



A Public–Private Insurance Model for Disaster Risk Management: An Application to Italy

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Abstract

This paper proposes a public–private insurance model for earthquakes and floods in Italy in which the insurer and the government co-operate in risk financing. Our model departs from the existing literature by describing an insurance scheme intended to relieve the financial burden that natural events place on governments, while at the same time assisting individuals and protecting the insurance business. Hence, the business aims at maximizing social welfare rather than profits. Given the limited amount of data available on natural risks, expected losses per individual are estimated through risk-modeling. In order to evaluate the insurer’s loss profile, spatial correlation among insured assets is included. Our findings suggest that, when not supported by the government, private insurance might either financially over-expose the insurer or set premiums so high that individuals would fail to purchase policies. This evidence is stronger for earthquake risks, but it is considerable for floods too. We found that jointly managing the two perils alleviates the burden on public capitals by lowering the amount of capitals required and by keeping the probability of additional capital injections into the insurance reserves relatively low.

Keywords Disaster risk management · Insurance · Earthquakes · Floods · Italy

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1 Introduction

Natural risks pose a broad range of social, financial and economic issues, with potentially long-lasting effects. Historically, governments have mostly addressed the

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financial effects of natural events on an ad-hoc basis, but countries are now increasingly focusing on proactive planning before a disaster strikes (World Bank 2014). Among others, OECD, G20 (OECD 2012), the World Bank and GFDRR (World Bank 2014) claim that governments should guide citizens towards recovery by implementing both risk reduction and financial protection. In particular, the World Bank (World Bank 2014) argues that “absent a sustainable risk financing strategy, [...], a country with an otherwise robust disaster risk management approach can remain highly exposed to financial shocks, either to the government budget or to groups throughout society”. While guaranteeing social assistance, governments should at the same time encourage private initiatives in prevention and financial protection. In particular, since private insurance is the main risk financing tool for businesses and households, the OECD (OECD 2012) recommends that governments “assess their availability, adequacy and efficiency to the population and within the economy, as well as their costs and benefits relative to other types of possible risk reduction measures”.

Unfortunately, a series of challenges hinders the development of the natural risk insurance. First of all, spatial correlation creates the potential for enormous losses at the aggregate level, and insurers therefore need to access a large amount of capital in order to offer the cover and meet solvency constraints (Kousky and Cooke 2012). As a consequence, they are often forced to drive up premiums, which could become so high that it would not be rational for individuals to purchase the policy. Large insurers can significantly reduce the probability of insolvency by pooling risks from more independent regions or by transferring a portion of their portfolio through reinsurance. However, while lowering premiums for regions with a higher risk, this solution might raise those of regions with a lower risk and, especially in a competitive market, low risk-individuals might fail to purchase, therefore leaving the company with an extremely risky pool. As shown by Charpentier and Le Maux (2014), the free market does not necessarily provide an efficient level of natural-catastrophe insurance, but government-supported insurance allows losses from disasters to be spread more equally among policyholders thanks to the government’s easier access to credit.

Climate change also exacerbates these issues: the Geneva Association (Geneva Association 2013) warns that return periods and correlation among claims for several high-loss extreme events are “ambiguous rather than simply uncertain”, and raises concerns about the future sustainability of insurance business on natural risks. Social assistance policies may also hinder the development of private markets and increase the financial burden of natural disasters on public finances due to charity hazard (World Bank 2014).

Against this background, a number of economies have established various forms of public–private co-operation to support the insurance business, and several countries have decided to enter the market by establishing a public–private company entirely devoted to insuring citizens’ properties against natural disasters at a discounted price (Consortio de Compensación de Seguros 2008). This work proposes a public–private insurance scheme. Our model departs from the existing literature by addressing a public–private partnership, which therefore modifies the fundamental hypotheses of traditional insurance. Our work contributes to the existing literature in several aspects:

- The purpose of the insurance is social assistance, and premium collection serves solely to risk management and to guarantee quick compensation to the damaged population. Therefore, rates do not include any profit load and are commensurate to citizens' demand.
- The government is introduced in the insurance model as a social guarantor that contributes to reserves and provides public funds in case reserves are not sufficient for claim compensation.
- We accounted for spatial correlation by applying the Hoeffding's bound for so-called $(r - 1)$ -dependent random variables (Hoeffding 1963), and identified a sufficiently large threshold r such that two municipalities that are at least r -km far away from each other are independent.
- Premiums are risk-based on municipality hazard and individual structural typology, thus guaranteeing social fairness.
- Merging portfolios is beneficial if risks are uncorrelated, as floods and earthquakes are likely to be. We studied whether the benefits from risk diversification counteract the negative impact of spatial correlation by analysing multi-hazard policies.

We investigate the insurability of natural risks and apply the proposed model to Italy. Italy is an interesting case study as it is highly exposed to natural risks, especially earthquakes and floods, but only a few people insure their properties (Maccaferri et al. 2012). Most of the population expects support from the government instead. Each natural event is evaluated by public authorities when it occurs and social assistance depends on the decisions of the parties in charge and is therefore commensurate with the financial resources available at the time. In recent years public debate has increasingly shifted towards natural risk management and planning, although at the moment no initiative has been undertaken. We find evidence of the need of the government's intervention in natural risk insurance in Italy. The evidence is stronger for earthquakes, but flood insurance might benefit of a public intervention as well. We explore different insurance policies and identify the best typology for each peril. We find that the amount of public capitals needed for flood and earthquake management can be lowered by jointly managing the two perils with a multi-hazard policy.

The paper is organized as follows. Section 2 discusses the international experience. Section 3 discusses the private insurance model and presents the public–private partnership insurance model. Section 4 presents the application of the model to floods and earthquakes in Italy and illustrates the data used, the model estimation with the parameter assumptions, and presents the results. Section 5 concludes. Further technical details are in the Appendix.

2 International Experience

When insurance is properly designed, it communicates risk to the population, fosters adaptive responses and risk reduction, improves economic stability and protects the well-being of the community (Hudson et al. 2016; Kousky et al. 2018; Kunreuther and Lyster 2016; Kunreuther and Pauly 2006; Linnerooth-Bayer et al. 2019; Lotze-Campen and Popp 2012). Unfortunately, natural risks are often unbearable for private insurers

and the free market faces market failures. Understanding the natural phenomenon and the expected losses is challenging. In particular, spatial correlation among insured properties is a major threat to financial stability (see, e.g., Woodard et al. 2012). In addition, several frictions generate low take up rates among the population: lack of trust in the institution, liquidity constraints, poor risk perception, poor policy understanding and charity hazard (Chivers and Flores 2002; Cole et al. 2013; Gurenko et al. 2006; Marshall 2018; Raschky and Weck-Hannemann 2007). Moreover, private insurers are affected by state regulations, market competition (Grossi et al. 2005) and social or political decisions that may result in moral hazard and adverse selection (Kunreuther and Pauly 2009). Coordinating government's and insurers' actions can prevent these drawbacks.

There is widespread agreement on the benefits of public–private partnerships in natural disasters insurance (Kunreuther 2006b; Shukla et al. 2019; World Bank 2012). Government-supported initiatives are able to distribute risks and losses over the entire population and over time (Kunreuther and Pauly 2006), and are more flexible than private insurers' ones as they are not tied to profit goals (Penning-Rowsell 2015). Moreover, they help strengthening the resilience of a community by promoting the development of the insurance sector and allowing faster recovery (Hallegatte and Przulski 2010). However, a public intervention is beneficial only if it solves market failures that the private sector is not able to cope with (Bruggeman et al. 2010). For this reason, market failures should be properly identified and targeted.

Public–private partnerships in disaster insurance can be grouped into two macro-categories: government-supported insurance, and public reinsurance. Government-supported insurance companies are established when the risk is so high that insurers are not able to provide coverage at affordable prices. Public reinsurance compensates the lack of private reinsurance, and aims at fostering the growth of the insurance sector. In this paper we restrict our attention to the first category.

A government-supported insurance is a private company supported by the government and the private insurers operating in the country. In government-supported insurance, private insurers primarily provide technical knowledge and expertise, while the government supports the company by offering guarantee or providing a prearranged facilitated access to credit. In addition, these companies may also access contingent credit lines from international organizations (e.g., the World Bank). The main existing government-supported insurances and their characteristics are presented in Table 7 in the Appendix.

The public–private partnership offers some important advantages with respect to a fully public insurance company. First of all, insurance business requires knowledge and expertise that are not freely accessible. In a public–private partnership, the private sector provides them. Moreover, the transaction and administration costs that public companies have to bear in a free market can be higher than those of the private ones (see, e.g., Marshall 2018; Michel-Kerjan 2010). In the existing partnerships, private insurers are intermediaries between citizens and the government-supported company. They underwrite policies and transfer risks and premiums to the government-supported insurer in exchange for a low fee. Alternatively, a public-owned company may lower

the transaction and administration expenses by establishing a monopoly.¹ However, many governments enter a partnership with private insurers with the aim of creating a self-sustaining environment over time and exit the market as soon as possible. For this reason, in many partnerships private insurers share the covered risks with the public-supported company through co-insurance. Partnerships support the growth and development of the private insurance sector, while public monopolies inhibit it. Moreover, some countries ban insurance monopolies (e.g., EU countries).

Since the goal of the public–private partnership is to provide affordable policies, government-supported insurances apply low rates. However, this might cause some issues. First of all, low premiums can compete with the few private insurers that offer the policy, generating a crowding out effect and weakening the private sector (McAneney et al. 2016). Therefore, private insurers should co-operate in rating and premiums should be arranged accordingly. Moreover, governments often apply flat premium rates that include a subsidy to individuals but fail to create risk-reflecting reserves. If rates are not actuarially sound, the government-supported insurer is exposed to a high risk of reserve depleting. For this reason, risk-based premiums should be preferred to flat ones and public financial support is necessary. In the next sections, we propose a public–private insurance model describing a government-supported insurance with risk-based premiums.

3 Insurance Models

In this section we present our proposed public–private insurance model. We begin with the definition of the maximum premiums that individuals are willing to pay for a coverage in Sect. 3.1. These premiums are risk-based on the hazard exposure and on the property’s structural typology. Then, we discuss the solvency and reserve constraints that the insurer is required to meet in Sect. 3.2. In Sect. 3.3 we present the private insurance model and define the risk-based premiums that the insurer applies for each policy. Comparing the property-owners willingness to pay and the insurance constraints, we discuss whether the private insurer is able to provide the coverage at affordable prices and, if so, we identify the maximum profit that he can charge. Lastly, Sect. 3.4 presents the public–private insurance model and discusses how the government’s intervention relaxes the insurer’s constraints. We identify the corresponding risk-based premiums and quantify the risk of public capital injections into the insurance reserve.

3.1 Homeowner’s Willingness to Pay

Here we discuss the demand side, define the utility function of the owners and compute the maximum premium that they are willing to pay.

Let us consider a single peril insurance (i.e., related to earthquakes or floods only) for a specific country. Let the time t be discrete and expressed in years. A homeowner

¹ Comparative analyses on Switzerland and Germany have shown that monopolies lower these costs by eliminating the need for insurance brokers and agents and allowing the public company to keep a simpler service (Kirchgaessner 2007; Ungern-Sternberg 2001).

i has an $m_{i,t}$ square metres property. The N_{ind} individuals gather in municipalities, thus any i belongs to a generic Italian municipality c .

