Production and Financial Networks in Interplay:
Crisis Evidence from Supplier-Customer and Credit Registers

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Abstract

We show that bank credit shocks to firms propagate upstream and downstream along the production network, with stronger effects for upstream than downstream propagation. Our identification strategy relies on: (i) administrative datasets from Spain on supplier-customer transactions and bank loans; (ii) a standard operationalization of bank credit-supply shocks during the Global Financial Crisis; and (iii) a general equilibrium model of an interfirm production network economy with financial frictions that is structurally estimated. Our results indicate that the network propagation leads to a 50% increase in the aggregate effects of bank credit supply shocks on GDP growth, with equally important first-order versus higher-order network effects.

JEL Codes: D85; E44; E51; G01; G21.

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1 Introduction

The production and financial networks of a modern economy are complex interrelated structures. On its real side, goods and services are produced as part of a dense web of specialized units, each of them relying on inputs from their upstream suppliers to produce outputs, which are then routed downstream towards other production units and consumers (see e.g. Acemoglu et al. (2012); Carvalho (2014)). On its financial side, moreover, these production firms are also connected to financial intermediaries (banks) through a similarly complex network of credit flows that shape the decisions of non-financial firms (see e.g. Diamond (1984); Holmstrom and Tirole (1997); Amiti and Weinstein (2018)), while banks themselves are interconnected as well through interbank claims (see e.g. Allen and Gale (2000); Elliott et al. (2014); Cabrales et al. (2017)).

The real and financial networks are, of course, intimately inter-related, and hence should be studied as such in order to gain a proper understanding of how modern economies actually work – in particular, if we want to understand how financial shocks propagate, and hence to be able to analyze their aggregate macroeconomic effects. Indeed, this is the view that, in the aftermath of the Global Financial Crisis, became widely adopted by academics and policy-makers alike. They largely came to accept that the role of those networks, and the *interactions among them*, had not been suitably recognized and, because of this, there was a failure in foreseeing its deep impact and wide span (Acemoglu et al. (2015); Freixas et al. (2015); Bernanke (2013, 2018)).

Despite this emergent consensus, however, the existing empirical research on the issue has mostly considered each of those networks in isolation, which in part can be attributed to a lack of reliable and comprehensive matched datasets on production and financial networks. A single notable exception is Costello (2020), which is the closest paper to ours and shows first-order downstream propagation of financial shocks (see below for a detailed discussion of this paper and the literature). This paper, however, relies on non-administrative data and, importantly, the analysis is carried out in reduced (non-structural) form. Moreover, it abstracts from general equilibrium effects and only accounts for propagation effects that involve either direct suppliers or direct customers.

In this light, our main contribution in this paper can be described in a nutshell as follows: to provide an *integrated* analysis of real and financial networks that is *theoretically founded* and shows, *empirically*, that such an integration can lead to *large amplification effects*. More specifically, we analyze a general equilibrium model that describes the production part of the economy as an interfirm network, with firms being subject to differential financial shocks. We provide the closed-form expressions for how those financial shocks impinge on equilibrium outcomes of the real side of the economy, thus accounting for all first- and higher-order propagation effects unfolding throughout the production network.[1] We show that these expressions can be taken to our data, by exploiting matched administrative datasets on the production and financial networks

[1] To advance our terminology at this point, we clarify that the first-order (network) propagation effects embody the consequences experienced by a firm when some of its direct customers or/and direct suppliers is affected by a financial (bank-credit supply) shock. In contrast, the higher-order effects experienced by a firm are those derived from bank shocks that affect its indirect customers or suppliers that are at network path lengths of more than one link. By upstream propagation we refer to the transmission of financial shocks from customers to suppliers, while by downstream propagation we refer to the transmission of financial shocks from suppliers to customers.

from Spain during the Global Financial Crisis. We estimate the effects of shock propagation on the real economy and find that those operating both downstream and upstream – as well as those taking place at first and all higher orders – are important and display substantial heterogeneity across propagation channels (e.g. stronger effects for upstream vs. downstream propagation).

As mentioned, our empirical identification strategy matches several administrative datasets from Spain (a bank-dominated economy), which include universal information on the following transactions: (a) the VAT Register of the Spanish Treasury covering the VAT firm-to-firm transactions in Spain; (b) the Credit Register held by the Spanish central bank (Banco de España), which collects information on all the bank loans to companies; (c) the Spanish Mercantile Register, which provides information on the balance sheets of the Spanish companies, which are legally obliged to report; and (d) supervisory bank balance sheet information, which includes the funding of each bank in the interbank market. These matched data allow us to construct the production network and identify the bank credit (financial) shocks to firms.

Regarding the production network, we rely on the fact that when a firm sells a product or service to another one, there is a VAT tax associated to the sale. Hence, by having access to all annual VAT transactions (above a threshold of only 3,005 euros), we can basically construct the whole weighted production network of Spain. And, in order to identify the financial shocks to firms stemming from banks (whose propagation occurs along the production network) we exploit the wide cross-section variability in exposure to the Global Financial Crisis that followed Lehman Brothers' failure in mid-September 2008. Specifically, we pursue the approach of Amiti and Weinstein (2018), itself following Khwaja and Mian (2008).[2] Their methodology, widely used in the literature, estimates a bank credit supply shock as the change in credit, cleaned by time-varying observed and unobserved fundamentals at the firm level, proxying e.g. for firm-level credit demand through firm-time fixed effects. Moreover, as a complementary exercise, we replicate the analysis with a different bank shock formulation (also extensively used in the literature), which is based on the ex-ante bank funding exposure to the interbank market, a market sharply affected by the Global Financial Crisis (see e.g. Iyer et al. (2014)). We show that the two alternative approaches lead to similar firm-level credit-supply negative effects, and find that these effects are significant only during the financial crisis but not before.[3]

At the level of firm-to-firm transactions (link level), we can exploit the full granularity enjoyed by our data across all supplier-customer pairs, which allows us to benefit from the wide variability in the changes of sales (purchases) observed among all suppliers (customers) of the same customer (supplier). In this way, we are able to account not only for observed, but also for

[2]For a related bank-level shock, see Chodorow-Reich (2014) and Jiménez et al. (2014). This approach exploits firms with at least two bank relationships at the same time. In Spain, during the sample period, approximately 75% of the credit came from firms with at least two such bank relationships. We get similar results when we rely on banks' exposure to the interbank network to identify bank credit supply shocks, which is an approach that does not depend on firms that necessarily have at least two bank relationships.

[3]This is expected since, before the collapse of Lehman Brothers, firms could much more easily switch from more affected to less affected banks, thereby reducing substantially the effect of the credit supply shocks; instead, during the crisis, no such flexibility existed. We show, therefore, that bank firm-level shocks were binding during the crisis but not before. As regards identification, it is important to highlight that the fact that Spain is a bank-dominated economy makes it reasonable to abstract from financial channels that involve other financial intermediaries – e.g. the shadow banking system – which could be important, for example, in the USA.

unobserved, heterogeneity through customer (supplier) fixed effects.[4] And complementing the link-level analysis, we also analyze the bank shock propagation at the node level. In this case, the focus is on the individual firms (rather than individual links) as they experience upstream or downstream propagation of the bank shocks hitting other firms in the production network.

The results based on reduced-form specifications are the following. At the link level, for a one standard deviation increase of the negative bank shock, and focusing first on within-supplier variation, we find that the upstream propagation of a bank shock experienced by a customer of a firm leads on average to a reduction of 2.36 percentage points (pp) in the growth rate of the firm's sales to that customer. Second, and relying instead on within-customer variation, we find that the downstream propagation of a bank shock hitting a supplier of a firm has an average reduction in the growth rate of the sales from that supplier to the firm of 1.09 pp. The stronger effect estimated for upstream propagation when compared to downward propagation is an interesting result, to which we return for the explanation in our structural analysis.

At the node-level analysis, we analyze upstream or downstream propagation of the bank shocks hitting, respectively, their direct customers or suppliers (first-order effects). This exercise requires some procedure of shock aggregation, and a natural way to do it is to weigh the shock originating in each customer (or supplier) by the share the latter commands on the firm's sales (or costs in intermediate inputs). And doing this, one can then estimate the effect of propagation as the average impact of the aggregate weighted shock affecting the direct (first-order) customers and suppliers of a firm on the growth of its own total sales. Our estimates indicate that while the upstream propagation effect is strong (though half of the link level analysis), this is not the case for downstream propagation for firm-level sales.[5]

The link- and node-level propagation effects just described suggest that shock propagation becomes weaker when it operates downstream or/and different shocks are combined at the firm (node) level. Intuitively, one may conjecture that this is probably a reflection of how the forces of substitution – in particular, among inputs, customers, or suppliers – work in each case (i.e. upstream *versus* downstream or link *versus* node levels). However, in order to have a clearer understanding of this issue, we need to go beyond the reduced-form approach and address it within a suitable theoretical framework where such substitution trade-offs can be properly described and analyzed.

The theoretical framework should not only reflect the possibilities afforded by the various substitution dimensions but must also include other features that seem comparably important in this context. One is that the propagation of bank shocks can involve many network orders (i.e. potentially long network paths). For, indeed, it is such long-range propagation that makes the process a truly systemic phenomenon and has the potential of bringing about substantial amplifying consequences. That is, not only do we need to analyze first-order effects but also those of higher network order, deriving all of these effects from the model and then testing them with our granular datasets.

[4]Although we control for firm unobservables via fixed effects, we also show that firm observed characteristics do not differ ex-ante across firms with stronger versus weaker bank credit shocks.

[5]In addition, we find that upstream propagation of first-order effects (financial shocks to customers) doubles the direct bank credit shocks to the firm, either at the link or node level.

Another feature of fundamental economic importance to be displayed by the model concerns the operation of markets for goods (intermediate and final) and production factors (in particular, labor and capital). For, as firms react to bank shocks (directly or indirectly), their responses will interact through these markets, hence reshaping market-clearing prices. In general, these market forces represent an additional important channel through which financial shocks can affect the economy via the production network.

In view of all former considerations, we propose and analyze a general equilibrium model that describes the production part of the economy as an interfirm network, its nodes being the firms and the directed links representing the flow of intermediate inputs. The production function of each firm follows a Cobb-Douglas structure, combining labor, capital, and a CES-aggregate of required intermediate inputs. We assume constant returns to scale and posit (as in Baqaee and Farhi (2019)) that firms determine their prices by applying a fixed markup over their marginal cost. The representative household consumes some goods produced in the economy, supplies factors of production –labor (elastically) and physical capital (inelastically)–, and owns all firms in the economy (including the rents associated with the borrowing expenses). Finally, following Bigio and La'O (2020), we formalize the financial shocks originating in the (non-modeled) financial part of the economy as price distortions (or wedges) affecting the price firms pay for their inputs (see also Liu (2019)).

Our main theoretical contribution is to provide a fully specified solution of the model that can be used to arrive at closed-form expressions of how financial shocks impinge on equilibrium effects through the production network, then taking these expressions to the data to estimate the effects of financial shock propagation on the real economy. Our model is similar to (but more general than) the parametric one studied in Bigio and La'O (2020).[6] Such a parametric approach enables us to derive closed-form solutions for equilibrium outcomes while still accounting for a rich set of equilibrium effects. More specifically, we determine how the financial shocks hitting individual firms in the economy ripple through its production network, affecting the bilateral sales between suppliers and customers (link-level analysis), the overall sales of any given firm (node-level analysis), and the whole economy as well (its real GDP).

At the link level, we derive the explicit expressions for the effects of shock propagation on equilibrium outcomes. They describe the change of the logarithm of sales s_{ji} of any given firm j to one of its customers i that (when normalized by the total sales of customer i) results from the *joint* operation of two spillover channels: exposure to financial shocks by the first-order supplier and the customer and the (higher-order) network propagation of shocks hitting other firms in the economy.[7] Crucially, such a purely bilateral focus already reveals the importance of accounting for shock propagation that involves full supply chains of any length along the production network. Moreover, as we also show, certain structural parameters of the model (such as e.g. the elasticity of substitution across intermediate inputs) play a key role in the propagation of shocks.

At the node (firm) level, we derive analogous expressions capturing the *combined* impact on

[6]The higher generality of our model derives from the fact that, unlike in Bigio and La'O (2020), we allow for non-unitary elasticity of substitution across intermediate inputs.

[7]Naturally, in addition to the two (spillover) network channels of shock propagation, we find that the direct financial shock experienced by the firm also matters.

total sales of each firm resulting from its *weighted* exposure to the financial shocks experienced by all its direct (first-order) customers and suppliers and, through these, to the firm's higher-order customers and suppliers. Compared to the link-level analysis, we find that the propagation pattern of the shocks at the node level is more intricate as it combines some "pure" upstream propagation with a non-separable "blend" of both downstream and upstream propagation.

Finally, our theoretical analysis of the model derives the equilibrium expression that captures the impact of firm-level financial shocks on the real GDP, fully accounting for the underlying structure of the production network. This result enables us to quantify the overall effect of (bank-to-firm) financial shocks on the economy and conduct various counterfactual exercises. Specifically, we use it to quantify the precise contribution of the production network propagation to the total effect of financial shocks on GDP – i.e., the impact that has to be added to that which would have obtained if network-based propagation could have been blocked – and we also disentangle the contribution of the first-order propagation of shocks hitting direct customers and suppliers from the contribution induced by higher-order propagation.

Once the theory is developed as described, we conduct a structural estimation of the model at both the link and node levels by bringing the induced equilibrium equations to our data. This entails, in particular, relying on the theory to determine what variables must enter into the estimated equations, how the different constituent effects have to be measured, and what is the functional form that brings all of them together.

We start by summarizing the main results obtained at the link level. Concerning first-order effects alone (i.e. those that involve a bank shock hitting a direct customer or direct supplier), the sign and magnitudes formerly obtained in the reduced form are quite well aligned with those derived from the structural estimation.[8] That is, we obtain reductions of 1.92 pp or 1.09 pp in the growth rates of a firm's sales or purchases if it has, respectively, one of its customers or suppliers being hit by a one standard deviation of a credit shock. We also find that, as in the reduced-form estimation, in the structural approach the upstream effect is estimated to be larger (by 65%) than the downstream one. But in this case, the model provides a clear-cut condition for this to happen: in facing downstream propagation of shocks, the customers have some ability (sufficient, but not too large) to offset their effect by substituting for inputs that become more expensive. More precisely, what is required for this to be the case is that the elasticity of substitution across intermediate inputs must lie between 1 and 2. And, indeed, this is consistent with the elasticity that is identified by our structural analysis at the link level, whose value is estimated to be equal to 1.56.

There are other additional crucial effects derived from our structural estimation that could not been obtained from the reduced-form approach. For example, we are able to estimate the importance of downstream propagation of all orders (i.e., including all higher-order effects) that, for any given link, typically impinge on each customer and each supplier. These different effects are fully identified and aggregated by the model, and therefore their impact on the sales for any given supplier-customer link can be estimated. As it turns out, this impact is not only

[8]Note that the fact that we obtain very similar results in the reduced and structural estimation of these effects is not *a priori* obvious as higher-order effects could have changed the estimated effects of first-order effects – however, we find that this does not happen in the data.

significant but also of comparable in (absolute) magnitude to that resulting from the (first-order) propagation of a credit shock hitting any of the two firms connected by the link, the supplier or the customer. The model, moreover, has a sharp prediction on their sign: it predicts that the impact of such higher-order shock propagation is positive (i.e. increases sales) if it impinges on the customer (due to substitution when it faces increased costs from other suppliers), but is negative if it affects the supplier (because it increases the costs of its intermediate inputs). We find that this theoretical prediction is fully supported by the evidence. Overall, therefore, the results obtained from our structural estimation suggest that the reduced-form approach misses a very substantial part of the shock propagation (around half of it), hence significantly underestimating the true extent of the phenomenon.

At the node level, effects are more complex than those estimated at the link level. For, in this case, one needs a suitable aggregation of the effects that flow into the node through *all* paths that connects it to *all* its different suppliers and customers, direct and indirect at *any* order. To provide a detailed procedure for such an aggregation is precisely what the theory does for us in a rigorous manner. Conceptually, we obtain two different effects of how financial shocks propagate through the network. Firstly, when a financial shock hits a firm, this induces a negative demand effect for its intermediate inputs, which propagates upstream. Secondly, there is a concatenation of an initial downstream-propagation phase and a subsequent pure upstream one – i.e., a mixture of downstream and upstream propagation, which we call bidirectional and can be intuitively understood as follows. When a shock hits any given firm, it affects the costs of that firm and, indirectly, the costs of firms positioned downstream in the network; consequently, those firms (which are the direct and indirect customers of the firm originally hit by the shock) respond by substituting away from their affected suppliers and toward those less affected, thus creating a cascading effect that propagates upstream as a demand shock.

Despite such complex considerations, our data allows us to compute the required magnitudes that aggregate the two types of indirect shocks and then we can estimate the strengths of the induced bidirectional and purely upstream propagation effects. We find that the effects associated to both propagation channels are sizable, a combined increase of one standard deviation in the shocks flowing through each of them giving rise to a sizable decrease of 2.1 pp in the average growth rate of firms' sales (amounting to a decrease in the average growth rate of 11%). We also find that the pure-upstream component is three times larger than the bidirectional one, again highlighting that the effects for firm sales propagating upstream are stronger.

The reason why bidirectional propagation at the node level is weaker than that proceeding upstream was essentially discussed already for the link-level analysis: since the bidirectional propagation involves a downstream component, its impact is mitigated by an elasticity of substitution that was estimated to lie between 1 and 2. In fact, this elasticity can also be estimated at the node level and, given the key role played by this parameter in our analysis, it is important to note that, when doing so, we arrive at an estimate of 1.35, which is quite close to the value of 1.56 derived from our link-level estimation. In this respect, therefore, we find that link- and node-level analyses deliver a consistent understanding of shock propagation. We also argue that both perspectives are also complementary and therefore comparably important. For, on the one hand, node-level analysis is key to conduct full (aggregate) counterfactual exercises of the short we shall outline shortly. Link-level analysis, on the other hand, allows us to get a sharper and more accurate grasp of the problem in the following three senses: (i) some substitution effects are only clear at the link level (e.g., the differential effects on a given firm of the financial shocks experienced by its various suppliers, including an opposite sign for customer's vs. supplier's higher order effects); (ii) we can control better for unobservables via different fixed effects and hence attain more accurate estimates (notably, the elasticity of substitution across intermediate inputs); (iii) we can fully separate upstream vs. downstream propagation (and then horse-race each other) while, in general, at the node level downstream propagation is inextricably coupled to an upstream component.

Finally, our paper turns to the issue of whether network propagation of financial shocks plays an important role in economy-wide outcomes, once we take into account all general equilibrium effects encompassed by our model. To address this point, we first evaluate the effects of financial shocks on (the log of) real GDP by relying on a first-order approximation of the equilibrium equations derived from the model and the estimated values of two key parameters, i.e. the elasticity of substitution among intermediate inputs and the coefficient that maps the theoretical to the empirical financial shocks.[9] Then we address the following counterfactual question on our Spanish data: What would have been the effect of the banking crisis on the Spanish GDP in the absence of the production network propagation of the bank credit supply shocks hitting the Spanish firms? This allows us to quantify the "aggregate effect" of network propagation by comparing the GDP outcomes induced with, and without, such blocking of production linkages. We find that, while the level of GDP is reduced by a percentage lying between 2.36% and 3.96% when the overall impact of bank credit shocks to firms is taken into account, the GDP only falls between 1.74% and 2.25% in the absence of (credit shocks via) input-output linkages. That is, an increase of around 50% in the total effect can be attributed to the network propagation. We also calculate that close to half of this impact is from network effects of higher order – i.e. from shocks that, for each firm, originate in other firms that are *not* direct suppliers or direct customers.

Related literature

The fast-growing literature studying the phenomenon of shock propagation in large economies has mostly evolved by studying separately the real and the financial networks. In the first case, the main focus has been on the supply chains that underlie the production of the non-financial firms of the economy and the role of the network structure in the propagation and aggregation of (for the most part) productivity shocks.[10] In the second case, financial networks, the analysis has mainly centered on the banks alone as the main actors, the links among them typically conceived as reflecting some form of financial flows.[11] In comparison with these two largely unconnected branches of the literature, our contribution considers both the real and financial sides of the economy and focuses the analysis on the interaction between them.

[9]For other parameters of the model that we cannot identify from the data, we either calibrate them to standard values used in the literature or explore how results change when the parameters vary within a natural grid.

[10]See for instance Acemoglu et al. (2012); Barrot and Sauvagnat (2016); Baqaee (2018); Carvalho et al. (2020).

[11]See for instance Allen and Gale (2000); Freixas et al. (2000); Iyer and Peydro (2011); Niepmann and Schmidt-Eisenlohr (2013); Elliott et al. (2014); Cabrales et al. (2017).

There is a rich literature that has explored whether credit-supply shocks may lead to significant real effects on the production side of the economy, but its analysis of the problem abstracts from the role played by the production network of the economy as a propagation structure of those shocks. As a representative sample of its more theoretical branch we can refer to e.g. Holmstrom and Tirole (1997); Stein (1998); Gertler and Kiyotaki (2010), while for its empirical branch we can mention e.g. Khwaja and Mian (2008), Chodorow-Reich (2014), Greenstone et al. (2014), Jiménez et al. (2012, 2017), Amiti and Weinstein (2018), and Galaasen et al. (2020). Only recently, we find a few papers that are closer to ours, in that they also aim at understanding the process by which financial shocks propagate through the real production network. To the best of our knowledge, the following two papers are the most related.[12]

The first paper is by Costello (2020), who studies downstream propagation of shocks through their influence on the trade credit that firms extend to their customers. Relying on data obtained from a third-party trade credit information platform, this paper documents that firms with greater exposure to a large decline in finance reduce their trade credit to customers, and consequently induce negative effects on employment.[13] In contrast with this paper, we use administrative registers and focus on the effects of financial shocks on sales at the firm-to-firm (link) and firm (node) levels. The analysis in Costello (2020) is non-structural, while we directly estimate the parameters governing the link- and node-level equations derived from our general equilibrium model (recovering also the elasticity of substitution across intermediate inputs that is important for understanding the results as well as for quantifying the GDP effects). The advantage of our approach, relative to the one taken in Costello (2020), is that it enables us to account for the general equilibrium and higher-order network effects of the shocks and interpret the estimates as structural parameters of the model. And in contrast with Costello (2020), we show that: (a) besides downstream propagation, upstream propagation is also important, with even higher economic effects; (b) in addition to first-order effects, also higher-order effects (e.g. bank shocks to suppliers of suppliers) do matter; (c) complex bidirectional propagation (i.e. the non-separable combination of downstream and upstream propagation) matters as well. This type of propagation of financial shocks has not yet been studied in the literature.

