Production and Financial Networks in Interplay

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April 30, 2025

Abstract

We show that bank credit shocks to firms propagate upstream and downstream along the production network, with stronger effects for upstream than for downstream propagation. Our identification strategy relies on: (i) administrative datasets from Spain on the universe of both supplier-customer transactions and bank loans; (ii) standard bank credit supply shocks during the Global Financial Crisis; and (iii) a general equilibrium model of an interfirm production network economy with financial frictions, which we estimate structurally. We find that the impact of bank credit shocks to firms on aggregate GDP growth increases by close to 50% due to network propagation. Moreover, the combined bank shocks to customers and suppliers *far* on the production network are as important as the bank shocks hitting *direct* customers and suppliers.

JEL Codes: D85; E44; E51; G01; G21.

Keywords: networks; supply chains; shock propagation; credit supply; real effects of finance.

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1 Introduction

The production and financial networks of a modern economy are complex, interrelated structures. On its real side, goods and services are produced by firms as part of a dense web of specialized units, each of them relying on inputs from their upstream suppliers to produce outputs, which are then routed downstream towards other firms and consumers (see Acemoglu et al. (2012); Carvalho (2014)). On its financial side, moreover, these firms are connected to banks through credit flow relationships, forming a similarly complex bank-firm credit network (see, e.g., Diamond (1984); Holmstrom and Tirole (1997); Amiti and Weinstein (2018)), while banks themselves are interconnected through interbank claims (see, e.g., Allen and Gale (2000); Iyer and Peydro (2011); Elliott et al. (2014); Cabrales et al. (2017)).

The real and financial networks are closely interrelated and hence should be studied as such, in particular to understand how bank shocks propagate through the production network, affecting micro and macro real outcomes. Indeed, this is the view that, in the aftermath of the Global Financial Crisis, became widely adopted by academics and policymakers alike, who came to accept that the role of these networks, and the interactions among them, had not been properly understood, including their deep impact and wide span (Acemoglu et al. (2015); Freixas et al. (2015); Bernanke (2013, 2018)). However, empirical research has mostly studied these networks in isolation, in part due to a lack of reliable and comprehensive matched datasets on the production and financial networks.

In this paper, we study one part of the interplay between production and financial networks. Specifically, we analyze a general equilibrium model that describes the production part of the economy as an inter-firm network, with firms being subject to financial shocks. We focus on a specific type of financial shock: bank credit supply shock, which we will refer to simply as *bank shock*. We provide closed-form expressions for how these shocks affect equilibrium outcomes of the real side of the economy, accounting for all first- and higher-order propagation effects unfolding throughout the production network.[1] We show that these expressions can be applied to the data by exploiting administrative datasets on the production and bank credit networks during the Global Financial Crisis. We estimate the propagation effects of bank shocks to firms through the production network. Our findings reveal substantial propagation effects, both downstream and upstream, including both first-order and higher-order impacts. Furthermore, we show that upstream propagation is stronger than downstream propagation. The results also indicate a *large amplification* of the impact of bank shocks to firms on the GDP growth due to the production network propagation, with first-order and higher-order propagation contributing nearly equally to this amplification.

Our empirical strategy exploits two administrative datasets from Spain, which include universal information on (a) customer-supplier transactions from the VAT (Value Added Tax) Register

[1] Direct shock is the bank shock hitting that firm, while indirect shock refers to bank shocks hitting firms directly or indirectly connected to the firm via the production network. First-order propagation effects embody the consequences experienced by a firm when some of its direct customers (or/and direct suppliers) are affected by a bank shock, while higher-order effects are those derived from bank shocks that affect its indirect customers (or suppliers) that are at network path lengths of more than one link, e.g., suppliers' suppliers. By upstream propagation, we refer to the transmission of bank shocks from customers to suppliers, while downstream propagation refers to the transmission of bank shocks from suppliers to customers.

of the Spanish tax authority and (b) bank loans to firms from the Credit Register held by the Spanish central bank. These data allow us to construct the production network, as well as the bank-firm credit network that we use to identify bank shocks to firms. We further match these data with the administrative Spanish Mercantile Register, which provides information on the balance sheets and profits of firms, and with supervisory bank balance sheet information, which, in particular, includes the funding of each bank in the interbank market.

Regarding the production network, when a firm sells a product or service to another one, there is a VAT associated with the sale. Hence, by having access to all annual VAT transactions, we can construct the whole weighted production network of Spain. To identify the bank shocks to firms, we exploit the wide cross-section variability in exposure to the Global Financial Crisis that followed Lehman Brothers' failure in mid-September 2008. Specifically, we pursue the approach of Amiti and Weinstein (2018), itself following Khwaja and Mian (2008), which is widely used in the literature and basically identifies bank-level credit supply as the change in credit cleaned by firm fixed effects, which proxy for firm observed and unobserved fundamentals, including firm-level credit demand. As a complementary exercise, we replicate the analysis with a different bank-level shock, also extensively used in the literature, which is based on the ex-ante bank funding exposure to the interbank market, a market sharply affected by the Global Financial Crisis (see, e.g., Iyer et al. (2014)). We aggregate these bank-level shocks into a bank shock at the firm level, using as weights the credit of each bank to a firm prior to the crisis. We use this firm-level bank shock measure throughout the paper and show that these bank shocks reduce firm-level credit availability during the financial crisis but not before.[2]

We start the analysis with a reduced-form approach and study first-order propagation. We first examine the effects of bank shocks on firm-to-firm transactions, which allows us to exploit the full granularity enjoyed by our data – in particular, the wide variability in the changes of sales (purchases) observed among all suppliers (customers) of the same customer (supplier). We account not only for observed, but also for unobserved, heterogeneity through customer (supplier) fixed effects. We then analyze the bank shock propagation at the firm level to better understand whether there are real corporate effects. In the production network, firm-to-firm transactions function as links and firms as nodes; accordingly, we refer to the former as link-level analysis and to the latter as node-level analysis.

At the link level, focusing on within-supplier variation, the upstream propagation of a one standard deviation negative bank shock experienced by a customer leads to an average reduction of 2.36 percentage points (pp) in the supplier–customer's sales growth. Differently, by exploiting within-customer variation, we find that the downstream propagation of a bank shock hitting a supplier implies a reduction of 1.09 pp in the supplier-customer's sales growth. The stronger effect for upstream compared to downstream propagation is an interesting result that we explain in our subsequent structural analysis.[3]

[2]Before Lehman Brothers' failure, firms could easily switch from more to less affected banks, thereby mitigating the effects of bank shocks; instead, during the crisis, no such flexibility existed.

[3]Not only do we show upstream and downstream propagation of bank shocks through the production network, but we also show that these bank shocks are in large part emanating from the interbank market, and also that these bank shocks to customers (or suppliers) reduce overall customers' (or suppliers') bank debt, in turn propagating upstream (or downstream) through the production network. Consistent again with financial frictions, we also show

In the node-level analysis, we study the upstream and downstream propagation of bank shocks hitting, respectively, their direct customers or suppliers. This requires shock aggregation, and one way to do it is to weigh the shock originating in each customer (or supplier) by its share in the supplier's sales (or customer's costs in intermediate inputs). We find that, for firm sales, the first-order upstream propagation effect is significant, though only half as strong as the link-level estimate. In contrast, we do not find evidence of significant first-order downstream propagation. Moreover, the first-order upstream propagation effect is twice as large as the impact of the direct bank shock to a firm on the overall firm's sales.[4]

The previous results of link- and node-level propagation suggest that shock propagation becomes weaker when it operates downstream or at the node level. Intuitively, one may conjecture that this is a reflection of how the forces of substitution among inputs, customers, or suppliers work in each case—i.e., upstream *versus* downstream or link *versus* node levels. Nevertheless, to have a better understanding of this issue, we need to go beyond the reduced-form approach and address it within a theoretical framework where we can analyze such substitution trade-offs.

The theoretical framework should also include other important features. One is that the propagation of bank shocks can involve potentially long network paths. It is such higher-order propagation that makes the process a truly systemic phenomenon and has the potential of bringing about substantial amplifying consequences. Hence, we need to analyze not only first-order effects, but also higher-order effects. Another feature is that, as firms react to bank shocks, their responses interact through both the markets for goods and the markets for production factors, thus reshaping market-clearing prices.

We, therefore, analyze a general equilibrium model that describes the production part of the economy as an interfirm network, its nodes being the firms and the directed links representing the flow of intermediate inputs. The production function of each firm follows a Cobb-Douglas structure that combines labor, capital, and a CES aggregate of required intermediate inputs. We assume constant returns to scale and posit that firms determine their prices by applying a fixed markup over their marginal cost. The representative household consumes some of the goods produced in the economy, supplies the factors of production, and owns all firms in the economy. Finally, we model bank shocks as financial frictions that manifest as input price wedges.

Our main theoretical contribution is to provide a fully specified solution of the model in closed-form expressions that capture how bank shocks propagate through the production network. We then bring these expressions to the data to estimate the effects of bank shock propagation on the real economy. Our model is a generalization of the parametric model studied in Bigio and La'O (2020) in that we allow for non-unitary elasticity of substitution across intermediate inputs. A parametric approach enables us to derive explicit solutions for equilibrium outcomes while still accounting for a rich set of equilibrium effects. Specifically, we determine how the bank shocks hitting individual firms ripple through the production network, affecting the bilateral sales between suppliers and customers, the overall sales of any given firm, and the real GDP of

that the network propagation effects of bank shocks are stronger for more leveraged and/or smaller firms.

[4]For upstream propagation, not only do sales go down, but also trade credit. Moreover, it is interesting to note that if we analyze the change of trade credit over sales, we find that trade credit from suppliers to customers partially substitutes the reduction of bank credit to customers.

the economy. That is, we study how bank shocks to firms propagate through the production network, focusing on the link, node, and aggregate economy levels.

At the link level, we derive explicit expressions for the effects of shock propagation on equilibrium outcomes. They describe the change in sales of any given firm to one of its customers resulting from the *joint* operation of two spillover channels. First, exposure to bank shocks by a direct supplier and customer, i.e., first-order propagation. Second, the higher-order network propagation of bank shocks hitting other firms in the economy. Crucially, this bilateral focus already reveals the importance of accounting for shock propagation along full supply chains of any length in the production network. In particular, we show that the elasticity of substitution across intermediate inputs plays a key role in the propagation of shocks.

At the node level, we derive analogous expressions that capture the *combined* impact on the sales of each firm resulting from its *weighted* exposure to the bank shocks hitting its first-order customers and suppliers, as well as the bank shocks impacting its higher-order customers and suppliers. Compared to the link-level analysis, the propagation pattern of the shocks at the node level is more complex, as it combines some "pure" upstream propagation with a non-separable "blend" of both downstream and upstream propagation.

Finally, we derive the equilibrium expression that captures the impact of bank shocks on the real GDP, fully accounting for the underlying structure of the production network. This result enables us to quantify the overall effect of bank shocks on the economy and conduct various counterfactual exercises.

We perform a structural estimation of the model at both the link and node levels by bringing the induced equilibrium equations to our data. This entails, in particular, relying on the theory to determine what variables enter into the empirical equations, how to measure the different effects, and the functional forms that bring all of them together.

At the link level, the results on first-order effects that we derive from the structural estimation are very similar to those obtained in the reduced form. It is worth noting that the fact that we obtain very similar results in the reduced and structural estimation of these effects is not *a priori* obvious, as, in principle, controlling for higher-order effects could have changed the estimated effects of first-order effects.

We estimate reductions of 1.92 pp or 1.09 pp in the growth rates of a firm's sales (or purchases) if one of its direct customers (or suppliers) is hit by one standard deviation bank shock.[5] The upstream effect is 65% larger than the downstream one. The model provides a clear-cut understanding of why this occurs. In facing downstream propagation of shocks, the customers must have some significant ability, but not too high, to offset their effect by substituting for inputs that become more expensive. More precisely, what the theory requires is that the elasticity of substitution across intermediate inputs must lie between 1 and 2. This is, in fact, consistent with the elasticity that we identify with our structural estimation at the link level, which leads to an estimated value equal to 1.56.

[5] We also find that it is easier to substitute an input with inputs provided by alternative suppliers from the same sector than with inputs provided by suppliers from other sectors.

We also find significant estimates for the higher-order effects bearing on both the customer and the supplier for any given link. In fact, the impact of such higher-order propagation is comparable in magnitude to that resulting from the first-order propagation. Therefore, these results obtained from our structural estimation suggest that the reduced-form approach misses a substantial part of the shock propagation, around half of it; hence, greatly underestimating the extent of the production network propagation. In addition, the model predicts opposite effects of the higher-order shocks depending on whether they influence the customer or the supplier, which is also a prediction consistent with the empirical results.

At the node level, the theory provides a precise way to aggregate the effects of bank shocks that flow into any given node through *all* paths that connect it to *all* its different suppliers and customers, direct and indirect, at *every* order. We show that such aggregate effects embody two different channels of propagation. First, when a bank shock hits a firm, this induces a negative demand effect for its intermediate inputs that propagates upstream. Second, there is a "blend" of downstream and upstream propagation that we term bidirectional and describe as follows. When a shock hits any given firm, it first affects the costs of that firm and, indirectly, the costs of firms positioned downstream in the network. These firms then respond by substituting away from their affected suppliers toward those less affected, in turn creating a cascading effect that propagates upstream as a demand shock.

We find that the effects associated with both propagation channels are sizable. A combined increase of one standard deviation in the bank shocks through both of them decreases by 2.1 pp the average growth rate of firm sales, which corresponds to an 11% decrease in the average growth rate. We also find that the upstream component is three times larger than the bidirectional one. The reason is that, as bidirectional propagation involves a downstream component, its impact is affected by the possibility that firms substitute intermediate inputs. In fact, from the node-level analysis, we can also recover this elasticity and we estimate it to be 1.35, which is quite close, and not statistically different, to the value of 1.56 that we derive from our link-level estimation. In this respect, therefore, we find that link- and node-level analyses provide a consistent understanding of shock propagation.

Finally, we quantify the impact of the production network propagation of bank shocks on the aggregate economy, accounting for general equilibrium effects. To do this, we first evaluate the effects of bank shocks to firms on the logarithm of the real GDP by relying on a first-order approximation of the equilibrium equations derived from the model and its estimated parameters. Second, we rely on our data to ask the counterfactual question: What would have been the effect of the banking crisis on the GDP in the absence of propagation of bank shocks along the production network? We quantify the "aggregate effect" of bank shock propagation by comparing GDP outcomes with and without the linkages induced by the interfirm network.