First, we consider the case in which the a negative event has an annual probability $1 - \pi_c(0)$ to hit the municipality c and ruin the i -th individual property ($i \in c$) at time t causing a loss $l_{i,t}$ per square metre. This loss affects his wealth $w_{i,t}$, that we assume equal to the house value for simplicity. However, the individual may buy an insurance coverage and pay a premium $p_{i,t}$ per square metre to get a reimbursement $x_{i,t}$ per square metre in case that the event occurs. Let us define $x_{i,t}$ as a function of the loss $l_{i,t}$ per square metre:

$$x_{i,t} = \begin{cases} 0, & \text{with probability } \pi_c(0), \\ x(l_{i,t}), & \text{with probability } 1 - \pi_c(0), \quad 0 \leq x(l_{i,t}) \leq l_{i,t}, \end{cases} \quad (1)$$

with $i \in c$ and

$$x(l_{i,t}) = \begin{cases} 0 & \text{if } l_{i,t} \leq D/m_{i,t}, \\ l_{i,t} - D/m_{i,t} & \text{if } D/m_{i,t} < l_{i,t} < E + D/m_{i,t}, \\ E & \text{if } l_{i,t} \geq E + D/m_{i,t}, \end{cases} \quad (2)$$

where D and E are the deductible and the maximum coverage provided per square metre by the insurer.²

The homeowner's (expected) utility of not being insured is traditionally expressed as the sum of two components representing the case of no events occurring during the year and a unique loss scenario:

$$U_{i,t,\text{not insured}} = \pi_c(0)u(w_{i,t}) + (1 - \pi_c(0))u(w_{i,t} - l_{i,t}m_{i,t}). \quad (3)$$

Similarly, the (expected) utility of purchase is defined as:

$$U_{i,t,\text{insured}} = \pi_c(0)u(w_{i,t} - p_{i,t}m_{i,t}) + (1 - \pi_c(0))u(w_{i,t} - p_{i,t}m_{i,t} - l_{i,t}m_{i,t} + x(l_{i,t})m_{i,t}). \quad (4)$$

Therefore, assuming rational behaviour, we can assume that the homeowner will buy an insurance coverage for his property if and only if his utility of purchasing is greater than or equal to that of not purchasing the policy: i.e., if and only if $U_{i,t,\text{insured}} \geq U_{i,t,\text{not insured}}$.

Now, extending the previous model by considering any possible loss level, hence any possible phenomenon intensity $\zeta \geq 0$, we can define the probability (density) $\pi_c(\zeta)$ that c will experience a ζ -intensity event in a year and that the homeowner i living in municipality c will suffer a loss $l_{i,t}(\zeta)$ per square metre, expressed as a function of ζ . In case he is owning a residential insurance coverage, his claim value

² It is common in insurance contracts to express the deductible with respect to the total value of the property. For this reason, in Eq. (4), $D/m_{i,t}$ could be interpreted as a deductible per square metre.

will be then:

$$x_{i,t} = \begin{cases} 0, & \text{with probability } \pi_c(0), \\ x(l_{i,t}(\zeta)), & \text{with probability} \\ & \text{(density) } \pi_c(\zeta), \quad 0 \leq x(l_{i,t}(\zeta)) \leq l_{i,t}, \end{cases} \quad \text{with } i \in c, \quad (5)$$

with

$$x(l_{i,t}(\zeta)) = \begin{cases} 0 & \text{if } l_{i,t}(\zeta) \leq D/m_{i,t}, \\ l_{i,t}(\zeta) - D/m_{i,t} & \text{if } D/m_{i,t} < l_{i,t}(\zeta) < E + D/m_{i,t}, \\ E & \text{if } l_{i,t}(\zeta) \geq E + D/m_{i,t}, \end{cases} \quad (6)$$

and the previous insuring condition becomes:

$$\begin{aligned} &\pi_c(0) \cdot u(w_{i,t}) + \int_0^\infty \pi_c(\zeta) \cdot u(w_{i,t} - l_{i,t}(\zeta)m_{i,t})d\zeta \\ &\leq \pi_c(0) \cdot u(w_{i,t} - p_{i,t}m_{i,t}) + \int_0^\infty \pi_c(\zeta) \cdot u(w_{i,t} - p_{i,t}m_{i,t} - l_{i,t}(\zeta)m_{i,t} \\ &\quad + x(l_{i,t}(\zeta))m_{i,t})d\zeta. \end{aligned} \quad (7)$$

We still set $w_{i,t}$ equal to the house value and assume for simplicity that it corresponds to the reconstruction cost, equal to RC per square metre. We assume that the premium $p_{i,t}$ is fixed at $t = 0$ and neither varies with respect to time (i.e., $p_{i,t} = p_i$), that the probability distribution of $l_{i,t}$ depends on the structural typology but does not depend on time, and that there are no inhabited square metres (i.e., $m_{i,t} = m_i$). According to the traditional literature on insurance purchasing decisions, we assume the individual to be risk-averse³ and, in order to compute the maximum allowable premium, we perform an analysis per square metre. In other words, we set $m_i = 1$ and we represent the individual’s preferences by means of the (per square metre) utility function $u(x) = \ln(x + 1)$.

We can compute the maximum premium p_i^H that homeowners are willing to pay by solving the equality:

$$\pi_c(0) \cdot \ln \frac{(RC + 1)}{(RC - p_i^H + 1)} + \int_0^\infty \pi_c(\zeta) \ln \frac{(RC - l_{i,t}(\zeta) + 1)}{(RC - p_i^H - l_{i,t}(\zeta) + x(l_{i,t}(\zeta)) + 1)} d\zeta = 0. \quad (8)$$

This equality states that the individual is indifferent between the two decisions of purchasing the policy or not, and allows us to derive the risk-based maximum premium

³ The standard assumptions of perfect information and rationality of individuals are often considered inadequate (Goda et al. 2015; Kunreuther and Pauly 2004; Skees et al. 1999): common shared information between insurer and insured is questionable (Cooper and Hayes 1987; Kunreuther and Pauly 1985) and individuals have limited cognitive capacity (Goda et al. 2015; Kahneman 2003) and imperfect rationality (Kunreuther 1996). However, governments have the ability to influence citizens’ behaviour through risk education and regulations. Since we are investigating whether the government’s intervention into the insurance market is necessary, we keep the traditional assumptions.

p_i^H that the individual i is willing to pay per structural typology and municipality. Indeed, premiums p_i^H for $i = 1, \dots, N_{ind}$ are risk-based and depend on the risk exposure of the municipality and on the structural typology.

3.2 Insurer

At $t = 0$ the insurer creates a reserve W , that will be increased every year by the annual premiums p_i collected from the N_{ind} individuals, $i = 1, \dots, N_{ind}$. Assume for simplicity that all the premiums are paid at the beginning of the year, while claims are paid when experienced. Hence, a minimum capital requirement W_d should be fixed, so that the insurer will have to inject a capital amount W_d in $t = 0$ and refill the fund at the end of the year t if it will decrease below this threshold. So, at the beginning (b) of the year $t = 0$ the initial reserve W_0^b is created and at the end (e) of the year it will be decreased of the total amount of reimbursement paid during the year:

$$W_0^b = W_d + \sum_{i=1}^{N_{ind}} p_i m_i, \quad W_0^e = W_0^b - \sum_{i=1}^{N_{ind}} x_{i,0} m_i.$$

Since claims $x_{i,t}$ may incur at any random discrete time period t and more events may happen close in time, the minimum capital requirement W_d is necessary to guarantee money availability for reimbursement with a sufficiently high probability. Thus, if $W_0^e < W_d$ the insurer will inject the amount $W_{r,0} = W_d - W_0^e$ in the reserve. At any subsequent time t , the reserve is:

$$W_t^b = W_{t-1} + \sum_{i=1}^{N_{ind}} p_i m_i, \quad W_{t-1} = \max(W_{t-1}^e; W_d), \quad W_t^e = W_t^b - \sum_{i=1}^{N_{ind}} x_{i,t} m_i.$$

The insurer is legally required to meet some solvency constraint and hence needs to set W_d such that the probability of not being able to promptly pay the claims (“insolvency” probability) is lower than certain small value $\epsilon_1 > 0$. Considering the worst case scenario $W_{t-1} = W_d$, we therefore have:

$$Prob \left\{ Y_t > W_d + \sum_{i=1}^{N_{ind}} p_i m_i \right\} < \epsilon_1, \quad (9)$$

where

$$Y_t = \sum_{i=1}^{N_{ind}} x_{i,t} m_i. \quad (10)$$

Let us assume that a negative event, if it occurs, hits any building within a municipality. Moreover, assume that square metre losses $l_{i,t}$ are equal for all the individuals within the same municipality, and so does $x_{i,t}$. Consider the N_{cities} municipalities in the country and indicate the total number of inhabited squared metres in the municipality c as $M_c = \sum_{i \in c} m_i$. We can also compute the total amount of claims in the time

period t as:

$$Y_t = \sum_{i=1}^{N_{ind}} x_{i,t} m_i = \sum_{c=1}^{N_{cities}} \sum_{i \in c} x_{i,t} m_i = \sum_{c=1}^{N_{cities}} X_{c,t} M_c, \quad \text{with} \quad \sum_{i \in c} x_{i,t} m_i = X_{c,t} M_c.$$

We assume that every policy can generate at most one claim per year and per individual; since reconstructing or restoring a building requires long time, this hypothesis is reasonable. Therefore, claim occurrence per year and per municipality can be modelled as a Bernoulli random variable $\bar{X}_{c,t} \sim Ber(q_c)$ (Olivieri and Pitacco 2010) with $q_c = Prob(\zeta > \zeta_{D/m_{i,t}})$ and $\zeta_{D/m_{i,t}}$ such that $l_{i,t}(\zeta_{D/m_{i,t}}) = D/m_{i,t}$. Therefore:

$$Y_t = \sum_{c=1}^{N_{cities}} \bar{X}_{c,t} \sum_j M_{j,c} x(l_{c,t,j}) = \sum_{c=1}^{N_{cities}} \bar{X}_{c,t} a_{c,t}, \tag{11}$$

where j indicates the structural typology, $M_{j,c}$ is the number of squared metres of properties of type j in municipality c , and $a_{c,t} = \sum_j M_{j,c} x(l_{c,t,j})$.

A major issue in natural disasters insurance is the presence of spatial correlation between individual risks. There is no physical bound for energy propagation and this means that we cannot consider municipalities as perfectly independent among each other, especially in the earthquakes' case. By the way, natural phenomena hit neighbour cities, but far enough municipalities fairly never experience the same event. Therefore, it could be found a certain distance r in kilometres such that municipalities whose centroids are at least r km far are independent. This assumption corresponds to the (Hoeffding 1963, Section 5d)'s definition of $(r - 1)$ -dependence.

We partition the set of municipalities in N_{gr} groups $g = 1, \dots, N_{gr}$ of independent units such that $Y_t = Y_t^1 + Y_t^2 + Y_t^3 + \dots + Y_t^{N_{gr}}$, where each among $Y_t^1, Y_t^2, Y_t^3, \dots, Y_t^{N_{gr}}$ refers to units in the same group. In other words, we create the groups in such a way that all the municipalities within a group are at least r km apart from each other. The number n_g of municipalities in group g varies with g , and the amount of claims for each group is the sum of n_g independent and bounded random variables $Y_t^g = \sum_{c \in g} \bar{X}_{c,t} a_{c,t}$.

Assuming that the hazard distribution is time-invariant (hence, removing the subscript t from the notation), we get

$$E[Y_t] = E[Y] = \sum_{g=1}^{N_{gr}} E[Y^g], \quad \text{and} \quad E[Y_t^g] = E[Y^g] = \sum_{c \in g} E[\bar{X}_{c,t} a_{c,t}]. \tag{12}$$

We can now compute the minimum capital requirement W_d by applying the bound in eq. (5.2) of Hoeffding (1963) for the weighted sum of N_{gr} dependent and bounded

random variables:

$$\begin{aligned} & \text{Prob}\left\{Y_t > N_{\text{cities}}\phi + E[Y]\right\} \\ & < \sum_{g=1}^{N_{gr}} w_g e^{-h_1\phi} E\left[e^{\frac{h_1}{n_g}(Y_t^g - E[Y^g])}\right], \quad \phi \in \mathbb{R}, \quad h_1 > 0, \quad w_g = \frac{n_g}{N_{\text{cities}}}. \end{aligned} \quad (13)$$

In the following, we describe how we choose the value of ϕ in Eq. (13). First, we set

$$W_d + \sum_{i=1}^{N_{ind}} p_i m_i = N_{\text{cities}}\phi + E[Y], \quad (14)$$

and

$$\begin{aligned} \epsilon_1 &= \sum_{g=1}^{N_{gr}} w_g e^{-h_1\phi} E\left[e^{\frac{h_1}{n_g}(Y_t^g - E[Y^g])}\right] \\ &= e^{-h_1\phi} \sum_{g=1}^{N_{gr}} w_g e^{-\frac{h_1}{n_g}E[Y^g]} E\left[e^{\frac{h_1}{n_g}\sum_{c \in g} \bar{X}_{c,t} a_{c,t}}\right] \\ &= e^{-h_1\phi} \sum_{g=1}^{N_{gr}} w_g e^{-\frac{h_1}{n_g}E[Y^g]} \mathcal{M}_{Y_t^g}\left(\frac{h_1}{n_g}\right) \\ &= e^{-h_1\phi} \sum_{g=1}^{N_{gr}} w_g e^{-\frac{h_1}{n_g}E[Y^g]} \prod_{c \in g} \mathcal{M}_{\bar{X}_{c,t} a_{c,t}}\left(\frac{h_1}{n_g}\right), \end{aligned} \quad (15)$$

where $\mathcal{M}_{Y_t^g}\left(\frac{h_1}{n_g}\right)$ is the moment generating function of Y_t^g evaluated at $\frac{h_1}{n_g}$. The last step in Eq. (15) is motivated by the fact that Y_t^g is the sum of various random variables $\bar{X}_{c,t} a_{c,t}$, which are independent since the index c is restricted to $c \in g$. Hence, we obtain $\mathcal{M}_{Y_t^g}\left(\frac{h_1}{n_g}\right) = \prod_{c \in g} \mathcal{M}_{\bar{X}_{c,t} a_{c,t}}\left(\frac{h_1}{n_g}\right)$, where $\mathcal{M}_{\bar{X}_{c,t} a_{c,t}}\left(\frac{h_1}{n_g}\right)$ is the moment generating function of each random variable $\bar{X}_{c,t} a_{c,t}$, still evaluated at $\frac{h_1}{n_g}$. Finally, we get ϕ by solving Eq. (15) with respect to it:

$$\phi = \frac{1}{h_1} \ln\left(\frac{\sum_{g=1}^{N_{gr}} w_g e^{-\frac{h_1}{n_g}E[Y^g]} \prod_{c \in g} \mathcal{M}_{\bar{X}_{c,t} a_{c,t}}\left(\frac{h_1}{n_g}\right)}{\epsilon_1}\right), \quad (16)$$

and compute W_d by substituting this value of ϕ in Eq. (14).