The second paper is by Cortes et al. (2019), who uses firm-to-firm transaction data from Brazil to estimate indirect effects of state-owned bank shocks. Methodologically, however, this paper differs from ours in several key respects. First, it only considers first-order propagation, while

[12] Another more distantly related paper is Alfaro et al. (2021), which investigates the propagation of credit shocks through *industry-level input-output data*. We outline here three important differences. First, they analyze reduced-form estimates while we show that such reduced-form estimation may miss about half of the overall propagation effects, hence substantially underestimating the extent of shock propagation. Second, and relatedly, they do not investigate higher-order propagation effects, but our findings suggest that these high-order effects are as crucial as first-order effects. Third, their reliance on industry-aggregated data raises identification concerns that our transaction-level data at the firm and supplier-customer level can handle in a significantly more effective manner, including the recovery of the elasticity of substitution across intermediate inputs, which plays a key role in understanding the results of our paper. In addition, we also refer to the paper by Dewachter et al. (2020), which complements our research by studying a dynamic Keynesian model that also displays an interplay of financial and production networks and is applied to Belgium data similar to ours. Their concern is quite different from ours in that their main focus is on how bank concentration and its effect on bank competition bears on macroeconomic volatility.

[13] Related to this, Demir et al. (2018) show that a negative shock to the cost of import financing gets propagated from liquidity-constrained firms to their customers (see also Jacobson and von Schedvin (2015)).

we also analyze the transmission of shocks through higher-order linkages. Second, it considers bank credit shocks by state-owned banks, while we consider bank credit supply shocks from all banks. Note that there is a large literature showing that there state-owned banks generate large inefficiencies (see e.g. La Porta et al. (2002)), and hence changes in credit through these government banks may not identify bank shocks appropriately. Third, due to data limitations, Cortes et al. (2019) only exploit transactions between firms working with different banks while we exploit all transactions. And fourth, in contrast to their paper, our approach is theory-based in that we propose and study a general equilibrium model of the problem, and then using it for the estimation.

In sum, our contribution to the literature can be schematically summarized as follows: we use administrative matched datasets on both supplier-customer transactions and bank loans; we present new theoretical results that, in closed-form, describe the different channels of shock propagation and provides a coherent way of aggregating their effects at the firm and economy levels; we structurally estimate the equilibrium equations both for link-level and node-level outcomes; we provide the quantification of the effects of financial shocks on GDP and conduct counterfactual analyses. These new features of our approach also generate novel results. By way of example, we can list the following: (a) we show that both downstream and upstream propagation yields significant effects, as it is also the case for both first- and higher-order propagation at different network distances; (b) we find interesting manifestations of heterogeneity across various effects, as for example between a stronger upstream propagation and a weaker downstream one for firm sales; (c) we show that as financial shocks propagate along the production network, they end up having an important aggregate impact on the GDP of the economy, with a significant contribution from higher-order effects.

2 Datasets

In this section we describe the administrative datasets for the Spanish economy that we use in our analysis. They cover both firm-to-firm transactions from VAT register and the bank-firm lending relationships from the credit registry. We also use administrative firm-level and supervisory bank-level data, the latter including the interbank credit information.

We use the confidential administrative VAT register. Spanish corporations are subject to Value Added Tax (VAT) and, as a part of an annual tax declaration to the Spanish tax agency (Agencia Estatal de Administración Tributaria, AEAT), report all annual paid and received transactions with third parties exceeding the amount of 3,005 euros (M.347 form).[14] We have access to this confidential dataset of all firm-to-firm transactions subject to VAT in years 2008 and 2009, and use them to construct the empirical counterpart of the firm level production network embedded in the theoretical model. In the next paragraphs we describe how we have processed the raw data to get the final dataset on firm transactions that we exploit in the empirical analysis.

For each bilateral transaction between two VAT-liable enterprises, the dataset contains two observations: the value of the transaction reported by the supplier and the value of the same

[14] More information available at: https://www.agenciatributaria.gob.es.

transaction reported by the customer. To construct the firm level network of transactions we need to assign a single value to each reported annual transaction. For that purpose, there is no ambiguity when the values reported by the supplier and the customer coincide. However, there may be a discrepancy between the supplier's and the customer's declaration of the same transaction. When the discrepancy is small relative to the higher reported value, we select the value reported by the supplier. When the difference is relatively large, which is the case for 0.01% of observations, we choose the smaller of the two declared values in order to be more conservative.

In our analysis we restrict ourselves to transactions where both the seller and the customer are publicly limited or limited liability companies (which applies to almost 95% of all non-financial firms), both are firms (IAE code starts with 1), and neither is from the financial sector.[15] We end up with a dataset containing information on 13,822,286 transactions between 867,013 firms in 2008 and 12,003,117 transactions between 861,350 firms in 2009.[16]

We use the administrative loan-level data for non-financial companies from the Spanish Credit Register (CIR), which is maintained by Banco de España in its role of banking supervisor (and central bank). The CIR contains very detailed loan level data since 1984 on all loan commitments above 6,000 euro granted by any bank operating in Spain. We aggregate the different loans between a firm and a bank in each period, thus using data given at the bank-firm-time level. Even though the CIR is updated on a monthly basis, given the annual frequency of other datasets that we use in the paper, we record the credit data annually. The CIR also provides information about loan characteristics such as the type of instrument, currency, maturity, degree of collateralization, default status, or the amount drawn and committed by the firm. In this paper, we focus on commercial and industrial (C&I) loans granted by depository financial institutions (i.e., with a bank license). For a more detailed description of the CIR see, for instance, Jiménez et al. (2020).

Other administrative datasets that we use in the analysis pertain to the balance sheets and income statements of non-financial companies and banks. At the non-financial firm level, we exploit information on firms' characteristics that is available at a yearly frequency from the Spanish Mercantile Register — an administrative database that contains available information on firms' financial statements (required by law to be submitted to the commercial registry) as well as on their income corporate tax returns. The data cover around 90% of firms in the non-financial market economy for all size categories, including both turnover and number of employees. The correlation between micro-aggregated employment and output growth and the National Accounts counterparts is above 0.90.

Moreover, we rely on supervisory bank-level data, which is based on information from the December reports that banks have to submit to the supervisor: Banco de España. We obtain information on banks' overall interbank funding positions, balance-sheet variables, and profit and loss account data. This information allows us to have, for each bank, how much it borrows

[15] The IAE code (Impuesto sobre Actividades Económicas) is the code used by the tax agency to classify the main economic activity of a tax payer. A firm is taken to belong to the financial sector if its main activity, according to the IAE classification, is one of the following: (i) financial institution, (ii) insurance company, (iii) financial, insurance and real-estate service provider. Our raw dataset covers the period 2008–2014.

[16] An annual transaction is an annual total sale from firm i to firm j (or, equivalently, an annual total purchase from firm i by firm j). See *Table A10* for additional summary statistics.

overall from the interbank market. On average each bank borrows 1.7 billion euros from the interbank market, 28% of total bank assets, with an inter-quantile range going from 2% to 53%.

3 Identification of financial shocks

Our main empirical challenge is to estimate how shocks originating in the financial system impinge, and then propagate, on the real production network. This section explains the strategy that we pursue for the identification of these shocks.

We start with the empirical formulation of the collection of financial shocks hitting firms, as well as its suppliers and customers. Our identification of these shocks follows a standard approach in the empirical literature. In our baseline specification, we follow Amiti and Weinstein (2018) and construct bank-credit shocks as follows. We estimate, for each bank, a credit supply factor identified as the bank fixed effect at a bank-firm-level weighted least square regression of credit growth (in percentage changes) on bank- and firm-fixed effects. This regression exploits the variability generated by the global financial crisis. Thus, if we denote by $CreditGrowth_{ib}$ the growth rate of total lending to firm i from bank b (in percent changes) and by ν_i and ι_b firm and bank level fixed effects respectively, we estimate the following regression (for 2009 and 2008) using a weighted least square (WLS) procedure:[17]

$$CreditGrowth_{ib}{=}\nu_i + \iota_b + \epsilon_{ib}. \tag{1}$$

Armed with the estimated bank-level shocks, we follow the AW approach to identify credit supply shocks at the firm level (θ^{AW}). Briefly, what we do is to compute firm-specific credit supply shocks as the weighted average of the bank-specific factors ι_b estimated in (1), using pre-crisis credit exposure of the firm to each particular bank as weights.[18] Moreover, we switch the sign of the estimated supply shocks so that higher values reflect a lower credit supply. We refer to the shock so defined as the continuous AW shock. In Appendix A (Table A4), we consider robustness exercises that exploit a binary version of the continuous AW shock that takes value one if the AW shock for the firm is above the median across all firms (and zero otherwise), thus implying that a firm experienced a higher AW shock and hence this firm was exposed to more financially constrained banks during the crisis, i.e. those banks that reduced their credit supply more.

Next, we analyze whether the estimated financial shocks at the firm level that stem from the bank supply side are orthogonal to pre-crisis observable firm characteristics (see Table A1 in the Appendix for summary statistics). That is, we want to test whether firm i that works with the more financially constrained banks is similar to other firms j that works with the less constrained banks. To do so, in Table A2 we explore a relevant range of observed firm characteristics for both types of groups.[19] It shows that the firms exposed to negative bank credit supply shocks and

[17] In particular, using the Amiti and Weinstein (2018) approach, credit growth is defined as $L_{ib,2009}/L_{ib,2008} - 1$, where L_{ib} denotes the borrowing by firm i from bank b. Moreover, the weights used for estimating Eq. (1) by WLS are those described by the authors.

[18] Using the Amiti and Weinstein (2018) terminology, the firm shock that is being computed is the sum of the common shock and the firm level bank shock.

[19] In Table A2, which focuses on what banks lend to each firm, we show bank characteristics at the firm level

those not exposed were not different prior to the global financial crisis. The first four columns of the table point to identical numbers for the firm characteristics for the two groups of firms (that is, they are not related to bank variables), while its fifth column reports the t-statistic of the differences in averages of the firm characteristics in each group.

The aforementioned statistic, however, is sample-size dependent, as it was noted by Imbens and Wooldridge (2009). This would make the rejection of the null hypothesis more likely as the number of observations increases. To avoid the problem, these authors propose to test the null of no differences in means between the two groups through a scale-and-sample-size-free estimator. The proposed estimator is labeled the normalized difference and scales the difference in means of each variable in the two samples by the square root of the sum of the variances. Imbens and Rubin (2015) suggested a heuristic threshold of 0.25 for the statistic (in absolute value) to judge whether the differences should be considered significant or not. As column 6 of Table A2 shows, no such firm variable is greater than 0.01 in absolute value. This provides, therefore, support to the claim that the estimated effects of financial shocks on firms are not driven by differential firm observable fundamentals (e.g. credit demand shocks).[20]

Importantly, when analyzing differences in pre-crisis bank characteristics, we find that banks that before the crisis relied more on the interbank market (or are smaller) reduced more the supply of credit, and hence their associated firms may have experienced a credit supply constraint (as columns 5 and 6 of Table A2 suggest). We arrive at similar conclusions from a linear probability regression of firm exposure to financially constrained banks on all firm characteristics and four-digit NACE $\times$ province fixed effects (which control for crisis differences across different industries and locations), as reported in column 7 of Table A2. The estimation results show that the only two statistically significant variables are the two bank variables (the net interbank position of the firm's average bank and the corresponding bank size).[21]

The fact that the banks which became more acutely constrained during the crisis were also those borrowing more heavily from the interbank market before the crisis is not specific to our case but is a general feature of financial crises – indeed, this is why researchers have used the net interbank position to identify bank credit supply shocks to firms (e.g. for Portugal and Italy, as in Iyer et al. (2014); Ippolito et al. (2016); Cingano et al. (2016)). Here, therefore, we also consider it as an alternative to the Amiti and Weinstein (2018) approach to singling out banks that experience (stronger) credit-supply shocks. More specifically, we use the bank's net exposure to interbank funding before Lehman's collapse. This is also a natural way of bringing into our analysis the other key financial network that has been considered in the literature: the interbank network (see e.g. Allen and Gale (2000)).

It is worth noting that one of the assumptions of the AW approach is that a firm's demand is the same regardless of the bank or type of loan the company applies for. For example, Ivashina

computed as a pre-crisis weighted average.

[20]Note that in some regressions we will also control for unobservables via e.g. firm, customer or supplier, fixed effects.

[21]If we visually analyze each bank's credit growth and bank characteristics, we find that banks that reduce credit growth the most are banks with higher interbank market and size and somewhat lower capital, though this latter variable is not significant in univariate or multivariate tests. Given the confidentially of the data, we cannot report these bank by bank results.

et al. (2022) shows that different banks offer different types of credit, so the proportional credit demand assumption in AW may not be true. In section 4, as a robustness check, we relax this assumption by allowing firms to have different demands for credit depending on their types of loan (asset-based loans, cash-flow loans, trade-finance agreements or leases, using the classification established by those authors), or different measures of specialization of the bank (e.g. in the real estate sector using its relative concentration of loans, or even whether the firm is in the province or/and industry where the bank is most specialized). To implement this exercise, in Eq. (1) we interact the firm fixed effects with the corresponding categorical variables.

Our two different identification approaches (overall credit supply shocks or those stemming from the interbank market) lead to similar identification effects on firm-level credit availability that are negative and significant only during the crisis and not before (see Appendix Table A3). That is, the induced (negative) effects caused by banks are significant in 2009, but not in 2007 or 2008. This is intuitive since before the financial crisis that followed the failure of Lehman Brothers in mid-September 2008, firms could switch much more easily from more to less constrained banks, thereby substantially reducing the effects of credit shocks. In this respect, an important consideration to bear in mind that supports our identification strategy is that Spain is a bank-dominated economy. Hence we can safely abstract from other financial intermediaries (such as, say, the shadow banking system), which would be crucial in other economies (e.g., in the US).

4 Reduced-form evidence and first-order propagation

In this section we explore the first-order network propagation of bank credit shocks originating in the financial network based on reduced-form regressions, both at the link-level (supplier-customer level) and at the node-level (firm-level).

We start with the link-level analysis. To explore both upstream and downstream propagation of bank shocks to firms, we include all firms and their customers and their suppliers that are affected by bank shocks.[22] We consider a specification that allows us to estimate the impact of credit shocks hitting customer i on sales of firm j (upstream propagation), and the impact of credit supply shocks hitting supplier j on its sales to firm i (downstream propagation). That is, we focus on links of the form $j \rightarrow i$ and consider the following specification:

$$\Delta \log s_{ji} = a^u \theta_i^{AW} + a^d \theta_j^{AW} + \boldsymbol{b}\boldsymbol{x}_{ji} + \epsilon_{ji} \tag{2}$$

where the sub-index j (i) refers to a generic supplier (customer) and the dependent variable is measured in terms of the log changes over the crisis of s_{ji}, i.e. the sales from supplier j to customer i (or, equivalently, the purchases by customer i from supplier j).[23] The main regressors

[22] If a firm does not have credit with a bank prior to the crisis, then it is not clear whether bank shocks to this firm would be zero, or if this firm would need to get finance during the crisis, then its bank shocks could be the shock of previous banks from which this firm borrowed in the past or banks in the same location. We analyze therefore firms with suppliers and customers that borrowed before the crisis, which represent the largest part of the economy not only in terms of borrowing but also in terms of sales.

[23] We winsorize growth rates to be bounded by +200% and −100% in order to reduce the impact of outliers. As a robustness check, we also considered, with similar results, the following definition of the dependent variable:

of interest are the firm-level credit supply shocks θ_i and θ_j, estimated following the AW approach described in the previous section. Thus, the coefficients of interest a^u and a^d refer to the effect of customer's financial shock θ_i on the sales of firm j (upstream propagation) and the effect of supplier's financial shock θ_j on the purchases of firm i (downstream propagation), respectively.

As financial crises may affect differently firms with different fundamentals (e.g. of larger or smaller size) or in different industries (e.g. more or less cyclical) by mechanisms different from our model, we control, depending on the specification considered, for supplier, customer and supplier-customer characteristics in vector $\boldsymbol{x}_{ji}$: the size of the supplier (customer) in terms of its log of total assets, log of age, capital-to-asset ratio (own funds over total assets), working capital as a measure of liquidity (current assets minus current liabilities over total assets), and its ratio of short-term debt (less than 1 year) as a measure of its maturity structure. We also include unobserved factors captured by the product of province and industry dummies (at 2-digit NACE level) of suppliers (customers), the share of total sales of firm j associated to customer i. Supplier-customer variables we include are: the share of total purchases (sales) of customer (supplier) j directed to firm i, and dummies indicating whether both firms share the same main bank or operate in the same province-industry pair. Finally, in some specifications, we also include a large set of dummies capturing specific trends in industries and zip codes in the form of (industry/zip code of a firm) $\times$ (industry/zip code of a customer/supplier) and the direct bank credit supply shock to firm i as an additional regressor (however, in the latter case, we cannot add firm i fixed effects, so we replace them by the set of firm's observed characteristics enumerated above).

Moreover, our firm-to-firm network data allow us to account for different configurations of fixed effects in our regressions in order to enhance identification. Since we are interested in identifying the impact of customers' (suppliers') credit shocks on supplier-customer sales, our most stringent specification includes supplier (customer) fixed effects so that identification is based on within-firm variation from multi-customer (multi-supplier) firms.[24] Intuitively, this identification strategy is based on the comparison of sales (purchases) of the same firm with different customers (suppliers) that are hit by different credit shocks – that is, identification is enhanced by accounting for firm-specific unobserved heterogeneity and thus isolating the credit shock associated to customers (suppliers).

Table 1 reports our estimates of equation (2) in the case of upstream propagation (to suppliers) of bank credit shocks to customers.[25] In column (1), the estimated impact of the direct bank-credit supply shocks on sales is negative and statistically significant, which corroborates that direct credit supply significantly affects firms' sales after accounting for indirect effects by means of customer fixed effects. Column (2) reports the estimated effect of customers' credit

$(s_{2009} - s_{2008})/(0.5(s_{2008} + s_{2009}))$, where s_t stands for the flows under consideration in year t. This formulation (which was originally proposed by Davis and Haltiwanger (1992) to study establishment-level data) allows us to account for both the extensive and the intensive margin.

[24]Note that 77% of suppliers have two or more customers in our sample, while 86% of customers have two or more suppliers.

[25]All shock variables are standardized to have zero mean and unit variance in order to make the estimated coefficients comparable. Also, equation (2) is estimated by weighted OLS, where the weights are the size of the firm-to-firm relationship captured by past sales or purchases between the two firms, and the standard errors are multi-clustered at the level of firm i, at the customer or supplier level, and at the bank level.

supply shocks (labeled in the table as "1^{st} order customer (bank) effect") on firm sales in our most stringent specification with firm fixed effects and customers' controls. The estimated first-order propagation effect is large and significant. The estimated effect is more than double of the direct effect in column (1), with one standard deviation reduction in customers' credit supply implying, on average, a reduction of 2.4 pp in firm-customer sales. This reduction represents a 19.5% of the mean value of the dependent variable (see Table A1 in Appendix A for summary statistics).

As a robustness check, columns (3) and (4) of Table 1 consider the alternative bank credit supply shock explained in the previous section, based on the interbank market position of the banks, as an instrument to the baseline AW shock. When we use such a measure of interbank-funding exposure as a source of identification, not only do our main findings on upstream propagation remain robust, but also the magnitude of the estimated effects is even larger.[26] In addition, columns (5) and (6) investigate whether a firm affected by a negative bank credit supply shock from its customers reduces its sales due to a restriction of total bank credit (i.e. it experiences a fall in total bank debt). Specifically, we consider the customer reduction of bank debt as the regressor of interest –instrumented with the credit supply customer shock– and find that firms reduce their sales (column (6)) due to a fall in total bank debt of their customers induced by a negative bank credit shock that these firms are not able offset through other financing sources (1st stage in column (5)). We interpret this result as evidence in favour of the bank credit channel explaining the first-order (upstream) propagation of bank credit supply shocks.

Turning to downstream propagation, Table 2 reports our estimates from equation (2) in the case of credit shocks propagated from suppliers to customers. The estimated effects also point to a statistically significant effect of suppliers' credit shocks on the sales to their customers. However, according to our estimates, first-order downstream propagation is smaller in magnitude than upstream propagation. For, as we see in column (2) of Table 2, one standard deviation reduction in suppliers' credit supply implies, on average, a reduction of 1.09 pp in sales to their customers, which represents a reduction of 9.1% of the mean value of the dependent variable (see Table A1 in the online Appendix A). In columns (3) to (6) of Table 2, we show that the alternative bank credit supply shock and the bank credit channel work as they did for the case of upstream propagation. Finally, Table A4 in the Online Appendix shows that if we use the discrete version of the bank shocks, the main results of Tables 1 and 2 remain essentially unchanged.

As already mentioned, for robustness, we have also studied the implications of a relaxation of the assumption of equal firm credit demand in the AW methodology. Tables A5, A6 and A7 of the Appendix replicate the main results of Tables 1, 2 and A4 (columns (1) and (2)) when we allow that firms may have different demands depending on the type of loan or the bank's specialization.

Specifically, Table A5 considers bank shocks constructed at the firm level from an equation analogous to Eq. (1) but introducing a firm fixed effect for each of the credit types of operations

[26] First-stage effective F statistic showed in the tables is based on Montiel Olea and Pflueger (2013) and it is robust to heteroskedasticity, serial correlation, and clustering. Its value is above the critical value of 23.109, for a confidence level alpha of 5% and a percentage of worst-case bias of 10%, in almost all the cases, always with a worst-case bias of 30%. Table A4 in the Appendix corroborates these results when we use discrete AW shocks.

being considered. That is, we group commercial credit into four categories: asset-based loans, cash-flow loans, trade finance agreements, and leases. For the results shown in Table A6, we have considered the possibility that, instead of firms having a single credit demand for all banks, they may have different credit demands depending on whether the bank's main specialization is in the same industry and/or the same province as the firm operates. We have constructed, therefore, an indicator that has four different categories. One category has the bank's main industry being the same as the firm's industry but their provinces not coinciding; a second category with the bank's main province being the same as the firm's province but their industries being different; a third category where both the industry and province of the firm and the bank coincide; and a fourth category in which neither the sector nor the province coincide. For the results shown in Table A7, we have allowed in Eq. (1) that firms may have a different credit demand for banks specialized (versus not) in real estate. Then, for the results in Tables A5 to A7, the estimated equations used to extract the bank supply shocks include the interaction of the different categorical variables thus constructed with the firm fixed effects. The results shown in Tables A5 to A7 are quite similar to those obtained before, which suggests that the AW assumption that firms have a similar credit demand for all banks is a good approximation in our context (consistently the AW shocks from baseline regression are very highly correlated with those AW shocks of Tables A5 to A7).

The link-level analysis above provides evidence that firms sell less to the customers that are hit by (negative) financial (bank-credit) shocks, and similarly suppliers hit more by credit shocks sell less to their customers. While this may be the case for firm-to-firm sales it is not immediate that these results should translate to firms' total sales. For example, a firm might be able to undo a particular negative shock from a particular supplier or customer by resorting to other suppliers or customers for its inputs or sales. In order to address this issue, we move from the link-level analysis to the node-level analysis. For each firm in the sample, we construct aggregate variables capturing the credit shocks experienced by *all* of its direct suppliers (suppliers' shock) and the credit shocks experienced by *all* of its customers (customers' shock). The suppliers' shock of firm i is a weighted average of the bank credit shocks hitting the direct suppliers of i, where weights are equal to the intermediate inputs cost shares of each supplier. These shares essentially capture the importance of each supplier for i. We denote this variable as $SupShock$. Analogously, the customers' shock of firm i is a weighted average of bank credit shocks hitting direct customers of i, where weights are equal to the sales shares corresponding to each customer. We denote this variable as $CustShock$.