The results show that, while the level of GDP falls between 2.36% and 3.96% when we take into account the impact of bank shocks to firms through the overall network, it only falls between 1.74% and 2.25% in the absence of propagation via input-output linkages. That is, we conclude that an increase of close to 50% in the total effect is due to the production network propagation of bank shocks. Moreover, around half of this impact is due to higher-order network effects.

Related literature. As mentioned, most of the literature has evolved by studying the real and the financial networks separately. On the real side, the main focus has been on the supply chains that underlie the production of firms, and the role of the network structure in the propagation and aggregation of, predominantly, productivity shocks.[6] On financial networks, the analysis has primarily been on the banks, the links among them typically reflecting some form of financial flows.[7] Moreover, there is a rich literature that has studied how bank shocks can lead to firm-level real effects,[8] but it abstracts from the production network propagation of those bank shocks. Our main contribution is to integrate bank shocks into the real side of the economy and study their propagation on the production network.

Only recently, a few papers have also analyzed how financial shocks propagate through the real production network. To the best of our knowledge, the following papers are the most related. One is by Costello (2020), who study downstream propagation of shocks through their influence on the trade credit that firms extend to their customers. Relying on data from a third-party trade credit information platform, this paper documents in a reduced-form analysis that firms with greater exposure to a large decline in finance reduce their trade credit to customers, and consequently induce negative effects on employment.[9] In another paper, Cortes et al. (2019) use *payment* data from Brazil across firms associated with different banks and estimate the first-order propagation effects of state-owned bank shocks in a reduced-form firm-to-firm analysis.

Compared with the previous two papers, our contribution to the literature is as follows. We use administrative matched datasets on the universe of both supplier-customer transactions and bank loans; we present new theoretical results that, in closed form, describe the different channels of shock propagation and provide a coherent way of aggregating their effects at the firm and economy levels; we structurally estimate the equilibrium equations both for link-level and node-level outcomes; we provide the quantification of the effects of bank shocks on GDP and conduct counterfactual analyses. These new features of our approach also generate novel results: (a) both upstream and downstream propagation, as well as first- and higher-order propagation, yield large economic effects; (b) interesting heterogeneity arises across various effects, e.g., stronger upstream vs. downstream propagation for firm sales; (c) bank shocks, as they propagate through the production network, generate a substantial impact on the economy's GDP, with higher-order effects contributing as strongly as first-order effects.[10]

[6] See e.g. Acemoglu et al. (2012); Barrot and Sauvagnat (2016); Baqaee (2018); Carvalho et al. (2020).

[7] See e,g, Allen and Gale (2000); Freixas et al. (2000); Iyer and Peydro (2011); Niepmann and Schmidt-Eisenlohr (2013); Elliott et al. (2014); Acemoglu et al. (2015); Cabrales et al. (2017).

[8] See e.g. Holmstrom and Tirole (1997); Stein (1998); Gertler and Kiyotaki (2010), Khwaja and Mian (2008), Chodorow-Reich (2014), Greenstone et al. (2014), Jiménez et al. (2012, 2014, 2017), Amiti and Weinstein (2018).

[9] Related to it, Demir et al. (2024) show that a negative shock to the cost of import financing propagates from liquidity-constrained firms to their customers; see also Jacobson and von Schedvin (2015).

[10] See online Appendix A for a more comprehensive differentiation with each of those papers and with other less related ones. A more distantly related paper is Alfaro et al. (2021), which studies the propagation of bank shocks through *industry-level* input-output network. Three key differences are: First, they rely on reduced-form estimates, while we show that such estimation may be biased and miss about half of the overall propagation effects. Second, and relatedly, they do not study higher-order propagation effects, while we show that higher-order effects are as crucial as first-order effects. Third, industry-aggregated data raises identification concerns, including the recovery of the elasticity of substitution across intermediate inputs. Dewachter et al. (2020), using data from Belgium, complements our research by studying a Keynesian model that also displays an interplay of financial and production networks, but their focus is on bank concentration and macroeconomic volatility.

2 Datasets

In this section we describe the administrative datasets for the Spanish economy that we use in our analysis. They cover both customer-supplier transactions from the VAT register and the bank-firm lending relationships from the credit registry. We also use administrative firm-level and bank-level data, the latter including interbank borrowing information.

We use the confidential administrative VAT register. Firms in Spain are subject to VAT. As a part of the VAT declaration, firms report annually to the Spanish tax agency all annually paid and received transactions with third parties exceeding the amount of 3,005 euros.[11] We have access to this confidential dataset of all firm-to-firm transactions subject to VAT during the years 2008 and 2009 and use it to construct the empirical counterpart of the production network considered in the theory. For each bilateral transaction between two VAT-liable enterprises, the dataset contains two observations: the value of the transaction reported by the supplier and the value of the same transaction reported by the customer. To construct the firm-level network of transactions, we need to assign a single value to each reported annual transaction. For that purpose, there is no ambiguity when the values reported by the supplier and the customer coincide. However, there may be a discrepancy between the supplier's and the customer's declarations of the same transaction. When the discrepancy is small relative to the higher reported value, we select the value reported by the supplier. When the difference is relatively large, which is the case for 0.01% of observations, we choose the smaller of the two declared values to be more conservative. In our analysis, we restrict the sample to firms that are not in the financial sector, and to transactions where both the seller and the customer are publicly limited or limited liability companies.[12] We end up with a dataset containing information on 13,810,158 transactions between 867,013 firms in 2008 and 11,988,607 transactions between 861,350 firms in 2009. An annual transaction is the annual total sales from firm i to firm j; or, equivalently, the annual total purchases by firm j from firm i. See Table A15 for additional summary statistics.

We also use the confidential administrative loan-level dataset for non-financial firms from the Spanish Credit Register (CIR), which is maintained by Banco de España in its role of banking supervisor and central bank. The CIR contains very detailed loan-level data since 1984 on all loan commitments above 6,000 euros granted by any bank operating in Spain. We aggregate the different loans between a firm and a bank in each period, thus analyzing the data at the bank-firm-time level. Although the CIR is updated monthly, given the annual frequency of the other datasets that we use in the paper, we record the credit data annually. The CIR also provides information about loan characteristics such as the loan volume, the type of credit instrument, currency, maturity, degree of collateralization, and default status. In this paper, we focus on loans granted by depository financial institutions to non-financial firms, including commercial and industrial loans and leasing. For a more detailed description of the CIR, as well as the banking system and crisis in Spain, see, for instance, Jiménez et al. (2020).

[11]These transactions are reported under the M.347 form. More information is available at the Spanish tax agency (Agencia Estatal de Administración Tributaria, AEAT): https://www.agenciatributaria.gob.es. Exports are not subject to VAT taxes. There are no VAT taxes in two small Spanish cities located in Africa, or in the Canary Islands.

[12]Almost 95% of all non-financial firms are publicly limited or limited liability companies. Our raw dataset covers the period 2008–2010.

Other administrative datasets that we use in the analysis contain the balance sheets and income statements of non-financial firms and banks. At the non-financial firm level, we exploit information on firms' characteristics that is available at a yearly frequency from the Spanish Mercantile Register, which is an administrative database that contains available information on firms' financial statements as well as on their corporate income tax returns. The data cover around 90% of the firms in the non-financial market economy for all size categories, including both turnover and number of employees. The correlation between micro-aggregated employment and output growth and the National Accounts counterparts is above 90%.

We also rely on supervisory bank-level data based on information from the December reports that banks have to submit to the supervisor, Banco de España. We obtain information on banks' overall interbank funding positions, balance-sheet variables, and profit and loss account data. This information allows us to have, for each bank, how much it borrows overall from the interbank market. On average, each bank borrows 1.7 billion euros from the interbank market, which represents, on average, 28% of total bank assets, with an inter-quantile range going from 2% to 53%.

3 Identification of bank shocks

Our main empirical challenge is to estimate how shocks originating in the banking system propagate through the production network. This section explains the strategy that we pursue for the identification of these bank shocks.

The identification of bank shocks follows a standard approach in the empirical literature. In our baseline specification, we follow Amiti and Weinstein (2018), hereafter abbreviated as AW, and construct bank shocks as follows. We estimate, for each bank, a credit supply factor identified as the bank fixed effect at a bank-firm-level weighted least square cross-sectional regression of credit growth on the bank and firm fixed effects. This regression exploits cross-sectional bank variability generated by the Global Financial Crisis. Thus, if we denote by $CreditGrowth_{ib}$ the growth rate of total lending to firm i from bank b in percent changes and by ν_i and ι_b the firm and bank fixed effects, respectively, we estimate the following regression over the year 2008 using a weighted least square (WLS) procedure:[13]

$$CreditGrowth_{ib} = \nu_i + \iota_b + \epsilon_{ib} \tag{1}$$

Using these estimated bank-level shocks, we apply the AW approach to identify bank shocks at the firm level (θ_i^{AW}). In this paper, we are interested in this firm-level measure of bank credit shocks. Specifically, we compute the firm-specific bank shocks as the weighted average of the bank-specific fixed effects ι_b estimated in (1) using *pre-crisis credit exposure* of the firm to each particular bank as weights.[14] Moreover, we switch the sign of the estimated bank shocks so that

[13] In particular, we follow the same regression, variables and weights as Amiti and Weinstein (2018). Note that we use firm fixed effects to control for firm credit demand as our credit regression is cross-sectional. We also show that the AW bank fixed effects aggregated at the firm level do not matter before the crisis for firm overall credit outcomes, differently from the crisis period.

[14] Note that these weights essentially capture the ex-ante importance of each bank for firm i just before the crisis. Moreover, using the Amiti and Weinstein (2018) terminology, the firm shock that is being computed is the sum of the common shock and the firm-level bank shock.

higher values reflect a negative bank shock, i.e., lower credit supply.[15] We refer to this shock as the AW bank shock when we wish to emphasize that the bank shock is estimated using the AW approach. In the online Table A4, we consider robustness exercises that exploit a binary version of the AW bank shock that takes the value one if the shock for the firm is above the median across all firms and zero otherwise. A higher AW bank shock indicates that a firm is more likely to be exposed to financially constrained banks during the crisis.

Next, we analyze whether the estimated bank shocks at the firm level are orthogonal to pre-crisis observable firm characteristics. That is, we want to test whether firms that, before the crisis, worked with banks that during the crisis will be more financially constrained are similar to other firms that work with less constrained banks. To do so, in Table A2 we explore a relevant range of observed firm characteristics for both types of groups. We find that the firms exposed to negative bank shocks and those not exposed were not different in observables prior to the Global Financial Crisis. The first four columns of the table point to very similar numbers for the firm characteristics for the two groups of firms, while the fifth column reports the t-statistic of the differences in averages of the firm characteristics in each group.

The t-statistic is, however, sample-size dependent as Imbens and Wooldridge (2009) note, which would make the rejection of the null hypothesis more likely as the number of observations increases. To avoid this problem, Imbens and Wooldridge (2009) propose to test the null of no differences in means between the two groups using a scale-and-sample-size-free estimator, labeled as the normalized difference. This estimator scales the difference in the means of each variable in the two samples by the square root of the sum of the variances. Imbens and Rubin (2015) suggested a heuristic threshold of 0.25 for the absolute value of the statistic to judge whether the differences should be considered significant or not. As column 6 of Table A2 shows, all the values of the firm variables are not higher than 0.01 in absolute value. This therefore supports the claim that the estimated effects of bank shocks on firms are not driven by different firm observable fundamentals.[16]

Importantly, when analyzing differences in pre-crisis bank characteristics, we find that banks that relied more on interbank borrowing or were smaller before the crisis reduce the supply of credit more after the start of the banking crisis (see columns 5 and 6 of Table A2).[17] Therefore, the firms that borrowed more from these banks before the crisis (e.g., via bank-firm relationship lending) may have experienced a stronger credit supply constraint during the crisis. We arrive at similar conclusions from a linear probability regression of firm exposure to financially constrained banks on all firm characteristics and four-digit NACE $\times$ province fixed effects, which control for crisis differences across industries and locations, as reported in column 7 of Table A2. The

[15]It is worth noting that a key assumption of the AW approach is that a firm's demand is the same regardless of the bank or type of loan the firm applies for. In Section 4, we show robust results relaxing this assumption.

[16]Note that in some regressions, we will also control for unobservables via, e.g., firm, customer or supplier, fixed effects. It is also important to note that the unconditional correlation between the bank shock to the firm and the bank shocks of its direct customers (or suppliers) is 0.06 (or 0.07). Moreover, this correlation falls to 0.02 (or 0.02) respectively, after controlling for the industry and location of the firm and customer or supplier, which we use in the paper. We also calculate several metrics related to firms' position in the network and find no significant differences in the network position measures between firms exposed versus not exposed to financially constrained banks.

[17]In Table A2, we also show bank characteristics at the firm level computed as a pre-crisis weighted average of the credit each bank lent to the firm.

results show that the only two statistically significant variables are the two bank variables: the net interbank borrowing position of the firm's average bank and the corresponding bank size.

The fact that the banks that became more acutely constrained during the crisis were also those borrowing more heavily from the interbank market before the crisis is not specific to our case, but it is a general feature of financial crises. This is why researchers have used the net interbank borrowing position to identify bank shocks to firms (see, e.g., Iyer et al. (2014); Ippolito et al. (2016); Cingano et al. (2016)). We also use this measure in robustness tests to define bank shocks after the start of the Global Financial Crisis. More specifically, we rely on the bank's net interbank borrowing before Lehman's collapse. This is also a natural way to bring into our analysis the other key financial network that research has considered key in the literature: the interbank network (see, e.g., Allen and Gale (2000)).

The bank shocks – that we aggregate at the firm-level – have negative and significant effects on firm-level credit availability only during the crisis and not before, as we show in Table A3. That is, the induced negative effects caused by banks are significant in 2009 but not in 2007 or over the whole 2008. This is intuitive since before the financial crisis that followed the failure of Lehman Brothers in mid-September 2008, firms could switch much more easily from more to less constrained banks (see, e.g., Jiménez et al. (2012)), thus massively reducing the effects of bank shocks at the firm level. In this respect, an important consideration to bear in mind that supports our identification strategy is that Spain is a bank-dominated economy. Hence, we can safely abstract from other financial intermediaries, such as the shadow banking system, which may be crucial in other economies.