If $W_d < 0$, the insurer will actually set it equal to 0 and keep an insolvency probability even lower than the desired level ϵ_1 : $\epsilon_1^* \leq \epsilon_1$. Here, ϵ_1^* is defined as the

threshold value for ϵ_1 for which one gets $W_d = 0$ in the procedure reported above. Therefore, by doing this, we bind the capital requirement to non-negative values, i.e., to $W_d \geq 0$.

In order to guarantee a desired maximum probability ϵ_2 to inject further capital at any time t (i.e., to refill the fund with additional capital $W_{r,t} = W_d - W_t^e$), the insurer needs to set a premium sufficiently high. More precisely, one imposes

$$Prob\left\{W_d - W_t^e > 0\right\} = Prob\left\{W_d - W_{t-1} - \sum_{i=1}^{N_{ind}} p_i m_i + Y_t > 0\right\} < \epsilon_2. \tag{17}$$

Considering the worst case scenario $W_{t-1} = W_d$, Eq.(17) can be rewritten as:

$$Prob\left\{Y_t > \sum_{i=1}^{N_{ind}} p_i m_i\right\} < \epsilon_2. \tag{18}$$

Given a sufficiently low desired probability ϵ_2 , we can define the minimum total amount of premiums by applying again the Hoeffding (1963) inequality. We follow the same steps as in Eqs. (13)–(15):

$$\begin{aligned}
 & Prob\left\{Y_t > N_{cities}\gamma + E[Y]\right\} \\
 & < e^{-h_2\gamma} \sum_{g=1}^{N_{gr}} w_g e^{-\frac{h_2}{n_g} E[Y^g]} \prod_{c \in g} \mathcal{M}_{\bar{X}_{c,t} a_{c,t}}\left(\frac{h_2}{n_g}\right), \quad \gamma \in \mathbb{R}, \quad h_2 > 0, \tag{19}
 \end{aligned}$$

and set

$$\gamma = \frac{1}{h_2} \ln \left(\frac{\sum_{g=1}^{N_{gr}} w_g e^{-\frac{h_2}{n_g} E[Y^g]} \prod_{c \in g} \mathcal{M}_{\bar{X}_{c,t} a_{c,t}}\left(\frac{h_2}{n_g}\right)}{\epsilon_2} \right).$$

Finally, we compute the minimum total amount of premiums that allows the guaranteed probability to refill the fund to be equal to ϵ_2 , i.e., $\sum_{i=1}^{N_{ind}} p_i^G m_i$, as:

$$\sum_{i=1}^{N_{ind}} p_i^G m_i = N_{cities}\gamma + E[Y]. \tag{20}$$

It is worth mentioning that Eq. (20) only establishes the value of $\sum_{i=1}^{N_{ind}} p_i^G m_i$, but not the values assumed by each premium p_i^G . This issue is discussed in the next two subsections.

3.3 Private Insurance Model

Since the seminal papers by Ehrlich and Becker (1972) and Mossin (1968), several premium setting models have been presented in the literature. These models describe policies offered by the private sector and set premiums by comparing the risk-averse individual's willingness to pay and the profit maximization problem faced by the insurer. In the free market, insurer's profit maximization is subject to survival and/or stability constraints that require low ruin probability (Goda et al. 2015). Therefore, a private insurer sets the premium equal to

$$p_i^{PI} = p_i^G + profit_i, \quad profit_i = PL \cdot p_i^{PI} \geq 0, \quad (21)$$

where $profit_i$ is the profit loading on the i -th policy and PL is the fraction of profit loading that the insurer charges on premiums. Recalling the definition of p_i^H in Sect. 3.1, we can compute p_i^G for fixed ϵ_1 and ϵ_2 as

$$p_i^G = \kappa p_i^H, \quad \kappa = \frac{\sum_{i=1}^{N_{ind}} p_i^G m_i}{\sum_{i=1}^{N_{ind}} p_i^H m_i}. \quad (22)$$

However, if $p_i^{PI} > p_i^H$ the homeowner does not buy the policy and the risk remains uncovered. Therefore, the maximum profit load that the insurer can charge is $\max(profit_i) = p_i^H - p_i^G$, which implies $0 \leq \max(PL) \leq 1 - \kappa$, hence $\kappa \leq 1$. If $p_i^G > p_i^H$, then the private sector is not able to provide a coverage at a price that would meet the demand, while if $p_i^G = p_i^H$, then the insurer does not have incentives to provide the coverage as this implies $profit_i = 0$. As a consequence, the private insurer offers the policy only if $p_i^G < p_i^H$. In this case, the minimum capital requirement can be computed from Eq. (14) as

$$W_d^{PI} = \max \left(N_{cities} \phi + E[Y] - \sum_{i=1}^{N_{ind}} p_i^G m_i; 0 \right). \quad (23)$$

Note that $p_i^G < p_i^H$ is a necessary but not sufficient condition for the insurer to offer policies: the profit should be adequate as well.

3.4 Public–Private Insurance Model

This section presents a public–private insurance model for natural disasters where homeowners, insurers and government cooperate in risk financing. The goal of the government is maximizing social well-being, while financially protecting the insurer. The government therefore forces insurers to set the lowest premium possible given both the demand and the solvency constraints. As a consequence, rates do not include profit loading and the premium is:

$$p_i^{PPI} = \min(\kappa, 1) \cdot p_i^H. \quad (24)$$

It is worth remarking that, differently from the private insurance model in Sect. 3.3, for the public–private insurance model both cases $\kappa \leq 1$ and $\kappa > 1$ have to be considered. Indeed, a public–private insurance is needed sometimes just because the profit for a private insurance is not sufficiently high (i.e., $\kappa > 1$), so the private insurance is not offered by the private insurer. Equations (20), (22) and (24) imply that

$$\begin{aligned} \sum_{i=1}^{N_{ind}} p_i^{PPI} m_i &= \min(\kappa, 1) \sum_{i=1}^{N_{ind}} p_i^H m_i \\ &= \min\left(1, \frac{1}{\kappa}\right) (N_{cities}\gamma + E[Y]) \\ &= N_{cities}\gamma^{PPI} + E[Y]. \end{aligned} \tag{25}$$

It follows straightforwardly from Eq. (25) that $\gamma^{PPI} \leq \gamma$, which in turn implies that

$$\epsilon_2^{PPI} = \frac{\sum_{g=1}^{N_{gr}} w_g e^{-\frac{h_2}{n_g} E[Y^g]} \prod_{c \in g} \mathcal{M}_{\bar{x}_{c,t} a_{c,t}}\left(\frac{h_2}{n_g}\right)}{e^{h_2 \gamma^{PPI}}} \geq \epsilon_2. \tag{26}$$

This means that the premium p_i^{PPI} may increase the probability of additional capital injection into the reserves, setting it higher than the upper bound ϵ_2 that the insurer is able to manage. We assume that the government provides $W_{r,t}$ whenever the reserve falls below the minimum capital requirement W_d , and therefore bears the risk of further capital injections into the reserve. The minimum capital requirement W_d^{PPI} corresponding to the set of premiums p^{PPI} is then given by:

$$\begin{aligned} W_d^{PPI} &= \max\left(N_{cities}\phi + E[Y] - \sum_{i=1}^{N_{ind}} p_i^{PPI} m_i; 0\right) \\ &= N_{cities}\phi^{PPI} + E[Y] - \sum_{i=1}^{N_{ind}} p_i^{PPI} m_i, \end{aligned} \tag{27}$$

with

$$\phi^{PPI} = \frac{W_d^{PPI} + \sum_{i=1}^{N_{ind}} p_i^{PPI} m_i - E[Y]}{N_{cities}} \geq \phi. \tag{28}$$

Since ϵ_1 decreases as ϕ increases, the corresponding guaranteed insolvency probability will be at most equal to the level desired by the private insurer:

$$\epsilon_1^{PPI} = \frac{\sum_{g=1}^{N_{gr}} w_g e^{-\frac{h_1}{n_g} E[Y^g]} \prod_{c \in g} \mathcal{M}_{\bar{x}_{c,t} a_{c,t}}\left(\frac{h_1}{n_g}\right)}{e^{h_1 \phi^{PPI}}} \leq \epsilon_1. \tag{29}$$

Appendix A discusses the relationship between ϵ_1^{PPI} and ϵ_2^{PPI} and shows that, if one assumes the same value for h_1 and h_2 (i.e., $h_1 = h_2 = h$), then the model implies

that the guaranteed insolvency probability is lower than the guaranteed probability of injecting additional public capitals into reserves. Moreover, the same appendix proves that W_d^{PPI} is directly proportional to the number of municipalities, and inversely related to the parameter h , whose value is determined by the bounds ϵ_1 and ϵ_2 and the overall risk distribution.

4 Application to Italy

4.1 Data

The insurance models of Sect. 3 have been applied to the Italian residential building stock. Information on Italian real estate have been collected from three datasets: the number of buildings per municipality, number of storeys, material and year of construction in “Mappa dei rischi dei comuni italiani” by ISTAT; the average number of apartments per municipality in the 2015 census by ISTAT; the average apartment’s square metres and the reconstruction cost ($RC = 1500$) in Agenzia delle Entrate (2015).

Expected losses have been estimated by means of catastrophe risk models that compute expected monetary losses by combining four fundamental components of risk: hazard, exposure, vulnerability, and loss (Grossi et al. 2005; Mitchell-Wallace et al. 2017). In particular, we referred to the model in Asprone et al. (2013) for earthquake losses, where the authors represent the hazard probability density (i.e., $\pi(\zeta)$) through the peak ground acceleration and assume it to be constant. Preliminary analyses on the most recent hazard maps (Gruppo di Lavoro MPS 2004; Meletti and Montaldo 2007) showed that a power law distribution is an excellent fit. Therefore, we referred to this distribution for loss estimation and for the insurance models. In addition to the hazard maps and the real estate datasets, additional information were necessary for earthquake loss estimation: stratigraphic and topographic amplification factors (Colombi et al. 2010) that have been kindly provided by INGV; the series of regulations that led to the progressive re-classification of risk-prone areas from 1974⁴ to 2003.⁵ We estimated earthquake losses for 6404 over 7904 municipalities in Italy. We were not able to include Sardinia in the analysis, as the region is not exposed to earthquakes and hazard maps are not available for the area. Expected losses have been computed per five structural typologies identified by the material (masonry, reinforced concrete, and other) and the year of construction. Buildings constructed before the anti-seismic regulation are gravity loaded, otherwise are seismic loaded. Masonry buildings are gravity loaded only.