With these two variables in place, we estimate the following regression at the firm-level:

$$\Delta \log s_i = a\theta_i^{AW} + a^u CustShock_i + a^d SupShock_i + \boldsymbol{b}\boldsymbol{x}_i + \epsilon_i, \tag{3}$$

where $\Delta \log s_i$ refers to the log change between 2008 and 2009 in the total sales of firm i, $\boldsymbol{x}_i$ is a vector of firm-specific characteristics those described for the link-level analysis, but as we do not (cannot) control for customer or supplier fixed effects, we control for a set of dummies capturing specific trends in industries and geographical areas in the form of industry-province (or zip-code) fixed effects.

Table 3 presents the estimated effects. In column (1) we report the impact of direct bank credit shocks, which is negative and statistically significant, in line with the findings in Table 1 as well as with the vast literature on the bank lending channel that documents significant real effects of credit shocks. The results in column (2) of Table 3 show that the impact of customers' credit shock on total sales (upstream propagation) is also negative and significant, but smaller in magnitude than the link-level estimates in Table 1. Specifically, recall that a one standard deviation credit shock to customers reduces the firms' sales to them by 19.5% of the average sales growth in the sample at the link level, while this reduction is only of around 11% at the node (firm) level. Intuitively, this reduction suggests that firms are able to partially undo the customers' shocks by resorting to other customers – but this substitution is limited since we still observe a negative impact in total sales. Column (2) also shows that firms' sales are not significantly affected by credit shocks to their direct suppliers. This result may be due to the substitutability across intermediate input providers, as we explore below through the lens of our theoretical model. Finally, columns (3) and (4) of Table 3 refer to the impact of bank credit shocks on employment growth at the node (firm) level, where we obtain results that are parallel to those obtained for sales. That is, we find a significant and non-negligible negative impact of bank credit shocks on employment growth, either from direct bank credit shocks to the firm or from bank credit shocks to its direct customers (but as for sales, a negligible effect of the credit shocks hitting direct suppliers).

Unanswered questions and the need for theoretical guidance

The reduced-form estimates presented above point to strong effects on sales from suppliers to customers and *vice versa* due to the propagation of financial shocks. This type of reduced-form evidence, however, is silent about important issues that are still unanswered by the literature. First, higher-order propagation (from shocks that hit e.g. suppliers of suppliers) may also have important effects on both firm-to-firm sales and firms' total sales. Second, the reduced-form approach ignores general equilibrium effects that could be quite important. Third, some of the estimated propagation effects are difficult to explain, which in turn makes us wonder what are the mechanisms at work. By way of a simple example, consider the question that arose when comparing upstream and downstream first-order propagation of credit shocks: Why is it that the former are larger than the latter? May this depend on different mechanisms or/and degrees of substitutability operating in each case?

In the remaining part of the paper we propose a model that, once it is taken to our rich network data, allows us to shed light on these and other important questions.

5 A model of financial-shock propagation along the production network

We ended the preceding section by pointing to the wide range of yet unanswered questions that require a theoretical framework to be properly addressed. To formulate such a framework is the objective of the present section. This will enable us to undertake the following tasks:

(a) establish an operational formal connection between the financial and real sides of the economy;

(b) evaluate the network effects (of first- and higher-order) induced on any given firm by the financial shocks;

(c) provide an interpretation of the empirical results and estimate the structural parameters of the model;

(d) suitably quantify the aggregate effects of financial shocks.

Our model closely aligns with the parametric framework postulated by Bigio and La'O (2020). By adopting a parametric approach, we are able to derive analytical solutions for equilibrium outcomes, thus encompassing a diverse range of equilibrium effects. Our framework, however, is more general than that Bigio and La'O's parametric model as it allows for non-unitary elasticity of substitution across intermediate inputs. The main contribution is to provide a fully specified solution of the model that can be used to arrive at closed-form expressions of how financial shocks impinge on equilibrium outcomes through the production network. These expressions will then be applied to our Spanish data in order to estimate the effects of shock propagation in the global financial crisis. Since the proposed theoretical framework is in many respects standard, we now present its different components in a quite compact manner, focusing in detail only on those features that are less common or more pertinent to the empirical analysis. When formal details and proofs are needed, they are relegated to the Online Appendix B.

5.1 Production

The production side of the economy consists of a given set of firms, N, each of them producing a single good with a technology displaying constant returns to scale. The production possibilities of a typical firm i are described by a nested formulation of the production function of the form:[27]

$$y_i = f_i(k_i, \ell_i, M_i) = \zeta_i k_i^{\rho} \ell_i^{\beta} M_i^{\alpha} \tag{4}$$

where y_i stands for the output of firm i, k_i for the physical capital used, ℓ_i for its labor input, and M_i is the following CES aggregate of the intermediate inputs:

$$M_i = \left(\sum_{j \in N} g_{ji}^{\frac{1}{\sigma}} z_{ji}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{5}$$

where z_{ji} stands for the amount of intermediate input j used by firm i. Thus, all production functions $f_i(\cdot)$ display the nested CES form, with the strictly positive α, β, and ρ being input shares, [28] σ is the elasticity of substitution across intermediate inputs, and $\boldsymbol{\zeta} = (\zeta_i)_{i=1}^{n}$ is the vector of classical Hicks productivity parameters. The non-negative vector $(g_{ji})_{j \in N_i^+}$ reflects the *relative* intensity with which any given firm i uses different intermediate inputs and satisfies

[27] The same approach is used by Bernard et al. (2022).

[28] We follow the recent literature on production networks – see Barrot and Sauvagnat (2016); Carvalho et al. (2020); Bernard et al. (2022) – in assuming homogeneous input shares in our benchmark setting. In the appendix, we extend the model to allow for firm-specific parameters α, β and ρ. All results about propagation of financial shocks are proved for such a more general version of the model in the online Appendix B.

$\sum_{j \in N} g_{ji}=1$ for every i. Thus, the interfirm (technological) production structure of the economy is characterized by the (column-stochastic) adjacency matrix $\mathbf{G}=(g_{ji})_{j,i=1}^{n}$.

Firms are assumed to set their price optimally, given the underlying competition structure of the economy. To account for different such structures, we follow Baqaee (2018); Baqaee and Farhi (2019) and use a reduced-form approach postulating that every firm i sets its price by applying a markup μ_i to its marginal cost of production. As these authors explain, different forms of competition give rise to alternative markup values. Thus, these markups can be conceived as parameters of the model that embody the (non-explicitly modeled) competition structure of the economy.

5.2 Financial shocks

As in Bigio and La'O (2020) we assume that every firm i is required to pay in advance a share χ_i of its input expenditure, which is financed by borrowing at an interest rate R_i. Its net profit is then given by:

$$\begin{aligned}\pi_i=&p_i y_i-(1-\chi_i)\left(\sum_{j \in N_i^+} p_j z_{jk}+w\ell_i+rk_i\right)-\chi_i(1+R_i)\left(\sum_{j \in N_i^+} p_j z_{ji}+w\ell_i+rk_i\right)\\ =&p_i y_i-(1+\theta_i)\left(\sum_{j \in N_i^+} p_j z_{ji}+w\ell_i+rk_i\right),\end{aligned}$$

where we use the notational shorthand $\theta_i=\chi_i R_i$, and p_i denotes the price of good i, w is the wage, and r is the rental cost of capital. For convenience, we use as a benchmark what we call *normal conditions* characterized by $R_i=0$, while if the firm's borrowing cost rises to some $R_i>0$ we say that the firm is experiencing a financial shock of magnitude $\theta_i=\chi_i R_i$. A consequence of this shock is that the firm faces a "financial distortion" (or wedge) in its decision problem given by θ_i. In our context, such a shock originates in the banks servicing the firm and leads to a change in the terms at which the firm can obtain bank credit.

5.3 Consumption and Equilibrium

To close the model, we need to formalize the consumption side of the economy and then posit a suitable equilibrium notion.

First, concerning consumption, we assume that the consumption vector $\boldsymbol{c}=(c_1,c_2,...,c_n)$ is chosen by a representative household, which also provides firms with labor L (endogenously supplied) and K units of physical capital (which is assumed to be inelastically provided). Her objective is to maximize the following utility function:[29]

$$U(\boldsymbol{c})=\frac{c^{1-\delta}}{1-\delta}-\frac{L^{1+\eta}}{1+\eta} \tag{6}$$

subject to a budget constraint

$$\sum_i p_i c_i \leq E, \tag{7}$$

[29]This widely used utility function was introduced in MaCurdy (1981).

where $c = \prod_{i=1}^{n} c_i^{\gamma_i}$ is a composite consumption bundle, $\boldsymbol{\gamma} = (\gamma_i)_{i=1}^{m}$ is a vector of preference weights for every good i, δ modulates the income elaticity of labor supply (equal to $-\frac{\delta}{\eta}$), η is its inverse Frisch elasticity, and E is the household's income (or expenditure). The financial flows in the economic system are taken to be balanced, so the household's income (non-normalized expenditure) satisfies $E = wL + r + \sum_{i \in N} \pi_i + TR$, where $TR \equiv \sum_{i \in N} \theta_i \left(\sum_{j \in N_i^+} p_j z_{ji} + w\ell_i + rk_i \right)$ is a lump-sum transfer from the financial sector.

Finally, the equilibrium concept embodies the usual requirements of individual (firm and household) optimality and market clearing. Verbally, it can be described as follows.

Definition 1. *Given a vector of financial distortions $\boldsymbol{\theta} = (\theta_i)_{i=1}^{n}$, a* Market Equilibrium *(ME) is an array $\left\{ [(p_i^*)_{i=1}^{n}, w^*, r^*], [(c_i^*)_{i=1}^{n}, (y_i^*)_{i=1}^{n}, (z_{ij}^*)_{i,j=1}^{n}, (\ell_i^*)_{i=1}^{n}, (k_i^*)_{i=1}^{n}] \right\}$ that satisfies the following conditions:*

- *Each firm i minimizes production costs and applies to them a mark-up μ_i to set its price.*
- *The consumption plan maximizes the household's utility subject to its budget constraint.*
- *Markets for each intermediate input, capital, and labor clear.*

The existence of a market equilibrium follows from standard arguments, and its uniqueness relies on our Cobb-Douglas assumption on preferences and nested CES assumption on technologies.

5.4 Link-level implications of the model

In this subsection, we explore the model implications for how a *financial* (credit) shock hitting a firm affects the bilateral interactions with any other firm directly related to it (be it a customer i or a supplier j). Following the terminology used in the previous section, if the credit shock in question hits the supplier, its effect on the link $j \to i$ is labeled as downstream, while if it hits the customer it is called upstream. In both cases, we center on the impact of a credit shock hitting the firms on the (equilibrium) log value of trade s_{ji} between j and i, normalized by the total sales s_i of customer firm i, i.e. we consider the change of $\log \frac{s_{ji}}{s_i}$. Our interest, therefore, is on how a shock to the supplier or customer affects the sales from j to i (or, equivalently, the purchases of i from j) once we net out the effect coming from the total change of the sales of i.[30]

The following expression, derived by Proposition 1 in the online Appendix B, provides a linear approximation of the total change in $\log \frac{s_{ji}}{s_i}$ induced by the *overall* vector of shocks in the economy:

$$\overbrace{d\log s_{ji}}^{\substack{\text{Change of sales} \\ \text{from j to i}}} - \overbrace{\mathrm{d}\log s_i}^{\substack{\text{Change in log sales} \\ \text{of customer } i}} = -(\sigma - 1) \overbrace{\theta_j}^{\substack{\text{Supplier} \\ \text{shock}}} - \overbrace{\theta_i}^{\substack{\text{Customer} \\ \text{shock}}} \\ -(\sigma-1)\,\alpha \overbrace{\mathbf{e}_j' \mathbf{G}' (\mathbf{I} - \alpha \mathbf{G}')^{-1} \boldsymbol{\theta}}^{\substack{\text{Higher order supplier} \\ \text{shock } (Net_j)}} + (\sigma - 1) \overbrace{\mathbf{e}_i' \mathbf{G}' (\mathbf{I} - \alpha \mathbf{G}')^{-1} \boldsymbol{\theta}}^{\substack{\text{Higher order customer} \\ \text{shock } (Net_i)}}. \tag{8}$$

[30] Even though our link-level analysis (in particular, the empirical one in Section 6.1 will always focus on such a normalized supplier-customer trades, in order to simplify exposition we sometimes omit an explicit reference to this normalization of bilateral trades.

In our empirical analysis, the right hand side of (8) will serve as a linear approximation of $\log\frac{s^1_{ji}}{s^1_i} - \log\frac{s^0_{ji}}{s^0_i}=\log\frac{s_{ji}(\boldsymbol{\theta})}{s_i(\boldsymbol{\theta})} - \log\frac{s_{ji}(\mathbf{0})}{s_{ji}(\mathbf{0})}$, where s^{t}_{ij} and s^{t}_i denote the respective values for bilateral trades and total sales over the crisis period (t=0 to 1). Equation (8) shows that the total effect can be decomposed into two parts, which we now describe in turn.

First, we have the propagation effect impinging on the sales s_{ji} of a supplier j to a customer i when a shock θ_j directly hits this supplier or, reciprocally, when a shock θ_i directly hits this customer. These are what we call the *first-order* effects. Importantly, note that equation (8) implies that the effects of customers' and suppliers' shocks are asymmetric. The intuitive reason for this asymmetry can be explained as follows. When firm i is directly hit by a credit shock θ_i, buying inputs becomes costlier and therefore its demand for *all* its inputs (including j) decreases. This effect is captured by $-\theta_i$. Instead, when supplier j is affected by a negative shock, the price of input j is affected. Hence the extent and direction to which firm i adjusts its purchases from j (relative to its sales s_i) depend on the degree of substitutability i enjoys across its intermediate inputs. More specifically, if $\sigma>1$, so that intermediate inputs are substitutes then firm i spends relatively less on input j when j is affected by the negative shock, while the opposite happens otherwise (i.e. $\sigma<1$). Moreover, we also learn from (8) that if $1<\sigma<2$, the downstream effect originating in a supplier's credit shock is smaller than the upstream effect induced by a customer's credit shock.

Second, equation (8) also includes (in its second line) higher-order effects that are labeled higher-order supplier shock (Net_j) and higher-order customer shock (Net_i). These are the network-based aggregates of the credit shocks hitting the direct and indirect suppliers of all orders of j and i, respectively. Again, there is an asymmetry in the effects of these two shocks.

On the one hand, Net_j captures the effect on the prices of j´s intermediate inputs (i.e. on the relevant price index) of the shocks hitting all of j´s direct and indirect suppliers of any order. Thus, a higher Net_j implies an increase in the marginal cost for j, which translates into a higher price of j at the equilibrium. Then, as it happened for the first-order supplier's shock, the effect of such a rise in Net_j on the sales s_{ji} is negative or positive depending on whether σ is higher or smaller than 1.

On the other hand, the customer's indirect shock Net_i captures the change in the price index of the intermediate inputs of i due to financial shocks. Interestingly, we find from (8) that the effect of a change in this index is of a sign opposite to that of the supplier's indirect shock Net_j. Intuitively, the reason is that higher Net_i implies that suppliers of i are hit on average by larger credit shocks (direct and indirect). Viewing those suppliers as an aggregate substitute for input j ($\sigma>1$), a higher Net_i implies that it is less attractive to firm i to substitute away from j to other suppliers, implying a positive effect on sales from j to i. But when this happens, the reaction to an increase in Net_j must be *negative*, as explained before. [31]

[31] Finally, note that equation (8) does not feature the propagation effects coming from customers of customers. The reason is that such upstream propagation is absorbed by the normalization factor d$logs_i$, which controls for all the *aggregate* effects impinging on the customer i. Thus, our present link-level analysis focuses on the *relative* effects affecting supplier j among all other suppliers of i.

5.5 Node-level implications of the model

We now turn our attention to the node (firm) level analysis. This entails *aggregating all direct and indirect effects* of financial shocks impinging on each *individual firm*, thus accounting exhaustively for the different channels through which the firm may be affected, at equilibrium, by the shocks hitting other firms in the economy.

In our ensuing analysis, the following notation will prove useful. Let $\mathbf{M}$ and $\mathbf{T}$ stand for diagonal matrices with elements $\frac{1}{\mu_i}$ and $\frac{1}{1+\theta_i}$ on the main diagonal, respectively, where μ_i denotes the mark-up of firm i and θ_i the credit shock hitting it. Then let $\boldsymbol{v}(\boldsymbol{\theta}) \equiv (\mathbf{I} - \alpha\mathbf{GMT}(\boldsymbol{\theta}))^{-1}\boldsymbol{\gamma}$, where $\boldsymbol{\theta}=(\theta_1,\theta_2,...,\theta_n)$ and recall that $\boldsymbol{\gamma}$ captures the relative preferences of the consumer for different consumption goods. Intuitively, in the absence of shocks, $\boldsymbol{v}(\mathbf{0})$ is a variation of the standard centrality notion proposed by Bonacich (1987), aggregating the number of suitably weighted downstream paths that connect i to the consumer along the production network.

The following expression, proven by Proposition 2 in the online Appendix B, provides a linear approximation of the total change in the sales of firm i relative to the GDP (the total consumer's income E)[32] induced by the overall vector of shocks in the economy:

$$\mathrm{dlog}\left(\frac{s_i}{E}\right) = -\alpha \boldsymbol{e}_i'(\mathbf{I} - \alpha\mathbf{V}^{-1}\mathbf{GMV})^{-1}\mathbf{V}^{-1}\mathbf{GMV}\boldsymbol{\theta} + (1-\sigma)\boldsymbol{e}_i'\boldsymbol{\Lambda}\boldsymbol{\theta}, \tag{9}$$

where $\mathbf{V}=diag(\boldsymbol{v}(\mathbf{0}))$ and

$$\boldsymbol{\Lambda} \equiv \mathbf{V}^{-1}(\mathbf{I} - \alpha\mathbf{GM})^{-1}\left(diag(\alpha\mathbf{GMV1}) - \alpha\mathbf{GMVG}'\right)\left(\mathbf{I} - \alpha\mathbf{G}'\right)^{-1}.$$

We first note that, in the steady state, the matrix $\mathbf{H} \equiv \alpha\mathbf{V}^{-1}\mathbf{GMV}$ has its elements $h_{ji}=\frac{s_{ji}}{s_j}$ essentially capture the relative importance of firm i among all direct customers of j (see Lemma 3 in online Appendix B). This information is observed in our dataset and hence can be directly used in our empirical analysis of the crisis. Therefore, we can also compute the matrix $\boldsymbol{\Lambda}$, which can be written in the following more convenient form:

$$\boldsymbol{\Lambda}=(\mathbf{I} - \mathbf{H})^{-1}\left(diag(\mathbf{H1}) - \mathbf{HG}'\right)\left(\mathbf{I} - \alpha\mathbf{G}'\right)^{-1},$$

which then also allows us to rewrite (9) as follows:

$$\overbrace{\mathrm{dlog}\left(\frac{s_i}{E}\right)}^{\substack{\text{Change of Domar weight}\\ \text{of firm i}}} = - \overbrace{\boldsymbol{e}_i'(\mathbf{I} - \mathbf{H})^{-1}\mathbf{H}\boldsymbol{\theta}}^{\substack{\text{Upstream}\\ \text{shock}}} + (1-\sigma) \overbrace{\boldsymbol{e}_i'\boldsymbol{\Lambda}\boldsymbol{\theta}}^{\substack{\text{Bidirectional}\\ \text{shock}}}. \tag{10}$$

The term $\boldsymbol{e}_i'(\mathbf{I}-\mathbf{H})^{-1}\mathbf{H}\boldsymbol{\theta}$ captures the demand effect that stems from the shocks that impinge on the firms' purchases of the inputs used in their production processes. When a firm is hit by a financial shock its demand for inputs decreases (as inputs are more expensive), and this effect propagates upstream through the network as captured by the upstream operator $(\mathbf{I} - \mathbf{H})^{-1}\mathbf{H}$. We label the network-based aggregate shocks given by $(\mathbf{I} - \mathbf{H})^{-1}\mathbf{H}\boldsymbol{\theta}$ as upstream shocks.

Matrix $\boldsymbol{\Lambda}$ captures a complex propagation process that *jointly* includes upstream and downstream components. In view of this feature, we label this type of propagation as *bidirectional.* Informally, the composite process induced can be explained as follows. First of all, shocks $\boldsymbol{\theta}$

[32] A firm's sales as a share of GDP is also known as its Domar weight.

impinge directly on firms' production costs and, indirectly, on the costs of firms downstream in the network. Such an initial phase of shock propagation is captured by the term $(\mathbf{I}-\alpha\mathbf{G}')^{-1}\boldsymbol{\theta}$. Naturally, the resulting changes in production costs will typically translate into corresponding changes in the prices of the goods being produced downstream, eventually affecting the demand for them. These substitution effects affecting demands are captured by the entries of the vector $(diag(\mathbf{H1})-\mathbf{HG}')(\mathbf{I}-\alpha\mathbf{G}')^{-1}\boldsymbol{\theta}$. The adjustments entailed then act as demand shocks that propagate upstream to all direct and indirect suppliers of the affected goods, as captured by the operator $(\mathbf{I}-\mathbf{H})^{-1}$.

An interesting point arising from equation (10) is that, for every firm i, its equilibrium sales s_i (measured in relative terms, as a fraction of the total expenditure E – or income – of the consumer) do not *directly* depend on its own shock θ_i. The reason is that such a dependence (which will generally occur because the shock affects the cost of the firm) materializes only indirectly. That is, it has an effect on i's sales through market-mediated channels that account for how all other firms react (directly and indirectly) to any change in the cost of its production, and how prices correspondingly adjust to clear *all* markets. This is visible from (10) as the logarithm of sales of firm i is affected by θ_i through bidirectional propagation.[33]

5.6 Aggregate implications of the model

So far, we have focused our theoretical analysis on the microeconomic implications of shock propagation at the node/firm or link levels. It is important, however, to understand and quantify what is the aggregate relevance of the phenomenon when such microeconomic effects are suitably aggregated into economy-wide magnitudes. To do this analytically within our theoretical framework is the objective of the present section. The empirical counterpart of this analysis is carried out in Section 6.3, where we conduct a quantitative assessment of the aggregate effects that shock propagation had in Spain during the financial crisis.