4 Reduced-form evidence

In this section, we explore the first-order propagation of bank shocks on the production network based on reduced-form regressions, both at the link level and at the node level. See Table A1 in the online Appendix A for summary statistics and the definition of the main variables that we use in the paper.

Link-level analysis. To explore both upstream and downstream propagation of bank shocks to firms, our analysis focuses on firms, with some customers or/and some suppliers, that prior to the crisis borrowed from banks.[18] For each link ji we estimate how the bank shock hitting customer i influences its purchases from firm j, as well as how the bank shock of supplier j affects its sales to customer i. Thus, here we study the first-order propagation effects of bank shocks across firms. Specifically, let s_{ji} stand for the sales of firm j to firm i, and denote by θ_j^{AW} and θ_i^{AW} the AW bank shocks hitting j and i, respectively. Furthermore, we use the notation $\boldsymbol{x}_{ji}$ to represent a vector of supplier, customer, and supplier-customer controls. We analyze the

[18]If a firm did not receive credit from a bank prior to the crisis then we cannot identify whether it is subject to a bank shock. We therefore consider only firms with suppliers and customers that borrowed before the crisis, which represent the largest part of the economy, not only in terms of borrowing but also in terms of sales. Relatedly, in some regressions, we control for the weighted average of suppliers that experience bank shocks and the number of different sectors from which the firm sources inputs with suppliers that experience bank shocks.

following cross-sectional specification:

$$\Delta \log s_{ji} = a^u \theta_i^{AW} + a^d \theta_j^{AW} + \boldsymbol{x}_{ji} + \epsilon_{ji} \tag{2}$$

where the sub-index j (i) refers to a generic supplier (customer), and the dependent variable is measured in terms of the log changes over the crisis.[19] The main regressors of interest are the firm-level bank shocks θ_i^{AW} and θ_j^{AW}, estimated following the AW approach described in the previous section. Thus, the coefficients of interest a^u and a^d refer to the upstream propagation effect of customer's bank shock θ_i^{AW} on the sales of firm j to firm i, and the downstream propagation effect of supplier's financial shock θ_j^{AW} on the purchases of firm i from firm j, respectively.

The effects of financial crises can differ depending on firm characteristics beyond the specific channels that we analyze in this paper. For example, there may be stronger effects for smaller firms, firms located in areas with a strong real estate boom, or in more cyclical industries. Therefore, we control, depending on the specification considered, for supplier, customer, and supplier-customer characteristics in the vector $\boldsymbol{x}_{ji}$. In particular, we include the size of the supplier (customer) in terms of its log of total assets, log of age, equity to asset ratio, current assets minus current liabilities over total assets as a measure of liquidity, its ratio of short-term debt (less than 1 year) as a measure of its maturity structure, and the legal form of the supplier (customer). We also add unobserved factors captured by the product of province and industry dummies at the 2-digit NACE level of suppliers (customers). Concerning supplier-customer variables, we consider the share of sales of supplier j directed to customer i, the share of purchases of customer i from supplier j, and dummies indicating whether both firms share the same main bank or operate in the same province-industry pair. Finally, in some specifications, we also add a large set of dummies capturing common trends in industries and provinces of the firm and of its customer or supplier; specifically, we include industry/province of a firm $\times$ industry/province of a customer/supplier fixed effects and unobserved customer/supplier fixed effects in the form of industry $\times$ zip code fixed effects.

Moreover, our granular firm-to-firm network data allow us to include even further configurations of fixed effects in our regressions to enhance identification. Since we are interested in identifying the impact of customers' (suppliers') bank shocks on supplier-customer sales, we include supplier (customer) fixed effects, so that identification is based on within-firm variation from multi-customer (multi-supplier) firms.[20] Intuitively, this identification strategy is based on the comparison of sales (purchases) of the same firm with different customers (suppliers) that are hit by different bank shocks.

To fully exploit the richness of the link-level data, we, therefore, estimate two versions of equation (2), one version in which we use j fixed effects that absorb θ_j^{AW}, and another in which we use i fixed effects that absorb θ_i^{AW}. Specifically, for any given firm with suppliers and/or customers in our data, we consider the following two separate samples from the perspective of this firm, and run two different sets of regressions. First, when estimating upstream propagation,

[19] We analyze the percentage change in log sales over the whole 2009 versus those of the whole year 2008. In the online Appendix A, we also consider the year 2010. We winsorize growth rates to be bounded by +200% and −100% to reduce the impact of outliers. As a robustness check, we use a measure inspired by Davis and Haltiwanger (1992) that accounts for both the extensive and intensive margins, yielding similar results.

[20] 77% of observed suppliers have two or more customers, while 86% of customers have two or more suppliers.

we consider the sample that includes all customers i buying from firm j. We then analyze the upstream propagation of customers' bank shocks within that same supplier by including supplier j fixed effects. Thus, a^u captures the extent to which firm j decreases its sales towards customers more affected by negative bank shocks relative to those less affected, controlling for j's observed and unobserved characteristics, factors correlated with location and sector of both firm j *and* its customers, as well as the link-specific variables mentioned above. Second, when estimating downstream propagation, we consider the sample that includes all suppliers j selling to any given firm i, and using a strategy analogous to the one for upstream propagation, we estimate the downstream coefficient of interest a^d.[21] We now discuss our findings.

Upstream propagation. Table 1 reports our estimates of equation (2).[22] In column (1), we report the estimated impact of the direct bank shocks hitting firm j on the bilateral sales to its customers. It is negative and statistically significant, which corroborates that direct credit supply reduction from firm j's banks significantly affects its sales after accounting for customer fixed effects. Column (2) displays the estimated first-order propagation effect of customer bank shocks in our most stringent specification, which includes firm j fixed effects and customers' controls. This is the most important result in Table 1. The estimated effect is negative and it is also statistically and economically significant. It exceeds twice the magnitude of the direct effect reported in column (1), and a one standard deviation increase of a customer bank shock implies, on average, a reduction of 2.4 pp in firm-customer sales. This reduction represents 19.5% of the mean value of the dependent variable.

As a robustness check, columns (3) and (4) of Table 1 consider the alternative estimation approach in which we use the bank's interbank market net borrowing as an instrument to the baseline AW bank shock. When we use the measure of interbank funding exposure as a source of identification, not only do our main findings on upstream propagation remain robust, but the magnitude of the estimated effects is even larger.[23]

Columns (5) and (6) moreover investigate whether a firm affected by a negative bank shock from its customers reduces its sales due to the customers' reduction of total bank credit. Specifically, we consider the customers' reduction of bank debt as the regressor of interest, instrumented with the customers' AW bank shocks. We find that, on average, a firm reduces its sales due to a fall in the total bank debt of its customer, induced by a customer bank shock.[24] We interpret this

[21] In unreported regressions, we obtain very similar results when estimating equation (2) in a single regression. See also Table 4, columns (5) and (6), below.

[22] We standardize all shock variables to have zero mean and unit variance to make the estimated coefficients comparable. Also, we estimate equation (2) by weighted OLS, where the weights are the size of the firm-to-firm relationship captured by past sales or purchases between the two firms, and we multi-cluster the standard errors at the level of firm i, at the customer or supplier level, and at the main bank level.

[23] The larger coefficient is not surprising as interbank borrowing is one of the only two bank and firm observable variables that are significant in explaining our main bank shocks in Table A2, see the previous section. Moreover, as an economic explanation, bear in mind that there was a massive dry-up shock in the interbank market during the crisis, so this channel was crucial to reducing bank liquidity for banks, hence causing a credit crunch. Note as well that the first-stage effective F statistic shown in the tables is based on Montiel Olea and Pflueger (2013), and it is robust to heteroskedasticity, serial correlation, and clustering. Its value is above the critical threshold of 23.109, for a confidence level of 5% and a percentage of worst-case bias of 10% in almost all the cases, always with a worst-case bias of 30%. Table A4 in the online Appendix A corroborates these results when we use a binary version of AW bank shocks.

[24] Note also that the first stage does not suffer from weak instrument problems as shown in column (5).

result as further evidence in favor of the claim that bank credit is a key channel in the upstream propagation of bank shocks.

Downstream propagation. The estimated effects in Table 2 imply a statistically significant negative effect of suppliers' bank shocks on the sales to their customers. However, according to our estimates, first-order downstream propagation is smaller in magnitude than first-order upstream propagation. As we see in column (2) of Table 2, a one standard deviation increase in the supplier bank shock implies, on average, a reduction of 1.09 pp in sales to its customers, which represents a reduction of 9.1% of the mean value of the dependent variable. In columns (3) to (6) of the same table, we show that the interbank borrowing exposure and the total bank credit reduction work as they did for the case of the upstream propagation. Finally, we note that Table A4 in the online Appendix A shows that if we use the binary version of the bank shocks, the main results of Tables 1 and 2 remain essentially unchanged.

Financial channel. The evidence provided so far is consistent with the importance of a financial channel in the production network propagation. First, our empirical evidence on upstream and downstream propagation through the production network is based on the bank shocks. Second, we show that these bank shocks are, in large part, emanating from the interbank market. Third, we show that these bank shocks to customers (or suppliers) reduce overall customers' (or suppliers') bank debt, thus propagating upstream (or downstream) through the production network.

To explore further the financial channel of our main results, in the online Appendix A, we report on a heterogeneity analysis that investigates how propagation effects depend on firm leverage and size, as these firm variables are proxies of financial frictions at the firm level. Table A9 shows that the upstream propagation of bank shocks is significantly stronger for more leveraged firms and smaller firms. The results are similar for downstream propagation, though we find that the effects are insignificant at conventional levels despite even larger estimated coefficients, compared to those obtained for upstream propagation, in the case of more leveraged firms as well as more leveraged and smaller firms.

Further robustness. In online Appendix A, we analyze the implications of relaxing the assumption of equal firm credit demand in the AW methodology. Tables A5, A6, and A7 replicate some key columns of Tables 1, 2, and A4, allowing firms to have different credit demands depending on the type of loan as in Ivashina et al. (2022), on the bank's specialization, or on a measure of weak bank as in Bentolila et al. (2017).

Specifically, Table A5 considers bank shocks constructed at the firm level using an equation similar to (1), with the addition of firm fixed effects for each of the four credit types analyzed: asset-based loans, cash-flow loans, trade-finance agreements, and leases. In Table A6, Panel A, we explore the possibility that firms may have different credit demands based on whether a bank's primary specialization matches the firm's industry and/or the province where it operates.[25]

[25] We have constructed, therefore, an indicator that has four different categories. One category has the bank's main industry being the same as the firm's industry but their provinces not coinciding; a second category with the bank's main province being the same as the firm's province but their industries being different; a third category where both the industry and province of the firm and the bank coincide; and a fourth category in which neither the sector nor the province coincide.

Instead, in Panel B of Table A6 we allow in (1) that firms may have a different credit demand for banks specialized in real estate. Finally, in Table A7 we modify (1) to allow for the possibility that firms may have different credit demand to weak banks by introducing an interaction between firm and weak-bank fixed effects, based on the weak-bank variable defined in Bentolila et al. (2017).

The results shown in Tables A5 to A7 are quite similar to those in our main Tables 1 and 2 and in Table A4. This suggests that the AW assumption that firms have an identical credit demand for all banks and loan types yields robust results.[26] In fact, the AW bank shocks that we use in our baseline regressions turn out to be highly correlated with the bank shocks that we use in Tables A5 to A7.[27]

In sum, the link-level analysis provides evidence that firms sell less to customers that are hit by negative bank shocks, and similarly, suppliers hit by bank shocks sell less to their customers. While these results arise for firm-to-firm sales, it is not immediate that they should translate to firms' *total* sales. For example, a firm might be able to undo a negative shock from a particular supplier or customer by resorting to other suppliers or customers. To address this issue, we move from the link-level analysis to the following node-level analysis.

Node-level analysis. For each firm in the sample, we construct aggregate variables capturing the bank shocks experienced by *all* of its direct suppliers, which we label as supplier bank shock and denote by $\theta_i^{d,AW}$, and the bank shocks experienced by *all* of its direct customers, which is labeled as customer bank shock and denoted by $\theta_i^{u,AW}$. The supplier bank shock of firm i is a weighted average of the bank shocks hitting the direct suppliers of i, where the weights are equal to the ex-ante shares of intermediate inputs cost accounted for by each supplier. These shares essentially capture the ex-ante importance of each supplier for i. Analogously, the customer bank shock of firm i is a weighted average of bank shocks hitting direct customers of i, where the weights in this case are equal to the ex-ante shares of total sales corresponding to each customer.

With these two variables in place, we estimate the following cross-sectional regression at the firm level:

$$\Delta \log s_i = a\theta_i^{AW} + a^u\theta_i^{u,AW} + a^d\theta_i^{d,AW} + \boldsymbol{x}_i + \epsilon_i, \tag{3}$$

where $\Delta \log s_i$ refers to the log change between the whole year 2008 and the whole year 2009 in the total sales of firm i, $\theta_i^{u,AW}$ is the customer bank shock, $\theta_i^{d,AW}$ is the supplier bank shock, and θ_i^{AW} is the direct bank shock experienced by firm i. We are mainly interested in estimating the coefficients of $\theta_i^{u,AW}$ and $\theta_i^{d,AW}$. As we cannot control for firm fixed effects in this firm-level regression, we control for more firm observables than in the link level, in particular for $\boldsymbol{x}_i$,

[26] Regarding Table A5, note that we do not claim that the critique by Ivashina et al. (2022) does not apply to our setting at the loan level; rather, we find that, similarly to Ivashina et al. (2022), when we group all bank loans, the effects that we estimate under either firm or firm-loan type fixed effects are very similar.

[27] Moreover, Table A8 shows that the downstream propagation results are very similar if we do, *versus* we do not, control for whether the firm and the supplier share the same province. The fact that we obtain very similar results independently of whether we control or not for the same firm-supplier province fixed effects suggests that, for the downstream propagation results, bank shocks to suppliers (*given all our controls*) are not correlated with firm-supplier-specific demand shocks related to local firms. A correlation different from zero would have, for example, been the case with local real estate problems affecting both the local economy and, therefore, supplier-customer relationships and local banks. In this case, however, the estimated coefficients of Table A8 versus those of Tables 1-2-A4 would have been different.

which includes: the vector of firm-specific observable characteristics described for the link-level analysis, in both levels and interactions; a dummy that proxies for whether the firm is audited or not (i.e., the firm has information, or not, in the mercantile register); a measure of firm's performance (ROA) as well as a measure of the firm's credit risk (its loan default status); and a set of dummies capturing specific trends in industries and geographical areas in the form of industry-province, zip-code and main-bank fixed effects (i.e., the main bank that the firm was borrowing before the crisis).