Flood losses have been estimated by adapting the approach in Apel et al. (2006) to the Italian case study. We represented flood hazard through flood frequency and depth, which have been fitted on the records from the AVI database by National Research Council (CNR) (Guzzetti and Tonelli 2004). Expected losses have then been estimated

⁴ Law n. 64, 2 Feb 1974 “Provvedimenti per le costruzioni con particolari prescrizioni per le zone sismiche”.

⁵ O.P.C.M. 3274 2003 “Primi elementi in materia di criteri generali per la classificazione sismica del territorio nazionale e di normative tecniche per le costruzioni in zona sismica”.

by means of a selection of depth-percent damage curves from the existing literature and the Italian flood risk maps (EU Directive 2007/60/CE). The model is described in Appendix B. We considered three structural typologies defined on the number of storeys of the building (one, two, three or more) and estimated losses for 7772 municipalities. Flood maps are not available for Marche region and for some parts of Sardinia and therefore the relative municipalities are not included in the analysis.

Further details on loss estimation are provided in a technical report version of this article, available at <https://arxiv.org/abs/2006.05840>. Some summary statistics on the estimated losses are presented in Table 1. As shown, seismic risk produces the highest expected losses at national level, but floods generate losses per square metre even higher than earthquakes. This happens because of the different extent of the areas exposed to the two perils: while almost all the Italian territory is exposed to earthquakes, floods affect a limited area.

4.2 Estimation

The presented models have been estimated on Italian residential risks of floods and earthquakes. In addition, we investigated multi-hazard policies covering both the perils. In fact, merging portfolios of different risks is beneficial if risks are uncorrelated, as floods and earthquakes are likely to be (Cesari and D’ Aurizio 2019; Tarvainen et al. 2006). The models can be easily extended to the multi-hazard analysis and the details are provided in the technical report version of this article. For multi-hazard policies, we restricted the attention to the municipalities for which both seismic and flood data are available, therefore we have $N_{cities} = 6217$.

We considered four policies: deductible 0 or 200 euro, and maximum coverage equal to 1200 or 1500 euro per square metre. Note that deductible equal to 0 indicates that no deductible applies to reimbursement, while maximum coverage equal to 1500 per square metre indicates that no maximum coverage applies to the policy. Therefore, the policy with $D = 0$ and $E = 1500$ provides full coverage of the risk. We estimated both the private insurance model and the public–private insurance one for each policy.

First, the maximum premium that the i -th individual is willing to pay p_i^H was calculated by solving the equality in Eq. (8). The computations for flood and earthquakes policies are fully described in Appendix C.1 and C.2. We then moved to the insurer’s side.

In order to apply the models described in Sect. 3, $\mathcal{M}_{\bar{X}_{c,t}a_{c,t}}\left(\frac{h}{n_g}\right)$ should be defined and some distributional assumption should be introduced. The choice of the best distributional form depends on the scope of the coverage, and the analysis might rather compare multiple significant scenarios represented by alternative distributional hypotheses. An informative choice, which allows some simplifications in the computations, consists in focusing on the expected value of claims, and thus assuming that Y_t is a weighted sum of Bernoulli random variables:

$$Y_t = \sum_{c=1}^{N_{cities}} \bar{X}_{c,t} \sum_j M_{j,c} \int_{\zeta_{D/m_{i,t}}}^{\infty} \pi_c(\zeta | \zeta > \zeta_{D/m_{i,t}}) x[l_{j,c,t}(\zeta)] d\zeta = \sum_{c=1}^{N_{cities}} \bar{X}_{c,t} a_{c,t}$$

Table 1 Estimated expected losses per structural typology

	Structural typology	Nr. buildings ($u = 1000$)	Mean loss/ m^2 (€)	Max loss/ m^2 (€)	Max municipal loss (Mln €)	Total loss (Mln €)
Earthquakes	Masonry	6975.98	1.913	12.69	109.54 Roma	3615.87
	Reinf. conc. (gravity)	2853.96	2.233	10.53	216.79 Roma	2223.61
	Reinf. conc. (seismic)	636.92	0.570	3.83	3.54 Roma	130.70
	Other (gravity)	1406.21	0.599	4.03	7.16 Roma	233.76
	Other (seismic)	260.88	0.480	3.22	0.43 Roma	30.73
	Total					6234.67
Floods	One storey	2083.39	0.232	19.61	7.93 S.Michele T.	105.75
	Two storeys	5981.26	0.180	15.16	36.53 Ferrara	536.14
	Three or more storeys	4123.05	0.137	11.56	18.24 Rimini	234.01
	Total					875.90

The first column indicates the structural typology (j); the second column reports the number of buildings per structural typology in Italy ($u = 1000$ means that the unit corresponds to 1000 buildings); the third and fourth columns report the mean and the maximum expected losses per square metre; the fifth column reports the maximum expected loss at the municipal level and the corresponding municipality; the last column shows the total expected loss of each structural typology in Italy

where a_c is a constant. In this case, the moment generating function of $\bar{X}_{c,t}a_c$ is:

$$\mathcal{M}_{\bar{X}_{c,t}a_c} \left(\frac{h}{n_g} \right) = \mathcal{G}_{\bar{X}_{c,t}a_c} \left(e^{\frac{h}{n_g}} \right) = \left[1 + \left(e^{\frac{h}{n_g}a_c} - 1 \right) q_c \right], \quad h = h_1, h_2,$$

where $\mathcal{G}_{\bar{X}_{c,t}a_c}$ is the probability generating function of the random variable $\bar{X}_{c,t}a_c$. In Appendix D we show that, under the Bernoulli distribution assumption above, the bounds in Eqs. (13) and (19) can be further optimized with respect to h_1 and h_2 , respectively, then they can be simplified to

$$\begin{aligned} \text{Prob} \left\{ Y_t > N_{cities}\phi + E[Y] \right\} &< \sum_{g=1}^{N_{gr}} w_g e^{-\frac{2\phi^2 n_g^2}{b_g^2}}, \quad \text{Prob} \left\{ Y_t > N_{cities}\gamma + E[Y] \right\} \\ &< \sum_{g=1}^{N_{gr}} w_g e^{-\frac{2\gamma^2 n_g^2}{b_g^2}}, \end{aligned}$$

where $b_g = \sum_{c \in g} a_c$. Therefore, we have

$$\epsilon_1 = \sum_{g=1}^{N_{gr}} w_g e^{-\frac{2\phi^2 n_g^2}{b_g^2}}, \quad \text{and} \quad \epsilon_2 = \sum_{g=1}^{N_{gr}} w_g e^{-\frac{2\gamma^2 n_g^2}{b_g^2}}. \tag{30}$$

Given ϵ_2 , we compute $\sum_{i=1}^{N_{ind}} p_i^G m_i$ via Eq. (20).

We can now estimate the private insurance model. We set ϵ_1 and ϵ_2 and compute the premiums and the minimum capital requirement as in Eqs. (22) and (23). If $p_i^G < p_i^H$, then the maximum profit load can also be calculated.

For the public–private insurance model, we compute the premium in Eq. (24). Then, we find the minimum capital requirement in Eq. (27), from which we get the corresponding ϕ^{PPI} . We compute the parameter γ^{PPI} as follows:

$$\gamma^{PPI} = \gamma \min \left(1, \frac{1}{\kappa} \right) + \frac{E[Y]}{N_{cities}} \min \left(0, \frac{1 - \kappa}{\kappa} \right).$$

We calculate ϵ_1^{PPI} and ϵ_2^{PPI} respectively by substituting ϕ with γ^{PPI} and γ with γ^{PPI} in Eq. (30).

Note that results rely on the $(r - 1)$ -dependence assumption and, therefore, on the distribution of the municipalities in the n_g groups. As a distance r allows for several grouping solutions, each model has been estimated 100 times, each time on a different set of groups of municipalities. Models’ results have been averaged, and the associated standard deviation has been reported. We assumed the geographical distance r beyond which two municipalities are independent is equal to 50 km in the case of earthquakes. We set this value based on earthquake’s impact maps by INGV, which were constructed by collecting population questionnaires on a voluntary basis. We considered the maps of recent major earthquakes with magnitude $Mw > 5$

(L'Aquila, 06-07-09 April 2009; Emilia, 20–29 May 2012; Amatrice, 24 August 2016) and computed the distance between the areas with average intensity VI-VII of the EMS scale where at least 5 questionnaires were collected. In order to identify r in the case of floods, we considered the events from 1900 to 1998 in the AVI database and computed the great-circle distance between the centroids of the municipalities flooded during each event. Since the database does not contain information on losses and we wanted to create groups of independent municipal losses, we fixed r equal to the 98-th percentile of the obtained distances. Therefore, we assumed $r = 148.02917 \simeq 150$ km. For multi-hazard policies, we also assumed $r = 150$ km.

4.3 Results

4.3.1 Maximum Premiums that Individuals are Willing to Pay

The maximum premiums that individuals are willing to pay have been computed for both earthquakes and floods. It can be easily proved that p_i^H for multi-hazard policies is equal to the sum of the maximum premiums of the two single hazard policies.⁶ Results are summarized in Tables 2 and 3 where the minimum, median, mean and maximum premiums per square metre are reported per structural typology and policy. As one can notice, the ranges of values of premiums per square metres are considerably larger for flood policies. This reflects the high variability in flood exposure of the Italian municipalities. The difference between the average and median values of flood premiums p_i^H suggests that most of the properties are located in flood low-risk areas. It is worth noticing that on average, premiums p_i^H are higher for floods, but median premiums are higher for earthquakes. This suggests that the majority of buildings are more likely to be damaged by an earthquake than by a flood, but areas highly exposed to floods are expected to produce highest annual losses than areas highly exposed to earthquakes. Overall, the maximum total amount that individuals are willing to pay for a specific flood policy exceeds the maximum total amount for the corresponding earthquake one.

4.3.2 Earthquake Policies

In this section we investigate the insurability of earthquake policies. In the context of natural disasters, two forces affect the market. On one hand, risk aversion drives individuals to buy policies at a premium higher than their expected loss. The stronger is the risk aversion, the higher is the amount of premiums that the insurer is able to collect and, in turn, the lower is the additional capital needed to satisfy the solvency constraint ϵ_1 . On the other hand, spatial correlation between insured assets inflates loss volatility and bumps the tail of the aggregate loss distribution, thereby increasing the amount of capital corresponding to ϵ_1 . The combined effect of individuals' risk aversion and correlation between the assets determines the ability of the private insurer to provide coverage.

⁶ The proof is provided in the technical report version of the article.

Table 2 Maximum premiums (€) that property-owners are willing to pay for earthquake policies

	$D = 0$ $E = 1500$	$D = 0$ $E = 1200$	$D = 200$ $E = 1500$	$D = 200$ $E = 1200$
Masonry				
Min	0.075	0.062	0.041	0.041
Median	3.698	3.077	3.124	2.724
Mean	4.461	3.910	3.975	3.544
Max	50.182	40.042	31.387	30.926
Reinf. conc. (gravity)				
Min	0.460	0.460	0.034	0.034
Median	6.291	6.244	4.023	3.701
Mean	6.620	6.582	4.413	4.106
Max	32.261	32.261	30.471	30.471
Reinf. conc. (seismic)				
Min	0.034	0.034	0.007	0.007
Median	1.244	0.995	0.837	0.979
Mean	2.005	1.676	1.351	1.350
Max	10.226	10.226	8.922	8.683
Other (gravity)				
Min	0.027	0.027	0.008	0.008
Median	1.393	1.259	1.110	1.076
Mean	1.902	1.712	1.536	1.424
Max	10.200	10.197	9.269	9.124
Other (seismic)				
Min	0.012	0.012	0.011	0.011
Median	1.138	1.076	0.902	0.878
Mean	1.745	1.535	1.278	1.205
Max	10.153	10.153	7.810	7.696
$\sum_{i=1}^{N_{ind}} p_i^H m_i$ (Mln €)	10735.78	9725.082	8837.312	8221.215

The table shows the minimum, the median, the average and the maximum premium per square metre at the municipal level per each structural typology (rows) and coverage limit (columns). The last row reports the sum of the maximum premiums that individuals are willing to pay for their properties. This last value is in million euros