Our discussion here will be particularly interested in the effect of financial shocks on the real GDP of the economy, and on how its different markets (for goods, labor, and capital) shape the response to them. In our model, the real GDP equals the total consumption $c{=}\sum_i c_i$. Therefore, our aim boils down to tracing how shocks affect the growth rate of this magnitude. In Proposition 3, included in the online Appendix B, we show that such an aggregate effect can be linearly approximated through the following equations:

$$\mathrm{dlog}c{=}-\boldsymbol{\gamma}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{\theta}-\frac{\beta}{1-\alpha}\mathrm{dlog}w-\frac{\rho}{1-\alpha}\mathrm{dlog}r, \tag{11}$$

where

$$\mathrm{dlog}w{=}\frac{\eta}{1+\eta}\frac{\beta}{wL}\boldsymbol{s}'\mathbf{M}\mathrm{dlog}\boldsymbol{s}-\frac{1-\delta}{1+\eta}\mathrm{dlog}c, \tag{12}$$

$$\mathrm{dlog}r{=}\frac{\rho}{rK}\boldsymbol{s}'\mathbf{M}\mathrm{dlog}\boldsymbol{s} \tag{13}$$

[33] Shock θ_i affects $\log s_i$ also trough upstream propagation whenever i is an indirect customer of itself, which may happen due to cycles in the production network.

and

$$\mathrm{dlog}\,\boldsymbol{s} = -[\mathbf{I}-\mathbf{H}]^{-1}\mathbf{H}\boldsymbol{\theta} + (1-\sigma)\boldsymbol{\Lambda}\boldsymbol{\theta}. \tag{14}$$

Equation (11) describes the (additively separable) channels through which financial shocks affect GDP. The first term on the right-hand side of (11) captures the effect operating through the production structure of the economy, while the last two terms capture the effects mediated through the wage w and capital return r determined *endogenously* through the operation of the labor and capital markets.

Equation (12) describes the operation of the labor market, whose equilibrium determines the wage as a function of the configuration determined in the markets for intermediate and final goods. This interaction across markets depends on the network structure of the economy (as given by $\mathbf{G}$, which underlies $\boldsymbol{s}$), the competition structure of the economy (reflected by $\mathbf{M}$), the pre-distortion labor income (wL), the labor elasticity of production (β) and – since labor is supplied elastically – consumer responses to changes in wage and income levels, as governed by η (inverse Frisch elasticity of labor) and δ (which modulates the income elasticity).

The capital-market channel is captured by (13). As for the labor market, the equilibrium in the capital market depends on the network structure, the competition structure, the pre-distortion capital income and, in this case, the capital elasticity of production. Note that, since capital is supplied inelastically, there is no feedback from the consumption level c, as was the case with the labor-market channel.

Solving the system of equations given by (11), (12) and (13),[34] we get (see Corollary 2 in the online Appendix B) the following closed-form expression for the effect of financial shocks on the real GDP:

$$\begin{aligned}\mathrm{dlog}\,c = &-\left(1-\frac{1-\delta}{1+\eta}\frac{\beta}{1-\alpha}\right)^{-1}\boldsymbol{\gamma}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{\theta}\\ &-\frac{1}{1-\alpha}\left(1-\frac{1-\delta}{1+\eta}\frac{\beta}{1-\alpha}\right)^{-1}\left(\frac{\eta\beta^2}{(1+\eta)wL}+\frac{\rho^2}{rK}\right)\boldsymbol{s}'\mathbf{M}\Big(-[\mathbf{I}-\mathbf{H}]^{-1}\mathbf{H}\boldsymbol{\theta}+(1-\sigma)\boldsymbol{\Lambda}\boldsymbol{\theta}\Big).\end{aligned} \tag{15}$$

The previous expression captures the wide range of effects that are involved in shaping the aggregate impact of direct credit shocks and their propagation on the GDP of the economy. As explained, these effects embody mechanisms of very different sorts: some are network-based while others are market-based or preference-based. These mechanisms are governed by key *preference* and *technological elasticities*, which interact with observable market magnitudes and the production structure of the economy in the complex non-linear manner displayed in 15.

One of the sources of this complexity derives from the fact that a complete analysis of the phenomenon of shock propagation requires considering *all possible paths* that connect every financial shock to every firm in the economy, both upstream and downstream. For, as we briefly discussed in the Introduction, it is precisely such full-fledged network-based propagation that has been highlighted as underlying the severity of the Great Recession.

[34] This system of equations is an ex-ante structural result in the language of Baqaee and Farhi (2019).

In Section 6.3, we shall rely on our model and data to provide a quantitative assessment of how important it actually was in the Spanish case. Specifically, we shall be comparing the overall aggregate effects on the whole economy that follow from our model – as given by (15) – with the counterfactual prediction induced from it under the assumption that the shock propagation unfolding through the network could have been blocked. As a simple way of rendering such counterfactual exercise operational, we shall focus on what is the prediction of our model if we posit a value of $\alpha{=}0$ for the share of intermediate goods in the production technology, thus effectively breaking all interfirm linkages in the production network of the economy.

We are also interested in the question of how important is the fully unrestricted propagation assumed by our model, when compared to the short-range first-order propagation that would apply if it involved only firms that are directly connected in the production network, i.e. if we had only one-step downstream propagation from a direct supplier or one-step upstream propagation from a direct customer. This can be formalized through the following restricted formulation of (15) where only such first-order propagation is allowed:

$$\begin{aligned} \mathrm{dlog}c = & -\left(1-\frac{1-\delta}{1+\eta}\frac{\beta}{1-\alpha}\right)^{-1}\boldsymbol{\gamma}'\left(\mathbf{I}+\alpha\mathbf{G}'\right)\boldsymbol{\theta} \\ & -\frac{1}{1-\alpha}\left(1-\frac{1-\delta}{1+\eta}\frac{\beta}{1-\alpha}\right)^{-1}\left(\frac{\eta\beta^2}{(1+\eta)wL}+\frac{\rho^2}{rK}\right)\boldsymbol{s}'\mathbf{M}(-\mathbf{H}\boldsymbol{\theta}+(1-\sigma)\boldsymbol{\Lambda}_{approx}\boldsymbol{\theta}), \end{aligned} \tag{16}$$

where $\boldsymbol{\Lambda}_{approx} \equiv (\mathbf{I}+\mathbf{H})(diag(\mathbf{H1})-\mathbf{HG}')(\mathbf{I}+\alpha\mathbf{G}')$. A comparison of the aggregate implications of higher-order propagation will be obtained in Section 6.3 for the Spanish case by comparing the effect on GDP induced by (15) by that resulting from (16).

6 Structural evidence and general equilibrium propagation

Our rich dataset allows us to bring the model to the empirical evidence, testing its predictions and estimating empirical counterparts of the structural equations (8) and (10) that embody, respectively, our link- and node-level analyses.

6.1 Link-level structural evidence

In order to take equation (8) to the data we proceed as follows. First, given the credit supply shocks θ_i^{AW} identified through the Amiti-Weinstein procedure described in Section 3, we map them to the corresponding firm-specific financial shocks θ_i contemplated by the theory (see Subsection 5.2) through a scale parameter. That is, we posit that $\theta_i{=}\xi\theta_i^{AW}$, where ξ is to be estimated. Second, to operationalize the higher-order shocks specified in the theory, Net_i, we need empirical counterparts of $\mathbf{G}$ and α, in addition to the aforementioned θ_i^{AW}. We directly obtain the matrix $\mathbf{G}$ from the observed pre-crisis firm-to-firm transactions using Lemma 3 (see online Appendix B) and calibrate the parameter α (the share of intermediate inputs) to be 0.48 based on standard estimation techniques of production functions at the firm level (Wooldridge

(2009)).[35] Then, we can readily compute the empirical higher-order AW-shocks Net_i^{AW}, which are mapped again to the theoretical ones through the scale parameter ξ to arrive at the linear expression $Net_i{=}\xi Net_i^{AW}$.

Given such an operationalization of our theory, we consider the following regression as an empirical counterpart of equation (8):

$$\Delta\log\left(\frac{s_{ji}}{s_i}\right)=\lambda_F^d\theta_j^{AW}+\lambda_F^u\theta_i^{AW}+\lambda_H^dNet_j^{AW}+\lambda_H^uNet_i^{AW}+\boldsymbol{b}\boldsymbol{x}_{ji}+\varepsilon_{ji} \tag{8R}$$

with λ_F^d, λ_F^u, λ_H^d, and λ_H^u being the parameters to be estimated, and the superindices d and u indicate propagation effects that operate downstream or upstream, respectively, while the subindices F and H refer to effects that are first- or higher-order, respectively.

An important point to make is that the aforementioned parameters have a structural interpretation based on the one-to-one mapping between equations (8) and (8R). This mapping is induced by the following relationships: $\lambda_F^d=-(\sigma-1)\xi$, $\lambda_F^u=-\xi$, $\lambda_H^d=-(\sigma-1)\alpha\xi$, and $\lambda_H^u=(\sigma-1)\xi$, where recall that σ denotes the elasticity of substitution across intermediate inputs, a key parameter in our analysis. Thus, even though this elasticity is neither observed in the data, we can identify it from the estimation of (8R) – specifically, $\sigma-1$ can be recovered from the ratio of λ_F^d and λ_F^u, i.e. the estimated coefficients associated to θ_j^{AW} and θ_i^{AW}.[36] Also observe that we can identify ξ directly from (8R) as the estimate of $-\lambda_F^u$, although its precise value is not as interesting as that of σ since, in essence, it merely plays the role of a scale parameter. Finally, note that, in order to account for non-modeled factors as explained in earlier sections (such as location, industry, or size) that may have affected how different firms responded to the financial crisis, our regressions also include a set of control variables $\boldsymbol{x}_{ji}$ in the empirical equation (8R). These are the same variables considered in the reduced-form regressions discussed in Section 4, including in some regressions firm fixed effects so that we do not control then for the direct bank shock.

Table 4 presents the result of estimating different variants of equation (8R). Columns (1) and (2) pertain to the effects of upstream propagation from customers to suppliers. In column (1) we do not include the terms corresponding to higher-order propagation (Net), while in column (2) we do incorporate higher-order effects. The results indicate that one standard deviation of customer (bank) shock leads, on average, to a reduction in firm-customer sales of approximately 2 pp, in line with the reduced-form estimate in Table 1.[37] In column (2) we also find that the estimated effect of higher-order upstream shocks is significant and of a magnitude similar to that of the first-order effects but with opposite sign. More concretely, we find that an increase in one standard deviation of customer's indirect (bank) shocks implies, on average, an increase of 1.8 pp of the dependent variable, which represents 15% of the mean value of the dependent

[35]Note also that this value is very similar to that estimated by Levinsohn and Petrin (2003) in a sample of Chilean firms.

[36]Note that $\sigma-1$ can also be recovered as the ratio of the estimated coefficients for Net_i^{AW} (λ_H^u) and θ_i^{AW} (λ_F^u). However, our preferred strategy for estimating σ is based on the ratio between θ_j^{AW} and θ_i^{AW} because these values depend less on estimated objects and thus reduce the possible measurement error.

[37]Indeed, note that the estimated value of Column (1) in Table 4 is very similar to that of column (2) in Table 1. Even though the dependent variables of the regressions reported in Table 4 and Table 1 are different, we obtain similar estimates as this difference is mostly irrelevant once we control for customer fixed effects.

variable. This result corroborates one of the main predictions of equation (8), reflecting the role of substitution among intermediate inputs in the model when the corresponding elasticity of substitution is higher than 1. We will return to this point below, when discussing at more length the value of σ actually following from our estimation.

Columns (3) and (4) of Table 4 then turn to downstream propagation from suppliers' credit shocks to firm purchases. In line with the results for upstream propagation, we find that first-order downstream propagation, when not controlling for higher-order propagation, is very similar to that of Table 2 from our reduced-form specification. Also we see that higher-order downstream propagation is statistically significant and large in economic terms. Specifically, a one standard deviation increase in the supplier's indirect shocks leads, on average, to a reduction of 2.0 pp in supplier-firm purchases, while a one standard deviation increase in the first-order supplier shock leads, on average, to just 1.0 pp reduction.

Comparing columns (1) and (2) for upstream propagation and columns (3) and (4) for downstream propagation we find that the estimated effect of direct customer (supplier) credit shocks remains almost unchanged once we control for the higher-order propagation effects of shocks predicted by the theory. This suggests that the estimates resulting from the reduced form approach in Section 4 are not biased due to the omission higher-order propagation and general equilibrium effects. However, our results indicate that not accounting for the higher-order effects predicted by the theory may lead to substantial bias – underestimation (or overestimation) – of the total downstream (or, respectively, upstream) propagation effect of financial shocks. For, indeed, the magnitude of the estimated impact of higher-order propagation is similar to, or even larger than, the magnitude of the direct propagation.

As explained in Section 5.4, the fact that upstream and downstream effects of higher-order propagation are of opposite sign is a prominent prediction of the model (recall equation (8)). However, the specific sign pattern predicted depends crucially on σ. In particular, the pattern observed on in columns (2) and (4) – negative downstream and positive upstream – requires that $\sigma > 1$. Our structural estimation of the model favors the identification of the elasticity of substitution through the relationship $\sigma = 1 + \lambda_F^d / \lambda_F^u$. In this regard, column (5) of Table 4 reports jointly the main model's predictions at the link level showing non-standardized coefficients. In view of the estimates displayed in column (5), this expression gives rise to a value for the elasticity of substitution of 1.56 (estimated with standard error of 0.45), which is well in line with the theoretical requirement and suggests that intermediate inputs are substitutes. Such an estimate, which is based on annual data,[38] is in line with the estimates by Carvalho et al. (2020), whose approach is closest to ours and reports estimates of σ ranging from 1.1 to 1.3.

Finally, from column (5) we cannot reject the following null hypotheses at conventional confidence levels. First, the sign of the effect of higher-order upstream shocks is opposite to that of the first-order upstream shocks. Second, the sign of the effect of higher-order downstream shocks is the same as that of the first-order downstream shocks. Third, the magnitude of higher-

[38]Research based on different time horizons reports estimates that have inputs behaving as complements in the short run but substitutes in the long run. For instance, Boehm et al. (2016) estimates a range between 0.20 and 0.62 using quarterly data, while Peter et al. (2022) report estimates that use a seven-year period and lie in the range between 1.8 and 4.4.

order upstream and first-order downstream propagation effects is similar and displays an opposite sign.

6.2 Node-level structural evidence

As we have explained in our discussion of the theory (see Section 5), the effects arising at the node (firm) level are substantially more complex than those estimated at the link level. For, at the node level, one needs a suitable aggregation of the effects that, at all orders, flow into any given node through all paths that connect it to its many different (direct and indirect) suppliers and customers; indeed, to aggregate in a suitably coherent and systematic manner all such paths for every firm in the economy is one of the primary contributions of the theory.

From a conceptual viewpoint, we have found it useful to distinguish two different kinds of shock propagation in our model. On the one hand, financial shocks propagate in a purely upstream manner as demand shocks to the suppliers, direct and indirect. This is what, in the theory (see equation (10)), was captured by the matrix $\mathbf{U} \equiv -(\mathbf{I}-\mathbf{H})^{-1}\mathbf{H}$, its ith row being the vector $\mathbf{U}_i = -\boldsymbol{e}_i'(\mathbf{I}-\mathbf{H})^{-1}\mathbf{H}$ reflecting how firm i is affected, directly and indirectly, by the shock θ_j hitting every firm j. On the other hand, the model also identifies another bidirectional type of propagation involving the concatenation of downstream propagation (affecting the costs of all direct and indirect customers of firms that are hit by a financial shock) followed by a chain of upstream propagation. As just explained, these subsequent upstream propagation chains induce "demand shocks" to the direct and indirect suppliers of each of the firms affected along the initial chains of downstream propagation. In the theory, such a bidirectional propagation is captured by the matrix $\boldsymbol{\Lambda}$, which we now mnemonically rename as $\mathbf{B}$ for "bidirectional" (as the previous notation $\mathbf{U}$ was meant to point to "upstream"), its ith row again denoted by $\mathbf{B}_i$.

The entries of the matrices $\mathbf{U}$ and $\mathbf{B}$ can be constructed from our data. $\mathbf{U}$ is a composition of powers of the matrix $\mathbf{H}$ (whose entries h_{ij} reflect the share of the sales of firm i that are channeled to its customer j), while $\mathbf{B}$ involves powers of the matrices $\mathbf{G}$ (whose entries are input cost shares) and $\mathbf{H}$. Thus, if we posit again a linear mapping $\theta_i = \xi\theta_i^{AW}$ from the empirical AW-shock θ_i^{AW} hitting each firm i to its corresponding theoretical shock θ_i, the equation in (10) that, according to our theory, governs the effect of all shocks on each of firm i of the economy has the following empirical counterpart:

$$\Delta \log\left(\frac{s_i}{E}\right) = \lambda_U \mathbf{U}_i \boldsymbol{\theta}^{AW} + \lambda_B \mathbf{B}_i \boldsymbol{\theta}^{AW} + \boldsymbol{b}\boldsymbol{x}_i + \varepsilon_i. \tag{10R}$$

Here $\lambda_U = \xi$ and $\lambda_B = (1-\sigma)\xi$ are parameters to be estimated that capture the upstream and bidirectional propagation effects predicted by the theory, while $\boldsymbol{x}_i$ stands for the set of observable and unobservable covariates that account for the heterogeneity not contemplated by the theory, as explained in previous sections, that can have some impact on firm sales.

Table 5 reports the results of estimating equation (10R). In column (1) we only account the upstream shock $\mathbf{U}_i\boldsymbol{\theta}^{AW}$, while in column (2) we only account for the bidirectional shock $\mathbf{B}_i\boldsymbol{\theta}^{AW}$. In both cases, the estimated coefficients are negative and significant. In column (3), we account for both shocks simultaneously and find evidence of sizable propagation, both upstream and bidirectional, negatively affecting firms' sales, as predicted by the theory if $\sigma > 1$. In particular, we find that an increase in one standard deviation of the upstream shock to a firm leads, on average, to a decrease in sales of 1.6 pp, while one standard deviation of the bidirectional shock reduces its sales, on average, by 0.5 pp. In economic terms, given that the average growth of firms' sales is -20.0% (see Table A1), these numbers amount to a joint reduction of 11% in average firm growth due to the financial shocks and their propagation through the production network.

Another interesting prediction of the model derived in Section 5.5 is that financial shocks of firm i should not matter once when we control for how this firm is affected by the propagation of all network shocks in the economy. In order to test it, we proceed in two steps. First, we show in column (4) that the effect of a direct shock is negative and statistically significant. This indicates that, when the issue is studied ignoring general equilibrium considerations, the intuitive dependence on own shock does arise. Second, in column (5), we focus on a specification that adds to the one considered in column (3) – that accounts for all general-equilibrium market adjustments – the direct credit shock as an additional regressor. We find that the estimation of this second specification delivers a non-significant coefficient, whose absolute value decreases by 54%.

Let us also note that by estimating equation (10R) we can obtain an alternative estimate of the structural parameter σ. For this purpose, column (6) in Table 5 mirrors column (3) but showing non-standardized coefficients. From (10R), $1-\sigma$ is identified equal to the ratio of the bidirectional effect (λ_B) and the upstream effect (λ_U), scaled by α. Using this approach, we obtain $\sigma = 1.35$ (with a standard error of 0.47), which is in line with our previous estimate of $\sigma = 1.56$ (with a standard error of 0.45) in the previous section. As explained, our preferred estimate of σ is the one obtained from our link-level regression. The reason is twofold: first, in this case we can control for firm observed and unobserved characteristics, thus enhancing identification; second, the estimation of σ using the node-level regression given by (10R) relies more heavily on network measures, for which measurement error may be more substantial than for the approach that relies on the link-level regression given by 8R.

Finally, we mention two other reasons why, more generally, it is important to conduct the analysis of the problem not only at the node level but at the link level as well. One of them is that some substitution effects are only really clear at the link level – for example, when we need to exploit the variability of how a firm reacts to the shocks experienced by its different suppliers. Another reason is that, as we showed in our link-level analysis of Section 6.1 , it is only at this level that we can fully separate upstream vs. downstream (see regression (8R)) and therefore compare their relative strengths. Instead, in the node-level analysis, our theory prescribes a regression of the form given in (10R), in which it is not possible to separately identify the importance of downstream propagation, as it is inextricably confounded with the upstream propagation in a bidirectional mixture.

6.3 Aggregate effects

Once the model's mechanisms and predictions are validated in the data – both at the link- and the node-levels – we now turn to quantify the extent to which network propagation plays an important role in aggregate economy-wide outcomes. To this end, we rely on the theory to take into account all the general-equilibrium effects induced by the shocks and then aggregate those effects on the log of the real GDP as given by a first-order approximation of the equilibrium equation (15). To bring this equation to the data and quantify the induced effects, we rely on the following calibration strategy:

(a) The parameters of the production function (α, β, ρ) are estimated using standard production function estimation techniques at the firm-level, as explained above. Specifically, we estimate $\alpha = 0.483$, $\beta = 0.317$ and $\rho = 0.2$, in line with available estimates in the literature (see, for instance, Levinsohn and Petrin (2003)).

(b) The entries of the diagonal matrix $\mathbf{M}$, which captures markups, are estimated using the model-implied relationship $\mu_i = \alpha\frac{\tilde{\omega}}{\omega}$, where $\tilde{\omega} \equiv \frac{s_{ji}}{\sum_q s_{qi}}$ and $\omega \equiv \frac{s_{ji}}{s_i}$. Crucially, note that we observe the values s_{ji} and s_i in our firm-to-firm data for the baseline (pre-crisis) year. Online Appendix B provides more details on the markup estimation procedure.

(c) The aggregate compensation to employees (wL) and capital (rK) are calculated from the market clearing conditions $wL = \beta \boldsymbol{s}'\mathbf{M1}$ and $rK = \rho \boldsymbol{s}'\mathbf{M1}$. Also, note that a vector of baseline-year sales $\boldsymbol{s}$ is observed in the data.

(d) Matrix $\mathbf{G}$ has as its entries the input cost shares of every firm, which (as shown in Lemma 3) can be calibrated as $g_{ij} = \tilde{\omega}_{ij}$ using baseline year observations. By definition, the entries of the matrix $\mathbf{H}$ are equal to $\frac{s_{ij}}{s_i}$, which are directly observed in the data.

(e) The vector of financial shocks θ is mapped to the data by combining our estimated shocks θ_i^{AW} and our estimate of ξ obtained from the link-level analysis – see column (5) in Table 4, which implies that $\xi = -\lambda_F^u = 42.411$.

(f) Our calibrated value of σ is 1.56, which is also based on the link-level regression estimates – see again column (5) of Table 4, with $\sigma = 1 + \lambda_F^d/\lambda_F^u$, as explained above.

(g) Finally, notice that the only parameters in (15) that we do not directly recover from our model and data are those in the utility function of the consumer, η and δ. These parameters are important because they govern the reaction of labor supply to changes in wages and also modulate the sensitivity of consumption decisions to the shocks. Specifically, η is the inverse Frisch elasticity of labor supply, while δ is the risk aversion parameter determining the effect of income on labor supply (income elasticity of labor supply is equal to $\frac{-\delta}{\eta}$). In view of the difficulty of pinpointing with confidence specific values for these parameters, we rely on the literature – in particular, we follow Gottlieb et al. (2021) – and choose the following range:[39] $(\eta, \delta) \in [0.25, 0.5] \times [0.5, 1.5]$.

[39] This strategy contrasts with that of other papers that select a single combination of these two parameters within this range (e.g. Alfaro et al. (2021); Bigio and La'O (2020)).

We first calculate the overall impact of bank credit shocks to firms on the level of real GDP, which we estimate to be between -2.36% and -3.96%, depending on the values for η and δ.[40]. To benchmark these results, we note that Spain's economy grew at the average rate of 3.5% in the period 1995–2007 García-Santana et al. (2020), while the decrease in the Spanish GDP between 2008 and 2009 was 3.76 %.