Table 3 presents the propagation effects estimated from regression 3. In column (1), we report the impact of direct bank shocks, which is negative and statistically significant. This result is in line with the findings in Table 1 as well as with the vast literature on the bank lending channel that documents significant real effects of bank shocks to firms. The results in column (2) show that the impact of the customer bank shock on sales is also negative and significant. This first-order propagation effect is, however, smaller in magnitude than the corresponding effect estimated at the link level in Table 1. Specifically, one standard deviation of the bank shocks to customers reduces the firms' sales to them by 19.5% of the average sales growth in the sample at the link level, while this reduction is only around 11% at the node level. Intuitively, this reduction suggests that firms are able to partially undo the customers' shocks by resorting to other customers. Column (2) also shows that firms' sales are not significantly affected by bank shocks to their direct suppliers. This result may be due to the substitutability of intermediate input across suppliers, which is an issue that we explore below through the lens of our theoretical model. Finally, columns (3) and (4) of Table 3 refer to the impact of bank credit shocks on employment growth at the firm level, where we obtain results that are similar to those obtained for sales. That is, we find a significant, negative impact of bank shocks on employment growth, either from direct bank shocks to the firm or from bank shocks to its direct customers, while there is a negligible effect of the bank shocks hitting direct suppliers.[28]

4.1 Unanswered questions and the need for theoretical guidance

The reduced-form estimates presented above suggest strong propagation effects of bank shocks from customers to direct suppliers and *vice versa*, i.e. from suppliers to direct customers. However, this type of reduced-form evidence is silent about important issues that are still unanswered by the literature. First, the higher-order propagation of bank shocks hitting indirect suppliers or customers may also have important effects on both firm-to-firm sales and firms' total sales. Second, the reduced-form approach ignores general equilibrium effects that could be quite important as well. Third, some of the estimated propagation effects are difficult to explain, which in turn makes us wonder what mechanisms are at work. For example, consider the question that arose when comparing upstream and downstream first-order propagation of bank shocks in the link-level analysis: Why is it that the former is larger than the latter? May this depend on

[28]In Table A10 of the online Appendix A, we analyze firm-level change in trade credit extended from suppliers to customers and change in trade credit over sales. Results suggest that negative bank shocks to customers reduce trade credit from suppliers to their customers. Note that the estimated coefficient is lower than the change in sales, so in columns (3) and (4) of Table A10 we analyze the change of trade credit over sales. We find that trade credit from suppliers to customers increases over sales when customers suffer a negative bank shock. These results suggest that trade credit partially substitutes the reduction of bank credit.

different mechanisms or/and degrees of substitutability operating in each case? In the remaining part of the paper we propose a model that, once it is taken to our rich network data, allows us to shed light on these and other important questions.

5 A model of bank-shock propagation along the production network

We ended the preceding section by pointing to the wide range of yet unanswered questions that require a theoretical framework to be properly addressed. The objective of the present section is to formulate such a framework. This will enable us to undertake the following tasks: (a) establish an operational connection between the financial and real sides of the economy, (b) evaluate the network effects induced on any given firm by the bank shocks, (c) provide an interpretation of the empirical results, (d) estimate the underlying structural parameters of the model, and (e) suitably quantify the aggregate effects of bank shocks.

Our model is closely aligned with the parametric framework postulated by Bigio and La'O (2020). By adopting a parametric approach, we are able to derive analytical solutions for equilibrium outcomes, thus encompassing a diverse range of equilibrium effects. The framework, however, is more general than Bigio and La'O's parametric model as it allows for non-unitary elasticity of substitution across intermediate inputs. Our main contribution is to provide closed-form expressions of how bank shocks affect equilibrium outcomes through the production network. We then use these expressions to estimate the effects of shock propagation through the production network in the Global Financial Crisis. Since the proposed theoretical framework is in many respects standard, we present its different components in a quite compact manner, focusing in detail only on those features that are less common or more pertinent to the empirical analysis. When formal details and proofs are needed, they are relegated to the online Appendix B.

5.1 Production

The production side of the economy consists of a given set of firms, N, each of them producing a single good with a technology displaying constant returns to scale. The production of a typical firm i is described by the production function:[29]

$$y_i = f_i(k_i, \ell_i, M_i) = \zeta_i k_i^{\rho} \ell_i^{\beta} M_i^{\alpha}, \tag{4}$$

where y_i stands for the output of firm i, k_i for the physical capital used, ℓ_i for its labor input. M_i is the following CES aggregate of the intermediate inputs:

$$M_i = \left(\sum_{j \in N} g_{ji}^{\frac{1}{\sigma}} z_{ji}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{5}$$

[29] The same formulation is used, for instance, in Bernard et al. (2022).

where z_{ji} stands for the amount of intermediate input j used by firm i. Thus, $f_i(\cdot)$ displays the nested CES form, with the strictly positive α,β, and ρ being input shares,[30] σ is the elasticity of substitution across intermediate inputs, and $\boldsymbol{\zeta}=(\zeta_i)_{i=1}^n$ is the vector of Hicks productivity parameters. The non-negative vector $(g_{ji})_{j\in N_i^+}$ reflects the *relative* intensity with which any given firm i uses different intermediate inputs in the set N_i^+ and satisfies $\sum_{j\in N} g_{ji}=1$ for every i. The interfirm (technological) production structure of the economy is then characterized by the (column-stochastic) adjacency matrix $\mathbf{G}=(g_{ji})_{j,i=1}^n$.

5.2 Bank shocks

As in Bigio and La'O (2020), we assume that every firm i is required to pay in advance a share χ_i of its input expenditure, which is financed by borrowing at an interest rate R_i. Its net profit is then given by:

$$\begin{aligned}\pi_i=&p_iy_i-(1-\chi_i)\left(\sum_{j\in N_i^+} p_jz_{jk}+w\ell_i+rk_i\right)-\chi_i(1+R_i)\left(\sum_{j\in N_i^+} p_jz_{ji}+w\ell_i+rk_i\right)\\ =&p_iy_i-(1+\theta_i)\left(\sum_{j\in N_i^+} p_jz_{ji}+w\ell_i+rk_i\right),\end{aligned}$$

where we use the notational shorthand $\theta_i\equiv\chi_iR_i$, and p_i denotes the price of good i, w is the wage, and r is the rental cost of capital. For convenience, we use as a benchmark what we call *baseline conditions* characterized by $R_i=0$, while if the firm's borrowing cost rises to some $R_i>0$ we say that the firm is experiencing a bank shock of magnitude $\theta_i=\chi_iR_i$. A consequence of this shock is that the firm faces a "financial distortion" (or wedge) in its decision problem given by θ_i. In our context, such a shock originates in the banks servicing the firm and leads to a change in the terms at which the firm can obtain bank credit. In Section 6, we explain how we map these shocks to the empirically identified bank shocks described in Section 3.

5.3 Consumption, prices, and equilibrium

To close the model, we need to formalize the consumption side of the economy, specify how prices are set, and then posit a suitable equilibrium notion.

First, concerning consumption, we assume that the consumption vector $\boldsymbol{c}=(c_1,c_2,...,c_n)$ is chosen by a representative household, which also provides firms with labor L and inelastically supplies K units of physical capital. Her objective is to maximize the following utility function:[31]

$$U(\boldsymbol{c})=\frac{c^{1-\delta}}{1-\delta}-\frac{L^{1+\eta}}{1+\eta} \tag{6}$$

[30] We follow the recent literature on production networks – see Barrot and Sauvagnat (2016); Carvalho et al. (2020); Bernard et al. (2022) – in assuming homogeneous input shares in our benchmark setting. In the online Appendix B, we extend the model to allow for firm-specific parameters α, β and ρ. We prove all the results about the propagation of bank shocks for such a more general version of the model.

[31] This widely used utility function was introduced in MaCurdy (1981).

subject to a budget constraint

$$\sum_i p_i c_i \leq E, \tag{7}$$

where $c=\prod_{i=1}^n c_i^{\gamma_i}$ is a composite consumption bundle, $\boldsymbol{\gamma}=(\gamma_i)_{i=1}^m$ is a vector of preference weights for every good i, δ modulates the income elaticity of labor supply (equal to $-\frac{\delta}{\eta}$), η is its inverse Frisch elasticity, and E is the household's income. The financial flows in the economic system are taken to be balanced, so the household's income satisfies $E=wL+r+\sum_{i\in N}\pi_i+TR$, where $TR\equiv\sum_{i\in N}\theta_i\left(\sum_{j\in N_i^+}p_j z_{ji}+w\ell_i+rk_i\right)$ is a lump-sum transfer from the financial sector.

Second, concerning prices, we follow Baqaee (2018) and Baqaee and Farhi (2019) in using a reduced-form approach postulating that every firm i sets its price by applying a markup μ_i to its marginal cost of production. These markups can be understood as parameters of the model that embody the competition structure of the economy, which is not explicitly modeled.

Finally, the equilibrium concept can be described as follows.

Definition 1. *Given a vector of financial distortions $\boldsymbol{\theta}=(\theta_i)_{i=1}^n$, a* Market Equilibrium *(ME) is an array $\left\{[(p_i^*)_{i=1}^n,w^*,r^*],[(c_i^*)_{i=1}^n,(y_i^*)_{i=1}^n,(z_{ij}^*)_{i,j=1}^n,(\ell_i^*)_{i=1}^n,(k_i^*)_{i=1}^n,L^*]\right\}$ that satisfies the following conditions:*

- *Each firm i minimizes production costs and applies to them a mark-up μ_i to set its price.*
- *The consumption plan maximizes the household's utility subject to its budget constraint.*
- *Markets for each intermediate input, capital, and labor clear.*

The existence of a market equilibrium follows from standard arguments, and its uniqueness relies on our Cobb-Douglas assumption on preferences and nested CES assumption on technologies.

5.4 Link-level implications of the model

In this subsection, we center on the impact of bank shocks hitting the firms on the equilibrium log value of trade s_{ji} between j and i, normalized by the total sales s_i of customer firm i, i.e. we consider the change of $\log\frac{s_{ji}}{s_i}$. Our interest, therefore, is in how a shock to the supplier j (supplier bank shock) or customer i (customer bank shock) affects the sales from j to i once we net out the effect coming from the total change of the sales of i.[32]

The following expression, derived by Proposition 1 in the online Appendix B, provides a linear approximation of the total change in $\log\frac{s_{ji}}{s_i}$ over the crisis period induced by the *overall* vector of bank shocks in the economy:

$$\begin{aligned}\overbrace{d\log s_{ji}}^{\substack{\text{Change of sales}\\\text{from j to i}}} - \overbrace{d\log s_i}^{\substack{\text{Change in log sales}\\\text{of customer } i}} &= -(\sigma-1)\overbrace{\theta_j}^{\substack{\text{Supplier}\\\text{bank shock}}} - \overbrace{\theta_i}^{\substack{\text{Customer}\\\text{bank shock}}}\\ &\quad -(\sigma-1)\,\alpha\,\overbrace{\mathbf{e}_j'\mathbf{G}'(\mathbf{I}-\alpha\mathbf{G}')^{-1}\boldsymbol{\theta}}^{\substack{\text{Higher-order supplier}\\\text{bank shock } (Net_j)}} + (\sigma-1)\,\overbrace{\mathbf{e}_i'\mathbf{G}'(\mathbf{I}-\alpha\mathbf{G}')^{-1}\boldsymbol{\theta}}^{\substack{\text{Higher-order customer}\\\text{bank shock } (Net_i)}}.\end{aligned} \tag{8}$$

[32] Even though our link-level analysis in this Section and Section 6.1 will focus on such a normalized supplier-customer trades, to simplify exposition we sometimes omit an explicit reference to this normalization.

Equation (8) shows that the total effect can be decomposed into two parts, which we now describe in turn.

First, we have the propagation effect influencing the sales s_{ji} of a supplier j to a customer i when a shock θ_j directly hits the supplier or, reciprocally, when a shock θ_i directly hits the customer. These are what we call the *first-order* effects. Importantly, note that equation (8) implies that the effects of customer bank shock and supplier bank shock are asymmetric. The intuitive reason for this asymmetry can be explained as follows. When firm i is hit by a bank shock θ_i, buying inputs becomes costlier and therefore its demand for *all* its inputs (including j) decreases. This effect is captured by $-\theta_i$. Instead, when supplier j is affected by a bank shock, the price of input j is affected. Hence, the extent and direction to which firm i adjusts its purchases from j depend on the degree of substitutability i enjoys across its intermediate inputs. More specifically, if $\sigma > 1$, so that intermediate inputs are substitutes, then firm i spends relatively less on input j when j is affected by the negative shock, while the opposite happens when $\sigma < 1$.

