \omega_2), (L, \omega_2)\}$.

4 An Application

Let the sender be a firm, which is interested to sell a product to a consumer. The quality of the good can be either high (H) or low (L). This is initially known by the firm, not by the consumer. The firm can operate in one of two different markets (ω_1 and ω_2), which the consumer fails to distinguish properly when he has to take into account firms' behavior and priors beliefs (i.e., the consumer puts them in the same analogy class). Prices are exogenously given in the two markets (q_{ω_1} and q_{ω_2}, respectively), because of competition, technology, or regulation. The firm has to choose the packaging of the product, or some other conspicuous characteristic, between a more costly option (x, with cost c_x) and a less costly one (y, with cost c_y). The consumer, after observing the packaging, and updating his belief on quality based on the observed packaging but not on the market under consideration, has to choose whether to exert effort (which has a cost of c) and acquire the knowledge of the actual quality of the product (e.g., by carefully reading the product label and processing the information in it) or abstain from it, relying on believed quality for his following decision.[7] Indeed, the consumer has then to choose whether to buy or not one unit of the product ($\overline{a}$ and $\underline{a}$, respectively).

The profit of the firm $V(t, \omega, z, a)$ is equal to $q_\omega - c_z$ if $a = \overline{a}$, and equal to 0 otherwise. We note that the single crossing condition is violated. The utility of the consumer $U(t, \omega, z, a)$ is equal to $u(t, z) - q_\omega$ if $a = \overline{a}$, and equal to 0 otherwise. It is easy to check that the property of increasing differences in actions and states is always satisfied in this example if $q_{\omega_1} \neq q_{\omega_2}$. In particular, it holds as stated in Section 3 for $q_{\omega_1} > q_{\omega_2}$, and with the reverse ordering, if $q_{\omega_1} < q_{\omega_2}$.

[7] The interpretation provided here suggests that the cost to acquire information might differ depending on ω, e.g., because mandatory labeling leads to more transparent information in one market than in the other. This does not change the quality of our results, provided that the difference in the acquisition costs is not too large.

5 Results

We say that *ordinary separation* (*reverse separation*) occurs in $\omega \in \{\omega_1, \omega_2\}$ when the H-type and the L-type of S, conditional on ω, choose signals x and y (y and x), respectively. Similarly, we say that *pooling on* x (*on* y) occurs in $\omega \in \{\omega_1, \omega_2\}$ when both the H-type and the L-type of S, conditional on ω, choose signal x (or y). When we say that separation occurs, we mean that either ordinary or reverse separation occurs in at least one between ω_1 and ω_2.

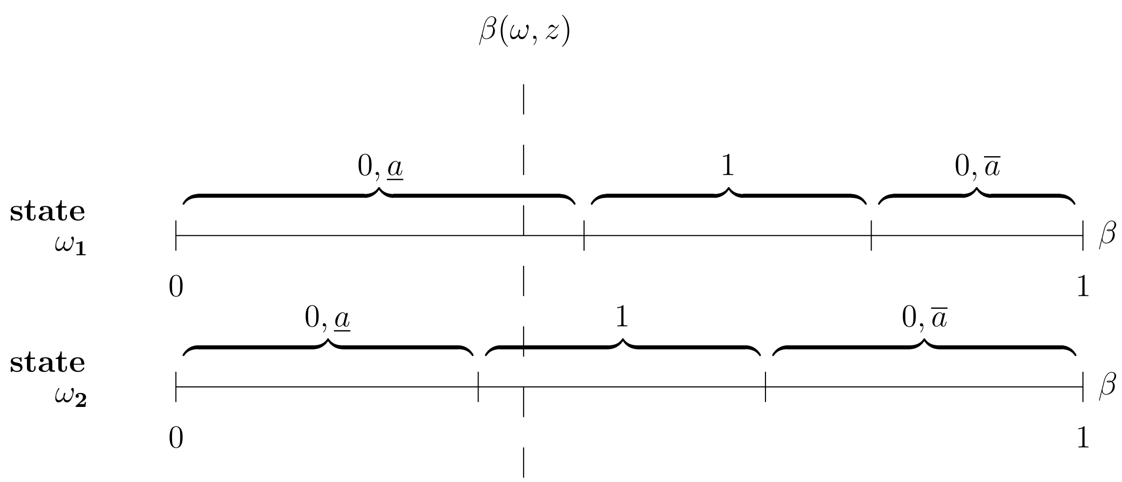

Figure 1: Receiver's optimal behavior as a function of beliefs β. We note that for certain beliefs, e.g., $\beta(\omega, z)$ in the picture, the receiver's optimal action changes between ω_1 and ω_2.