Next, we address the following question: How large was the role of network-based shock propagation on the overall impact of the financial crisis on the Spanish GDP? In practice, the way in which we address this question is by setting the share of intermediate inputs, α, to zero. This blocks all shock propagation channeled through the production network and therefore helps to quantify what part of the aggregate effect of shocks can be attributed to their propagation.

We find the estimated effect of bank-credit shocks to firms on GDP in the counterfactual scenario without input-output linkages to be in the range $[-1.74\%, -2.25\%]$. We thus conclude that, on average, network propagation led to a 50% increase in the aggregate effect on the Spanish GDP, as compared to the hypothetical case where the direct bank-credit shocks experienced by firms had not triggered any propagation through the production network.[41]

Finally, we examine whether aggregate network effects are primarily driven by first-order propagation or higher-order connections. In particular, we consider our calibration strategy above and we take to the data equation (16). Our results indicate that, on average, the first-order network effects amplify the impact of shocks on GDP by 26% (this number ranges from 13% to 47% depending on the values of parameters δ and η in the range we consider), thus accounting for 52% of the total network amplification. In other words, according to our findings, both first- and higher-order propagation are equally important in the amplification of the real effects of financial shocks through input-output linkages.

7 Concluding remarks

Despite the fact that both academics and policy-makers have often argued that networks are important to understand the real effects of financial shocks, evidence on it has been scant mainly due to unavailability of *matched* networks that suitably represent the customer/supplier trade flows and bank-firm loans. In this paper, we contribute to addressing the problem by studying two matched administrative datasets from a bank-dominated economy, Spain, on supplier-customer transactions stemming from the Treasury's Value Added Tax (VAT) Register, and on bank-firm loans gathered from the Credit Register of the Spanish Central Bank. Moreover, we use a standard operationalization of bank credit-supply shocks during the Global Financial Crisis, and importantly, a general equilibrium model of an interfirm production network economy with financial frictions that we structurally estimate. We show that bank credit shocks to firms propagate upstream and downstream along the production network, with stronger effects for

[40]The lower bound (-2.36%) is reached for $(\eta, \delta) = (0.5, 1.5)$, while the upper bound (-3.96%) is achieved at $(\eta, \delta) = (0.25, 0.5)$

[41]This number ranges from 33% to 76% depending on the values of parameters δ and η. We have calculated the aforementioned average by considering a 20×20 grid of uniformly distributed values of δ and η in the range considered.

upstream than downstream propagation. Furthermore, our results indicate that the network propagation leads to a 50% increase in the aggregate effects of bank credit supply shocks on GDP growth, with equally important first-order versus higher-order network effects.

References

Acemoglu, D., Carvalho, V. M., Ozdaglar, A., and Tahbaz-Salehi, A. (2012). The network origins of aggregate fluctuations. *Econometrica*, 80(5):1977–2016.

Acemoglu, D., Ozdaglar, A., and Tahbaz-Salehi, A. (2015). Systemic risk and stability in financial networks. *American Economic Review*, 105(2):564–608.

Alfaro, L., García-Santana, M., and Moral-Benito, E. (2021). On the direct and indirect real effects of credit supply shocks. *Journal of Financial Economics*, 139(3):895–921.

Allen, F. and Gale, D. (2000). Financial contagion. *Journal of Political Economy*, 108(1):1–33.

Amiti, M. and Weinstein, D. E. (2018). How much do idiosyncratic bank shocks affect investment? Evidence from matched bank-firm loan data. *Journal of Political Economy*, 126(2):525–587.

Baqaee, D. R. (2018). Cascading failures in production networks. *Econometrica*, 86(5):1819–1838.

Baqaee, D. R. and Farhi, E. (2019). Productivity and misallocation in general equilibrium. *The Quarterly Journal of Economics*, 135(1):105–163.

Barrot, J.-N. and Sauvagnat, J. (2016). Input specificity and the propagation of idiosyncratic shocks in production networks. *The Quarterly Journal of Economics*, 131(3):1543–1592.

Bernanke, B. (2018). The real effects of the financial crisis. *Brookings Papers on Economic Activity*, 20.

Bernanke, B. S. (2013). A century of US central banking: Goals, frameworks, accountability. *Journal of Economic Perspectives*, 27(4):3–16.

Bernard, A. B., Dhyne, E., Magerman, G., Manova, K., and Moxnes, A. (2022). The origins of firm heterogeneity: A production network approach. *Journal of Political Economy*, Forthcoming.

Bigio, S. and La'O, J. (2020). Distortions in production networks. *The Quarterly Journal of Economics*, 135(4):2187–2253.

Boehm, C. E., Flaaen, A., Pandalai-Nayar, N., et al. (2016). The role of global supply chains in the transmission of shocks: Firm-level evidence from the 2011 tōhoku earthquake. *Working paper.*

Bonacich, P. (1987). Power and centrality: A family of measures. *American Journal of Sociology*, 92(5):1170–1182.

Cabrales, A., Gottardi, P., and Vega-Redondo, F. (2017). Risk sharing and contagion in networks. *The Review of Financial Studies*, 30(9):3086–3127.

Carvalho, V. M. (2014). From micro to macro via production networks. *Journal of Economic Perspectives*, 28(4):23–48.

Carvalho, V. M., Nirei, M., Saito, Y. U., and Tahbaz-Salehi, A. (2020). Supply Chain Disruptions: Evidence from the Great East Japan Earthquake. *The Quarterly Journal of Economics.*

Chodorow-Reich, G. (2014). The employment effects of credit market disruptions: Firm-level evidence from the 2008–9 financial crisis. *The Quarterly Journal of Economics*, 129(1):1–59.

Cingano, F., Manaresi, F., and Sette, E. (2016). Does credit crunch investment down? New evidence on the real effects of the bank-lending channel. *The Review of Financial Studies*, 29(10):2737–2773.

Cortes, G., Silva, T., and Doornik, B. (2019). Credit shock propagation in firm networks: evidence from government bank credit expansions. *Working paper, Central Bank of Brazil, Research Department.*

Costello, A. M. (2020). Credit market disruptions and liquidity spillover effects in the supply chain. *Journal of Political Economy*, 128(9):3434–3468.

Davis, S. J. and Haltiwanger, J. (1992). Gross job creation, gross job destruction, and employment reallocation. *The Quarterly Journal of Economics*, 107(3):819–863.

Demir, B., Javorcik, B., Michalski, T., and Ors, E. (2018). Financial constraints and propagation of shocks in production networks. *Working paper.*

Dewachter, H., Tielens, J., and Hove, J. (2020). The production network sources of bank influence. Technical report, Working Paper.

Diamond, D. W. (1984). Financial intermediation and delegated monitoring. *The Review of Economic Studies*, 51(3):393–414.

Elliott, M., Golub, B., and Jackson, M. O. (2014). Financial networks and contagion. *American Economic Review*, 104(10):3115–53.

Freixas, X., Laeven, L., and Peydró, J.-L. (2015). *Systemic Risk, Crises, and Macroprudential Regulation.* MIT Press.

Freixas, X., Parigi, B. M., and Rochet, J.-C. (2000). Systemic risk, interbank relations, and liquidity provision by the central bank. *Journal of Money, Credit and Banking*, 32(3):611–638.

Galaasen, S., Jamilov, R., Juelsrud, R., and Rey, H. (2020). Granular credit risk. *Working paper.*

García-Santana, M., Moral-Benito, E., Pijoan-Mas, J., and Ramos, R. (2020). Growing like spain: 1995–2007. *International Economic Review*, 61(1):383–416.

Gertler, M. and Kiyotaki, N. (2010). Financial intermediation and credit policy in business cycle analysis. In *Handbook of Monetary Economics*, volume 3, pages 547–599. Elsevier.

Gottlieb, C., Onken, J., and Valladares-Esteban, A. (2021). On the measurement of the elasticity of labour. *European Economic Review*, 139:103879.

Greenstone, M., Mas, A., and Nguyen, H.-L. (2014). Do credit market shocks affect the real economy? Quasi-experimental evidence from the great recession and 'normal' economic times. *Working paper.*

Holmstrom, B. and Tirole, J. (1997). Financial intermediation, loanable funds, and the real sector. *the Quarterly Journal of Economics*, 112(3):663–691.

Imbens, G. W. and Rubin, D. B. (2015). *Causal inference in statistics, social, and biomedical sciences.* Cambridge University Press.

Imbens, G. W. and Wooldridge, J. M. (2009). Recent developments in the econometrics of program evaluation. *Journal of Economic Literature*, 47(1):5–86.

Ippolito, F., Peydró, J.-L., Polo, A., and Sette, E. (2016). Double bank runs and liquidity risk management. *Journal of Financial Economics*, 122(1):135–154.

Ivashina, V., Laeven, L., and Moral-Benito, E. (2022). Loan types and the bank lending channel. *Journal of Monetary Economics*, 126:171–187.

Iyer, R. and Peydro, J.-L. (2011). Interbank contagion at work: Evidence from a natural experiment. *The Review of Financial Studies*, 24(4):1337–1377.

Iyer, R., Peydró, J.-L., da Rocha-Lopes, S., and Schoar, A. (2014). Interbank liquidity crunch and the firm credit crunch: Evidence from the 2007–2009 crisis. *The Review of Financial Studies*, 27(1):347–372.

Jacobson, T. and von Schedvin, E. (2015). Trade credit and the propagation of corporate failure: an empirical analysis. *Econometrica*, 83(4):1315–1371.

Jiménez, G., Mian, A., Peydró, J.-L., and Saurina, J. (2020). The real effects of the bank lending channel. *Journal of Monetary Economics*.

Jiménez, G., Ongena, S., Peydró, J.-L., and Saurina, J. (2012). Credit supply and monetary policy: Identifying the bank balance-sheet channel with loan applications. *American Economic Review*, 102(5):2301–26.

Jiménez, G., Ongena, S., Peydró, J.-L., and Saurina, J. (2014). Hazardous times for monetary policy: What do twenty-three million bank loans say about the effects of monetary policy on credit risk-taking? *Econometrica*, 82(2):463–505.

Jiménez, G., Ongena, S., Peydró, J.-L., and Saurina, J. (2017). Macroprudential policy, countercyclical bank capital buffers, and credit supply: evidence from the spanish dynamic provisioning experiments. *Journal of Political Economy*, 125(6):2126–2177.

Khwaja, A. I. and Mian, A. (2008). Tracing the impact of bank liquidity shocks: Evidence from an emerging market. *American Economic Review*, 98(4):1413–42.

La Porta, R., Lopez-de Silanes, F., and Shleifer, A. (2002). Government ownership of banks. *The Journal of Finance*, 57(1):265–301.

Levinsohn, J. and Petrin, A. (2003). Estimating production functions using inputs to control for unobservables. *The Review of Economic Studies*, 70(2):317–341.

Liu, E. (2019). Industrial policies in production networks. *The Quarterly Journal of Economics*, 134(4):1883–1948.

MaCurdy, T. E. (1981). An empirical model of labor supply in a life-cycle setting. *Journal of political Economy*, 89(6):1059–1085.

Montiel Olea, J. L. and Pflueger, C. (2013). A robust test for weak instruments. *Journal of Business & Economic Statistics*, 31(3):358–369.

Niepmann, F. and Schmidt-Eisenlohr, T. (2013). Bank bailouts, international linkages, and cooperation. *American Economic Journal: Economic Policy*, 5(4):270–305.

Peter, A., Ruane, C., et al. (2022). The aggregate importance of intermediate input substitutability. *Princeton University, memo*.

Stein, J. C. (1998). An adverse-selection model of bank asset and liability management with implications for the transmission of monetary policy. *RAND Journal of Economics*, 29(3):466–486.

Wooldridge, J. M. (2009). On estimating firm-level production functions using proxy variables to control for unobservables. *Economics letters*, 104(3):112–114.

Tables

TABLE 1

LINK-LEVEL: PROPAGATION OF BANK CREDIT SUPPLY SHOCKS THROUGH THE NETWORK OF CUSTOMERS. REDUCED FORM

Upstream propagation (indirect shocks via bank credit supply shocks to first-order customers)

Dependent Variable: Δlog(sales from supplier to customer)			IV. Instrument: Bank Net Interbank Borrowing		IV. Instrument: Bank Shock	
			1st Stage	2º Stage	1st Stage	2º Stage
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank Credit Supply) Shock	-0.703*					
	(0.412)					
1st Order Customer (Bank) Effect		-2.358**		-6.568**	3.445***	
		(1.181)		(2.844)	(0.571)	
Customer (Bank) Net Interbank Borrowing			8.912***			
			(0.933)			
Customer Reduction of Bank Debt						-0.739***
						(0.265)
Customer:						
Controls	-	Yes	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	Yes	Yes	Yes
Fixed Effects	Yes	No	No	No	No	No
Firm:						
Controls	Yes	-	-	-	-	-
Spatial*Industry Fixed Effects	Yes	-	-	-	-	-
Fixed Effects	No	Yes	Yes	Yes	Yes	Yes
Firm*Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
First Stage Effective F statistic	-	-	65.28	-	36.09	-
R-squared	0.404	0.474	-	-	-	-
Observations	1,119,169	1,119,169	1,119,169	1,119,169	1,119,169	1,119,169

Notes: This table reports estimates from WLS results. See Section 4. Observations are at the level of firm-customer, i.e. link-level. The dependent variable is the change in the log of sales form supplier to customer between 2008 and 2009 for all columns but (3) and (5). In column (4) the firm bank shock is instrumented with the firm financial shock derived from the (weighted) average net interbank borrowing of the firm across all its banks before the crisis (column (3)). In column (6) the reduction in bank debt between 2008 and 2009 is instrumented with the firm financial shock (column (5)). Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). All shocks are standardized. For the list of controls, see Section 4. First stage effective F statistic is based on Montiel Olea and Pfleuger (2013) and it is robust to heteroskedasticity, serial correlation, and clustering. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE 2

LINK-LEVEL: PROPAGATION OF BANK CREDIT SUPPLY SHOCKS THROUGH THE NETWORK OF SUPPLIERS. REDUCED FORM

Downstream propagation (indirect shocks via bank credit supply shocks to first-order suppliers)

Dependent Variable: Δlog(sales from supplier to customer)			IV. Instrument: Bank Net Interbank Borrowing		IV. Instrument: Bank Shock	
			1st Stage	2º Stage	1st Stage	2º Stage
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank Credit Supply) Shock	-2.678**					
	(1.050)					
1st Order Supplier (Bank) Effect		-1.086**		-4.500***	2.094***	
		(0.546)		(1.537)	(0.499)	
Supplier (Bank) Net Interbank Borrowing			9.227***			
			(0.623)			
Supplier Reduction of Bank Debt						-0.519*
						(0.310)
Supplier:						
Controls	-	Yes	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	Yes	Yes	Yes
Fixed Effects	Yes	No	No	No	No	No
Firm:						
Controls	Yes	-	-	-	-	-
Spatial*Industry Fixed Effects	Yes	-	-	-	-	-
Fixed Effects	No	Yes	Yes	Yes	Yes	Yes
Firm*Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
First Stage Effective F statistic	-	-	97.53	-	12.40	-
R-squared	0.358	0.483	-	-	-	-
Observations	1,114,421	1,114,421	1,114,421	1,114,421	1,114,421	1,114,421

Notes: This table reports estimates from WLS results. See Section 4. Observations are at the level of the firm-supplier, i.e. link-level. The dependent variable is the change in the log of sales from supplier to customer between 2008 and 2009 for all columns but (3) to (5). In column (4) the firm bank shock is instrumented with the firm financial shock derived from the (weighted) average net interbank borrowing of the firm across all its banks before the crisis (column (3)). In column (6) the reduction in bank debt between 2008 and 2009 is instrumented with the firm financial shock (column (5)). Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). All shocks are standardized. For the list of controls, see Section 4. First stage effective F statistic is based on Montiel Olea and Pfleuger (2013) and it is robust to heteroskedasticity, serial correlation, and clustering. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE 3

NODE-LEVEL: FIRM-LEVEL EFFECTS OF BANK SUPPLY SHOCKS THROUGH THE PRODUCTION NETWORK ON FIRM SALES AND EMPLOYMENT. REDUCED FORM

Dependent Variable:	Δlog(sales)		Δlog(employment)	
	(1)	(2)	(3)	(4)
Direct (Bank Credit Supply) Shock	-0.888*	-0.821*	-0.552**	-0.536**
	(0.520)	(0.490)	(0.241)	(0.239)
1st Order Customer (Bank) Effect		-2.223***		-0.488***
		(0.373)		(0.091)
1st Order Supplier (Bank) Effect		0.374		-0.031
		(0.346)		(0.069)
Firm Controls	Yes	Yes	Yes	Yes
Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes
R-squared	0.351	0.353	0.091	0.091
Observations	196,171	196,171	196,171	196,171

Notes: This table reports estimates from WLS. See Section 4. Observations are at the level of the firm (node-level). The dependent variables are the change, between 2008 and 2009, in the log of aggregate sales to all customers (columns (1) and (2)) and the log of total employment (columns (3) and (4)). Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction), and to construct the first order customer (supplier) bank effect we aggregate it using the lagged sales (purchases) between the firm and all its direct customers (suppliers) as weights. As we cannot control for firm fixed effects, we control for spatial, industry and main bank fixed effects. All shocks are standardized. For the list of firm controls, see Section 4. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the level of the main bank). In each column, the word Yes indicates that the set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE 4

LINK-LEVEL: PROPAGATION OF A BANK SUPPLY SHOCK THROUGH THE NETWORK OF CUSTOMERS/SUPPLIERS. STRUCTURAL FORM

Dependent Variable: Δlog(sales from supplier to customer/sales of customer)	Upstream propagation		Downstream propagation		Joint estimation Non-standardized
	(1)	(2)	(3)	(4)	(5)
1st Order Customer (Bank) Effect	-1.923**	-2.002**			-42.411**
	(0.900)	(0.924)			(20.237)
Higher Order Customer (Bank) Effect		1.796***			51.176***
		(0.632)			(18.511)
1st Order Supplier (Bank) Effect			-1.086**	-1.055**	-23.950*
			(0.546)	(0.526)	(12.621)
Higher Order Supplier (Bank) Effect				-2.045*	-47.837*
				(1.065)	(28.554)
Supplier/Customer:					
Controls	Yes	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes
Firm:					
Fixed Effects	Yes	Yes	Yes	Yes	Yes
Firm*Supplier/Customer Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes
R-squared	0.477	0.478	0.493	0.493	0.485
Observations	1,119,169	1,119,169	1,114,421	1,114,421	2,233,590

This table reports estimates from WLS results. See Section 6. Observations are at the level of the firm-customer (columns (1) and (2)) or firm-supplier (columns (3) and (4)), i.e. link-level. The dependent variable is the change in the sales from supplier to customer, minus the change in the log of total sales of the customer between 2008 and 2009. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). For the definition of higher order bank shock effects, see Section 5 and 6 of the paper. All variables are standardized but those of column (5). For the list of controls, see Section 6. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE 5

NODE-LEVEL: FIRM-LEVEL EFFECTS OF BANK SUPPLY SHOCKS THROUGH THE PRODUCTION NETWORK. STRUCTURAL FORM

						Non-standardized
Dependent Variable: Δlog(sales)	(1)	(2)	(3)	(4)	(5)	(6)
Upstream (1st & Higher Order Effects)	-1.810***		-1.611***		-1.641***	-32.891***
	(0.326)		(0.333)		(0.350)	(6.802)
Bidirectional (Up & Down 1st & Higher Order Effects)		-1.435***	-0.527*		-0.439	-11.441*
		(0.333)	(0.300)		(0.367)	(6.504)
Direct (Bank Credit Supply) Shock				-0.888*	-0.411	
				(0.520)	(0.537)	
Firm Controls	Yes	Yes	Yes	Yes	Yes	Yes
Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.358	0.357	0.358	0.351	0.358	0.358
Observations	196,171	196,171	196,171	196,171	196,171	196,171

Notes: This table reports estimates from WLS. See Section 6. Observations are at the level of the firm (node-level). The dependent variables is the change, between 2008 and 2009, in the log of firm-level aggregate sales to all customers. *E* from the theory Section 5 is spanned by fixed effects. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction), and for aggregation (upstream and bidirectional) see Section 5 and 6 of the paper. As we cannot control for firm fixed effects, we control for spatial and industry fixed effects. All variables are standardized but those of column 5. For the list of firm controls, see Section 6. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the level of the main bank). In each column, the word Yes indicates that the set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

Appendix A: Additional results — For online publication

A.1. Additional Tables

TABLE A1

SUMMARY STATISTICS

			Mean	S.D.	P25	Median	P75
	Link Level						
Upstream propagation							
Δlog(sales from supplier to customer)	The log of change of firm's sales to its customer between 2008 and 2009	%	-12.062	61.407	-53.513	-13.894	17.599
Direct (Bank Credit Supply) Shock	(Bank supply) Shock of a firm is computed as (following Amiti & Weinstein (2018)) the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction), where the firm-level shock is aggregated using the lagged December 2007 credit between the firm and each bank as weights		0.151	0.050	0.125	0.152	0.176
1st Order Customer (Bank) Effect	Direct (bank credit supply) shock of the customer of a firm		0.151	0.047	0.125	0.151	0.176
Discrete Direct (Bank Credit Supply) Shock	A binary variable that takes the value of one when the Direct Shock is above its median and zero otherwise	0/1	0.544	0.498	0.000	1.000	1.000
Discrete 1st Order Customer (Bank) Effect	A binary variable that takes the value of one when the 1st Order Customer Shock is above its median and zero otherwise	0/1	0.497	0.500	0.000	0.000	1.000
Higher Order Customer (Bank) Effect	A network aggregate of shocks hitting suppliers of the customer of any order (see equations (8) and (8R) and Section 5 and 6)		0.120	0.035	0.096	0.117	0.140
Customer Reduction of Bank Debt	The (negative) change in overall bank credit of the customer between 2008 and 2009	%	8.563	61.479	-7.496	7.449	27.897
Customer (Bank) Net Interbank Borrowing	The net interbank position (interbank deposits minus interbank loans) of the customer's weighted average banks, where weights are based on the lagged credit	0.0x	0.025	0.019	0.011	0.023	0.036
Δlog(sales from supplier to customer/sales of customer)	The log of change of firm's sales to its customer between 2008 and 2009 minus the log of change of the customer's sales	%	5.214	63.099	-33.604	0.973	35.927
Downstream propagation							
Δlog(sales from supplier to customer)	The log of change of a supplier's sales to the firm between 2008 and 2009	%	-11.932	60.414	-52.008	-12.730	16.381
Direct (Bank Credit Supply) Shock	(Bank supply) Shock of a firm is computed as (following Amiti & Weinstein (2018)) the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction), where the firm-level shock is aggregated using the lagged December 2007 credit between the firm and each bank as weights		0.150	0.050	0.121	0.152	0.180
1st Order Supplier (Bank) Effect	Direct (bank credit supply) shock of the supplier of a firm		0.151	0.044	0.129	0.151	0.173
Discrete Direct (Bank Credit Supply) Shock	A binary variable that takes the vaule of one when the Direct Shock is above its median and zero otherwise	0/1	0.541	0.498	0.000	1.000	1.000
Discrete 1st Order Supplier (Bank) Effect	A binary variable that takes the vaule of one when the 1st Order Supplier Shock is above its median and zero otherwise	0/1	0.505	0.500	0.000	1.000	1.000
Higher Order Supplier (Bank) Effect	A network aggregate of shocks hitting suppliers of the supplier of any order (see equations (8) and (8R) and Section 5 and 6)		0.108	0.041	0.081	0.106	0.133
Supplier Reduction of Bank Debt	The (negative) change in overall bank credit of the supplier between 2008 and 2009	%	10.071	58.433	-6.275	8.696	28.814
Supplier (Bank) Net Interbank Borrowing	The net interbank position (interbank deposits minus interbank loans) of the supplier's weighted average banks, where weights are based on the lagged credit	0.0x	0.027	0.019	0.015	0.024	0.036
Δlog(sales from supplier to customer/sales of customer)	The log of change of a supplier's sales to the firm between 2008 and 2009 minus the log of change of the firm (customer)'s sales	%	6.280	62.476	-31.823	2.272	36.550
	Node Level						
Δlog(sales)	The log of change of firm' sales to all its customers between 2008 and 2009	%	-19.970	40.779	-40.314	-16.800	-0.585
Δlog(employment)	The log of change of firm' employment between 2008 and 2009	%	-8.967	30.296	-20.743	-0.358	0.000
Direct (Bank Credit Supply) Shock	(Bank supply) Shock of a firm is computed as (following Amiti & Weinstein (2018)) the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction), where the firm-level shock is aggregated using the lagged December 2007 credit between the firm and each bank as weights		0.148	0.061	0.109	0.149	0.185
1st Order Supplier (Bank) Effect	Aggregate 1st Order Supplier (Bank) Effect of all the firm's suppliers' direct bank shocks, weighted by the lagged sales from each supplier to the firm		0.063	0.043	0.030	0.058	0.088
1st Order Customer (Bank) Effect	Aggregate 1st Order Customer (Bank) Effect of all the firm's customers' direct bank shocks, weighted by the lagged sales from the firm to each customer		0.026	0.032	0.002	0.015	0.039
Upstream (1st & Higher Order Effects)	A network aggregate of shocks hitting customers of any order (see equations (10) and (10R) and Section 5 and 6)		0.034	0.049	0.000	0.008	0.054
Bidirectional (Up & Down 1st & Higher Order Effects)	A network aggregate of shocks hitting suppliers and customers of any order (see equations (10) and (10R) and Section 5 and 6)		0.041	0.046	0.004	0.026	0.064

Notes: This table reports the definition, mean, standard deviation and first, second and third quartiles of the variables used in the analysis. See Section 3 to 6 of the paper for a more in depth explanation.