Second, equation (8) also includes the higher-order supplier bank shock Net_j and the higher-order customer bank shock Net_i. These are the network-based aggregates of the bank shocks hitting the direct and indirect suppliers of all orders of j and i, respectively. Again, there is an asymmetry in the effects of these two shocks, which can be explained in a way similar to how we did it before. On the one hand, Net_j captures the effect of shocks hitting all of j's direct and indirect suppliers of any order on the prices of j's intermediate inputs. Thus, a higher Net_j implies an increase in the marginal cost for j, which translates into a higher price of j at the equilibrium. Then, as it happened for the supplier bank shock, the effect of such a rise in Net_j on the sales s_{ji} is negative or positive depending on whether σ is higher or smaller than 1. On the other hand, the higher-order customer bank shock Net_i captures the change in the price index of the intermediate inputs of i due to bank shocks. Higher Net_i implies that suppliers of i are hit on average by larger bank shocks (direct and indirect). Viewing those suppliers as an aggregate substitute for input j ($\sigma > 1$), a higher Net_i implies that it is less attractive to firm i to substitute away from j to other suppliers, implying a positive effect on sales from j to i. But when this happens, the reaction to an increase in Net_j must be *negative*, as explained before.[33]

5.5 Node-level implications of the model

We now turn our attention to the node-level analysis. This entails jointly accounting for the *direct* and *indirect* (first- and higher-order) effects of bank shocks hitting each *individual firm*. In our ensuing analysis, the following notation will be useful. Let $\mathbf{M}$ and $\mathbf{T}$ stand for diagonal matrices with elements $\frac{1}{\mu_i}$ and $\frac{1}{1+\theta_i}$ on the main diagonal, respectively, where μ_i denotes the mark-up of firm i and θ_i the financial shock hitting it. Then denote $\boldsymbol{v}(\boldsymbol{\theta}) \equiv (\mathbf{I} - \alpha \mathbf{GMT}(\boldsymbol{\theta}))^{-1} \boldsymbol{\gamma}$, where $\boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_n)$ and recall that $\boldsymbol{\gamma}$ captures the relative preferences of the consumer for different consumption goods. In the absence of shocks, $\boldsymbol{v}(\mathbf{0})$ is a variation of the standard centrality notion

[33] Note that equation (8) does not feature the propagation effects coming from customers of customers. The reason is that such upstream propagation is absorbed by the normalization factor $\mathrm{d}\log s_i$, which we include in our link-level analysis in order to focus on the *relative* effects affecting supplier j among all other suppliers of i. We study the effects of bank shocks on $\mathrm{d}\log s_i$ in our node-level analysis, which is undertaken in the next section.

proposed by Bonacich (1987), aggregating the number of suitably weighted downstream paths that connect i to the representative household along the production network.

The following expression, proven by Proposition 2 in the online Appendix B, provides a linear approximation of the total change in the sales of firm i relative to the household's income E[34] induced by the overall vector of shocks in the economy:

$$\mathrm{dlog}\left(\frac{s_i}{E}\right) = -\alpha \boldsymbol{e}_i'(\mathbf{I}-\alpha\mathbf{V}^{-1}\mathbf{GMV})^{-1}\mathbf{V}^{-1}\mathbf{GMV}\boldsymbol{\theta} + (1-\sigma)\boldsymbol{e}_i'\boldsymbol{\Lambda}\boldsymbol{\theta}, \tag{9}$$

where $\mathbf{V}{=}diag(\boldsymbol{v}(\mathbf{0}))$ and

$$\boldsymbol{\Lambda} \equiv \mathbf{V}^{-1}(\mathbf{I}-\alpha\mathbf{GM})^{-1}\left(diag(\alpha\mathbf{GMV1}) - \alpha\mathbf{GMVG}'\right)\left(\mathbf{I}-\alpha\mathbf{G}'\right)^{-1}.$$

We first note that the matrix $\mathbf{H} \equiv \alpha\mathbf{V}^{-1}\mathbf{GMV}$ with elements $h_{ji}{=}\frac{s_{ji}}{s_j}$ that essentially capture the relative importance of firm i among all direct customers of j (see Lemma 3 in the online Appendix B). This information is observed in our dataset and hence can be directly used in our empirical analysis. Therefore, we write $\boldsymbol{\Lambda}$ in the following more convenient form:

$$\boldsymbol{\Lambda}{=}(\mathbf{I}-\mathbf{H})^{-1}\left(diag(\mathbf{H1}) - \mathbf{HG}'\right)\left(\mathbf{I}-\alpha\mathbf{G}'\right)^{-1},$$

which then allows us to rewrite (9) as follows:

$$\overbrace{\mathrm{dlog}\left(\frac{s_i}{E}\right)}^{\substack{\text{Change of Domar weight}\\ \text{of firm i}}} = - \overbrace{\boldsymbol{e}_i'(\mathbf{I}-\mathbf{H})^{-1}\mathbf{H}\boldsymbol{\theta}}^{\substack{\text{Upstream}\\ \text{shock}}} + (1-\sigma) \overbrace{\boldsymbol{e}_i'\boldsymbol{\Lambda}\boldsymbol{\theta}}^{\substack{\text{Bidirectional}\\ \text{shock}}}. \tag{10}$$

Intuitively, the term $\boldsymbol{e}_i'(\mathbf{I}-\mathbf{H})^{-1}\mathbf{H}\boldsymbol{\theta}$ captures the demand effect due to bank shocks. When a firm is hit by a bank shock its demand for inputs decreases, as inputs become more expensive. This effect propagates upstream through the network, as captured by the upstream operator $(\mathbf{I}-\mathbf{H})^{-1}\mathbf{H}$. We label the network-based aggregate shocks given by $(\mathbf{I}-\mathbf{H})^{-1}\mathbf{H}\boldsymbol{\theta}$ as upstream shocks and note that they aggregate both first-order and higher-order shocks.

Matrix $\boldsymbol{\Lambda}$ captures a complex propagation process that *jointly* includes upstream and downstream, first-order and higher-order components. In view of this feature, we label this type of propagation as *bidirectional*. Informally, the composite process induced can be explained as follows. First of all, shocks $\boldsymbol{\theta}$ directly affect firms' production costs and, indirectly, the costs of firms downstream in the network. Such an initial phase of shock propagation is captured by the term $(\mathbf{I}-\alpha\mathbf{G}')^{-1}\boldsymbol{\theta}$. Naturally, the resulting changes in production costs will typically translate into corresponding changes in the prices of the goods being produced downstream, eventually affecting the demand for them. These price-induced substitution effects affecting demands are captured by the entries of the vector $(diag(\mathbf{H1})-\mathbf{HG}')(\mathbf{I}-\alpha\mathbf{G}')^{-1}\boldsymbol{\theta}$. The resulting adjustments then act as demand shocks, propagating upstream to all direct and indirect suppliers of the affected goods, as captured by the operator $(\mathbf{I}-\mathbf{H})^{-1}$.

An interesting point arising from equation (10) is that, for every firm i, $\mathrm{dlog}s_i$ does not *directly* depend on its bank shock θ_i. The reason is that such a dependence materializes only indirectly through market-mediated channels that account for how all other firms react to any

[34]Note that E coincides with the GDP of the economy. A firm's sales as a share of GDP is known as its Domar weight.

change in the cost of its production, and how prices correspondingly adjust to clear *all* markets. This is clear from (10), where the logarithm of sales of firm i is found to be affected by θ_i through bidirectional propagation.[35]

5.6 Aggregate implications of the model

So far, we have focused our theoretical analysis on the microeconomic implications of shock propagation at the node or link levels. It is important, however, to understand and quantify the aggregate relevance of the phenomenon when such microeconomic effects are suitably aggregated into economy-wide magnitudes. In this section, we carry out this analysis within our theoretical framework.

We are primarily interested in the effect of bank shocks on the real GDP of the economy. In our model, the real GDP equals the aggregate consumption c. Therefore, our aim boils down to tracing how shocks affect the growth rate of this magnitude. In Proposition 3, included in the online Appendix B, we show that such an aggregate effect can be linearly approximated through the following equations:

$$\mathrm{dlog} c = -\boldsymbol{\gamma}' \left[\mathbf{I} - \alpha \mathbf{G}'\right]^{-1} \boldsymbol{\theta} - \frac{\beta}{1-\alpha} \mathrm{dlog} w - \frac{\rho}{1-\alpha} \mathrm{dlog} r, \tag{11}$$

where

$$\mathrm{dlog} w = \frac{\eta}{1+\eta} \frac{\beta}{wL} \boldsymbol{s}' \mathbf{M} \mathrm{dlog} \boldsymbol{s} - \frac{1-\delta}{1+\eta} \mathrm{dlog} c, \tag{12}$$

$$\mathrm{dlog} r = \frac{\rho}{rK} \boldsymbol{s}' \mathbf{M} \mathrm{dlog} \boldsymbol{s}, \tag{13}$$

and

$$\mathrm{dlog} \boldsymbol{s} = -\left[\mathbf{I} - \mathbf{H}\right]^{-1} \mathbf{H} \boldsymbol{\theta} + (1-\sigma) \boldsymbol{\Lambda} \boldsymbol{\theta}. \tag{14}$$

Equation (11) describes the additively separable channels through which bank shocks affect the logarithm of GDP. The first term on the right-hand side of (11) captures the effect operating through the markets for intermediate inputs, and hence the production structure of the economy. The last two terms capture the effects mediated through the wage w and capital return r determined *endogenously* through the operation of the labor and capital markets.

Equation (12) describes the effect operating through the wage adjustment. This depends on the network structure of the economy (as given by $\mathbf{G}$, which underlies $\boldsymbol{s}$), the competition structure of the economy (reflected by $\mathbf{M}$), the pre-distortion labor income (wL), the labor elasticity of production (β) and household responses to changes in wage and income levels, captured by η and δ, respectively. Since capital is supplied inelastically, there is no feedback from the consumption level c in equation 13, as was the case with the labor-market channel.

Solving the system of equations given by (11), (12) and (13),[36] we get (see Corollary 2 in the online Appendix B) the following closed-form expression for the effect of bank shocks on the real

[35] Shock θ_i also affects $\log s_i$ through upstream propagation whenever i is an indirect customer of itself, which may happen due to cycles in the production network.

[36] This system of equations induces an ex-ante structural result in the language of Baqaee and Farhi (2019).

GDP:

$$\begin{aligned}\mathrm{dlog} c = & -\left(1-\frac{1-\delta}{1+\eta}\frac{\beta}{1-\alpha}\right)^{-1}\boldsymbol{\gamma}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{\theta} \\ & -\frac{1}{1-\alpha}\left(1-\frac{1-\delta}{1+\eta}\frac{\beta}{1-\alpha}\right)^{-1}\left(\frac{\eta\beta^2}{(1+\eta)wL}+\frac{\rho^2}{rK}\right)\boldsymbol{s}'\mathbf{M}\Big(-\left[\mathbf{I}-\mathbf{H}\right]^{-1}\mathbf{H}\boldsymbol{\theta}+(1-\sigma)\boldsymbol{\Lambda}\boldsymbol{\theta}\Big).\end{aligned} \tag{15}$$

The previous expression captures the wide range of effects that are involved in shaping the aggregate impact of direct bank shocks and their propagation on the GDP of the economy. As explained, these effects embody mechanisms of very different sorts: some are network-based, while others are market-based or preference-based. These mechanisms are affected by key *preference* and *technological elasticities*, which interact with observable market magnitudes and the production structure of the economy in the complex non-linear manner displayed in (15). One of the sources of this complexity derives from the fact that a complete analysis of the phenomenon of shock propagation requires considering *all possible paths* that connect every bank shock to every firm in the economy, both upstream and downstream. In fact, as we briefly discussed in the Introduction, it is precisely such full-fledged network-based propagation that has been highlighted as underlying the severity of the Great Recession.

In Section 6.3, we shall rely on our model and data to provide a quantitative assessment of how important the network propagation of bank shocks was in Spain. Specifically, we will compare the overall aggregate effects on the GDP, as given by (15), with the prediction induced from the model under the counterfactual assumption that the shock propagation unfolding through the network has been blocked.

We are also interested in the question of how important the higher-order network propagation effects are compared to the first-order effects. To do this, we note for later use that if we allow only for first-order propagation (15) becomes:

$$\begin{aligned}\mathrm{dlog} c = & -\left(1-\frac{1-\delta}{1+\eta}\frac{\beta}{1-\alpha}\right)^{-1}\boldsymbol{\gamma}'\left(\mathbf{I}+\alpha\mathbf{G}'\right)\boldsymbol{\theta} \\ & -\frac{1}{1-\alpha}\left(1-\frac{1-\delta}{1+\eta}\frac{\beta}{1-\alpha}\right)^{-1}\left(\frac{\eta\beta^2}{(1+\eta)wL}+\frac{\rho^2}{rK}\right)\boldsymbol{s}'\mathbf{M}(-\mathbf{H}\boldsymbol{\theta}+(1-\sigma)\boldsymbol{\Lambda}_{approx}\boldsymbol{\theta}),\end{aligned} \tag{16}$$

where $\boldsymbol{\Lambda}_{approx} \equiv (\mathbf{I}+\mathbf{H})(diag(\mathbf{H1})-\mathbf{HG}')(\mathbf{I}+\alpha\mathbf{G}')$.

6 Structural evidence and general equilibrium propagation

Our rich dataset allows us to estimate empirical counterparts of the structural equations (8) and (10) that embody, respectively, our link- and node-level analyses. In this section, we address each of these estimation exercises. We also quantify the role of network propagation in determining the impact of bank shocks on aggregate economy-wide outcomes.

6.1 Link-level structural evidence

To take equation (8) to the data we proceed as follows. First, we map the AW bank shocks θ_i^{AW} (see Section 3) to corresponding bank shocks θ_i contemplated by the theory (see Section 5.2). This mapping follows the form $\theta_i = \xi\theta_i^{AW}$, where ξ is to be estimated. Second, to operationalize the higher-order bank shocks specified in the theory, we additionally need empirical counterparts of $\mathbf{G}$ and α. We directly obtain the matrix $\mathbf{G}$ from the observed ex-ante firm-to-firm transactions using Lemma 3 (see online Appendix B). To obtain the parameter α (the share of intermediate inputs) we rely on standard methods for estimating production functions at the firm level in the Olley and Pakes (1996), Levinsohn and Petrin (2003) and Ackerberg et al. (2015) tradition, and obtain a value of 0.48, which is in line with previous estimates available in the literature.[37]

Given such an operationalization of our theory, we consider the following regression as an empirical counterpart of equation (8):

$$\Delta \log\left(\frac{s_{ji}}{s_i}\right) = \lambda_F^d \theta_j^{AW} + \lambda_F^u \theta_i^{AW} + \lambda_H^d Net_j^{AW} + \lambda_H^u Net_i^{AW} + \boldsymbol{x}_{ji} + \varepsilon_{ji} \tag{8R}$$

with λ_F^d, λ_F^u, λ_H^d, and λ_H^u being the parameters to be estimated. The superindices d and u indicate propagation effects that operate downstream or upstream, respectively, while the subindices F and H refer to effects that are first- or higher-order, respectively.