Figure 1 (left plot) compares the total amount of premiums $\sum_{i=1}^{N_{ind}} p_i^H m_i$ with the minimum amount $\sum_{i=1}^{N_{ind}} p_i^G m_i$ necessary for the private insurer to provide coverage for earthquakes at varying ϵ_2 . As discussed in Sect. 3.3, $\sum_{i=1}^{N_{ind}} p_i^G < \sum_{i=1}^{N_{ind}} p_i^H$ is a necessary condition for private insurers to be able to offer policies and this corresponds to the regions where the black lines representing $\sum_{i=1}^{N_{ind}} p_i^G m_i$ stand below the corresponding red lines indicating $\sum_{i=1}^{N_{ind}} p_i^H m_i$. In the case of earthquake policies, the condition is met at high probabilities of additional capital injections, namely $\epsilon_2 > 0.06$. However, these ϵ_2 values by far exceed the current regulation requirements and this therefore suggests that the private market is not able to provide earthquake

Table 3 Maximum premiums (€) that property-owners are willing to pay for flood policies

	$D = 0$ $E = 1500$	$D = 0$ $E = 1200$	$D = 200$ $E = 1500$	$D = 200$ $E = 1200$
1 storey				
Min	0.000	0.000	0.000	0.000
Median	0.584	0.468	0.584	0.468
Mean	17.587	14.110	17.587	14.109
Max	710.000	610.000	710.000	610.000
2 storeys				
Min	0.000	0.000	0.000	0.000
Median	0.085	0.069	0.085	0.069
Mean	4.586	3.750	4.584	3.749
Max	243.529	200.387	243.445	200.303
3 or more storeys				
Min	0.0000	0.0000	0.0000	0.0000
Median	0.078	0.063	0.078	0.063
Mean	4.249	3.464	4.247	3.462
Max	227.290	186.394	227.204	186.309
$\sum_{i=1}^{N_{ind}} p_i^H m_i$ (Mln €)	11346.93	10456.83	11345.6	10455.36

The table shows the minimum, the median, the average and the maximum premium per square metre at the municipal level per each structural typology (rows) and coverage limit (columns). The last row reports the sum of the maximum premiums that individuals are willing to pay for their properties. This last value is in million euros

coverage to the whole Italian population. This finding is even more significant if we consider that the premiums p_i^H are calculated under assumptions of rather favorable risk attitude of the property-owners. In fact, empirical evidence often suggests low risk-aversion of the homeowners and therefore the identified premiums p_i^H should be considered as best case scenario. Our finding is consistent with the current Italian state of the market.⁷

Since earthquake policies do not meet the condition $\sum_{i=1}^{N_{ind}} p_i^G m_i < \sum_{i=1}^{N_{ind}} p_i^H m_i$ for reasonably low values of ϵ_2 , a market failure may emerge and therefore a public–private insurance might be desirable. Results of the public–private insurance model for earthquakes residential risks in Italy are presented in Table 4. Since $p_i^G > p_i^H$, the earthquake premium of the i -th individual is set equal to p_i^H and $\epsilon_1^{PPI} = \epsilon_1$. It follows straightforwardly that $\epsilon_2^{PPI} > \epsilon_2$. In particular, we estimated that ϵ_2^{PPI} is equal to 0.112 for the policy ($D = 200, E = 1200$), to 0.095 for ($D = 200, E = 1500$), to 0.080 for ($D = 0, E = 1200$), and to 0.061 for ($D = 0, E = 1500$). The minimum amount of public capital necessary depends on the probability ϵ_1^{PPI} and is represented in Fig. 2. As shown in the figure, introducing a 200 euro-deductible reduces the overall minimum amount of reserves W_d^{PPI} if $\epsilon_1 < 0.004$ approximately,

⁷ Cesari and D' Aurizio (2019) (p. 42) reports that only 0.8% of the Italian housing stock is insured against earthquakes and insured homes are largely located in areas at medium-low seismic risk.

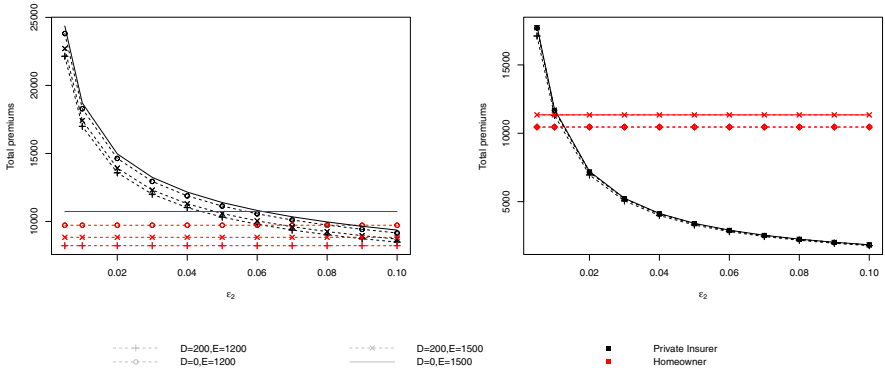
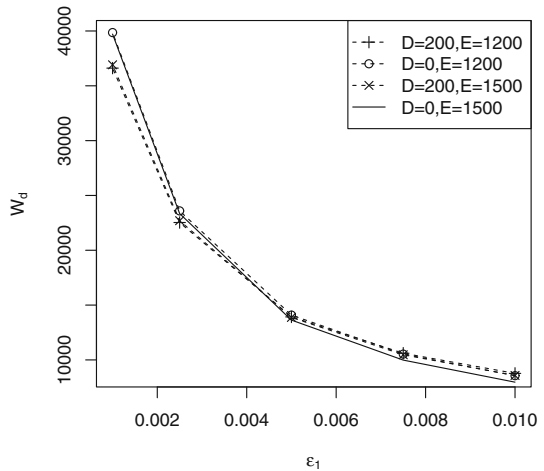


Fig. 1 Total amount of premiums $\sum_{i=1}^{N_{ind}} p_i^G m_i$ necessary for the insurer to respect the requirements (black lines) versus the probability ϵ_2 of injecting further capital into the reserves. The red lines indicate the total maximum premiums that individuals are willing to pay ($\sum_{i=1}^{N_{ind}} p_i^H m_i$). Left: earthquake insurance. Right: flood insurance

Fig. 2 Capital requirement W_d^{PPI} in million euros for earthquake policies



but substantially increases the probability of capital re-injection ϵ_2^{PPI} . For this reason, policies with $D = 0$ should be preferred. These two policies are associated to similar reserves W_d^{PPI} , but the full coverage leads to the lowest ϵ_2 . This happens because of individuals' increasing risk-aversion: property-owners appear reluctant to coverage limits⁸ and this negatively affects their willingness to pay, that in turn lowers their contribution to the reserves. Summing up, the full coverage appears the best policy for earthquake residential risk in Italy in a public–private partnership context.

⁸ Risk aversion has been here represented by means of the utility function $u(x) = \ln(x + 1)$, whose relative risk aversion coefficient is increasing in x .

Table 4 Public-private insurance scheme for earthquake risk management

Policy	ϵ_2^{PPI}	$\sum_{i=1}^{N_{ind}} P_i^{PPI} m_i$	$\frac{W_i^{PPI}}{b^{PPI}} (\epsilon_1^{PPI} = 0.01)$	$\frac{W_i^{PPI}}{b^{PPI}} (\epsilon_1^{PPI} = 0.005)$	$\frac{W_i^{PPI}}{b^{PPI}} (\epsilon_1^{PPI} = 0.001)$
200–1200	0.1123 (0.0027)	8221.2146 (0.0000)	8771.0878 (572.7528)	13,924.2026 (1085.3838)	36614.1756 (3065.7737)
0–1200	0.0804 (0.0024)	9725.0817 (0.0000)	8563.8104 (621.7234)	14087.9477 (1194.7769)	39,855.5188 (3359.5806)
200–1500	0.0946 (0.0025)	8837.3122 (0.0000)	8582.1299 (585.2765)	13,875.3617 (1106.3377)	36,916.6648 (3137.7732)
0–1500	0.0615 (0.0021)	10,735.7840 (0.0000)	7970.7257 (634.0185)	13,632.3427 (1215.5108)	39,716.0306 (3427.9691)

Policies are defined on deductible and maximum coverage (first column) and results are reported for different values of ϵ_2^{PPI} (second column). The table reports the sum of optimal premiums (column 3), the minimum capital requirement for three levels of insolvency probability (ϵ_1^{PPI}), namely 0.01 (column 4), 0.005 (column 5) and 0.001 (column 6). Results have been obtained on 100 samplings, reported values are mean and standard deviation (brackets). Quantities $\sum_{i=1}^{N_{ind}} P_i^{PPI} m_i$ and W_d^{PPI} (columns 3–6) are in million euros

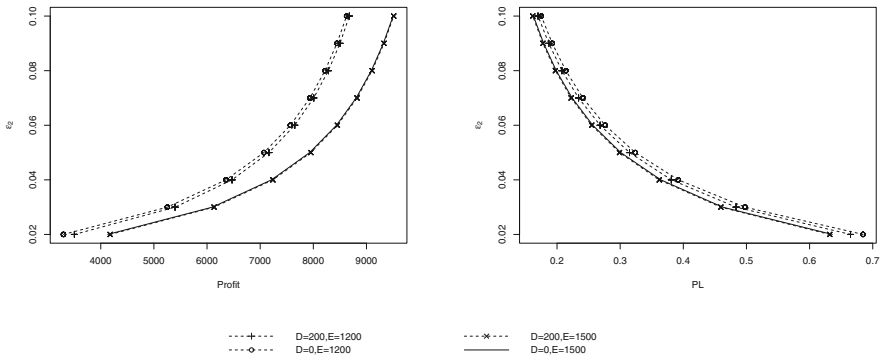


Fig. 3 Maximum profits in flood insurance per policy versus ϵ_2 . Left: total maximum profit $\sum_{i=1}^{N_{ind}} profit_i$ in million euros. Right: maximum profit load PL

4.3.3 Flood Policies

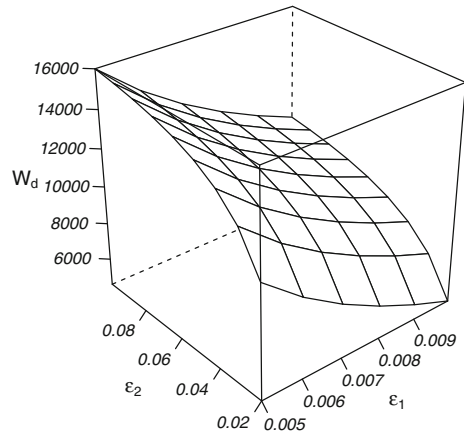
We now consider flood residential risks in Italy. As shown in the right plot of Fig. 1, flood policies meet the condition $\sum_{i=1}^{N_{ind}} p_i^G < \sum_{i=1}^{N_{ind}} p_i^H$ for ϵ_2 slightly higher than 0.01 (please notice that in this and in some of the following figures, a subset of curves overlaps). Therefore, the private market might be able to offer the policies. However, the condition is not sufficient and the business might not be profitable enough for the insurer. To this extent, Fig. 3 investigates the profits that the insurer can earn from the flood policies, and shows that for any ϵ_2 such that $\sum_{i=1}^{N_{ind}} p_i^G < \sum_{i=1}^{N_{ind}} p_i^H$, the policies ($D = 0, E = 1500$) and ($D = 200, E = 1500$) allow for the highest profits. It is worth noticing that these two policies are also associated to the lowest profit loads (right plot of Fig. 3). Identifying the optimal profit level concerns the strategic decisions of the insurance company and goes beyond the scope of this paper, but it might be argued that acceptably low values of ϵ_2 might not allow the insurer to reach the profit goal that he has set. As far as reserves are concerned, the minimum capital requirement W_d^{PI} depends on both the probabilities ϵ_1 and ϵ_2 . The value of the reserve associated to the policy ($D = 200, E = 1500$) is represented in Fig. 4.⁹

As discussed, if the probability ϵ_2 is set slightly higher than 0.01, then there is the potential for the private market to offer flood policies. Although this value might be acceptable, private insurers typically prefer lower probabilities of capital re-injection. Therefore, we investigated the public public–private partnership model too.

Results of the public–private model on flood residential risks are collected in Table 5. Three possible scenarios have been investigated for each policy. First, we considered $\epsilon_2^{PPI} = 0.02$, for which we have $\sum_{i=1}^{N_{ind}} p_i^G m_i < \sum_{i=1}^{N_{ind}} p_i^H m_i$ ($\kappa < 1$). In this case, the partnership substantially lowers the premiums and strongly increases the property-owners’ utility to buy the cover. However, the partnership also affects the minimum capital requirement, which in this case is very high. Then, we investigated the case in which $\kappa \simeq 1$, and we found that this corresponds to $\epsilon_2^{PPI} \approx 0.11$ for all the policies. In this scenario, the partnership requires premiums that are slightly

⁹ The corresponding plot for ($D = 0, E = 1500$) is similar and has been neglected.