TABLE A2

DIFFERENCE IN MEAN TESTS DEPENDING ON EX-ANTE LINKS WITH BANKS WITH STRONG NEGATIVE CREDIT SUPPLY

	Firms Exposed to Unconstrained Banks		Firms Exposed to Constrained Banks		Difference in Means	Normalized Differences	Dependent Variable: Bank Credit Supply Shock	
	Mean	S.D.	Mean	S.D.	t test	test	Coefficient	S.E.
Firm Characteristics								
Short Term Debt	49.57	(15.36)	49.90	(15.36)	4.39	0.02	0.000	(0.000)
Log(Age)	2.63	(0.34)	2.63	(0.34)	-2.57	-0.01	-0.001	(0.001)
Own Funds/Total Assets	31.56	(14.93)	31.36	(14.93)	-2.63	-0.01	0.000	(0.000)
Log(Total Assets)	7.57	(0.97)	7.58	(0.97)	1.21	0.00	0.002	(0.001)
Liquidity Ratio	16.25	(13.72)	16.11	(13.72)	-2.08	-0.01	-0.000	(0.000)
Average Bank Characteristics								
Log(Total Assets)	18.32	(0.77)	17.82	(0.69)	-138.55	-0.48	-0.090**	(0.037)
Own Funds/Total Assets	0.05	(0.01)	0.05	(0.01)	29.97	0.10	0.004	(0.054)
Net Interbank Borrowing	0.02	(0.01)	0.03	(0.01)	82.81	0.29	0.055**	(0.026)
ROA	0.01	(0.00)	0.01	(0.00)	40.09	0.14	0.020	(0.031)
NPL	0.03	(0.01)	0.03	(0.01)	28.73	0.10	-0.007	(0.035)
Loans/Deposits	0.62	(0.09)	0.63	(0.09)	7.70	0.03	0.013	(0.035)
% Construction & Real Estate	0.47	(0.05)	0.48	(0.06)	36.33	0.13	0.045	(0.040)
Savings Bank	0.53	(0.50)	0.41	(0.49)	-48.33	-0.17	-0.062	(0.055)
R-squared							0.212	
No. of Observations	80,884		85,999				166,883	

Notes: This table (in the first four columns) reports means and standard deviations of firm characteristics as of December 2007. Firms are classified in two groups. The first two columns refer to firms that ex-ante worked with unconstrained banks (its bank credit supply is below the median of the bank supply factor estimated following Amiti & Weinstein (2018), see below and Section 3), while the third and fourth columns refer to firms that worked with constrained banks (above the median). Column (5) reports the t-statistic of the differences in mean and column (6) shows the normalized difference test (a scale-and-sample-size-free estimator) proposed by Imbens and Wooldridge (2009), for which Imbens and Rubin (2015) suggested a heuristic threshold of 0.25 in absolute value for significant differences. Bank characteristics at the firm level are computed as a weighted average of the bank variables at the firm-bank level, using as weights the credit amount of each relationship. Columns (7) and (8) shows the results of a OLS regressions where the dependent variable is the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction), where the firm-level shock is aggregated using the lagged credit between the firm and each bank as weights, and the rest of the variables are standardized. Industry*province dummies are included. Coefficients are listed in the first row, robust standard errors are reported in the adjacent column which are corrected for clustering at the four-digit NACE, province and main bank. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A3

FIRM-LEVEL EFFECTS OF BANK SUPPLY SHOCKS

Dependent Variable:	ΔCredit		
	(1)	(2)	(3)
	2009	2008	2007
Direct (Bank Credit Supply) Shock	-0.604**	-0.214	0.297
	(0.285)	(0.612)	(0.353)
Firm Controls	Yes	Yes	Yes
Spatial & Industry Fixed Effects	Yes	Yes	Yes
R-squared	0.057	0.086	0.102
Observations	196,171	99,257	87,085

Notes: This table reports estimates from WLS. See Section 3. Observations are at the level of the firm (node-level). The dependent variable is the change in bank credit. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction), where the firm-level shock is aggregated using the lagged credit between the firm and each bank as weights. As we cannot control for firm fixed effects, we control for zip code fixed effects. All shocks are standardized. For the list of firm controls, see Section 3 and 4. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the level of the main bank). In each column, the word Yes indicates that the set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A4

LINK-LEVEL: PROPAGATION OF BANK CREDIT SUPPLY SHOCKS THROUGH THE NETWORK OF CUSTOMERS/SUPPLIERS. REDUCED FORM. DISCRETE SHOCK

Panel A. Upstream propagation (indirect shocks via bank credit supply shocks to first-order customers)

Dependent Variable: Δlog(sales from supplier to customer)			IV. Instrument: Bank Net Interbank Borrowing		IV. Instrument: Bank Shock	
			1st Stage	2º Stage	1st Stage	2º Stage
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank Credit Supply) Shock	-0.914*					
	(0.493)					
1st Order Customer (Bank) Effect		-2.620**		-5.219**	3.366***	
		(1.173)		(2.156)	(0.602)	
Customer (Bank) Net Interbank Borrowing			0.413***			
			(0.017)			
Customer Reduction of Bank Debt						-0.558***
						(0.265)
Customer:						
Controls	-	Yes	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	Yes	Yes	Yes
Fixed Effects	Yes	No	No	No	No	No
Firm:						
Controls	Yes	-	-	-	-	-
Spatial*Industry Fixed Effects	Yes	-	-	-	-	-
Fixed Effects	No	Yes	Yes	Yes	Yes	Yes
Firm*Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
First Stage Effective F statistic	-	-	91.77	-	31.88	-
R-squared	0.404	0.488	-	-	-	-
Observations	1,119,169	1,119,169	1,119,169	1,119,169	1,119,169	1,119,169

Panel B. Downstream propagation (indirect shocks via bank credit supply shocks to first-order suppliers)

Dependent Variable: Δlog(sales from supplier to customer)			IV. Instrument: Bank Net Interbank Borrowing		IV. Instrument: Bank Shock	
			1st Stage	2º Stage	1st Stage	2º Stage
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank Credit Supply) Shock	-2.902**					
	(1.452)					
1st Order Supplier (Bank) Effect		-1.180**		-4.684***	2.173***	
		(0.516)		(1.720)	(0.434)	
Supplier (Bank) Net Interbank Borrowing			0.848***			
			(0.037)			
Supplier Reduction of Bank Debt						-0.453**
						(0.094)
Supplier:						
Controls	-	Yes	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	Yes	Yes	Yes
Fixed Effects	Yes	No	No	No	No	No
Firm:						
Controls	Yes	-	-	-	-	-
Spatial*Industry Fixed Effects	Yes	-	-	-	-	-
Fixed Effects	No	Yes	Yes	Yes	Yes	Yes
Firm*Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
First Stage Effective F statistic	-	-	1,121.63	-	11.07	-
R-squared	0.358	0.483	-	-	-	-
Observations	1,114,421	1,114,421	1,114,421	1,114,421	1,114,421	1,114,421

Notes: This table reports estimates from WLS results. See Section 4. Observations are at the level of the firm-customer (Panel A) or firm-supplier (Panel B), i.e. link-level. The dependent variable is the change in the log of sales from supplier to customer between 2008 and 2009 for all columns but (3) and (5). In column (4) the firm bank shock is instrumented with the firm financial shock derived from the (weighted) average net interbank borrowing of the firm across all its banks before the crisis (column (3)). In column (6) the reduction in bank debt between 2008 and 2009 is instrumented with the firm financial shock (column (5)). The continuous bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). Here we discretize this variable based on the median of the distribution. All shocks are standardized. For the list of controls, see Section 4. First stage effective F statistic is based on Montiel Olea and Pfleuger (2013) and it is robust to heteroskedasticity, serial correlation, and clustering. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A5

LINK-LEVEL: PROPAGATION OF BANK CREDIT SUPPLY SHOCKS THROUGH THE NETWORK. REDUCED FORM.
BANK SHOCKS COMPUTING USING A FIRM VARYING DEMAND BY LOAN TYPE

Dependent Variable: Δlog(sales from supplier to customer)	Upstream propagation			Downstream propagation		
	Continuous Shock		Discrete Shock	Continuous Shock		Discrete Shock
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank Credit Supply) Shock	-1.094**			-3.072**		
	(0.442)			(1.470)		
1st Order Customer (Bank) Effect		-2.445**	-2.923**			
		(1.233)	(0.098)			
1st Order Supplier (Bank) Effect					-0.990***	-1.377***
					(0.372)	(0.508)
Customer/Supplier:						
Controls	-	Yes	Yes	-	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	-	Yes	Yes
Fixed Effects	Yes	No	No	Yes	No	No
Firm:						
Controls	Yes	-	-	Yes	-	-
Spatial*Industry Fixed Effects	Yes	-	-	Yes	-	-
Fixed Effects	No	Yes	Yes	No	Yes	Yes
Firm*Customer/Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.405	0.474	0.474	0.358	0.484	0.483
Observations	1,119,169	1,119,169	1,119,169	1,114,421	1,114,421	1,114,421

Notes: This table reports estimates from WLS results. See Section 4. Observations are at the level of firm-customer/supplier, i.e. link-level. The dependent variable is the change in the log of sales from supplier to customer between 2008 and 2009. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018), but allowing firm-loan type fixed effects (where loan types are asset-based loans, cash flow loans, trade finance agreements, and leases following Ivashina, Laeven, and Moral-Benito (2022)), as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. All shocks are standardized. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A6

LINK-LEVEL: PROPAGATION OF BANK CREDIT SUPPLY SHOCKS THROUGH THE NETWORK. REDUCED FORM.
BANK SHOCKS COMPUTING USING A FIRM VARYING DEMAND BY INDUSTRY AND PROVINCE OF THE FIRM VS BANK'S SPECIALIZATION

Dependent Variable: Δlog(sales from supplier to customer)	Upstream propagation			Downstream propagation		
	Continuous Shock		Discrete Shock	Continuous Shock		Discrete Shock
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank Credit Supply) Shock	-1.209**			-2.623**		
	(0.570)			(1.191)		
1st Order Customer (Bank) Effect		-2.665**	-1.680*			
		(1.261)	(0.957)			
1st Order Supplier (Bank) Effect					-0.896**	-0.937***
					(0.409)	(0.459)
Customer/Supplier:						
Controls	-	Yes	Yes	-	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	-	Yes	Yes
Fixed Effects	Yes	No	No	Yes	No	No
Firm:						
Controls	Yes	-	-	Yes	-	-
Spatial*Industry Fixed Effects	Yes	-	-	Yes	-	-
Fixed Effects	No	Yes	Yes	No	Yes	Yes
Firm*Customer/Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.404	0.474	0.474	0.358	0.483	0.484
Observations	1,119,169	1,119,169	1,119,169	1,114,421	1,114,421	1,114,421

Notes: This table reports estimates from WLS results. See Section 4. Observations are at the level of firm-customer/supplier, i.e. link-level. The dependent variable is the change in the log of sales from supplier to customer between 2008 and 2009. In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018), but allowing firm fixed effects to vary depending on whether the firm and the bank match in their industry and/or province (where the province or industry, NACE two digits, of the bank relates to its main province or industry computed in terms of total credit at December of 2007), as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. All shocks are standardized. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A7

LINK-LEVEL: PROPAGATION OF BANK CREDIT SUPPLY SHOCKS THROUGH THE NETWORK. REDUCED FORM.
BANK SHOCKS COMPUTING USING A FIRM VARYING DEMAND DEPENDING ON BANK'S SPECIALIZATION IN REAL ESTATE

Dependent Variable: Δlog(sales from supplier to customer)	Upstream propagation			Downstream propagation		
	Continuous Shock		Discrete Shock	Continuous Shock		Discrete Shock
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank Credit Supply) Shock	-0.992**			-3.017**		
	(0.471)			(1.417)		
1st Order Customer (Bank) Effect		-2.146*	-2.453**			
		(1.196)	(1.104)			
1st Order Supplier (Bank) Effect					-1.103**	-1.394***
					(0.556)	(0.497)
Customer/Supplier:						
Controls	-	Yes	Yes	-	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	-	Yes	Yes
Fixed Effects	Yes	No	No	Yes	No	No
Firm:						
Controls	Yes	-	-	Yes	-	-
Spatial*Industry Fixed Effects	Yes	-	-	Yes	-	-
Fixed Effects	No	Yes	Yes	No	Yes	Yes
Firm*Customer/Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.404	0.474	0.474	0.358	0.483	0.484
Observations	1,119,169	1,119,169	1,119,169	1,114,421	1,114,421	1,114,421

Notes: This table reports estimates from WLS results. See Section 4. Observations are at the level of firm-customer/supplier, i.e. link-level. The dependent variable is the change in the log of sales from supplier to customer between 2008 and 2009. In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use firm level shock estimated following Amiti & Weinstein (2018), but allowing firm fixed effects to vary depending on whether the bank is specialized in the real estate sector or not (in terms of total credit at December 2007), as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. All shocks are standardized. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A8

LINK-LEVEL: PROPAGATION OF A BANK SUPPLY SHOCK THROUGH THE NETWORK OF CUSTOMERS/SUPPLIERS: HETEROGENEOUS INPUT ELASTICITIES. STRUCTURAL FORM

Dependent Variable: Δlog(sales from supplier to customer/sales of customer)	Upstream propagation		Downstream propagation	
	(1)	(2)	(3)	(4)
1st Order Customer (Bank) Effect	-2.006**	-1.960**		
	(0.921)	(0.886)		
Higher Order Customer (Bank) Effect	2.409**	1.598*		
	(1.000)	(0.857)		
Customer Capital Input Effect		2.713*		
		(1.555)		
Customer Labor Input Effect		-1.378**		
		(0.692)		
1st Order Supplier (Bank) Effect			-1.077**	- 1.059**
			(0.547)	(0.515)
Higher Order Supplier (Bank) Effect			-3.326**	-2.713*
			(1.595)	(1.527)
Supplier Capital Input Effect				-4.062***
				(1.316
Supplier Labor Input Effect				-0.985
				(1.722)
Supplier/Customer:				
Controls	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	Yes	Yes	Yes	Yes
Firm:				
Fixed Effects	Yes	Yes	Yes	Yes
Firm*Supplier/Customer Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes
R-squared	0.478	0.478	0.493	0.494
No. of Observations	1,119,169	1,119,169	1,114,421	1,114,421

Notes: This table reports estimates from WLS. See Online Appendix and Section 5 and 6. Observations are at the level of the firm (node-level). The dependent variables is the change, between 2008 and 2009, in the log of firm-level aggregate sales to all customers. E from the theory Section 5 is spanned by fixed effects. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). As we cannot control for firm fixed effects, we control for spatial and industry, and main bank fixed effects. All variables are standardized. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A9

NODE-LEVEL: FIRM-LEVEL EFFECTS OF BANK SUPPLY SHOCKS THROUGH THE PRODUCTION NETWORK: HETEROGENEOUS INPUT ELASTICITIES. STRUCTURAL FORM.

Dependent Variable: Δlog(sales/E)	(1)	(2)	(3)	(4)	(5)
Upstream (1st & Higher Order Effects)	-1.810***		-1.534***	-1.531***	-1.506***
	(0.326)		(0.321)	(0.351)	(0.321)
Bidirectional (Up & Down 1st & Higher Order Effects)		-1.542***	-0.715**	-0.725*	-0.498*
		(0.344)	(0.304)	(0.402)	(0.292)
Direct (Bank Credit Supply) Shock				0.033	
				(0.599)	
Bidirectional Capital Input Effect					-2.274***
					(0.283)
Bidirectional Labor Input Effect					-0.443
					(0.475)
Firm Controls	Yes	Yes	Yes	Yes	Yes
Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes
R-squared	0.358	0.357	0.358	0.358	0.359
Observations	196,171	196,171	196,171	196,171	196,171

Notes: This table reports estimates from WLS. See Online Appendix and Section 5 and 6. Observations are at the level of the firm (node-level). The dependent variables are the change, between 2008 and 2009, in the log of firm-level aggregate sales to all customers. *E* from the theory Section 5 is spanned by fixed effects. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). See also Online Appendix and Section 5 and 6 of the paper for the definition of the variables. As we cannot control for firm fixed effects, we control for spatial and industry, and main bank fixed effects. All variables are standardized. For the list of firm controls, see Section 4. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the level of the main bank). In each column, the word Yes indicates that the set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A10

SUMMARY STATISTICS: SUPPLIER-CUSTOMER DATASET

	Num. Obs.	Mean	S.D.	P25	Median	P75
		Links				
Links by Year:						
2008	13,822,286	77,988	5,279,693	4,915	9,442	25,368
2009	12,003,117	71,560	4,986,726	4,783	8,934	23,326
Links Appearing in Both Years:						
2008	7,666,728	115,058	7,030,309	6,347	13,413	38,020
2009	7,666,728	92,372	5,085,995	5,475	10,921	30,021
		Nodes				
2008						
Number of Customers	696,370	20	336	2	4	13
Number of Suppliers	809,142	17	55	3	7	17
2009						
Number of Customers	692,811	17	309	1	4	11
Number of Suppliers	789,351	15	52	2	6	15

Notes: This table reports means, standard deviations and first/second/third quartiles of annual bilateral transactions for 2008 and 2009 (Links), as well as the number of suppliers/customers for years 2008 and 2009 (Nodes). A firm is a supplier (customer) if it has at least one customer (supplier) in the network in a given year. Link $i \to j$ between two firms appears in both years if i reports a sale to j (or j reports a purchase from i) in both 2008 and 2009.

A.2. Heterogeneous input elasticities

We note that equation (8) does not feature network propagation of effects coming from labor and capital. Intuitively, because $\mathbf{G}$ is column stochastic the effect changes in wages and the price of physical capital affect each firm in the same way as all firms have the same labor and capital input shares in the production function.

We now explore the implications of relaxing the assumption that α, β and ρ are common to all firms, both for the link-level analysis from Section 5.4 and node-level analysis from Section 5.5.

As we show in Proposition 1, when input shares are firm specific (8) becomes:

$$\begin{aligned}\mathrm{d}\log\left(\frac{s_{ji}}{s_i}\right) =& -\theta_i-(\sigma-1)\theta_j\\ &-(\sigma-1)\boldsymbol{e}_j'\mathbf{A}\mathbf{G}'\left(\mathbf{I}-\mathbf{A}\mathbf{G}'\right)^{-1}\boldsymbol{\theta}+(\sigma-1)\boldsymbol{e}_i'\mathbf{G}'\left(\mathbf{I}-\mathbf{A}\mathbf{G}'\right)^{-1}\boldsymbol{\theta}\\ &-(\sigma-1)\boldsymbol{e}_j'\left(\mathbf{I}-\mathbf{A}\mathbf{G}'\right)^{-1}(\boldsymbol{\beta}\mathrm{d}w+\boldsymbol{\rho}\mathrm{d}r)+(\sigma-1)\boldsymbol{e}_i'\mathbf{G}'\left(\mathbf{I}-\mathbf{A}\mathbf{G}'\right)^{-1}(\boldsymbol{\beta}\mathrm{d}w+\boldsymbol{\rho}\mathrm{d}r).\end{aligned} \tag{17}$$

The first two lines of (17) are analogous to (8). The only difference is in that the scalar α is replaced with diagonal matrix $\mathbf{A}\equiv diag(\alpha_i)$ in which i-th element is equal to α_i. The third line of the expression is new, and it captures how the change of the wage ($\mathrm{d}w$) and the change of price of capital ($\mathrm{d}r$) affect the production costs of supplier j and customer i, respectively. Intuitively, the effects of a change of the wage and the capital price is proportional to how much a firm relies on labor and capital in the production, respectively.

In empirical implementation we estimate sector specific parameters α, β and ρ using standard approach Wooldridge (2009). We assign these parameters to firms according to their respective sectors. While we do not observe $\mathrm{d}w$, $\mathrm{d}r$, we are able to calculate network measures $(\mathbf{I}-\mathbf{A}\mathbf{G}')^{-1}\boldsymbol{\beta}$ and $(\mathbf{I}-\mathbf{A}\mathbf{G}')^{-1}\boldsymbol{\rho}$. Therefore, when estimating equation (17) the theory implied effect of $\boldsymbol{e}_j'(\mathbf{I}-\mathbf{A}\mathbf{G}')^{-1}\boldsymbol{\beta}$ is equal to $-(\sigma-1)\mathrm{d}w$, and analogously $(\mathbf{I}-\mathbf{A}\mathbf{G}')^{-1}\boldsymbol{\rho}$. We report the results of the estimation in Table A8. We label estimated parameters of $\boldsymbol{e}_j'(\mathbf{I}-\mathbf{A}\mathbf{G}')^{-1}\boldsymbol{\beta}$ and $\boldsymbol{e}_j'(\mathbf{I}-\mathbf{A}\mathbf{G}')^{-1}\boldsymbol{\rho}$ as the *Supplier Labor Input Effect* and the *Supplier Capital Input Effect*, respectively. Similarly, the parameters of $\boldsymbol{e}_i'(\mathbf{I}-\mathbf{A}\mathbf{G}')^{-1}\boldsymbol{\beta}$ and $\boldsymbol{e}_i'(\mathbf{I}-\mathbf{A}\mathbf{G}')^{-1}\boldsymbol{\rho}$ are labeled as the *Customer Labor Input Effect* and the *Customer Capital Input Effect*, respectively. We find that our estimates of parameters reported in panel A of Table 3 are not affected.