The graphical illustration in Figure 1 helps follow the proofs of all propositions. If the cost c to acquire information about the sender's type is positive but sufficiently low,[8] then the segment $[0, 1]$ representing R's belief can be divided into three intervals. When the belief β is either close enough to 0, or close enough to 1, the expected benefit to acquire information on t is not worth its (positive) cost, which means that R optimally chooses $i = 0$, and $a = \underline{a}$

[8] More precisely, c must be lower than $\min_z \frac{[U(L,\omega,z,\underline{a})-U(L,\omega,z,\overline{a})][U(H,\omega,z,\overline{a})-U(H,\omega,z,\underline{a})]}{[U(L,\omega,z,\underline{a})-U(L,\omega,z,\overline{a})]+[U(H,\omega,z,\overline{a})-U(H,\omega,z,\underline{a})]}$.

if β is low, or $a = \overline{a}$ if β is high. When instead β is in a mid-range, the expected benefit to acquire information on t is worth its cost, so that R's optimal choice is $i = 1$ (for this it is necessary that c is not too large). For a given $\omega \in \{\omega_1, \omega_2\}$, the threshold between the left region and the central region in figure is denoted by $\beta_{01}^{\omega,z}$, while the threshold between the central region and the right region is denoted by $\beta_{10}^{\omega,z}$, and their values can be computed as follows:

$$\beta_{01}^{\omega,z} = \frac{c}{U(H,\omega,z,\overline{a}) - U(H,\omega,z,\underline{a})};$$

$$\beta_{10}^{\omega,z} = 1 - \frac{c}{U(L,\omega,z,\underline{a}) - U(L,\omega,z,\overline{a})}.$$

Importantly, since R exhibits analogical reasoning, he forms the same belief in ω_1 and ω_2 when observing a signal, as indicated by the vertical dashed line which gives $\beta(\omega, z)$ for the given the signal $z \in \{x, y\}$. At the same time, R can make different decisions in the two states, which for the case of signal z are $(0, \underline{a})$ in ω_1 and 1 in ω_2. This, together with the assumption of increasing differences in actions and states, implies that the left and right extrema of the interval where $i = 1$ is optimal are larger in ω_1 than they are in ω_2.[9] This feature turns out to be pivotal for several of the following findings.

We are ready to state and prove all our results. Proposition 1 gives a necessary condition for having reverse separation in ω_2 in an equilibrium profile.

Proposition 1. *When R exhibits analogical reasoning, if in equilibrium reverse separation occurs in ω_2, then ordinary separation must occur in ω_1.*

Proof. Consider a profile where reverse separation occurs in ω_2. This means that, in ω_2, the L-type of S chooses x and the H-type of S chooses y. If both types find their choice optimal,

[9] If R cannot distinguish in which state he finds himself, then the extrema must necessarily coincide, since R must take the same actions in ω_1 and ω_2. Also, if R can distinguish in which state he finds himself and can exploit this information in updating beliefs, then the extrema can be different in ω_1 and ω_2 (as in the case of analogical reasoning), but in addition R can form different beliefs in ω_1 and ω_2 when observing a signal z.

then R's choice in ω_2 must be: $i = 0$ and $a = \overline{a}$ when x is observed and $i = 1$ when y is observed. Indeed, if the L-type of S finds optimal to choose x, which is more costly than y, then R must respond in such a way that x pays more than y to the L-type. This only happens when R chooses $i = 0$ and $a = \overline{a}$ when x is observed, and not when y is observed. If this is the case, then the H-type of S prefers y over x only if R chooses $i = 1$ when y is observed, otherwise the H-type would prefer x in order to induce R to switch to $a = \overline{a}$.