The aforementioned parameters have a structural interpretation based on the one-to-one mapping between equations (8) and (8R). This mapping is induced by the following relationships: $\lambda_F^d = -(\sigma-1)\xi$, $\lambda_F^u = -\xi$, $\lambda_H^d = -(\sigma-1)\alpha\xi$, and $\lambda_H^u = (\sigma-1)\xi$, where we use the fact that the mapping $\theta_i = \xi\theta_i^{AW}$ directly implies $Net_i = \xi Net_i^{AW}$. From these relationships, σ, the elasticity of substitution across intermediate inputs, can be recovered from the estimation of (8R) as $\sigma = 1 + \lambda_F^d/\lambda_F^u$.[38] Note as well that we can identify ξ directly from (8R) as the estimate of $-\lambda_F^u$, although its precise value is not as interesting as that of σ since, in essence, it just plays the role of a scale parameter. Finally, our regressions estimating (8R) also include a set of control variables $\boldsymbol{x}_{ji}$ to account for non-modeled factors that may have affected how different firms responded to the financial crisis, such as location, industry, or size, over and above the channel that we are interested in this paper. These are the same variables considered in the reduced-form regressions discussed in Section 4, including firm-fixed effects in some regressions.

Table 4 presents the results of estimating different variants of equation (8R). Following our approach from Section 4, we start by estimating upstream and downstream propagation separately, and then we estimate both upstream and downstream together as in equation (8R).

[37]See, for instance, Levinsohn and Petrin (2003) in a sample of Chilean firms. Note also that, since firms' productivities are unobservable and likely correlated with input choices, standard OLS estimates of the production function parameters will be biased. In order to address this endogeneity, this strand of the literature considers a so-called control approach and uses a non-parametric function of input demand to control for unobserved productivity under the assumption of a strictly monotonic relationship between a firm's input demand and productivity. To be more concrete, given our interest in the share of intermediates parameter, we follow Levinsohn and Petrin (2003) and exploit the firms' material input demand to control for unobserved productivity at the firm level; also, estimation is based on the GMM approach outlined in Wooldridge (2009), which leads to more efficient estimators than the original two-step approach.

[38]The value of σ can also be recovered as $1 - \lambda_H^u/\lambda_F^u$. We have tested this overidentifying restriction and verified that both estimates are not statistically different.

Columns (1) and (2) present the estimated effects of upstream propagation from customers to suppliers. In column (1), we do not control for higher-order shocks (Net), while in column (2), we do incorporate them. The results indicate that one standard deviation of customer bank shock leads, on average, to a reduction in firm-customer sales of approximately 2 pp, in line with the reduced-form estimate in Table 1.[39] In column (2) we also find that the estimated effect of higher-order customer bank shocks is significant and of a magnitude similar to that of the first-order effects but with the opposite sign. More concretely, we find that an increase in one standard deviation of higher-order customer shocks implies, on average, an increase of 1.8 pp of the dependent variable, which represents 15% of the mean value of the dependent variable.

Columns (3) and (4) of Table 4 then turn to downstream propagation of supplier bank shocks.[40] In line with the results for upstream propagation, we find that the first-order downstream effect, when not controlling for higher-order shocks, is very similar to that of Table 2 from our reduced-form specification. Also, we find that the effect of higher-order supplier bank shocks is statistically significant and large in economic terms. Specifically, a one standard deviation increase in Net_j^{AW} leads, on average, to a reduction of 2 pp in supplier-firm purchases, while a one standard deviation increase in the supplier bank shock (θ_j^{AW}) leads, on average, to 1 pp reduction.

Comparing columns (1) and (2) for upstream propagation and columns (3) and (4) for downstream propagation, we find that the estimated effect of customer (supplier) bank shocks remains almost unchanged once we control for the higher-order shocks as prescribed by the theory. This suggests that the estimates for the first-order effects resulting from the reduced form approach in Section 4 are not biased due to the omission of higher-order shocks and general equilibrium effects. However, our results indicate that not accounting for the higher-order shocks leads to substantial bias – underestimation (or overestimation) – of the total downstream (or, respectively, upstream) propagation effect of bank shocks. Indeed, the magnitude of the estimated higher-order effects is similar to, or even larger than, the magnitude of the first-order effects.

Next, we estimate equation (8R) directly as a *single* regression, jointly capturing upstream and downstream propagation, using the joint samples of columns (2) and (4), with the same two sets of firm fixed effects. We report the results in column (5) of Table 4. Since, as discussed above, we use these estimates to recover the structural parameters of the model, we report non-standardized coefficients. From estimates displayed in column (5) we have $\lambda_F^d = -23.95$ and $\lambda_F^u = -42.41$, which leads to a value for the elasticity of substitution $\sigma = 1 + \lambda_F^d/\lambda_F^u$ of 1.56 (estimated with a standard error of 0.45).[41] This implies that the intermediate inputs are

[39] Indeed, note that the estimated value of column (1) in Table 4 is very similar to that of column (2) in Table 1. Even though the dependent variables of the regressions reported in Table 4 and Table 1 are different, we obtain similar estimates as this difference is mostly irrelevant once we control for customer fixed effects.

[40] We note that our estimation strategy relies on the assumption that the production function (and hence also $\mathbf{G}$) does not change in response to bank shocks. We, therefore, require that $\theta_i^{AW} \perp \mathbf{G}$. In particular, a bank shock hitting firm i does not affect g_{ji} and g_{ij}. This assumption may be violated, for instance, if, given our set of controls, there are firm $\times$ supplier-specific demand shocks that are systematically correlated with suppliers' bank shocks. Given the variation of our data, we cannot obtain identification of structural parameters by relaxing this assumption and allowing a nonparametric relation between θ_i^{AW} and g_{ji}. However, we can show, by controlling for a suitable set of fixed effects, that our results are not driven by e.g. the same location of firms versus their suppliers or customers (see footnote 27 and Table A8 in the online Appendix A).

[41] The alternative way to recover σ mentioned in Footnote 38 yields a value of 2.21 (with a standard error of

substitutes.[42] Note that the estimated value of σ rationalizes the finding that the first-order downstream effect is lower than the first-order upstream effect.[43] As a robustness check, in column (6), we again estimate equation (8R) but, unlike for column (5), we only control for firm fixed effects with a value of one if the firm is either a supplier or a customer. The results are similar to those of column (5), giving rise to an estimated $\sigma = 1.43$.

We now contrast our elasticity estimate with those from other studies. In doing so, it is essential to consider an important point emphasized by Ruhl et al. (2008) and Boehm et al. (2019): the elasticity of substitution is inherently linked to the time horizon and the nature of the shocks being considered. Bearing this in mind, we provide a brief review of the estimates of this elasticity obtained by several other papers using different approaches. Using a variation in imports and annual data from Hungary, Halpern et al. (2015) estimate the elasticity of substitution between imported and domestic intermediate inputs to be between 4 and 7. Relying on monthly data on imports from Japan to the USA and variation caused by the Tohoku earthquake, Boehm et al. (2019) estimate the elasticity of substitution between imported and domestic intermediate inputs to be between 0.201 and 0.624. By utilizing annual sector-level data from USA and plausible variation in input prices, Atalay (2017) estimates the annual elasticity of substitution between intermediate inputs between -0.13 and - 0.07. Peter et al. (2022) estimate, using sector-level data from India and exploiting a trade liberalization shock, a long-run elasticity of substitution between material input categories of 3.1. The following two papers are closest to ours, as they also rely on firm-level production network data to back up elasticities. Using partial annual data on the production network of Japan and the shock caused by the Tohoku earthquake, Carvalho et al. (2020) estimate the elasticity of substitution across intermediate inputs to be 1.18. Relying on monthly firm-level transaction data from India and disruptions caused by governmental responses to Covid-19 pandemic, Fujiy et al. (2024) find that the estimated elasticity of substitution across intermediate inputs lies within the range of 0.50 to 0.66.[44]

Finally, we note that using results from column (5), we can formally test several implications of the model regarding the sign and magnitude of the estimated coefficients from regression (8R). In particular, we cannot reject the following null hypotheses at conventional confidence levels. First, the sign of the effect of higher-order customer bank shock is opposite to that of the (first-order) customer bank shock. Second, the sign of the effect of the higher-order supplier bank

0.68). By performing a statistical test, we find that the two estimates are not statistically different (with p-value of 0.26). Our preferred strategy for estimating σ is based on the ratio between λ_F^d and λ_F^u because these values depend less on estimated and calibrated objects and this reduces the possible measurement error.

[42] Substituting an input with inputs provided by alternative suppliers from the same sector may be different than substituting it with inputs provided by suppliers from other sectors. We investigate this issue in the online Appendix A. We find that the across-sectors σ takes a value of 1.25 (1.28) across 2-digit (3-digit) NACE sectors. On the other hand, we find that the within-sector σ is equal to 1.95 (2.37) within the 2-digit (3-digit) NACE sector. The results of both exercises are reported in Table A11 in the online Appendix A. They confirm the intuition that it is easier to substitute inputs with inputs provided by alternative suppliers from the same sector than with inputs provided by suppliers from other sectors.

[43] In the online Appendix A, Table A12, we also estimate (8R) by considering 2010 instead of 2009 as the end year, thereby looking at the biannual changes. In this case, we estimate the elasticity of substitution to be $\sigma = 2.17$ with a standard error of 0.88. This result is in line with the intuition that, in the longer run, it becomes easier to substitute across suppliers.

[44] Even though they do not estimate the elasticity of substitution directly, Barrot and Sauvagnat (2016) argue that their results on the effects of supply chain disruption on firm-level outcomes are consistent with Leontief production function in the short run (quarterly data).

shock is the same as that of the (first-order) supplier bank shock. Third, the effects of higher-order upstream and first-order downstream propagation are similar in magnitude and display an opposite sign.

6.2 Node-level structural evidence

As we have explained in our discussion of the theory in Section 5, bank shocks affect firms' sales through two complex propagation channels. First, bank shocks propagate in a purely upstream manner as demand shocks. In equation (10), this is captured by the matrix $\mathbf{U} \equiv (\mathbf{I}-\mathbf{H})^{-1}\mathbf{H}$, with its ith row being the vector $\mathbf{U}_i = \boldsymbol{e}_i'(\mathbf{I}-\mathbf{H})^{-1}\mathbf{H}$, which reflects how firm i is affected, directly and indirectly, by the shock θ_k hitting each firm k. Second, the model also identifies another, bidirectional, type of propagation involving the concatenation of downstream propagation followed by a chain of upstream propagation. Downstream propagation affects the costs of all direct and indirect customers of firms experiencing a bank shock. The subsequent upstream propagation chains induce "demand shocks" to the direct and indirect suppliers of each of the firms affected along the initial chains of downstream propagation. In the theory, such a bidirectional propagation is captured by the matrix $\boldsymbol{\Lambda}$, which we now mnemonically rename as $\mathbf{B}$ for "bidirectional" (as, in an analogous manner, the previous notation $\mathbf{U}$ was meant to point to "upstream"), its ith row denoted by $\mathbf{B}_i$.

The entries of the matrices $\mathbf{U}$ and $\mathbf{B}$ can be constructed from our data. $\mathbf{U}$ is a composition of powers of the matrix $\mathbf{H}$, while $\mathbf{B}$ involves powers of the matrices $\mathbf{G}$ and $\mathbf{H}$. Thus, if we posit again a linear relationship $\theta_i = \xi\theta_i^{AW}$, mapping the empirical AW shock θ_i^{AW} hitting each firm i to its corresponding bank shock θ_i contemplated by theory, the theoretical equation (10) that aggregates the effects of all shocks hitting any given firm i in the economy has the following empirical counterpart:

$$\Delta \log\left(\frac{s_i}{E}\right) = \lambda_U \mathbf{U}_i \boldsymbol{\theta}^{AW} + \lambda_B \mathbf{B}_i \boldsymbol{\theta}^{AW} + \boldsymbol{x}_i + \varepsilon_i. \tag{10R}$$

Here $\lambda_U = -\xi$ and $\lambda_B = (1-\sigma)\xi$ are parameters to be estimated that capture the upstream and bidirectional propagation effects predicted by the theory, while $\boldsymbol{x}_i$ stands for the set of observable and unobservable covariates that account for the heterogeneity not contemplated by the theory.

Table 5 reports the results of estimates following from (10R). In column (1) we only account the upstream shock $\mathbf{U}_i\boldsymbol{\theta}^{AW}$, while in column (2) we only account for the bidirectional shock $\mathbf{B}_i\boldsymbol{\theta}^{AW}$. In both cases, the estimated coefficients are negative and significant. In column (3), we directly estimate equation (10R) accounting for both shocks simultaneously and find evidence of sizable propagation, both upstream and bidirectional. Furthermore, the effects in both cases are negative (i.e. reflect decreases in firms' sales), as the theory predicts if $\sigma > 1$. In particular, we find that an increase in one standard deviation of the upstream shock to a firm leads, on average, to a decrease in sales of 1.6 pp, while one standard deviation of the bidirectional shock reduces its sales, on average, by 0.5 pp. In economic terms, given that the average growth of firms' sales is -20% (see Table A1), these numbers amount to a joint reduction of 11% in average firm growth due to the bank shocks and their propagation through the production network.

Let us also note that, by estimating equation (10R), we can obtain an alternative estimate of the structural parameter σ. For this purpose, column (5) in Table 5 mirrors column (3) but shows non-standardized coefficients. From (10R), $\sigma - 1$ is equal to the ratio of the bidirectional effect (λ_B) and the upstream effect (λ_U). Using this approach, we obtain $\sigma = 1.35$ (with a standard error of 0.47), which is in line with our previous estimate of $\sigma = 1.56$ (with a standard error of 0.45) in Section 6.1. As explained, our preferred estimate of σ is the one obtained from our previous link-level regression. The reason is two-fold: first, in that case we could control for firm observed and unobserved characteristics, thus enhancing identification; second, the estimation of σ using the node-level regression given by (10R) relies more heavily on network measures, for which measurement error may be more substantial than for the approach undertaken by the link-level regression (8R).

Finally, recall that an interesting prediction of the model, derived in Section 5.5 (cf. (10)), is that a bank shock hitting firm i should not affect the growth rate of its sales once we control for how this firm is affected by the propagation of all bank shocks in the economy. To test it, we proceed in two steps. First, we show in Table 3 that the effect of a direct bank shock to the firm is negative and statistically significant. This suggests that, when we ignore general equilibrium considerations, the dependence on the own bank shock does arise. Second, in column (4) of Table 5, we focus on a specification that adds to the one considered in column (3) – that accounts for all general-equilibrium market adjustments – the direct bank shock as an additional regressor. We find that the estimation of this second specification delivers a non-statistically significant coefficient of the direct bank shock, whose absolute value decreases by approximately 50%.