By allowing firm specific values of parameters α, β and ρ equation (10) becomes:

$$\mathrm{d}\log\left(\frac{s_i}{E}\right)=-\boldsymbol{e}_i'\left(\mathbf{I}-\mathbf{H}\right)^{-1}\mathbf{H}\boldsymbol{\theta}+(1-\sigma)\boldsymbol{e}_i'\boldsymbol{\Lambda}\boldsymbol{\theta}+(1-\sigma)\boldsymbol{e}_i'\boldsymbol{\Lambda}(\boldsymbol{\beta}\mathrm{d}w+\boldsymbol{\rho}\mathrm{d}r), \tag{18}$$

where now, $\mathbf{H}=\mathbf{V}^{-1}\mathbf{G}\mathbf{A}\mathbf{M}\mathbf{V}$ and $\boldsymbol{\Lambda}=(\mathbf{I}-\mathbf{H})\left(diag\left(\mathbf{H}\mathbf{1}\right)-\mathbf{H}\mathbf{G}'\right)\left(\mathbf{I}-\mathbf{A}\mathbf{G}'\right)^{-1}$.

Relative to (10), equation (18) features bidirectional propagation coming from the changes in labor and capital markets. Since in this case firms are different with respect to intensity in which they use inputs, changes in labor and capital price affects different firms differently. Note that both upstream and bidirectional propagation of financial shocks in (18) depend on the firm specific values of parameter α_i which is captured by diagonal matrix $\mathbf{A}$. The intuition behind these two types of propagation remains the same as explained in the homogeneous case.

We estimate equation (9) and report the results in Table A9 in online Appendix A. We label estimated parameters of $\boldsymbol{e}_i'\boldsymbol{\Lambda}\boldsymbol{\beta}$ and $\boldsymbol{e}_i'\boldsymbol{\Lambda}\boldsymbol{\rho}$ as the *Bidirectional Capital Input Effect* and the *Bidirectional Labor Input Effect*, respectively.

Appendix B: Proofs — For online publication

Lemma 1. *The marginal cost of firm i is given by*

$$mc_i = (1+\theta_i)\frac{1}{\kappa_i} r^{\rho_i} w^{\beta_i} P_i^{\alpha_i}, \tag{19}$$

where $\kappa_i \equiv \zeta_i \rho_i^{\rho_i} \alpha_i^{\alpha_i} \beta_i^{\beta_i}$ *and* $P_i \equiv \left[\sum_{k\in N_i^+} g_{ki} p_k^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$.

Proof of Lemma 1. Given any feasible production plan $[\ell_i, k_i, (z_{ij})_{j=1}^n]$ and shock θ_i, firm i minimizes:

$$(1+\theta_i)\left(w\ell_i + rk_i + \sum_{j\in N_i^+} p_j z_{ji}\right), \tag{20}$$

subject to the technological constraint:

$$y_i \leq \zeta_i k_i^{\rho_i} \ell_i^{\beta_i} \left[\left(\sum_{j\in N_i^+} g_{ji}^{\frac{1}{\sigma}} z_{ji}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\alpha_i}.$$

The above constraint holds with equality. Hence the Langragian of this problem is:

$$\mathscr{L} = (1+\theta_i)\left(w\ell_i + rk_i + \sum_{j\in N_i^+} p_j z_{ji}\right) - \varphi_i \left[\zeta_i k_i^{\rho_i} \ell_i^{\beta_i} M_i^{\alpha_i} - y_i\right].$$

The first-order necessary conditions (FONC) which are also sufficient, given the postulated convexity conditions, read:

$$\begin{aligned}
&(1+\theta_i)p_j = \varphi_i \zeta_i k_i^{\rho_i} \ell_i^{\beta_i} \alpha_i M_i^{\alpha_i - 1} \frac{\partial M_i}{\partial z_{ji}} = \varphi_i \alpha_i y_i \frac{1}{M_i}\frac{\partial M_i}{\partial z_{ji}} = \varphi_i \alpha_i y_i \frac{1}{M_i}\left[\sum_k g_{ki}^{\frac{1}{\sigma}} z_{ki}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma}-1} g_{ji}^{\frac{1}{\sigma}} z_{ji}^{-\frac{1}{\sigma}},\\
&(1+\theta_i)w = \varphi_i \beta_i \frac{1}{\ell_i} y_i,\\
&(1+\theta_i)r = \varphi_i \rho_i \frac{1}{k_i} y_i.
\end{aligned} \tag{21}$$

From (21) follows directly that for any two interemediate inputs j and k used by firm i, we have:

$$\frac{p_j}{p_k} = \left[\frac{g_{ji}}{g_{ki}}\right]^{\frac{1}{\sigma}} \left[\frac{z_{ji}}{z_{ki}}\right]^{-\frac{1}{\sigma}} \Rightarrow z_{ki} = \left[\frac{p_j}{p_k}\right]^{\sigma} \frac{g_{ki}}{g_{ji}} z_{ji}.$$

Substituting z_{ji} from above in (5) we get:

$$M_i = \left[\sum_{k\in N_i^+, k\neq j} g_{ki}^{\frac{1}{\sigma}} \left[\left[\frac{p_j}{p_k}\right]^{\sigma} \frac{g_{ki}}{g_{ji}} z_{ji}\right]^{\frac{\sigma-1}{\sigma}} + g_{ji}^{\frac{1}{\sigma}} z_{ji}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \Rightarrow z_{ji} = g_{ji} p_j^{-\sigma} P_i^{\sigma} M_i, \tag{22}$$

where

$$P_i \equiv \left[\sum_{k \in N_i^+} g_{ki} p_k^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

is the price index of intermediate inputs firm i uses in the production. Using (21), the definition of P_i and (22) we can write the conditional demand for intermediate inputs, labor and capital:

$$\begin{aligned}
\ell_i\left(y_i\,;\,w,r,\boldsymbol{p},\theta_i\right) &= \varphi_i \frac{1}{1+\theta_i}\beta_i \frac{y_i}{w}, \\
k_i\left(y_i\,;\,w,r,\boldsymbol{p},\theta_i\right) &= \varphi_i \frac{1}{1+\theta_i}\rho_i \frac{y_i}{r}, \\
M_i\left(y_i\,;\,w,r,\boldsymbol{p},\theta_i\right) &= \varphi_i \frac{1}{1+\theta_i}\alpha_i \frac{y_i}{P_i}, \\
z_{ji}\left(y_i\,;\,w,r,\boldsymbol{p},\theta_i\right) &= \varphi_i \frac{1}{1+\theta_i}\alpha_i g_{ji} p_j^{-\sigma} P_i^{\sigma-1} y_i.
\end{aligned} \tag{23}$$

Substituting (23) in (20) we get that $mc_i = \varphi_i$. Then, to derive the expression for φ_i, substitute (23) in (4) to obtain:

$$y_i = \zeta_i \left(\frac{\varphi_i \rho_i y_i}{(1+\theta_i)r}\right)^{\beta_i} \left(\frac{\varphi_i \beta_i y_i}{(1+\theta_i)w}\right)^{\beta_i} \left(\frac{\varphi_i \alpha_i y_i}{(1+\theta_i)P_i}\right)^{\alpha_i} = \frac{\zeta_i}{1+\theta_i}\varphi_i y_i \left(\frac{\rho_i}{r}\right)^{\rho_i} \left(\frac{\beta_i}{w}\right)^{\beta_i} \left(\frac{\alpha_i}{P_i}\right)^{\alpha_i},$$

which gives:

$$mc_i = \varphi_i = \frac{1+\theta_i}{\zeta_i}\rho_i^{-\rho_i}\beta_i^{-\beta_i}\alpha_i^{-\alpha_i} r^{\rho_i} w^{\beta_i} P_i^{\alpha_i} = \frac{1+\theta_i}{\kappa_i} r^{\rho_i} w^{\beta_i} P_i^{\alpha_i},$$

as desired. □

Steady state and normalization

To facilitate the calibration, following Baqaee (2018); Baqaee and Farhi (2019), we define the steady state as a contingency in which there are no financial shocks $\theta_i = 0, \forall i$, and $\kappa_i = \mu_i$, $\forall i$.

From the Lemma 1 and the pricing rule we can write:

$$\log p_i = \log\left[\frac{1+\theta_i}{\kappa_i}\mu_i\right] + \rho_i \log r + \beta_i \log w + \frac{\alpha_i}{1-\sigma}\log\left[\sum_k g_{ki} p_k^{1-\sigma}\right],$$

which in the steady state reduces to:

$$\log p_i = \rho_i \log r + \beta_i \log w + \frac{\alpha_i}{1-\sigma}\log\left[\sum_k g_{ki} p_k^{1-\sigma}\right].$$

Clearly, $p_i = 1 \forall (i \in N)$, $w = 1$ and $r = 1$ satisfies this equation for every i.

The consumer's problem

Lemma 2. *Let* $\bar{p} = \prod_i \left(\frac{p_i}{\gamma_i}\right)^{\gamma_i}$ *denote the price index of consumption goods. The consumer chooses consumption plan such that*

$$\frac{p_i c_i}{\bar{p} c} = \gamma_j \text{ and } \frac{c^{-\gamma}}{L^\eta} = \frac{\bar{p}}{w}.$$

Proof of Lemma 2. The consumer solves the following problem:

$$\max_{\boldsymbol{c},L} \frac{1}{1-\delta}\left[\overbrace{\prod_i c_i^{\gamma_i}}^{c}\right]^{1-\delta} - \frac{L^{1+\eta}}{1+\eta}$$
$$s.t. \sum_i p_i c_i \leq E.$$

The monotonicity of preferences implies $\bar{p}c = E(\boldsymbol{\theta})$. The fact that $\frac{p_i c_i}{\bar{p} c} = \gamma_j$ is directly obtained by solving for the expenditure minimization problem, where $\bar{p}c$ is the resulting expenditure. $\frac{c^{-\gamma}}{L^\eta} = \frac{\bar{p}}{w}$ follows directly from the utility maximization problem with respect to c and L. □

Cost share, revenue share, sales share

We now introduce some additional notation. Let $s_{ji} \equiv p_j z_{ji}$ and $s_i = p_i y_i$. Furthermore, let $\tilde{\omega}_{ji}$ denote cost share on intermediate input j among intermediate inputs firm i uses in production, that is $\tilde{\omega}_{ji} \equiv \frac{s_{ji}}{\sum_\ell s_{\ell i}}$. Let $\omega_{ji} = \frac{s_{ji}}{s_i}$ denote the expenditure share in sales on input j (we'll refer to it as *revenue share*). Finally, let $h_{ji} \equiv \frac{s_{ji}}{s_j}$ denote the sales share.

Let us now relate the technological parameters $(g_{ji})_{ij}$ with cost shares $\tilde{\omega}_{ji}$, revenue shares ω_{ji}, and sales shares h_{ji}.

Lemma 3. *The following holds in the equilibrium:*

$$\tilde{\omega}_{ji} = g_{ji} p_j^{1-\sigma} P_i^{\sigma-1} = \frac{g_{ji} p_j^{1-\sigma}}{\sum_k g_{ki} p_k^{1-\sigma}}, \text{ or in terms of quantities, } \tilde{\omega}_{ji} = g_{ji}^{\frac{1}{\sigma}} \left[\frac{z_{ji}}{M_i}\right]^{\frac{\sigma-1}{\sigma}}, \tag{24}$$

$$\omega_{ji} = \frac{\alpha_i}{(1+\theta_i)\mu_i} \tilde{\omega}_{ji}, \tag{25}$$

and

$$h_{ji} = \omega_{ji} \frac{s_i}{s_j} = \frac{\alpha_i}{(1+\theta_i)\mu_i} \tilde{\omega}_{ji} \frac{s_i}{s_j}. \tag{26}$$

Proof of Lemma 3. The first equation in (24) follows directly from the fact that $s_{ji} = g_{ji} p_j^{1-\sigma} P_i^{\sigma-1} P_i M_i$. To prove that the second equation in (24) we note that the FONC of the cost minimization problem with respect to z_{ji} in (21) gives:

$$(1+\theta_i)p_j = mc_i \alpha_i y_i M_i^{\frac{1-\sigma}{\sigma}} g_{ji}^{\frac{1}{\sigma}} z_{ji}^{-\frac{1}{\sigma}} \Rightarrow \frac{(1+\theta_i) p_j z_{ji}}{mc_i \alpha_i y_i} = M_i^{\frac{1-\sigma}{\sigma}} g_{ji}^{\frac{1}{\sigma}} z_{ji}^{\frac{\sigma-1}{\sigma}} \overbrace{\Rightarrow}^{\text{by (23)}} \tilde{\omega}_{ji} = g_{ji}^{\frac{1}{\sigma}} M_i^{\frac{1-\sigma}{\sigma}} z_{ji}^{\frac{\sigma-1}{\sigma}}.$$

The equation (25) follows directly from (23) and the pricing rule $p_i = \mu_i mc_i$. Finally, (26) follows from equality $\frac{s_{ji}}{s_j} = \frac{s_{ji}}{s_i}\frac{s_i}{s_j}$, which concludes the proof. $\square$

Let $\mathbf{\Omega}$, $\tilde{\mathbf{\Omega}}$ and $\mathbf{H}$ be matrices with elements ω_{ij}, $\tilde{\omega}_{ij}$ and h_{ij} respectively. Recall that, $\mathbf{A}$, $\mathbf{M}$ and $\mathbf{T}$ stand for diagonal matrices with elements α_i, $\frac{1}{\mu_i}$ and $\frac{1}{1+\theta_i}$ on the main diagonal, respectively. Lemma 3 implies that in steady state:

$$\tilde{\mathbf{\Omega}} \overbrace{=}^{\text{steady state}} \mathbf{G},$$

$$\mathbf{\Omega} = \tilde{\mathbf{\Omega}}\mathbf{AMT} \overbrace{=}^{\text{steady state}} \mathbf{GAM},$$

$$\mathbf{H} = \mathbf{V}^{-1}\mathbf{\Omega}\mathbf{V} \overbrace{=}^{\text{steady state}} \mathbf{V}^{-1}\mathbf{GAMV},$$

where $\mathbf{V}$ are diagonal matrices with elements diagonal elements equal to entries of vector

$$\boldsymbol{v}(\boldsymbol{\theta}) = (v_i(\boldsymbol{\theta}))_{i=1}^n = (\mathbf{I} - \mathbf{GAMT})^{-1}\boldsymbol{\gamma}.$$

Effect of shocks on prices

Lemma 4. *In the steady state:*

$$\frac{\partial \log \boldsymbol{p}}{\partial \theta_k} = \left[\mathbf{I} - \mathbf{AG}'\right]^{-1}\left[\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta} + \frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho} + \boldsymbol{e}_k\right].$$

In the special case when $\alpha_i = \alpha$, $\beta_i = \beta$:

$$\frac{\partial \log \boldsymbol{p}}{\partial \theta_k} = \left[\mathbf{I} - \alpha\mathbf{G}'\right]^{-1}\boldsymbol{e}_k + \frac{\beta}{1-\alpha}\frac{\partial \log w}{\partial \theta_k}\mathbf{1} + \frac{\rho}{1-\alpha}\frac{\partial \log r}{\partial \theta_k}\mathbf{1}.$$

Proof. Consider $\frac{\partial \log p_i}{\partial \theta_k}$. From the expression for marginal cost of firm i (Lemma 1), we have:

$$\log p_i = \log(1+\theta_i) + \log \mu_i - \log \kappa_i + \beta_i \log w + \rho_i \log r_i + \alpha_i \log P_i.$$

Differentiating with respect to θ_k we get:

$$\frac{\partial \log p_i}{\partial \theta_k} = \frac{\partial \log(1+\theta_i)}{\partial \theta_k} + \beta_i \frac{\partial \log w}{\partial \theta} + \rho_i \frac{\partial \log r}{\partial \theta_k} + \alpha_i \frac{\partial \log P_i}{\partial \theta_k}.$$

To get the expression for $\frac{\partial \log P_i}{\partial \theta_k}$ we note that from the definition of P_i it follows that:

$$\frac{\partial \log P_i}{\partial \theta_k} = \frac{1}{\sum_\ell g_{\ell i} p_\ell^{1-\sigma}} \sum_j g_{ji} p_j^{-\sigma} \frac{\partial \log p_j}{\partial \theta_k}.$$

At the steady state:

$$\left.\frac{\partial \log P_i}{\partial \theta_k}\right|_{\boldsymbol{\theta}=0} = \sum_j g_{ji} \left.\frac{\partial \log p_j}{\partial \theta_k}\right|_{\boldsymbol{\theta}=0} = \boldsymbol{e}_i' \mathbf{G}' \frac{\partial \log \boldsymbol{p}}{\partial \theta_k}.$$

In what follows we omit $|_{\boldsymbol{\theta}=0}$ whenever it is clear that the derivatives are evaluated at the steady state. Finaly, evaluating at the steady state:

$$\frac{\partial \log p_i}{\partial \theta_k} = -\delta_{ki} + \beta_i \frac{\partial \log w}{\partial \theta_k} + \rho_i \frac{\partial \log r}{\partial \theta_k} + \alpha_i \sum_j g_{ji} \frac{\partial \log p_j}{\partial \theta_k},$$

where δ_{ki} denotes the Kroenecker's delta. Writing this expression for each price in vector notation gives:

$$\frac{\partial \log \boldsymbol{p}}{\partial \theta_k} = \left[\mathbf{I} - \mathbf{A}\mathbf{G}'\right]^{-1} \left[\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta} + \frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho} + \boldsymbol{e}_k\right].$$

In the special case when $\alpha_i = \alpha$, $\beta_i = \beta$ for all i the above expression becomes

$$\frac{\partial \log \boldsymbol{p}}{\partial \theta_k} = \left[\mathbf{I} - \alpha\mathbf{G}'\right]^{-1} \boldsymbol{e}_k + \frac{\beta}{1-\alpha}\frac{\partial \log w}{\partial \theta_k}\mathbf{1} + \frac{\rho}{1-\alpha}\frac{\partial \log r}{\partial \theta_k}\mathbf{1}.$$

□

From Lemma 4 it directly follows:

Corollary 1. *At the steady state:*

$$\frac{\partial \log \boldsymbol{P}}{\partial \theta_k} = \mathbf{G}' \left[\mathbf{I} - \mathbf{A}\mathbf{G}'\right]^{-1} \left[\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta} + \frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho} + \boldsymbol{e}_k\right],$$

and in homogeneous case:

$$\frac{\partial \log \boldsymbol{P}}{\partial \theta_k} = \frac{\beta}{1-\alpha}\frac{\partial \log w}{\partial \theta_k}\mathbf{1} + \frac{\rho}{1-\alpha}\frac{\partial \log r}{\partial \theta_k}\mathbf{1} + \mathbf{G}'\left[\mathbf{I} - \alpha\mathbf{G}'\right]^{-1}\boldsymbol{e}_k,$$

Link-level outcomes

Proposition 1. *The first order approximation of the change* $\log \frac{s_{ji}}{s_i} = \log w_{ji}$ *at the steady state is given with* (17)*. In the special case when all firms have equal input shares* α, β *and* ρ (17) *becomes* (8)*.*

Proof of Proposition 1. We use the following approximation:

$$d\log s_{ji} = \sum_{k \in N} \frac{\partial \log s_{ji}}{\partial \theta_k}\theta_k, \tag{27}$$

where derivatives are evaluated at point $\boldsymbol{\theta} = \mathbf{0}$.

From firms' pricing rule ($p_i = \mu_i mc_i$), Lemma 1, and (22) it directly follows that

$$p_j z_{ji} = g_{ji}(\mu_j mc_j)^{1-\sigma} P_i^{\sigma} M_i \Rightarrow p_j z_{ji} = g_{ji}(\mu_j(1+\theta_j)\kappa_j^{-1} r_j^{\rho_j} w^{\beta_j} P_j^{\alpha_j})^{1-\sigma} P_i^{\sigma} M_i,$$

which together with (23) implies:

$$p_j z_{ji} = g_{ji}(\mu_j(1+\theta_j)\kappa_j^{-1} r_j^{\rho_j} w^{\beta_j} P_j^{\alpha_j})^{1-\sigma} P_i^{\sigma-1} \mu_i^{-1}(1+\theta_i)^{-1}\alpha_i s_i.$$

Taking logs, and using $s_{ji} \equiv p_j z_{ji}$ we get:

$$\log s_{ji} = (1-\sigma)\log(1+\theta_j) - \log(1+\theta_i) + \log g_{ji} + (1-\sigma)(\log \mu_j - \log \kappa_j + \beta_j \log w + \rho_j \log r) + \log \alpha_i + \log s_i - \log \mu_i + (1-\sigma)\alpha_j \log P_j + (\sigma-1)\log P_i.$$

Differentiating with respect to θ_k we get:

$$\begin{aligned} \frac{\partial \log s_{ji}}{\partial \theta_k} = &- \frac{1}{1+\theta_i}\delta_{ki} - (\sigma-1)\frac{1}{1+\theta_j}\delta_{kj} + \frac{\partial \log s_i}{\theta_k} - \\ &(\sigma-1)\left[\beta_j \frac{\partial \log w}{\partial \theta_k} + \rho_j \frac{\partial \log r}{\partial \theta_k} + \alpha_j \frac{\partial \log P_j}{\partial \theta_k}\right] + (\sigma-1)\frac{\partial \log P_i}{\partial \theta_k}, \end{aligned} \tag{28}$$

where δ_{jk} is Kroeneker's delta.

Corollary 1 implies that we can write, (28) as:

$$\begin{aligned}\frac{\partial \log s_{ji}}{\partial \theta_k} =& -\frac{1}{1+\theta_i}\delta_{ki} - (\sigma-1)\frac{1}{1+\theta_j}\delta_{kj} + \frac{\partial \log s_i}{\theta_k} - \\ & (\sigma-1)\boldsymbol{e}_j'\left[\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta} + \frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho} + \mathbf{AG}'\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}\left(\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta} + \frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho} + \boldsymbol{e}_k\right)\right] + \\ & (\sigma-1)\boldsymbol{e}_i'\left[\mathbf{G}'\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}\left(\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta} + \frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho} + \boldsymbol{e}_k\right)\right],\end{aligned} \tag{29}$$

which can be simplified to

$$\begin{aligned}\frac{\partial \log s_{ji}}{\partial \theta_k} =& -\frac{1}{1+\theta_i}\delta_{ki} - (\sigma-1)\frac{1}{1+\theta_j}\delta_{kj} + \frac{\partial \log s_i}{\partial \theta_k} - \\ & (\sigma-1)\boldsymbol{e}_j'\left[\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}\left(\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta} + \frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho}\right) + \mathbf{AG}'\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}\boldsymbol{e}_k\right] + \\ & (\sigma-1)\boldsymbol{e}_i'\left[\mathbf{G}'\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}\left(\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta} + \frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho}\right) + \mathbf{G}'(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{e}_k\right].\end{aligned} \tag{30}$$

Using (30) in (27) and evaluating derivatives at $\boldsymbol{\theta}=\mathbf{0}$ gives (17), where we use the fact that $\mathbf{I}+\mathbf{AG}'(\mathbf{I}-\mathbf{AG}')^{-1}=(\mathbf{I}-\mathbf{AG}')^{-1}$.