If R finds optimal to choose $i = 0$ and $a = \overline{a}$ when x is observed in ω_2, then the H-type must be choosing x in ω_1. Otherwise, signal x would fully reveal the L-type, and hence R's optimal choice would become $i = 0$ and $a = \underline{a}$ when x is observed. By the some token, if R finds optimal to choose $i = 1$ when y is observed in ω_2, then the L-type must be choosing y in ω_1. Therefore, ordinary separation must occur in ω_1. $\square$

Proposition 2 provides a necessary condition for having ordinary separation in ω_1 in an equilibrium profile.

PROPOSITION **2.** *When R exhibits analogical reasoning, if in equilibrium ordinary separation occurs in ω_1, then either reverse separation or pooling on x must occur in ω_2.*

Proof. Consider a profile where ordinary separation occurs in ω_1. This means that the L-type of S chooses y in ω_1, and the H-type of S chooses x in ω_1. If both types find their choice optimal, then R's choice in ω_1 must be: $i = 1$ when x is observed, and $i = 0$ with $a = \underline{a}$ when y is observed. Indeed, if the H-type of S finds optimal to choose x, which is more costly than y, then R must respond in such a way that x pays more than y to the H-type. This only happens when R chooses $i = 0$ and $a = \underline{a}$ when y is observed, and not when x is observed. If this is the case, then the L-type of S prefers y over x only if R chooses $i = 1$ when x is observed, otherwise the L-type would prefer x in order to induce R to switch to $a = \overline{a}$.

If R finds optimal to choose $i = 1$ when x is observed in ω_1, then the L-type must be choosing x in ω_2. Otherwise, signal x would fully reveal the H-type, and hence R's optimal choice would become $i = 0$ and $a = \overline{a}$ when x is observed. If the L-type finds x optimal in ω_2, then R's choice in ω_2 must be: $i = 0$ and $a = \overline{a}$ when x is observed, and either (i) $i = 0$ and $a = \underline{a}$ or (ii) $i = 1$ when y is observed. Otherwise, the L-type of S would prefer y over x because y is less costly and he would get $a = \underline{a}$ anyway. In case (i), the H-type and the L-type both find optimal to choose x in ω_2, so that pooling on x occurs. In case (ii), the H-type finds optimal to choose y, while the L-type finds optimal to choose x, so that reverse separation occurs in ω_2. □

Proposition 3 clarifies in which states ordinary and reverse separation can occur.

Proposition **3.** *When R exhibits analogical reasoning and $c > 0$, ordinary separation can never occur in ω_2, and reverse separation can never occur in ω_1.*

Proof. Consider first a profile where ordinary signaling occurs in ω_2, i.e., the H-type of S chooses x in ω_2 and the L-type of S chooses y in ω_2. If both types find their choice optimal, then R's choice in ω_2 must be: $i = 0$ with $a = \underline{a}$ when y is observed, and $i = 1$ when x is observed. Indeed, if the H-type of S finds optimal to choose x, which is more costly than y, then R must respond in such a way that x pays more than y to the H-type. This only happens when R chooses $i = 0$ and $a = \underline{a}$ when y is observed, and not when x is observed. If this is the case, then the L-type of S prefers y over x only if R chooses $i = 1$ when x is observed, otherwise the L-type would prefer x in order to induce R to switch to $a = \overline{a}$.

If R finds optimal to choose $i = 1$ when x is observed in ω_2, then the L-type must be choosing x in ω_1. Otherwise, signal x would fully reveal the H-type, and hence R's optimal choice would become $i = 0$ and $a = \overline{a}$ when x is observed. If the L-type finds optimal to choose x in ω_1, then R must be choosing $i = 0$ with $a = \overline{a}$ when x is observed in ω_1, otherwise the L-type would prefer the less costly signal y. However this is not possible, since the assumption $U(t, \omega_2, z, \overline{a}) - U(t, \omega_2, z, \underline{a}) > U(t, \omega_1, z, \overline{a}) - U(t, \omega_1, z, \underline{a})$, for all t and z, implies that if $i = 1$ is optimal for R when x is observed in ω_2, then $i = 1$ is a fortiori optimal for R when x is observed in ω_1.