6.3 Aggregate effects

We now turn to quantifying the extent to which network propagation plays an important role in determining aggregate economy-wide outcomes. To this end, we rely on the theory to take into account the general equilibrium effects induced by the shocks and then aggregate them to assess their overall impact on the log of the real GDP through a first-order approximation of the equilibrium equation (15). To bring this equation to the data and arrive at a quantitative estimation of the aggregate effects, we rely on the following calibration strategy:

(a) The parameters of the production function (α, β, ρ) are estimated using standard production function estimation techniques at the firm level, as explained in Section 6.1. Specifically, we arrive at the following values: $\alpha = 0.483$, $\beta = 0.317$, and $\rho = 0.2$.

(b) The entries of the diagonal matrix $\mathbf{M}$, which captures markups, are estimated using the model-implied relationship $\mu_i = \alpha \frac{\tilde{\omega}_{ji}}{\omega_{ji}}$, where $\tilde{\omega}_{ji} \equiv \frac{s_{ji}}{\sum_{q \in N} s_{qi}}$ and $\omega_{ji} \equiv \frac{s_{ji}}{s_i}$. Crucially, note that we observe the values s_{ji} and s_i in our firm-to-firm data for the baseline year. The online Appendix B provides more details on the markup estimation procedure.

(c) The aggregate compensation to employees (wL) and capital (rK) are calculated from the market clearing conditions $wL = \beta \boldsymbol{s}'\mathbf{M}\mathbf{1}$ and $rK = \rho \boldsymbol{s}'\mathbf{M}\mathbf{1}$. Again we rely on the fact that the vector of baseline-year sales $\boldsymbol{s}$ is observed in the data.

(d) Matrix $\mathbf{G}$ has as its entries the input cost shares of every firm, which, as shown in Lemma 3, can be calibrated as $g_{ij} = \tilde{\omega}_{ij}$ using baseline year observations. By definition, the entries of the matrix $\mathbf{H}$ are equal to $\frac{s_{ij}}{s_i}$, which are also directly observed in the data.

(e) The vector of bank shocks $\boldsymbol{\theta}$ is mapped to the data by combining our estimated AW shocks θ_i^{AW} and our estimate of ξ obtained from the link-level analysis – see column (5) in Table 4, which implies that $\xi = -\lambda_F^u = 42.411$.

(f) The value used for the elasticity of substitution, $\sigma = 1.56$, is as obtained from the link-level regression estimates – see again column (5) of Table 4.

(g) Finally, notice that the only parameters in (15) that we do not directly recover from our model and data are those in the utility function of the representative household, η and δ. These parameters are important because they govern the reaction of the labor supply to changes in wages and income. In view of the difficulty of pinpointing with confidence specific values for these parameters, we rely on the literature – in particular, we follow Gottlieb et al. (2021) – and choose the following range: $(\eta, \delta) \in [0.25, 0.5] \times [0.5, 1.5]$.[45]

We first calculate the overall impact of bank shocks to firms on the level of real GDP. Depending on the values for η and δ, we estimate it to be between -2.35% and -3.94%.[46] To benchmark these results, we note that Spain's economy grew at the average rate of 3.5% in the thirteen years before the crisis, while the Spanish GDP *decreased* between the end of 2008 and the end of 2009 by 3.76% (see García-Santana et al. (2020)).

Next, we address the following counterfactual question: How large was the role of bank shock propagation through the production network on the overall impact of the banking crisis on the Spanish GDP? In practice, the way in which we answer this question is by setting the share of intermediate inputs, α, to zero. This blocks all shock propagation along the production network and, therefore, helps to quantify what part of the aggregate effect of shocks can be attributed to such propagation.

In the counterfactual scenario without input-output linkages, we find that the estimated effect of bank shocks on the logarithm of GDP is in the range $[-1.78\%, -2.25\%]$. We therefore conclude that, on average,[47] production network propagation amplified the impact of bank shocks on firms, increasing by close to 50% their aggregate effect on the Spanish GDP compared to the scenario where the direct bank shocks to firms are assumed not to trigger any propagation through the production network.

Finally, we use equation (16) to examine whether aggregate network effects are primarily driven by first-order or higher-order propagation. Our results indicate that, on average, the first-order network effects amplify the impact of shocks on GDP by 26% (this number ranges

[45] This strategy contrasts with that of other papers that select a single combination of these two parameters (e.g. Alfaro et al. (2021); Bigio and La'O (2020)).

[46] The upper bound (-2.35%) is reached for $(\eta, \delta) = (0.5, 1.5)$, while the lower bound (-3.94%) is achieved at $(\eta, \delta) = (0.25, 0.5)$

[47] Note that the amplification ranges from 33% to 75%, depending on the values of parameters δ and η. Thus, to calculate the indicated average we have considered a 20×20 grid of uniformly distributed values of δ and η in the specified range.

from 13% to 46% depending on the values of parameters δ and η in the range considered), thus accounting for 53% of the total network amplification. In other words, according to our findings, both first- and higher-order effects are almost equally important in the amplification of the real effects of bank shocks to firms through the production network.

7 Concluding remarks

Despite the fact that both academics and policy-makers have often argued that production and financial networks are important to understand the real effects of bank shocks, evidence on this has been scant – mainly because of the unavailability of matched data that suitably represent the customer-supplier trade flows and bank-firm loans. In this paper, we contribute to addressing this problem by studying two matched administrative datasets from a bank-dominated economy, Spain, on: (i) supplier-customer transactions, as they stem from the Treasury's Value Added Tax (VAT) Register; and (ii) bank-firm loans, gathered from the Credit Register of the Bank of Spain. Moreover, we use a standard operationalization of bank credit supply shocks during the Global Financial Crisis and, importantly, a general equilibrium model of an interfirm production network economy with financial frictions that we structurally estimate.

We show that bank shocks to firms propagate upstream and downstream along the production network, with stronger effects for upstream than for downstream propagation. Our results also show that network propagation leads to a close to 50% increase in the aggregate effects of bank credit shocks to firms on GDP growth, with equally important first-order versus higher-order network effects.

There may be other interesting channels at work that further reflect the interplay between production and financial networks. For example, banks' decisions on their supply of credit across different firms may depend on whether these firms are involved in customer-supplier relationships of any low-enough order. Or, adopting a dynamic perspective on the problem, certain shocks may induce firms or banks to a persistent "rewiring" of some of their relationships. We believe that a deeper investigation of these questions is a fruitful direction for further research.

References

Acemoglu, D., Carvalho, V. M., Ozdaglar, A., and Tahbaz-Salehi, A. (2012). The network origins of aggregate fluctuations. *Econometrica*, 80(5).

Acemoglu, D., Ozdaglar, A., and Tahbaz-Salehi, A. (2015). Systemic risk and stability in financial networks. *American Economic Review*, 105(2).

Ackerberg, D. A., Caves, K., and Frazer, G. (2015). Identification properties of recent production function estimators. *Econometrica*, 83(6).

Alfaro, L., García-Santana, M., and Moral-Benito, E. (2021). On the direct and indirect real effects of credit supply shocks. *Journal of Financial Economics*, 139(3).

Allen, F. and Gale, D. (2000). Financial contagion. *Journal of Political Economy*, 108(1).

Amiti, M. and Weinstein, D. E. (2018). How much do idiosyncratic bank shocks affect investment? Evidence from matched bank-firm loan data. *Journal of Political Economy*, 126(2).

Antràs, P., Chor, D., Fally, T., and Hillberry, R. (2012). Measuring the upstreamness of production and trade flows. *American Economic Review*, 102(3).

Atalay, E. (2017). How important are sectoral shocks? *American Economic Journal: Macroeconomics*, 9(4).

Baqaee, D. R. (2018). Cascading failures in production networks. *Econometrica*, 86(5).

Baqaee, D. R. and Farhi, E. (2019). Productivity and misallocation in general equilibrium. *The Quarterly Journal of Economics*, 135(1).

Barrot, J.-N. and Sauvagnat, J. (2016). Input specificity and the propagation of idiosyncratic shocks in production networks. *The Quarterly Journal of Economics*, 131(3).

Bentolila, S., Jansen, M., and Jiménez, G. (2017). When credit dries up: Job losses in the great recession. *Journal of the European Economic Association*, 16(3).

Bernanke, B. (2018). The real effects of the financial crisis. *Brookings Papers on Economic Activity*, 20.

Bernanke, B. S. (2013). A century of US central banking: Goals, frameworks, accountability. *Journal of Economic Perspectives*, 27(4).

Bernard, A. B., Dhyne, E., Magerman, G., Manova, K., and Moxnes, A. (2022). The origins of firm heterogeneity: A production network approach. *Journal of Political Economy*, 130(7).

Bigio, S. and La'O, J. (2020). Distortions in production networks. *The Quarterly Journal of Economics*, 135(4).

Boehm, C. E., Flaaen, A., and Pandalai-Nayar, N. (2019). Input linkages and the transmission of shocks: Firm-level evidence from the 2011 tōhoku earthquake. *Review of Economics and Statistics*, 101(1).

Bonacich, P. (1987). Power and centrality: A family of measures. *American Journal of Sociology*, 92(5).

Cabrales, A., Gottardi, P., and Vega-Redondo, F. (2017). Risk sharing and contagion in networks. *The Review of Financial Studies*, 30(9).

Carvalho, V. M. (2014). From micro to macro via production networks. *Journal of Economic Perspectives*, 28(4).

Carvalho, V. M., Nirei, M., Saito, Y. U., and Tahbaz-Salehi, A. (2020). Supply Chain Disruptions: Evidence from the Great East Japan Earthquake. *The Quarterly Journal of Economics*.

Chodorow-Reich, G. (2014). The employment effects of credit market disruptions: Firm-level evidence from the 2008–9 financial crisis. *The Quarterly Journal of Economics*, 129(1).

Cingano, F., Manaresi, F., and Sette, E. (2016). Does credit crunch investment down? New evidence on the real effects of the bank-lending channel. *The Review of Financial Studies*, 29(10).

Cortes, G., Silva, T., and Doornik, B. (2019). Credit shock propagation in firm networks: evidence from government bank credit expansions. *Working paper, Central Bank of Brazil, Research Department.*

Costello, A. M. (2020). Credit market disruptions and liquidity spillover effects in the supply chain. *Journal of Political Economy*, 128(9).

Davis, S. J. and Haltiwanger, J. (1992). Gross job creation, gross job destruction, and employment reallocation. *The Quarterly Journal of Economics*, 107(3).

Demir, B., Javorcik, B., Michalski, T. K., and Ors, E. (2024). Financial constraints and propagation of shocks in production networks. *Review of Economics and Statistics*, 106(2).

Dewachter, H., Tielens, J., and Hove, J. (2020). The production network sources of bank influence. Technical report, Working Paper.

Diamond, D. W. (1984). Financial intermediation and delegated monitoring. *The Review of Economic Studies*, 51(3).

Elliott, M., Golub, B., and Jackson, M. O. (2014). Financial networks and contagion. *American Economic Review*, 104(10).

Freixas, X., Laeven, L., and Peydró, J.-L. (2015). *Systemic Risk, Crises, and Macroprudential Regulation*. MIT Press.

Freixas, X., Parigi, B. M., and Rochet, J.-C. (2000). Systemic risk, interbank relations, and liquidity provision by the central bank. *Journal of Money, Credit and Banking*, 32(3).

Fujiy, B. C., Ghose, D., and Khanna, G. (2024). Production networks and firm-level elasticities of substitution. Technical report, The World Bank.

Galaasen, S., Jamilov, R., Juelsrud, R., and Rey, H. (2020). Granular credit risk. *Working paper*.

García-Santana, M., Moral-Benito, E., Pijoan-Mas, J., and Ramos, R. (2020). Growing like spain: 1995–2007. *International Economic Review*, 61(1).

Gertler, M. and Kiyotaki, N. (2010). Financial intermediation and credit policy in business cycle analysis. In *Handbook of Monetary Economics*, volume 3. Elsevier.

Gottlieb, C., Onken, J., and Valladares-Esteban, A. (2021). On the measurement of the elasticity of labour. *European Economic Review*, 139:103879.

Greenstone, M., Mas, A., and Nguyen, H.-L. (2014). Do credit market shocks affect the real economy? Quasi-experimental evidence from the great recession and 'normal' economic times. *Working paper*.

Halpern, L., Koren, M., and Szeidl, A. (2015). Imported inputs and productivity. *American economic review*, 105(12).

Holmstrom, B. and Tirole, J. (1997). Financial intermediation, loanable funds, and the real sector. *the Quarterly Journal of Economics*, 112(3).

Imbens, G. W. and Rubin, D. B. (2015). *Causal inference in statistics, social, and biomedical sciences*. Cambridge University Press.

Imbens, G. W. and Wooldridge, J. M. (2009). Recent developments in the econometrics of program evaluation. *Journal of Economic Literature*, 47(1).

Ippolito, F., Peydró, J.-L., Polo, A., and Sette, E. (2016). Double bank runs and liquidity risk management. *Journal of Financial Economics*, 122(1).

Ivashina, V., Laeven, L., and Moral-Benito, E. (2022). Loan types and the bank lending channel. *Journal of Monetary Economics*, 126.

Iyer, R. and Peydro, J.-L. (2011). Interbank contagion at work: Evidence from a natural experiment. *The Review of Financial Studies*, 24(4).

Iyer, R., Peydró, J.-L., da Rocha-Lopes, S., and Schoar, A. (2014). Interbank liquidity crunch and the firm credit crunch: Evidence from the 2007–2009 crisis. *The Review of Financial Studies*, 27(1).

Jacobson, T. and von Schedvin, E. (2015). Trade credit and the propagation of corporate failure: an empirical analysis. *Econometrica*, 83(4).

Jiménez, G., Mian, A., Peydró, J.-L., and Saurina, J. (2020). The real effects of the bank lending channel. *Journal of Monetary Economics*, 115.

Jiménez, G., Ongena, S., Peydró, J.-L., and Saurina, J. (2012). Credit supply and monetary policy: Identifying the bank balance-sheet channel with loan applications. *American Economic Review*, 102(5).

Jiménez, G., Ongena, S., Peydró, J.-L., and Saurina, J. (2014). Hazardous times for monetary policy: What do twenty-three million bank loans say about the effects of monetary policy on credit risk-taking? *Econometrica*, 82(2).