In the special case when $\alpha_i=\alpha$ and $\beta_i=\beta$ for all firms i (30) simplifies to:

$$\begin{aligned}\frac{\partial \log s_{ji}}{\partial \theta_k} =& -\frac{1}{1+\theta_i}\delta_{ki} - (\sigma-1)\frac{1}{1+\theta_j}\delta_{kj} + \frac{\partial \log s_i}{\partial \theta_k} - \\ & (\sigma-1)\left[\frac{\beta}{1-\alpha}\frac{\partial \log w}{\partial \theta_k} + \frac{\rho}{1-\alpha}\frac{\partial \log r}{\partial \theta_k} + \alpha\boldsymbol{e}_j'\mathbf{G}'\left(\mathbf{I}-\alpha\mathbf{G}\right)^{-1}\boldsymbol{e}_k\right] + \\ & (\sigma-1)\left[\frac{\beta}{1-\alpha}\frac{\partial \log w}{\partial \theta_k} + \frac{\rho}{1-\alpha}\frac{\partial \log r}{\partial \theta_k} + \boldsymbol{e}_i'\mathbf{G}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{e}_k\right] \\ =& -\frac{1}{1+\theta_i}\delta_{ki} - (\sigma-1)\frac{1}{1+\theta_j}\delta_{kj} + \frac{\partial \log s_i}{\theta_k} - (\sigma-1)\left[\alpha\boldsymbol{e}_j'\mathbf{G}'\left(\mathbf{I}-\alpha\mathbf{G}\right)^{-1}\boldsymbol{e}_k - \boldsymbol{e}_i'\mathbf{G}'\left(\mathbf{I}-\alpha\mathbf{G}'\right)^{-1}\boldsymbol{e}_k\right].\end{aligned}$$

It is easy to see that in this case (17) becomes (8). This concludes the proof. □

Node-level outcomes

Proposition 2. *A first order approximation of the change in* $\log\left(\frac{s_i}{E}\right)$ *at the steady state is given with* (18)*. In the special case when all firms have equal input shares* α*,* β *and* ρ (18) *becomes* (9)*.*

Proof of Proposition 2. We first provide expression for $\frac{\partial \log \boldsymbol{s}}{\partial \theta_k} - \frac{\partial \log E}{\partial \theta_k}$. Equations (18) and (9) then follow directly.

Market clearing condition for intermediate inputs read:

$$y_i = c_i + \sum_j z_{ij},$$

which be written as:

$$p_i y_i = p_i c_i + \sum_j p_i z_{ij} \Rightarrow s_i = \gamma_i E + \sum_j \omega_{ij} s_j,$$

and therefore:

$$\boldsymbol{s} = E(\mathbf{I} - \boldsymbol{\Omega})^{-1}\boldsymbol{\gamma} = E\boldsymbol{v}(\boldsymbol{\theta}).$$

Taking derivatives we get:

$$\begin{aligned}\frac{\partial \boldsymbol{s}}{\partial \theta_k} =& \frac{\partial E}{\partial \theta_k}(\mathbf{I} - \boldsymbol{\Omega})^{-1}\boldsymbol{\gamma} - (\mathbf{I} - \boldsymbol{\Omega})^{-1}\frac{\partial(\mathbf{I} - \boldsymbol{\Omega})}{\partial \theta_k}(\mathbf{I} - \boldsymbol{\Omega})^{-1}E\boldsymbol{\gamma} \\ =& \frac{\partial \log E}{\partial \theta_k}\boldsymbol{s} + (\mathbf{I} - \boldsymbol{\Omega})^{-1}\frac{\partial \boldsymbol{\Omega}}{\partial \theta_k}\boldsymbol{s} = \left[\frac{\partial \log E}{\partial \theta_k}\mathbf{I} + \overbrace{(\mathbf{I} - \boldsymbol{\Omega})^{-1}}^{\boldsymbol{\Psi}}\frac{\partial \boldsymbol{\Omega}}{\partial \theta_k}\right]\boldsymbol{s}.\end{aligned}$$

For a given firm i, we have:

$$\frac{\partial s_i}{\partial \theta_k} = \frac{\partial \log E}{\partial \theta_k}s_i + \sum_\ell \sum_j \psi_{i\ell}\frac{\partial \omega_{\ell j}}{\partial \theta_k}s_j,$$

and consequently

$$\frac{\partial \log s_i}{\partial \theta_k} = \frac{\partial \log E}{\partial \theta_k} + \frac{1}{s_i}\sum_\ell \sum_j \psi_{i\ell}\frac{\partial \omega_{\ell j}}{\partial \theta_k}s_j = \frac{\partial \log E}{\partial \theta_k} + \frac{1}{s_i}\sum_\ell \sum_j \psi_{i\ell}\omega_{\ell j}\frac{\partial \log \omega_{\ell j}}{\partial \theta_k}s_j.$$

Substituting the expression for $\frac{\partial \log \omega_{\ell j}}{\partial \theta_k}$ (from Proposition 1), the previous expression becomes:

$$\frac{\partial \log s_i}{\partial \log \theta_k} = \frac{\partial \log E}{\partial \theta_k} + \frac{1}{s_i}\left[\sum_\ell \sum_j \psi_{i\ell}\omega_{\ell j}s_j\left(-\frac{\partial \log(1+\theta_j)}{\partial \theta_k} + (1-\sigma)\left(\frac{\partial \log p_\ell}{\partial \theta_k} - \sum_r \tilde{\omega}_{rj}\frac{\partial \log p_r}{\partial \theta_k}\right)\right)\right].$$

To write the expression for $\frac{\partial \log \boldsymbol{s}}{\partial \theta_k}$, we consider the parts of the right hand side of the previous expression in the brackets separately.

First, we note that:

$$-\frac{1}{s_i}\left[\sum_\ell \sum_j \psi_{i\ell}\omega_{\ell j}s_j\frac{\partial \log(1+\theta_j)}{\partial \theta_k}\right] = -\frac{1}{v_i}\left[\sum_\ell \sum_j \psi_{i\ell}\omega_{\ell j}v_j\frac{\partial \log(1+\theta_j)}{\partial \theta_k}\right],$$

where we used $s_i = Ev_i$. For each i this can be written in the matrix notation as:

$$-\frac{1}{1+\theta_k}\mathbf{V}^{-1}\left[\mathbf{I} - \boldsymbol{\Omega}\right]^{-1}\boldsymbol{\Omega}\mathbf{V}\boldsymbol{e}_k = -\frac{1}{1+\theta_k}(\mathbf{I} - \mathbf{H})^{-1}\mathbf{H}\boldsymbol{e}_k,$$

since

$$\mathbf{V}^{-1}(\mathbf{I} - \boldsymbol{\Omega})^{-1}\boldsymbol{\Omega}\mathbf{V}\boldsymbol{e}_k = \left(\mathbf{V}^{-1}\boldsymbol{\Omega}\mathbf{V} + \mathbf{V}^{-1}\boldsymbol{\Omega}\mathbf{V}\mathbf{V}^{-1}\boldsymbol{\Omega}\mathbf{V} + \ldots\right)\boldsymbol{e}_k = \left(\sum_{i=1}^{\infty}\mathbf{H}^i\right)\boldsymbol{e}_k = (\mathbf{I} - \mathbf{H})^{-1}\mathbf{H}\boldsymbol{e}_k.$$

The expression:

$$(1-\sigma)\frac{1}{s_i}\left[\sum_\ell \sum_j \psi_{i\ell}\omega_{\ell j}s_j\frac{\partial \log p_\ell}{\partial \theta_k}\right],$$

for each $i \in N$ can be written as:

$$(1-\sigma)\mathbf{V}^{-1}\left[\mathbf{I}-\mathbf{\Omega}\right]^{-1} diag(\mathbf{\Omega V 1})\frac{\partial \log \boldsymbol{p}}{\partial \theta_k}.$$

Finally, consider:

$$-(1-\sigma)\frac{1}{s_i}\left[\sum_{\ell}\sum_{j}\psi_{i\ell}\omega_{\ell j}s_j\sum_{r}\tilde{\omega}_{rj}\frac{\partial \log p_r}{\partial \theta_k}\right],$$

and note that we can write this expression (for each i) in matrix notation as:

$$-(1-\sigma)\mathbf{V}^{-1}\left[\mathbf{I}-\mathbf{\Omega}\right]^{-1}\mathbf{\Omega V}\tilde{\mathbf{\Omega}}'\frac{\partial \log \boldsymbol{p}}{\partial \theta_k}.$$

Putting everything together we get:

$$\frac{\partial \log \boldsymbol{s}}{\partial \theta_k}=\frac{\partial \log E}{\partial \theta_k}-\frac{1}{1+\theta_k}[\mathbf{I}-\mathbf{H}]^{-1}\mathbf{H}\boldsymbol{e}_k+(1-\sigma)\mathbf{V}^{-1}\left[\mathbf{I}-\mathbf{\Omega}\right]^{-1}\left[diag(\mathbf{\Omega V 1})-\mathbf{\Omega V}\tilde{\mathbf{\Omega}}'\right]\frac{\partial \log \boldsymbol{p}}{\partial \theta_k}. \quad (31)$$

Plugging in the expression for $\frac{\partial \log \boldsymbol{p}}{\partial \theta_k}$ (from Lemma 4) in (31) gives:

$$\begin{aligned}\frac{\partial \log \boldsymbol{s}}{\partial \theta_k}&=\frac{\partial \log E}{\partial \theta_k}-\frac{1}{1+\theta_k}[\mathbf{I}-\mathbf{H}]^{-1}\mathbf{H}\boldsymbol{e}_k+\\ &(1-\sigma)\mathbf{V}^{-1}\left[\mathbf{I}-\mathbf{\Omega}\right]^{-1}\left[diag(\mathbf{\Omega V 1})-\mathbf{\Omega V}\tilde{\mathbf{\Omega}}'\right]\left[\mathbf{I}-\mathbf{A}\tilde{\mathbf{\Omega}}'\right]^{-1}\left[\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta}+\frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho}+\frac{1}{1+\theta_k}\boldsymbol{e}_k\right].\end{aligned} \quad (32)$$

Evaluating in the steady state ($\tilde{\omega}_{ij}=g_{ij}$, $\omega_{ij}=\frac{\alpha_j}{\mu_j}g_{ij}$, and $\boldsymbol{\theta}=\mathbf{0}$) the previous expression becomes:

$$\begin{aligned}&\frac{\partial \log \boldsymbol{s}}{\partial \theta_k}-\frac{\partial \log E}{\partial \theta_k}=-[\mathbf{I}-\mathbf{V}^{-1}\mathbf{GAMV}]^{-1}\mathbf{V}^{-1}\mathbf{GAMV}\boldsymbol{e}_k+\\ &(1-\sigma)\mathbf{V}^{-1}\left[\mathbf{I}-\mathbf{GAM}\right]^{-1}\left[diag(\mathbf{GAMV1})-\mathbf{GAMVG}'\right]\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}\cdot\\ &\left[\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta}+\frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho}+\boldsymbol{e}_k\right].\end{aligned}$$

Define $\mathbf{\Lambda}\equiv\mathbf{V}^{-1}\left[\mathbf{I}-\mathbf{GAM}\right]^{-1}\left[diag(\mathbf{GAMV1})-\mathbf{GAMVG}'\right]\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}$.

We can now write(evaluating at the steady state):

$$\frac{\partial \log \boldsymbol{s}}{\partial \theta_k}-\frac{\partial \log E}{\partial \theta_k}=-[\mathbf{I}-\mathbf{V}^{-1}\mathbf{GAMV}]^{-1}\mathbf{V}^{-1}\mathbf{GAMV}\boldsymbol{e}_k+(1-\sigma)\mathbf{\Lambda}\left[\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta}+\frac{\partial \log r}{\partial \theta_k}+\boldsymbol{e}_k\right].$$

In the special case $\alpha_i=\alpha$ and $\beta_i=\beta$ for all $i\in N$ we get:

$$\frac{\partial \log \boldsymbol{s}}{\partial \theta_k}-\frac{\partial \log E}{\partial \theta_k}=-\alpha[\mathbf{I}-\alpha\mathbf{V}^{-1}\mathbf{GMV}]^{-1}\mathbf{V}^{-1}\mathbf{GMV}\boldsymbol{e}_k+(1-\sigma)\mathbf{\Lambda}\boldsymbol{e}_k,$$

where we used the fact that in the symmetric case $(\mathbf{I}-\alpha\mathbf{G}')^{-1}\mathbf{1}=\frac{1}{1-\alpha}\mathbf{1}$, and $[diag(\mathbf{GMV1})-\mathbf{GMVG}']\mathbf{1}=\mathbf{0}$.

□

Auxiliary result

To obtain (10) from (9) we use the following result.

Lemma 5. *In steady state:*

$$\mathbf{V}^{-1}\left[\mathbf{I}-\mathbf{GAM}\right]^{-1}\left[diag(\mathbf{GAMV1})-\mathbf{GAMVG}'\right]\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}=\left[\mathbf{I}-\mathbf{H}\right]^{-1}\left[diag(\mathbf{H1})-\mathbf{HG}'\right]\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}.$$

Proof.

$$\begin{aligned}
&\mathbf{V}^{-1}\left[\mathbf{I}-\mathbf{GAM}\right]^{-1}\left[diag(\mathbf{GAMV1})-\mathbf{GAMVG}'\right]\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}=\\
&\mathbf{V}^{-1}\left[\mathbf{I}-\mathbf{GAM}\right]^{-1}\mathbf{VV}^{-1}\left[\mathbf{V}diag(\mathbf{V}^{-1}\mathbf{GAMV1})-\mathbf{VV}^{-1}\mathbf{GAMVG}'\right]\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}=\\
&\left[\mathbf{I}-\mathbf{H}\right]^{-1}\left[diag(\mathbf{H1})-\mathbf{HG}'\right]\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}.
\end{aligned}$$

□

Aggregation

We now examine the effect of the shocks on the real GDP. We consider the case with homogeneous α, β and ρ. We follow Baqaee and Farhi (2019) and choose as a numeraire the nominal GDP, so that $E=\bar{p}c=\sum p_i c_i=1$, where $\bar{p}\equiv\sum_{i\in N}\left(\frac{p_i}{\gamma_i}\right)^{\gamma_i}$ is the consumer price index, and c is the aggregate production of the consumption good (real GDP). Therefore the real GDP we can write

$$\frac{\partial\log c}{\partial\theta_k}=-\frac{\partial\log\bar{p}}{\partial\theta_k}=-\sum_{i\in N}\gamma_i\frac{\partial\log p_i}{\partial\theta_k},\tag{33}$$

and consequently

$$d\log c=-\sum_{i=1}^{n}\gamma_i d\log p_i.\tag{34}$$

Proposition 3. *The first order approximation of the effects of financial shocks on GDP is given by:*

$$\mathrm{d}\log c=-\boldsymbol{\gamma}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{\theta}-\frac{\beta}{1-\alpha}\mathrm{d}\log w-\frac{\rho}{1-\alpha}\mathrm{d}\log r,\tag{35}$$

where

$$\mathrm{d}\log w=\frac{\eta}{1+\eta}\frac{\beta}{wL}\boldsymbol{s}'\mathbf{M}\mathrm{d}\log\boldsymbol{s}-\frac{1-\delta}{1+\eta}\mathrm{d}\log c,\tag{36}$$

$$\mathrm{d}\log r=\frac{\rho}{rK}\boldsymbol{s}'\mathbf{M}\mathrm{d}\log\boldsymbol{s},\tag{37}$$

and

$$\mathrm{d}\log\boldsymbol{s}=-\left[\mathbf{I}-\mathbf{H}\right]^{-1}\mathbf{H}\boldsymbol{\theta}+(1-\sigma)\boldsymbol{\Lambda}\boldsymbol{\theta}.$$

Proof of Proposition 3. We first find the expression for $\frac{\partial\log c}{\partial\theta_k}$. From Lemma 4:

$$\frac{\partial\log\boldsymbol{p}}{\partial\theta_k}=\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{e}_k+\frac{\beta}{1-\alpha}\frac{\partial\log w}{\partial\theta_k}\mathbf{1}+\frac{\rho}{1-\alpha}\frac{\partial\log r}{\partial\theta_k}\mathbf{1}.$$

from where we can write

$$\frac{\partial\log c}{\partial\theta_k}=-\boldsymbol{\gamma}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{e}_k-\left(\frac{\beta}{1-\alpha}\frac{\partial\log w}{\partial\theta_k}+\frac{\rho}{1-\alpha}\frac{\partial\log r}{\partial\theta_k}\right).\tag{38}$$

Therefore,

$$d\log c = -\boldsymbol{\gamma}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{\theta} - \frac{\beta}{1-\alpha}\sum_{k\in N}\frac{\partial\log w}{\partial\theta_k}\theta_k - \frac{\rho}{1-\alpha}\sum_{k\in N}\frac{\partial\log r}{\partial\theta_k}\theta_k =$$

$$-\boldsymbol{\gamma}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{\theta} - \frac{\beta}{1-\alpha}\mathrm{d}\log w - \frac{\rho}{1-\alpha}\mathrm{d}\log r,$$

which delivers equation (35).[42].

Combining the firm's decision to purchase the capital (23) and the market clearing for the capital we get:

$$rK = \rho\sum_{i=1}^{n}\frac{s_i}{\mu_i} = \rho\boldsymbol{s}'\mathbf{M}\mathbf{1},$$

where we recall that K is the aggregate (innelastic) supply of capital.

Similarly, for labor, we get:

$$wL = \beta\sum_{i=1}^{n}\frac{s_i}{\mu_i} = \beta\boldsymbol{s}'\mathbf{M}\mathbf{1}.$$

Therefore, we can write

$$\frac{\partial\log r}{\partial\theta_k} = \frac{\partial\log(rK)}{\partial\theta_k} = \frac{\partial\log(rK)}{\partial(rK)}\frac{\partial(rK)}{\partial\theta_k} = \frac{\rho}{rK}\left[\sum_{i=1}^{n}\frac{s_i}{\mu_i}\frac{\partial\log s_i}{\partial\theta_k}\right] = \frac{\rho}{rK}\boldsymbol{s}'\mathbf{M}\frac{\partial\log\boldsymbol{s}}{\partial\theta_k},$$

from where we get (37).

We recall that from Proposition 2 $\mathrm{d}\log\boldsymbol{s} = -\left[\mathbf{I}-\mathbf{H}\right]^{-1}\mathbf{H}\boldsymbol{\theta} + (1-\sigma)\boldsymbol{\Lambda}\boldsymbol{\theta}$.

As for $\frac{\partial\log w}{\partial\theta_k}$, from market clearing condition we have:

$$\frac{\partial\log w}{\partial\theta_k} = \frac{\beta}{wL}\boldsymbol{s}'\mathbf{M}\frac{\partial\log\boldsymbol{s}}{\partial\theta_k} - \frac{\partial\log L}{\partial\theta_k}.$$

To calculate $\frac{\partial\log L}{\partial\theta_k}$, we note that from the representative consumer's problem we have:

$$\log L = \frac{1-\delta}{\eta}\log c + \frac{1}{\eta}\log w,$$

and therefore,

$$\frac{\partial\log L}{\partial\theta_k} = \frac{1-\delta}{\eta}\frac{\partial\log c}{\partial\log\theta_k} + \frac{1}{\eta}\frac{\partial\log w}{\partial\theta_k},$$

which in turn implies,

$$\frac{\partial\log w}{\partial\theta_k} = \frac{\eta}{1+\eta}\frac{\beta}{wL}\boldsymbol{s}'\mathbf{M}\frac{\partial\log\boldsymbol{s}}{\partial\theta_k} - \frac{1-\delta}{1+\eta}\frac{\partial\log c}{\partial\theta_k},$$

and therefore:

$$\mathrm{d}\log w = \frac{\eta}{1+\eta}\frac{\beta}{wL}\boldsymbol{s}'\mathbf{M}\mathrm{d}\log\boldsymbol{s} - \frac{1-\delta}{1+\eta}\mathrm{d}\log c,$$

[42]Equation (35) corresponds to a more general expression derived in equation (4) in Baqaee and Farhi (2019), labeled as *ex-ante effect of distortions*. While Baqaee and Farhi (2019) does not provide explicit expression for the ex-ante effect of distortions on the GDP, we are able to do it (see Corollary 2) thanks to our parametric assumptions.

which is exactly (36). This completes the proof.

□

Corollary 2.

$$\begin{aligned}\mathrm{d}\log c = &-\left(1-\frac{1-\delta}{1+\eta}\frac{\beta}{1-\alpha}\right)^{-1}\boldsymbol{\gamma}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{\theta}\\ &-\frac{1}{1-\alpha}\left(1-\frac{1-\delta}{1+\eta}\frac{\beta}{1-\alpha}\right)^{-1}\left(\frac{\eta\beta^2}{(1+\eta)wL}+\frac{\rho^2}{rK}\right)\boldsymbol{s}'\mathbf{M}\left(-\left[\mathbf{I}-\mathbf{H}\right]^{-1}\mathbf{H}\boldsymbol{\theta}+(1-\sigma)\boldsymbol{\Lambda}\boldsymbol{\theta}\right)\end{aligned} \tag{39}$$

and

$$\mathrm{d}\log L = \frac{1-\delta}{1+\eta}\mathrm{d}\log c + \frac{1}{1+\eta}\frac{\beta}{wL}\mathrm{d}\log \boldsymbol{s} \tag{40}$$

Proof. Follows directly from Proposition 3.

□

Corollary 3. *In the absence of production network*

$$\mathrm{d}\log c = -\left(1-\frac{1-\delta}{1+\eta}\beta\right)^{-1}\boldsymbol{\gamma}'\boldsymbol{\theta} \tag{41}$$

Proof. In counterfactual without network, we set $\alpha = 0$, which implies $\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1} = \mathbf{I}$. Moreover, $\alpha = 0$ implies $\mathbf{H} = 0$ which basically captures the idea that firms are selling only to the final consumer, and therefore $\mathrm{d}\log \boldsymbol{s} = 0$. Hence, (40) becomes (41). □

Recovering markups

From Lemma 3 it follows that at the steady state

$$\mu_i = \alpha_i \frac{\tilde{\omega}_{ji}}{\omega_{ji}}.$$

We observe $\tilde{\omega}_{ji}$ and ω_{ji} in our VAT data in period where no shocks arise. For firms that have more than one supplier, we calculate μ_i as the average across all suppliers.

$$\mu_i = \alpha_i \frac{1}{d_i^-}\sum_j \frac{\tilde{\omega}_{ji}}{\omega_{ji}},$$

where d_i^- is the in-degree of firm i.