Consider now a profile where reverse separation occurs in ω_1, i.e., the H-type chooses y in ω_1 and the L-type chooses x in ω_1. If both types find their choice optimal, then R's choice in ω_1 must be: $i = 1$ when y is observed, and $i = 0$ with $a = \overline{a}$ when x is observed. Indeed, if the L-type of S finds optimal to choose x, which is more costly than y, then R must respond in such a way that x pays more than y to the L-type. This only happens when R chooses $i = 0$ and $a = \overline{a}$ when x is observed, and not when y is observed. If this is the case, then the H-type of S prefers y over x only if R chooses $i = 1$ when y is observed, otherwise the H-type would prefer x in order to induce R to switch to $a = \overline{a}$.

If R finds optimal to choose $i = 0$ with $a = \overline{a}$ when x is observed in ω_1, then the H-type must be choosing x in ω_2. Otherwise, signal x would fully reveal the L-type, and hence R's optimal choice would become $i = 0$ and $a = \underline{a}$ when x is observed. If the H-type finds optimal to choose x in ω_2, then R must be choosing $i = 0$ with $a = \underline{a}$ when y is observed in ω_2, otherwise the H-type would prefer the less costly signal y. However this is not possible, since the assumption $U(t, \omega_2, z, \overline{a}) - U(t, \omega_2, z, \underline{a}) > U(t, \omega_1, z, \overline{a}) - U(t, \omega_1, z, \underline{a})$, for all t and z, implies that if $i = 1$ is optimal for R when y is observed in ω_1, then either $i = 1$ or $i = 0$ with $a = \overline{a}$ is optimal for R when y is observed in ω_2. □

The next two propositions state that the only cases that have not been ruled out by Propositions 1-3 can actually occur in equilibrium.

Proposition **4.** *When R exhibits analogical reasoning, if c is positive but sufficiently low, then there exist priors such that a profile where ordinary separation occurs in ω_1 and reverse separation occurs in ω_2 is an equilibrium.*

Proof. It is a matter of computation to verify that when

$$0 < c < \min_{\omega,z} \frac{[U(L,\omega,z,\underline{a}) - U(L,\omega,z,\overline{a})][U(H,\omega,z,\overline{a}) - U(H,\omega,z,\underline{a})]}{[U(L,\omega,z,\underline{a}) - U(L,\omega,z,\overline{a})] + [U(H,\omega,z,\overline{a}) - U(H,\omega,z,\underline{a})]},$$

there exist, for $\omega \in \{\omega_1, \omega_2\}$ and $z \in \{x, y\}$, threshold numbers $\beta_{01}^{\omega,z}$, $\beta_{10}^{\omega,z} \geq 0$, with $\beta_{01}^{\omega,z} < \beta_{10}^{\omega,z}$, such that R's optimal behavior is partitioned on the belief space as follows: $i = 0$ with

$a = \underline{a}$ when his belief belongs to $(0, \beta_{01}^{\omega,z})$, $i = 1$ when his beliefs belongs to $(\beta_{01}^{\omega,z}, \beta_{10}^{\omega,z})$, and $i = 0$ with $a = \overline{a}$ when his belief belongs to $(\beta_{10}^{\omega,z}, 1)$. Moreover, note that $\beta_{01}^{\omega_1,z} > \beta_{01}^{\omega_2,z}$ and $\beta_{10}^{\omega_1,z} > \beta_{10}^{\omega_2,z}$ for $z \in \{x, y\}$, due to the assumption that $U(t, \omega_2, z, \overline{a}) - U(t, \omega_2, z, \underline{a}) > U(t, \omega_1, z, \overline{a}) - U(t, \omega_1, z, \underline{a})$ for all t and z.

Therefore, if $\beta_{01}^{\omega_2,y} < \beta(\omega_1, y) = \beta(\omega_2, y) < \min\{\beta_{01}^{\omega_1,y}, \beta_{01}^{\omega_2,y}\}$ and $\max\{\beta_{10}^{\omega_2,x}, \beta_{10}^{\omega_1,x}\} < \beta(\omega_1, x) = \beta(\omega_2, x) < \beta_{10}^{\omega_1,x}$, then R optimally chooses as follows: in ω_1, $i = 0$ with $a = \underline{a}$ when y is observed and $i = 1$ when x is observed; in ω_2, $i = 1$ when y is observed and $i = 0$ with $a = \overline{a}$ when x is observed. Given this behavior by R, it is optimal for the L-type of S to choose y in ω_1 and x in ω_2, while for the H-type of S it is optimal to choose x in ω_1 and y in ω_2. Finally, note that, when S behaves in such a way, the Bayes' rule for R, taken into account that R is an analogical reasoner, implies that:

$$\beta(\omega_1, x) = \beta(\omega_2, x) = \frac{p_{\omega_1} p_{(H|\omega_1)}}{p_{\omega_1} p_{(H|\omega_1)} + (1 - p_{\omega_1})(1 - p_{(H|\omega_2)})}; \tag{1}$$

$$\beta(\omega_1, y) = \beta(\omega_2, y) = \frac{(1 - p_{\omega_1}) p_{(H|\omega_2)}}{p_{\omega_1}(1 - p_{(H|\omega_1)}) + (1 - p_{\omega_1}) p_{(H|\omega_2)}}. \tag{2}$$

To complete the proof, it is enough to note that priors p_{ω_1}, $p_{(H|\omega_1)}$ and $p_{(H|\omega_2)}$ can be chosen to have indeed $\beta_{01}^{\omega_2,y} < \beta(\omega_1, y) = \beta(\omega_2, y) < \min\{\beta_{01}^{\omega_1,y}, \beta_{01}^{\omega_2,y}\}$ and $\max\{\beta_{10}^{\omega_2,x}, \beta_{10}^{\omega_1,x}\} < \beta(\omega_1, x) = \beta(\omega_2, x) < \beta_{10}^{\omega_1,x}$. □

PROPOSITION **5.** *When R exhibits analogical reasoning, if c is positive but sufficiently low, then there exist priors such that a profile where ordinary separation occurs in ω_1 and pooling on x occurs in ω_2 is an equilibrium.*

Proof. The statement can be proven by adjusting the proof of Proposition 4. In particular, consider the case where $\beta(\omega_1, x) = \beta(\omega_2, x) < \beta_{01}^{\omega_2,x}$ and $\max\{\beta_{10}^{\omega_2,x}, \beta_{10}^{\omega_1,x}\} < \beta(\omega_1, x) = \beta(\omega_2, x) < \beta_{10}^{\omega_1,x}$, so that R's choice is: in ω_1, $i = 0$ with $a = \underline{a}$ when y is observed and $i = 1$ when x is observed; in ω_2, $i = 0$ with $a = \underline{a}$ when y is observed and $i = 0$ with $a = \overline{a}$ when x is observed. Given this behavior by R, the L-type of S finds optimal to

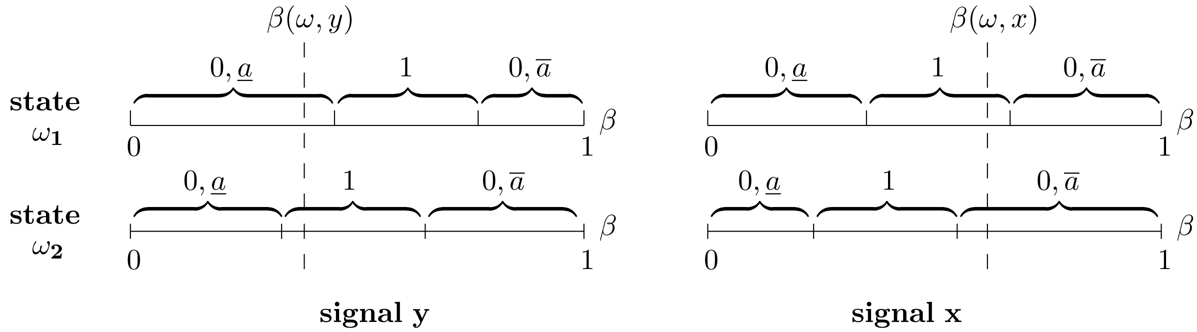

Figure 2: Receiver's behavior that is optimal and sustains a profile where ordinary separation takes place in ω_1 and reverse separation takes place in ω_2.

choose y in ω_1 and x in ω_2, while the H-type of S finds optimal to choose x in ω_1 and x in ω_2. As in the proof of Proposition 4, to complete the proof it is enough to note that priors p_{ω_1}, $p_{(H|\omega_1)}$ and $p_{(H|\omega_2)}$ can be chosen to have indeed $\beta(\omega_1, x) = \beta(\omega_2, x) < \beta_{01}^{\omega_2,x}$ and $\max\{\beta_{10}^{\omega_2,x}, \beta_{10}^{\omega_1,x}\} < \beta(\omega_1, x) = \beta(\omega_2, x) < \beta_{10}^{\omega_1,x}$. $\square$