Fig. 4 Capital requirement W_d^{PPI} in million euros for flood policies $D = 0, E = 1500$ with respect to ϵ_1 and ϵ_2



lower than the maximum that individuals are willing to pay. The higher amount of premiums collected lowers the capital requirement W_d^{PPI} . At last, we considered $\kappa > 1$. In this case we have $p_i^{PPI} = p_i^H$, while ϵ_2^{PPI} and W_d^{PPI} are slightly lower than the corresponding values in the scenario $\kappa \simeq 1$. These results suggest that when $\kappa < 1$ the public–private partnership is mostly beneficial to the property-owners, but might potentially over-stress public finances. If $\kappa \geq 1$, the partnership brings moderate benefits to property-owners, but appears more financially sustainable for the government. Therefore, a public–private partnership is more beneficial in the last two scenarios. We therefore restricted our attention to $\kappa \geq 1$.

We can notice that policies with deductibles $D = 200$ have both smaller W_d^{PPI} and smaller ϵ_2^{PPI} . This emerges due to a combination of risk aversion and loss distribution. Floods are high frequency-low intensity perils and mostly generate small claims on relatively low return times. Increasing risk aversion makes individuals extremely averse to high losses and less concerned about low damages that they can afford by their own. As it could be noted in Table 3, when applying the deductible $D = 200$, p_i^H remains substantially unchanged. By contrast, the maximum coverage $E = 1200$ increases both ϵ_2^{PPI} and W_d^{PPI} . This happens because this policy limit lowers the tail of the distribution of the insurer's aggregate loss, but the highest levels of risk remain to property-owners. Because of increasing risk aversion, the premium individuals are willing to pay is much lowered, and the amount of public funds needed much increased. Summing up, we conclude that the policy ($D = 200, E = 1500$) should be preferred for flood risk management.

4.3.4 Multi-Hazard Policies

Here we investigate whether benefits from risk diversification in multi-hazard policies counteract the negative impact of spatial correlation. First of all, the comparison between the premiums $\sum_{i=1}^{N_{ind}} p_i^G m_i$ and $\sum_{i=1}^{N_{ind}} p_i^H m_i$ (Fig. 5) shows that the private market is able to supply policies for $\epsilon_2 \approx 0.03$. This value is quite high, and the government's intervention could be considered necessary.

Table 5 Public–private insurance scheme for flood risk management

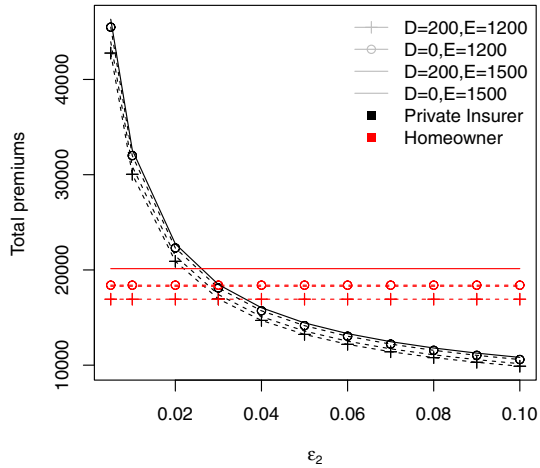
Policy	ϵ_2^{PPI}	$\sum_{i=1}^{N_{ind}} p_i^{PPI} m_i$	$W_{d,i}^{PPI}$ ($\epsilon_1^{PPI} = 0.01$)	$W_{d,i}^{PPI}$ ($\epsilon_1^{PPI} = 0.005$)	$W_{d,i}^{PPI}$ ($\epsilon_1^{PPI} = 0.001$)
$(D = 200, E = 1200)$	0.02000	6950.4158	4356.8281	10,165.1768	32,089.8977
	(0.0000)	(368.2914)	(525.6878)	(1147.8282)	(5060.4870)
	0.01130	10,421.1646	886.0792	6694.4280	28619.1488
	(0.0009)	(127.0803)	(624.3060)	(1030.9692)	(4833.0137)
	0.01126	10,455.3647	886.0792	6660.2279	28584.9488
	(0.0010)	(0.0000)	(624.3060)	(1067.9553)	(4779.9064)
$(D = 0, E = 1200)$	0.02000	7158.6717	4513.1478	10,549.1687	33,194.0574
	(0.0000)	(382.3598)	(543.4462)	(1197.9748)	(5217.8691)
	0.01181	10,445.1439	1226.6756	7262.6964	29,907.5852
	(0.0010)	(58.1938)	(681.6797)	(1091.3831)	(4944.6918)
	0.01179	10,456.8315	1226.6756	7251.0088	29,895.8976
	(0.0010)	(0.0000)	(681.6797)	(1107.3653)	(4926.9294)
$(D = 200, E = 1500)$	0.02000	7169.3174	4518.6147	10,559.2964	33,229.9405
	(0.0000)	(382.7245)	(544.0119)	(1197.9184)	(5227.0833)
	0.01066	11,198.8084	489.1237	6529.8055	29,200.4496
	(0.0007)	(290.3450)	(518.6516)	(1006.9280)	(5099.1912)
	0.01046	11,345.6026	489.1237	6383.0113	29053.6554
	(0.0010)	(0.0000)	(518.6516)	(1107.7612)	(4935.7903)

Table 5 continued

Policy	ϵ_2^{PPI}	$\sum_{i=1}^{N_{ind}} p_i^{PPI} m_i$	W_d^{PPI} ($\epsilon_1^{PPI} = 0.01$)	W_d^{PPI} ($\epsilon_1^{PPI} = 0.005$)	W_d^{PPI} ($\epsilon_1^{PPI} = 0.001$)
($D = 0, E = 1500$)	0.02000 (0.00000)	7216.7129 (385.3391)	4548.6469 (547.7111)	10630.5139 (1206.4844)	33453.8370 (5261.6497)
	0.01073 (0.0007)	11,222.5342 (267.5681)	542.8255 (545.0641)	6624.6926 (1026.6015)	29,448.0157 (5114.4159)
	0.01056 (0.0010)	11346.9258 (0.0000)	542.8255 (545.0641)	6500.3010 (1115.5769)	29,323.6241 (4968.3906)

Policies are defined on deductible and maximum coverage (first column) and results are reported for different values of ϵ_2^{PPI} (second column). The table reports the sum of optimal premiums (column 3), the minimum capital requirement for three levels of insolvency probability (ϵ_1^{PPI}), namely 0.01 (column 4), 0.005 (column 5) and 0.001 (column 6). Results have been obtained on 100 samplings and reported values are mean and standard deviation (brackets). Quantities $\sum_{i=1}^{N_{ind}} p_i^{PPI} m_i$ and W_d^{PPI} (columns 3-6) are in million euros

Fig. 5 Total amount of premiums $\sum_{i=1}^{N_{ind}} p_i^G m_i$ for multi-hazard policies necessary for the insurer to respect the requirements (black lines) versus the probability ϵ_2 of injecting further capital into the reserves. The red lines indicate the total maximum premiums that individuals are willing to pay ($\sum_{i=1}^{N_{ind}} p_i^H m_i$)



We therefore investigated the public–private insurance model for multi-hazard policies. Results are presented in Table 6 together with the corresponding single hazard policies, that have been re-estimated considering only the municipalities for which both earthquakes and floods data were available for the sake of comparability. As previously discussed, multi-hazard premiums p_i^H are given by the sum of the maximum premiums that individuals are willing to pay for floods and earthquakes. For the investigated values of ϵ_2 , we have $\sum_{i=1}^{N_{ind}} p_i^{PPI} m_i = \sum_{i=1}^{N_{ind}} p_i^H m_i$. It is worth noticing that the coverage limits strongly affect the composition of multi-hazard premiums $\sum_{i=1}^{N_{ind}} p_i^{PPI} m_i$. In particular, if $D = 200$ ($D = 0$), the multi-hazard premium $\sum_{i=1}^{N_{ind}} p_i^{PPI} m_i$ is mostly determined by the flood (earthquake) coverage.

Moreover, we observe that multi-hazard W_d^{PPI} is always lower than the sum of the minimum capital requirement of the two single peril’s policies. In particular, for $\epsilon_1^{PPI} = \epsilon_1 = 0.01$, the value of W_d^{PPI} is lower than the corresponding earthquake policies’ value. This is the effect of risk diversification. Moreover, the associated probability of further capital injection into the reserves, ϵ_2^{PPI} , is a bit higher than the corresponding probability for flood policies, but is much lower with respect to the earthquakes’ policies. Overall, this evidence suggests that the multi-hazard policies should be preferred.

As far as coverage limits concern, the full coverage policy results in the minimum probability ϵ_2^{PPI} . The policy is also associated with low values of W_d^{PPI} . As an alternative, the policy ($D = 200, E = 1500$) requires the lowest capital requirements for the lowest values of ϵ_1^{PPI} and a slightly higher probability ϵ_2^{PPI} .

5 Conclusion

Flood and earthquake risks in Italy have been investigated. We showed that the private market is not able to insure the whole residential risks and that the country may face a market failure. Due to spatial correlation among insured assets, the maximum

Table 6 Public-private insurance scheme for multi-hazard risk management

Policy	Hazard	ϵ_2^{PPI}	$\sum_{i=1}^{N_{ind}} p_i^{PPI} m_i$	W_c^{PPI} ($\epsilon_1^{PPI} = 0.01$)	W_d^{PPI} ($\beta^{PPI} = 0.005$)	W^{PPI} ($\epsilon_1^{PPI} = 0.001$)
200–1200	Mh	0.0301	16,928.0146 (0.0000)	13,115.2146 (1385.6749)	25,841.3207 (2452.3660)	72,628.2809 (9920.6266)
	Eq	0.1139	7907.1993 (0.0000)	14,607.3177 (937.1676)	23,814.2876 (2042.1647)	63,844.8490 (7738.9713)
	FI	0.0112	9020.8153 (0.0000)	849.7164 (628.9989)	6843.7589 (1075.1615)	29,718.0746 (4826.9196)
0–1200	Mh	0.0290	18,400.5163 (0.0000)	13,604.3658 (1473.2700)	27,092.5567 (2628.6721)	77,319.9737 (10450.1542)
	Eq	0.0866	9378.4333 (0.0000)	14,866.9860 (1011.3834)	24,737.9462 (2225.7351)	68,163.9598 (8560.2621)
	FI	0.0117	9022.0829 (0.0000)	1134.1985 (679.4467)	7360.6097 (1106.0026)	30,961.7685 (4977.1965)
200–1500	Mh	0.0272	18,286.2227 (0.0000)	12,587.5538 (1423.8134)	25,669.7775 (2511.9983)	73,690.8827 (10,233.5862)
	Eq	0.0991	8507.8142 (0.0000)	14,574.5766 (959.4709)	24,013.2261 (2092.0910)	64,997.8647 (7892.7980)
	FI	0.0105	9778.4086 (0.0000)	509.7442 (540.7445)	6625.7567 (1107.4550)	30,255.3593 (4985.8925)

Table 6 continued

Policy	Hazard	ϵ_2^{PPI}	$\sum_{i=1}^{N_{ind}} p_i^{PPI} m_i$	W_d^{PPI} ($\epsilon_1^{PPI} = 0.01$)	W_d^{PPI} ($\epsilon_1^{PPI} = 0.005$)	W_d^{PPI} ($\epsilon_1^{PPI} = 0.001$)
0–1500	Mh	0.0253	20,136.3991 (0.0000)	12,470.5695 (1497.2935)	26,187.5460 (2672.0534)	77,275.9819 (10648.3697)
	Eq	0.0703	10,356.8594 (0.0000)	14,443.3536 (1033.4068)	24,540.8081 (2275.2421)	68,897.8136 (8704.7545)
	FI	0.0106	9779.5398 (0.0000)	559.5089 (561.1034)	6733.6368 (1114.9029)	30,521.6830 (5018.5600)

Policies are defined on deductible and maximum coverage (first column) and results are reported for multi-hazard (Mh), earthquake (Eq), and flood (FI) policies (second column) and for different values of ϵ_2^{PPI} (third column). The table reports the sum of optimal premiums (column 4), the minimum capital requirement for three levels of insolvency probability (ϵ_1^{PPI}), namely 0.01 (column 5), 0.005 (column 6) and 0.001 (column 7). Results have been obtained on 100 samplings, reported values are mean and standard deviation (brackets). Quantities $\sum_{i=1}^{N_{ind}} p_i^{PPI} m_i$ and W_d^{PPI} (columns 4–7) are in million euros

premiums that individuals are willing to pay do not meet the insurer's solvency and capital constraints. This evidence is stronger for earthquake policies. Without the government, a private insurer would be forced to drive up premiums, which would not meet the demand. Therefore, we argued that the government's intervention in insurance is necessary to guarantee proper access to insurance to the population against floods and earthquakes.