Considered together, Propositions 1-5 imply that only two patterns of separation are possible in equilibrium: either ordinary separation in ω_1 and reverse separation in ω_2, or ordinary separation in ω_1 and pooling on x in ω_2. Figure 2 and Figure 3 provide examples where acquisition costs and priors are such that the separation patterns considered in, respectively, Proposition 4 and Proposition 5 can actually be sustained in equilibrium.

6 Discussion

The possible outcomes of signaling to a receiver who reasons analogically over two states and who can acquire information at a cost can be, as in the standard model, separation or pooling.[10] Separation can take place even if the single crossing condition does not hold and

[10]This paper focuses on separation, but is straightforward to see that pooling in both states can occur in equilibrium.

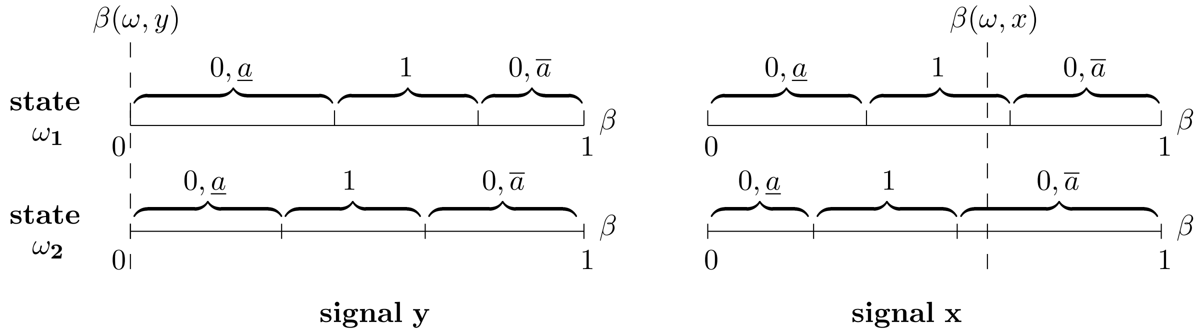

Figure 3: Receiver's behavior that is optimal and sustains a profile where ordinary separation takes place in ω_1 and pooling on x takes place in ω_2.

can be ordinary or reverse. Some combinations of separation and pooling are possible, but not all of them. Table 1 summarizes the feasible equilibrium outcomes.

Are analogical reasoning and costly acquisition of information necessary for these results?[11] It turns out that abandoning either of the two assumptions leads to the impossibility of a separation outcome.

No kind of separation is possible if we assume that the receiver is not an analogical reasoner but either cannot distinguish at all state 1 from state 2, or is perfectly aware of the state and updates beliefs according to Bayes' rule. In any state where separation occurs, R learns all relevant information if he is perfectly aware of the state, so he never acquires information, making a deviation by L profitable. If, instead, R cannot distinguish at all between states, separation requires that R does not choose $\overline{a}$ upon seeing y while acquiring information upon seeing x; but if R acquires information upon seeing x, L never sends x, so x separates perfectly H from L, making information acquisition worthless.

No kind of separation is possible even if we maintain analogical reasoning, but we make

[11] If the sender is forced to hold the same beliefs in the two states, and sends the same signals, separation is impossible: signals would be fully revealing, hence the receiver would acquire no information, making a deviation by L profitable.

		state 2		
		ordinary separation	reverse separation	pooling
state 1	ordinary separation		✓	✓
	reverse separation			
	pooling			✓

Table 1: Check marks denote combinations of outcomes that are feasibile in equilibrium.

costly acquisition ineffective, by either assuming that R always acquires information, or R never acquires information. In both cases all sender's types want to send the same signal (the least costly that grants $\overline{a}$, or if it does not exist just the least costly).

Lastly, even if we did not compare welfare across equilibria, it may be worth remarking that beyond the standard trade-off (between the cost of the signal and the value of the information transmitted) in our setting there is the additional cost of acquiring information (which is necessary to sustain separation). This diminishes the relative desirability of separation whenever pooling is attained without costly acquisition.

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