To this aim we proposed a public–private insurance model. Our model is intended to relieve the financial burden that natural events pose on governments, while at the same time supporting individuals and protecting the insurance business. We found that the best policy for earthquake risk management is the full coverage, while a deductible should be applied to flood policies.

Provided that the public intervention is necessary, the effectiveness of the insurance system depends on a number of conditions. First of all, it is essential to achieve a satisfactory understanding of the natural phenomenon and the extent of the losses to which it can lead. Secondly, actions to avoid low take up rates should be undertaken. Educating the population has often fostered the adoption of policies (Bogale 2015; Gan et al. 2014). In particular, raising awareness on natural disasters in quiet times is important, as the prolonged absence of major events leads to lowered attention and decreases policy's purchase (Gan et al. 2014; Gallagher 2014). If this is not sufficient, mandatory insurance purchase tackles the root problem (Kunreuther and Pauly 2006), but the obligation should be properly formulated and monitored (Dixon et al. 2006).¹⁰ However, mandatory requirement may not be well received by citizens and frequent monitoring might be expensive.

We found that the probability that the government will have to inject further capital into the insurance reserves among time might be moderate. To this aim, we investigated multi-hazard policies covering both earthquake and floods and found significant advantages in jointly managing the two perils: the amount of public capital necessary for multi-hazard policies is lower than the sum of the reserves necessary to separately manage the two. In addition, the government can lower the probability of public capital injections by means of risk mitigation (Kunreuther 2006a, 2015). Building codes and urban planning are powerful tools, provided that the government carefully coordinates its management goals with risk reduction objectives.¹¹ Nevertheless, risk reduction remains largely demanded to citizens, who often consider the investment not advantageous (Kleindorfer et al. 2005). To this aim, premium discounts to retrofitted properties are common incentives. Along with risk mitigation, reinsurance lines and insurance-linked securities also help limiting public capital injections by allowing the insurer to get rid of the highest layers of risk (OECD 2018). As natural risks evolve quickly, risk transfer tools are proving increasingly necessary for government-supported insurers

¹⁰ For example, in Turkey, property-owners are required to prove to have valid policy only when they want to buy or sell a house or to obtain a new account for water and electricity services. As argued by Başbuğ Erkan and Yılmaz (2015), this sporadic check does not enforce ongoing renewal of the insurance.

¹¹ The governments' management objectives strongly hindered risk reduction in many countries. For example, in France the flood risk mitigation measures were not implemented properly because of the urban growth goal of the local authorities (Vallet 2004). The Florida Catastrophe Insurance Fund was launched to encourage urban growth but this increased the risk exposure over time, powered by climate change (Seo 2004). In the UK, there is a heated debate on how the government is honouring its risk reduction commitments (Penning-Rowsell 2015; Surminski 2018; Surminski and Eldridge 2017).

to survive (Seo 2004). Further research might extend the model to additional layers of risk transfer such as reinsurance or CatBonds.

At last, in the presented analysis we did not take into account the effect of climate change. This is not relevant for earthquakes, but affects flood risk management. Flood losses are expected to increase in the future, as the amount of assets exposed to flooding is growing (Kovats et al. 2014). An in-depth examination of changing risks may bring important insights on the topic and will help decision makers in implementing the most effective protection strategies.

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Appendix A: Relationship Between the Bounds

In this section we discuss the mathematical relationship between the two guaranteed probabilities ϵ_1^{PPI} and ϵ_2^{PPI} in the public–private model. From Eqs. (25) and (27) we have

$$\begin{aligned}
 W_d^{PPI} &= N_{cities} \phi^{PPI} + E[Y] - \sum_{i=1}^{N_{ind}} p_i^{PPI} m_i \\
 &= N_{cities} \phi^{PPI} + E[Y] - E[Y] - N_{cities} \gamma^{PPI} \\
 &= N_{cities} (\phi^{PPI} - \gamma^{PPI}).
 \end{aligned} \tag{31}$$

Moreover, in a similar way as for Eq. (15), one gets

$$\begin{aligned}
 \epsilon_2^{PPI} &= \frac{\sum_{g=1}^{N_{gr}} w_g e^{-\frac{h_2}{n_g} E[Y^g]} \prod_{c \in g} \mathcal{M}_{\bar{X}_{c,t} a_{c,t}} \left(\frac{h_2}{n_g} \right)}{e^{h_2 \gamma^{PPI}}} \\
 &\geq \frac{\sum_{g=1}^{N_{gr}} w_g e^{-\frac{h_2}{n_g} E[Y^g]} \prod_{c \in g} \mathcal{M}_{\bar{X}_{c,t} a_{c,t}} \left(\frac{h_2}{n_g} \right)}{e^{h_2 \gamma}} = \epsilon_2,
 \end{aligned} \tag{32}$$

hence

$$\gamma^{PPI} = \frac{1}{h_2} \ln \left(\frac{\sum_{g=1}^{N_{gr}} w_g e^{-\frac{h_2}{n_g} E[Y^g]} \prod_{c \in g} \mathcal{M}_{\bar{X}_{c,t} a_{c,t}} \left(\frac{h_2}{n_g} \right)}{\epsilon_2^{PPI}} \right). \quad (33)$$

Similarly, one gets

$$\phi^{PPI} = \frac{1}{h_1} \ln \left(\frac{\sum_{g=1}^{N_{gr}} w_g e^{-\frac{h_1}{n_g} E[Y^g]} \prod_{c \in g} \mathcal{M}_{\bar{X}_{c,t} a_{c,t}} \left(\frac{h_1}{n_g} \right)}{\epsilon_1^{PPI}} \right). \quad (34)$$

Since $W_d^{PPI} \geq 0$, combining Eqs. (31), (33) and (34) we find

$$\frac{\left(\sum_{g=1}^{N_{gr}} w_g e^{-\frac{h_1}{n_g} E[Y^g]} \prod_{c \in g} \mathcal{M}_{\bar{X}_{c,t} a_{c,t}} \left(\frac{h_1}{n_g} \right) \right)^{\frac{1}{h_1}}}{\left(\sum_{g=1}^{N_{gr}} w_g e^{-\frac{h_2}{n_g} E[Y^g]} \prod_{c \in g} \mathcal{M}_{\bar{X}_{c,t} a_{c,t}} \left(\frac{h_2}{n_g} \right) \right)^{\frac{1}{h_2}}} \cdot \frac{(\epsilon_2^{PPI})^{\frac{1}{h_2}}}{(\epsilon_1^{PPI})^{\frac{1}{h_1}}} \geq 1. \quad (35)$$

In particular, if one sets $h_1 = h_2 = h$, then Eq. (31) becomes

$$W_d^{PPI} = \frac{N_{cities}}{h} \ln \left(\frac{\epsilon_2^{PPI}}{\epsilon_1^{PPI}} \right),$$

and Eq. (35) simplifies to:

$$\epsilon_2^{PPI} \geq \epsilon_1^{PPI}.$$

The inequality indicates that the guaranteed insolvency probability must be lower than the guaranteed probability of injecting additional public capitals into reserves, thus enforcing the government's role of social guarantor. Moreover, it implies that the minimum W_d^{PPI} value corresponds to $\epsilon_1^{PPI} = \epsilon_2^{PPI}$ and is equal to 0.

Appendix B: Flood Risk Assessment

Flood risk was assessed combining hazard, exposure, vulnerability and loss (Grossi et al. 2005; Mitchell-Wallace et al. 2017). Further details are provided in a technical report version of the article, available at <https://arxiv.org/abs/2006.05840>.

(i) **Hazard** Flood hazard was represented by flood frequency and depth probabilities.

Both distributions were estimated on floods events after 1900, collected in the AVI database and fitted by means of non-parametric techniques due to the small number of events recorded (795).¹² As far as flood frequency $P_c(\text{flood})$ concerns, its municipal probability was estimated through the following steps:

¹² Each event corresponds to multiple records in the database, one for each area that has been flooded.

1. Two clusters of municipalities were identified on the basis of the hydrological hazard index $P2$ in the database “Mappa dei rischi dei comuni italiani” by ISTAT. Specifically, we considered $P2 < 0.5$ and $P2 \geq 0.5$. Each clusters’ number of floods in a year was analyzed and the best fit was achieved by the negative binomial distribution for both.
2. Since each flood involved a certain number of municipalities within the cluster, the municipal probability of experiencing at least one flood in a year was estimated by multiplying the cluster flood frequency and the average ratio of flooded municipalities in the cluster.
3. A flood strikes several municipalities, but not all the properties in a municipality get flooded. We adjusted the municipal probability by the $P3$ index in the ISTAT database. The index indicates the percentage of municipal surface flooded in a 20–50 years probabilistic scenario.

The conditional probability density $f(\delta|flood)$ of a flood to reach a certain depth δ (conditional to the flood occurrence) was estimated on the maximum depth levels reported in the AVI database for a flood event. Unfortunately, this information was available for 475 events only. Therefore, we fitted the depth distribution at the national level. The Gamma distribution resulted in the best fit.

- (ii) **Exposure** Data on the building stock were obtained from the ISTAT database, and municipal buildings were classified in three groups j according to the number of storeys—one, two, and three or more. In addition, the average number of apartments per building from the 2015 Italian census (ISTAT) and the average apartment’s surface in (Agenzia delle Entrate 2015) were used to represent the Italian residential exposure. Combining the three information, we computed the total number $E_{j,c}$ of square metres of the j -th structural typology in the municipality c .
- (iii) **Vulnerability** Flood’s vulnerability was represented with a selection of depth-percent damage curves for each structural typology j : Appelbaum (1985); Arrighi et al. (2013); Debo (1982); Genovese (2006); Luino et al. (2009); Oliveri and Santoro (2000). These curves represent the damage of a structure as a percentage of its value and are functions of the flood depth. Selected curves per structure were then averaged into three curves $v_j(\delta)$. The curves were fitted through polynomial regressions.
- (iv) **Loss** We assumed that the property value is equal to its reconstruction cost (RC), taking its average value of 1500 euros per square metre, constant among all the municipalities (Agenzia delle Entrate 2015).

We estimated the expected losses per square metre, structural typology and municipality as

$$l_{j,c} = \frac{RC}{100} \cdot \int_0^\infty v_j(\delta) P_c(flood) f(\delta|flood) d\delta, \tag{36}$$

and the municipal flood losses L_c as

$$L_c = \sum_{j=1}^3 l_{j,c} \cdot E_{j,c}. \tag{37}$$

Our estimates are compatible to those presented by ANIA and Guy Carpenter (2011), which estimate that residential losses generated by river flooding amount to about 230 million euros per year and constitute nearly 8% of the total annual expected loss generated by both river floods and earthquakes. We estimated that the overall flood risk produces approximately 12% of the total annual expected loss due to earthquakes and floods. The model turned out to be robust with respect to the distributional choices and to the years chosen for the analysis.

Appendix C: Homeowners' Willingness to Pay

The next two subsections discuss the application of the equality in Eq. (8) to earthquake (C.1) and flood (C.2) policies. Again, further details are provided in the technical report version of this article.

C.1: Earthquake Policies

For earthquake policies, $\zeta = PGA$, which is the peak ground acceleration. The associated probability density $\pi_c(\zeta)$ is given by $\pi_c(PGA) = \left| \frac{d\lambda_c(PGA)}{d(PGA)} \right|$ (Asprone et al. 2013), where $\lambda_c(PGA)$ is its cumulative probability distribution function. The absence of seismic movements $\zeta = 0$ corresponds to the case of no seismic event happening in the year, thus we have $l_{i,t}(0) = 0$ and $x(l_{i,t}(\zeta)) = 0$. Therefore we can compute Eq. (8) as:

$$\int_0^{\infty} \pi_c(PGA) \ln \frac{(RC - l_{i,t}(PGA) + 1)}{(RC - p_i^H - l_{i,t}(PGA) + x(l_{i,t}(PGA)) + 1)} d(PGA) = 0. \quad (38)$$

$\lambda_c(PGA)$ is approximately power law-distributed and therefore:

$$\pi_c(PGA) = \begin{cases} \left| \frac{d(\lambda_c(PGA))}{d(PGA)} \right| = \alpha_c PGA^{-\beta_c}, & \text{if } PGA \geq PGA_{min_c} = e^{\frac{\ln(\frac{\alpha_c}{\beta_c - 1})}{\beta_c - 1}}, \\ 0, & \text{otherwise.} \end{cases}$$

In our data PGA_{min_c} take values ranging from $7.92e^{-09}$ to 0.002, which are small enough to include the case of no seismic loss. The loss function per structural typology $l_{j,t}(PGA)$ is obtained from the model in Asprone et al. (2013) as:

$$l_{j,t}(PGA) = \frac{1}{K_j} \sum_{k=1}^{K_j} \sum_{LS=1}^{N_{LSk}} RC_k(LS) \cdot [P_k(LS|PGA) - P_k(LS+1|PGA)],$$

where an average on K_j fragility models is computed. In the above, each model k is characterized by N_{LSk} limit states representing building's structural damage conditions, and $P_k(N_{LSk} + 1|PGA) = 0$. Concluding, Eq. (38) for earthquakes risk in

Italy becomes:

$$\int_{PGA_{min_c}}^{\infty} \alpha_c PGA^{-\beta_c} \ln \left(\frac{RC - l_{i,t}(PGA) + 1}{RC - p_i^H - l_{i,t}(PGA) + x(l_{i,t}(PGA)) + 1} \right) d(PGA) = 0. \quad (39)$$

C.2: Flood Policies

We refer to the flood risk assessment in Appendix B and apply Eq. (8) to the case of floods in Italy:

$$P_c(flood = 0) \ln \left(\frac{RC + 1}{RC - p_i^H + 1} \right) + P_c(flood > 0) \int_0^{\infty} f(\delta|flood) \cdot \ln \left(\frac{RC - \frac{RC}{100} v_j(\delta) + 1}{RC - p_i^H - \frac{RC}{100} \cdot v_j(\delta) + x \left[\frac{RC}{100} \cdot v_j(\delta) \right] + 1} \right) d\delta = 0. \quad (40)$$

We know that: (i) v_j is a non-negative non-decreasing function that becomes constant at level 100% corresponding to a certain depth δ_{max} ; (ii) there exists $\delta_D > 0$ such that $v_j(\delta_D) \cdot \frac{RC}{100} = D/m_{i,t}$; (iii) there exists $\delta_E > 0$ such that $v_j(\delta_E) \cdot \frac{RC}{100} = E + D/m_{i,t}$. Therefore, Eq. (40) can be rewritten as

$$P_c(flood = 0) \cdot \ln \left(\frac{RC + 1}{RC - p_i^H + 1} \right) + (P_c(flood > 0)) \cdot \left\{ \int_0^{\delta_{max}} f(\delta|flood) \cdot \ln \left(RC - \frac{RC}{100} \cdot v_j(\delta) + 1 \right) d\delta - \int_0^{\delta_D} f(\delta|flood) \cdot \ln \left(RC - p_i^H - \frac{RC}{100} \cdot v_j(\delta) + 1 \right) d\delta - \ln \left(RC - p_i^H - D + 1 \right) \cdot [F(\delta_E|flood) - F(\delta_D|flood)] - \int_{\delta_E}^{\delta_{max}} f(\delta|flood) \cdot \ln \left(RC - p_i^H - \frac{RC}{100} \cdot v_j(\delta) + E + 1 \right) d\delta - \ln \left(E - p_i^H + 1 \right) [1 - F(\delta_{max}|flood)] \right\} = 0.$$

Appendix D: Application of the Hoeffding's Bound to the Weighted Sum of Bernoulli Random Variables

Bernoulli variables are bounded in $[0, 1]$, implying that $0 \leq Y_t^g \leq \sum_{c \in g} a_c = b_g$.

According to Hoeffding (1963), the bounds in Eqs. (13) and (19) simplify for the case of bounded weighted random variables. Consider, for instance, the bound in Eq. (13):

$$\text{Prob} \left\{ Y_t > N_{cities} \phi + E[Y] \right\} < \sum_{g=1}^{N_{gr}} w_g e^{-h_1 \phi} E \left[e^{\frac{h_1}{n_g} (Y_t^g - E[Y^g])} \right], \quad \phi \in \mathbb{R}, \quad h_1 > 0.$$

According to Lemma 1 in Hoeffding (1963), since the final term in the right-hand side of the inequality is convex, we know that:

$$\begin{aligned} E \left[e^{\frac{h_1}{n_g} (Y_t^g - E[Y^g])} \right] &\leq e^{\frac{h_1}{n_g} E[Y^g]} \left[\frac{b_g - E[Y^g]}{b_g} + \frac{E[Y^g]}{b_g} e^{\frac{h_1}{n_g} b_g} \right] \\ &= e^{-\frac{h_1}{n_g} E[Y^g]} \left[1 + \frac{E[Y^g]}{b_g} \left(e^{\frac{h_1}{n_g} b_g} - 1 \right) \right] = e^{L(h_g)}, \end{aligned}$$

where $L(h_g) = -h_g p_g + \ln(1 + p_g (e^{h_g} - 1))$ and

$$h_g = \frac{h_1}{n_g} b_g, \quad \text{and} \quad p_g = \frac{E[Y^g]}{b_g}.$$

According to the proof of Theorem 2 in Hoeffding (1963), one gets

$$L(h_g) \leq \frac{1}{8} h_g^2 = \frac{1}{8} \left(\frac{h_1 b_g}{n_g} \right)^2,$$

hence the bound can be rewritten as

$$\text{Prob} \left\{ Y_t > N_{cities} \phi + E[Y] \right\} < \sum_{g=1}^{N_{gr}} w_g e^{-h_1 \phi} \left(e^{\frac{1}{8} \left(\frac{h_1 b_g}{n_g} \right)^2} \right) = \sum_{g=1}^{N_{gr}} w_g e^{-h_1 \phi + \frac{1}{8} \left(\frac{h_1 b_g}{n_g} \right)^2}. \quad (41)$$

In order to get the best possible upper bound, we minimize the right-hand side of the inequality (41) with respect to h_1 and we express the minimizer as a function of ϕ , thus obtaining

$$h_1 = \frac{4\phi n_g^2}{b_g^2}.$$

Substituting the parameter h_1 in Eq. (41), the Hoeffding's bound simplifies to

$$Prob\left\{Y_t > N_{cities}\phi + E[Y]\right\} < \sum_{g=1}^{N_{gr}} w_g e^{-\frac{2\phi^2 n_g^2}{b_g^2}}.$$

Similarly, the bound in Eq. (19) can be rewritten as

$$Prob\left\{Y_t > N_{cities}\gamma + E[Y]\right\} < \sum_{g=1}^{N_{gr}} w_g e^{-\frac{2\gamma^2 n_g^2}{b_g^2}}.$$

Appendix E: Main Existing Government-Supported Insurances

Table 7 Main existing government-supported insurances

Country, name, year	Peril	Position in the market	Policy details	Risk reduction
California, California Earthquake Authority (CEA) ^a , 1996	Earthquakes	Co-insurance with private insurers	California insurers are obliged to offer earthquake policies, but they can choose whether to co-operate with CEA or not. Policy purchase is voluntary. Risk-based premium. Possibility to choose deductibles and maximum coverage	Incentives for risk mitigation (discounted premiums), reinsurance
Iceland, National Catastrophe Insurance of Iceland ^b , 1975	Avalanches Earthquakes Floods Landslides Volcanic eruptions	Private insurers collect and transfer premiums, and receive a commission	All buildings and contents insured against fire are also insured against catastrophe risks. Fire insurance is compulsory. Flat premium set by law. 5% minimum deductible	Reinsurance, If the agency borrows funds, such loans are unconditionally guaranteed by the government
New Zealand, Earthquake Commission (EQC) ^c , 1993	Earthquakes floods hydrothermal activity natural landslides tsunamis volcanic eruptions	Co-insurance with private insurers	Specific building categories are covered compulsorily and automatically along with fire insurance. If a building is not insured for fire, it is also not covered for natural risks. Flat rates. The policy applies fixed maxima and deductibles	Reinsurance, Unlimited State guarantee
Spain, Consorcio de Compensacion de Seguros ^d , 1954	Atypical cyclonic storms Earthquakes Extraordinary floods Fall of meteorites Tsunamis Volcanic eruptions	Private insurers collect and transfer premiums, and receive a commission	Compulsorily included in personal accident policies, life insurance and some branches of property damage. Flat rates. Deductibles apply to most of the covers	Unlimited State guarantee
Taiwan, Taiwan Residential Earthquake Insurance Fund ^e , 2001	Earthquakes	Co-insurance with private insurers	Compulsorily attached to all residential fire insurance policies. Flat rates. The policy applies maxima	Reinsurance, State guarantee

Table 7 continued

Country, name, year	Peril	Position in the market	Policy details	Risk reduction
Turkey, Turkish Catastrophe Insurance Pool (TCIP) ^f , 2000	Earthquakes	Accredited insurance companies and agents arrange policies on behalf of the TCIP.	Compulsory for certain types of buildings and dwellings. Partially risk based rates (5 risk zones, 3 construction types). The policy applies maxima	Reinsurance, contingent credit line
US, National Flood Insurance Program ^g , 1968	Floods	It competes with private insurers	Residential buildings and contents policies are offered to those communities that participate in the program. Communities can participate in the program but are not forced. If they do, they undertake to adopt appropriate preventive measures. Members of a community involved are not forced to buy policies. Risk-based rates. The policy applies maxima and deductibles	Incentives for risk mitigation, government as lender of last resort

^aCalifornia Earthquake Authority, Audited Financial Statements 2018

^bGovernment of Iceland, ACT 55/1992 on The Natural Catastrophe Insurance of Iceland after changes to NTI's legislation in July 2018

^cNew Zealand Government, "Earthquake Commission Act 1993"; Civil Defence – New Zealand, "Government Financial Support", 2009

^dConsortio de Compensación de Seguros (2008, 2017); Machetti (2004)

^eGovernment of Taiwan, "Insurance Act", art 138-1, 1999; Government of Taiwan, "Enforcement Rules for Coinsurance and Risk Assumption Mechanism of Residential Earthquake Insurance", 2001; Government of Taiwan, "Taiwan Residential Earthquake Insurance Fund Articles of Incorporation", 2001; Government of Taiwan, "Regulations Governing Taiwan Residential Earthquake Insurance Fund", 2001; Government of Taiwan, "Enforcement Rules for the Risk-Spreading Mechanism of Residential Earthquake Insurance", 2008; Taiwan Residential Earthquake Insurance Fund (TREIF), Annual Report 2015

^fYazici (2005); World Bank (2011); Gurenko et al. (2006); Turkish Government, Law no: 4452 "Measures to be taken Against Natural Disasters and Authorization in Regards to Arrangements to be made in Overcoming the Damage Caused by Natural Disasters"; 27/08/1999; Turkish Government, Decree Law no: 587 "Decree Law Relating to Compulsory Earthquake Insurance"; 27/11/1999; Turkish Government, Law no: 6305 "Catastrophe Insurance Law", accepted 09/05/2012; Turkish Government, Tariff and instruction of compulsory earthquake insurance, Official Gazette 28512, 29 December 2012

^gAll-Hazard Authorities of the Federal Emergency Management Agency, "The National Flood Insurance Act of 1968" as amended 42 U.S.C 4001 et seq., sec 1366, Office of the General Counsel, August 1997; US Government, "Disaster Mitigation Act of 2000"; Public Law 106-390,30 October 2000; Federal Insurance and Mitigation Administration - FEMA, "FY 2016 Pre-Disaster Mitigation (PDM) Grant Program. Fact Sheet", FEMA, 2016; United States Code, Title 42. The Public Health and Welfare, Chapter 68. Disaster Relief, "Robert T. Stafford Disaster Relief and Emergency Assistance Act", Public Law 93-288, signed into law 23 November 1988, last amended April 2013

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