Jiménez, G., Ongena, S., Peydró, J.-L., and Saurina, J. (2017). Macroprudential policy, countercyclical bank capital buffers, and credit supply: evidence from the spanish dynamic provisioning experiments. *Journal of Political Economy*, 125(6).

Khwaja, A. I. and Mian, A. (2008). Tracing the impact of bank liquidity shocks: Evidence from an emerging market. *American Economic Review*, 98(4).

La Porta, R., Lopez-de Silanes, F., and Shleifer, A. (2002). Government ownership of banks. *The Journal of Finance*, 57(1).

Levinsohn, J. and Petrin, A. (2003). Estimating production functions using inputs to control for unobservables. *The Review of Economic Studies*, 70(2).

MaCurdy, T. E. (1981). An empirical model of labor supply in a life-cycle setting. *Journal of political Economy*, 89(6).

Montiel Olea, J. L. and Pflueger, C. (2013). A robust test for weak instruments. *Journal of Business & Economic Statistics*, 31(3).

Niepmann, F. and Schmidt-Eisenlohr, T. (2013). Bank bailouts, international linkages, and cooperation. *American Economic Journal: Economic Policy*, 5(4).

Olley, S. and Pakes, A. (1996). The dynamics of productivity in the telecommunications equipment industry. *Econometrica*, 64(6).

Peter, A., Ruane, C., et al. (2022). The aggregate importance of intermediate input substitutability. *Princeton University, memo.*

Ruhl, K. J. et al. (2008). The international elasticity puzzle. *NYU Mimeo.*

Stein, J. C. (1998). An adverse-selection model of bank asset and liability management with implications for the transmission of monetary policy. *RAND Journal of Economics*, 29(3).

Wooldridge, J. M. (2009). On estimating firm-level production functions using proxy variables to control for unobservables. *Economics letters*, 104(3).

Tables

TABLE 1

LINK-LEVEL: PROPAGATION OF BANK SHOCKS THROUGH THE NETWORK OF CUSTOMERS. REDUCED FORM

Upstream propagation (indirect shocks via bank shocks to first-order customers)

Dependent Variable: Δlog(sales from supplier to customer)			IV. Instrument: Bank Net Interbank Borrowing		IV. Instrument: Bank Shock	
			1st Stage	2º Stage	1st Stage	2º Stage
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank) Shock	-0.703*					
	(0.415)					
Customer (Bank) Shock (1st Order Effect)		-2.359**		-6.068**	3.445***	
		(1.181)		(2.628)	(0.571)	
Customer (Bank) Net Interbank Borrowing			8.917***			
			(0.933)			
Customer Reduction of Bank Debt						-0.739***
						(0.265)
Customer:						
Controls	-	Yes	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	Yes	Yes	Yes
Fixed Effects	Yes	No	No	No	No	No
Firm:						
Controls	Yes	-	-	-	-	-
Spatial*Industry Fixed Effects	Yes	-	-	-	-	-
Fixed Effects	No	Yes	Yes	Yes	Yes	Yes
Firm*Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
First Stage Effective F statistic	-	-	65.28	-	36.09	-
R-squared	0.404	0.474	-	-	-	-
Observations	1,119,169	1,119,169	1,119,169	1,119,169	1,119,169	1,119,169

Notes: This table reports estimates from WLS results. See Section 4. Observations are at the level of firm-customer, i.e. link-level. The dependent variable is the change in the log of sales form supplier to customer between the whole year 2008 and 2009 for all columns but (3) and (5). In column (4) the bank shock is instrumented with the shock derived from the (weighted) average net interbank borrowing of the firm across all its banks (that was borrowing from) before the crisis (column (3)) and corrected for the standardization of the instrumental variable. In column (6) the reduction in bank debt between 2008 and 2009 is instrumented with the bank shock (column (5)). Bank shock (to a firm) is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply more during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values imply a credit reduction). All shocks are standardized. See Section 3. For the list of controls, see Section 4. First stage effective F statistic is based on Montial Olea and Pfleuger (2013) and it is robust to heteroskedasticity, serial correlation, and clustering. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below, with clustering at the firm, main bank, and supplier or customer levels for columns (1) and (2), and for firm and main bank level for columns (3) to (6) due to the IV estimation. In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE 2

LINK-LEVEL: PROPAGATION OF BANK SHOCKS THROUGH THE NETWORK OF SUPPLIERS. REDUCED FORM

Downstream propagation (indirect shocks via bank shocks to first-order suppliers)

Dependent Variable: Δlog(sales from supplier to customer)			IV. Instrument: Bank Net Interbank Borrowing		IV. Instrument: Bank Shock	
			1st Stage	2º Stage	1st Stage	2º Stage
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank) Shock	-2.678**					
	(1.050)					
Supplier (Bank) Shock (1st Order Effect)		-1.086**		-3.656***	2.094***	
		(0.546)		(1.249)	(0.499)	
Supplier (Bank) Net Interbank Borrowing			9.227***			
			(0.623)			
Supplier Reduction of Bank Debt						-0.519*
						(0.310)
Supplier:						
Controls	-	Yes	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	Yes	Yes	Yes
Fixed Effects	Yes	No	No	No	No	No
Firm:						
Controls	Yes	-	-	-	-	-
Spatial*Industry Fixed Effects	Yes	-	-	-	-	-
Fixed Effects	No	Yes	Yes	Yes	Yes	Yes
Firm*Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
First Stage Effective F statistic	-	-	97.53	-	12.40	-
R-squared	0.358	0.483	-	-	-	-
Observations	1,114,420	1,114,420	1,114,420	1,114,420	1,114,420	1,114,420

Notes: This table reports estimates from WLS results. See Section 4. Observations are at the level of the firm-supplier, i.e. link-level. The dependent variable is the change in the log of sales from supplier to customer between the whole year 2008 and 2009 for all columns but (3) and (5). In column (4) the bank shock is instrumented with the shock derived from the (weighted) average net interbank borrowing of the firm across all its banks (that was borrowing from) before the crisis (column (3)) and corrected for the standardization of the instrumental variable. In column (6) the reduction in bank debt between 2008 and 2009 is instrumented with the bank shock (column (5)). Bank shock (to a firm) is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply more during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values imply a credit reduction). All shocks are standardized. See Section 3. For the list of controls, see Section 4. First stage effective F statistic is based on Montial Olea and Pfleuger (2013) and it is robust to heteroskedasticity, serial correlation, and clustering. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below, with clustering at the firm, main bank, and supplier or customer levels for columns (1) and (2), and for firm and main bank level for columns (3) to (6) due to the IV estimation. In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE 3

NODE-LEVEL: FIRM-LEVEL EFFECTS OF BANK SHOCKS THROUGH THE PRODUCTION NETWORK. REDUCED FORM. FIRM-LEVEL SALES AND EMPLOYMENT

Dependent Variable:	Δlog(sales)		Δlog(employment)	
	(1)	(2)	(3)	(4)
Direct (Bank) Shock	-0.888*	-0.821*	-0.552**	-0.536**
	(0.520)	(0.490)	(0.241)	(0.239)
Customer (Bank) Shock (1st Order Effect)		-2.223***		-0.488***
		(0.373)		(0.091)
Supplier (Bank) Shock (1st Order Effect)		0.374		-0.031
		(0.346)		(0.069)
Firm Controls	Yes	Yes	Yes	Yes
Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes
R-squared	0.351	0.353	0.091	0.091
Observations	196,171	196,171	196,171	196,171

Notes: This table reports estimates from WLS for columns (1) and (2) and OLS for the rest of the columns. See Section 4. Observations are at the level of the firm (node-level). The dependent variables are the change, between 2008 and 2009, in the log of aggregate sales to all customers (columns (1) and (2)) and the log of total employment (columns (3) and (4)). Bank shock (to a firm) is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced more credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values imply a credit reduction), and to construct the first order customer (supplier) we aggregate it using the lagged sales between the firms and all its direct customers (suppliers) as weights. See Section 3. As we cannot control for firm fixed effects, we control for industry*province and zip code fixed effects, in addition to firm (observable) controls and main bank fixed effects. All shocks are standardized. For the list of firm controls, see Section 4. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the level of the main bank). In each column, the word Yes indicates that the set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE 4

LINK-LEVEL: PROPAGATION OF BANK SHOCKS THROUGH THE PRODUCTION NETWORK. STRUCTURAL FORM

Dependent Variable: Δlog(sales from supplier to customer/sales of customer)	Upstream propagation		Downstream propagation		Joint estimation Non-standardized	Non-standardized
	(1)	(2)	(3)	(4)	(5)	(6)
Customer (Bank) Shock (1st Order Effect)	-1.924**	-2.002**			-42.411**	-35.510**
	(0.900)	(0.924)			(20.242)	(16.535)
Higher Order Customer (Bank) Shock		1.796***			51.176***	45.724**
		(0.632)			(18.520)	(19.771)
Supplier (Bank) Shock (1st Order Effect)			-1.086**	-1.055**	-23.950*	-15.149*
			(0.546)	(0.527)	(12.621)	(8.543)
Higher Order Supplier (Bank) Shock				-2.045*	-47.837*	-55.222*
				(1.066)	(28.554)	(32.630)
Supplier/Customer:						
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Firm:						
Controls	-	-	-	-	-	Yes
Spatial*Industry Fixed Effects	-	-	-	-	-	Yes
Fixed Effects	Yes	Yes	Yes	Yes	Yes	No
Fixed Effects as Supplier or Customer	No	No	No	No	No	Yes
Firm*Supplier/Customer Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.477	0.478	0.493	0.493	0.485	0.449
Observations	1,119,169	1,119,169	1,114,420	1,114,420	2,233,589	1,313,776

Notes: This table reports estimates from WLS results. See Section 6. Observations are at the link-level. Columns (1) and (2) are at the firm-customer level, with firm (supplier) fixed effects; columns (3) and (4) are at the firm-supplier level, with firm (customer) fixed effects. Column (5) uses all links appearing in both samples used in columns (2) and (4), including the two sets of firm fixed effects. In column (6) we consider all non-repeated links from columns (2) and (4), with one set of firm fixed effects, in which the firm dummy equals one if the firm appears as either supplier or customer. The dependent variable is the change in the log of sales from supplier to customer (normalized by the total sales of the customer) between the whole year 2008 and 2009. Bank shock (to a firm) is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced more credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values imply a credit reduction). See Section 3. For first-order and higher-order bank shock effects, see Section 6 of the paper. All variables are standardized but those of columns (5) and (6). For the list of controls and samples, see also Section 4. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below, with clustering at the firm, main bank, and supplier or customer levels in all columns but column (6), in which it is clustered for firm and main bank level as in this column there is no distinction between a firm acting as a supplier or as a customer. In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE 5

NODE-LEVEL: PROPAGATION OF BANK SHOCKS THROUGH THE PRODUCTION NETWORK. STRUCTURAL FORM

Dependent Variable: Δlog(sales)	(1)	(2)	(3)	(4)	Non-standardized (5)
Upstream (1st & Higher Order Effects)	-1.810***		-1.611***	-1.641***	-32.891***
	(0.326)		(0.333)	(0.350)	(6.802)
Bidirectional (Up & Down 1st & Higher Order Effects)		-1.435***	-0.527*	-0.439	-11.441*
		(0.333)	(0.300)	(0.367)	(6.504)
Direct (Bank) Shock				-0.411	
				(0.537)	
Firm Controls	Yes	Yes	Yes	Yes	Yes
Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes
R-squared	0.358	0.357	0.358	0.358	0.358
Observations	196,171	196,171	196,171	196,171	196,171

Notes: This table reports estimates from WLS. See Section 6. Observations are at the level of the firm (node-level). The dependent variable is the change, between the whole year 2008 and 2009, in the log of firm-level aggregate sales to all customers. *E* from the theory Section 5 is spanned by fixed effects. Bank shock (to a firm) is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced more credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values imply a credit reduction), see Section 3, and for aggregation (upstream and bidirectional) see Section 6 of the paper. As we cannot control for firm fixed effects, we control for industry*province and zip code fixed effects, in addition to firm (observable) controls and main bank fixed effects. All variables are standardized but those of column 5. For the list of firm controls, see Section 4. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the level of the main bank). In each column, the word Yes indicates that the set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

Appendix A: Additional results — For online publication

A.1. Additional Tables

TABLE A1

SUMMARY STATISTICS

			Mean	S.D.	P25	Median	P75
	Link Level						
Upstream propagation							
Δlog(sales from supplier to customer)	The log of change of firm's sales to its customer between 2008 and 2009	%	-12.062	61.407	-53.513	-13.894	17.599
Direct (Bank) Shock	Bank (supply) Shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction), where the firm-level shock is aggregated using the lagged December 2007 credit between the firm and each bank as weights		0.151	0.050	0.125	0.152	0.176
Customer (Bank) Shock (1st Order Effect)	Direct (bank credit supply) shock of the customer of a firm		0.151	0.047	0.125	0.151	0.176
Dummy Direct (Bank) Shock	A binary variable that takes the value of one when the Direct Shock is above its median and zero otherwise	0/1	0.544	0.498	0.000	1.000	1.000
Dummy Customer (Bank) Shock (1st Order Effect)	A binary variable that takes the vaule of one when the Customer (Bank) Shock (1st Order Effect) is above its median and zero otherwise	0/1	0.497	0.500	0.000	0.000	1.000
Higher Order Customer (Bank) Shock	A network aggregate of shocks hitting suppliers of the customer of any order (see equations (8) and (8R) and Section 5 and 6)		0.120	0.035	0.096	0.117	0.140
Customer Reduction of Bank Debt	The (negative) change in overall bank credit of the customer between 2008 and 2009	%	8.563	61.479	-7.496	7.449	27.897
Customer (Bank) Net Interbank Borrowing	The net interbank position (interbank deposits minus interbank loans) of the customer's weighted average banks, where weights are based on the lagged credit	0.0x	0.025	0.019	0.011	0.023	0.036
Δlog(sales from supplier to customer/sales of customer)	The log of change of firm's sales to its customer between 2008 and 2009 minus the log of change of the customer's sales	%	5.214	63.099	-33.604	0.973	35.927
Downstream propagation							
Δlog(sales from supplier to customer)	The log of change of a supplier's sales to the firm between 2008 and 2009	%	-11.932	60.414	-52.008	-12.730	16.381
Direct (Bank) Shock	Bank (supply) Shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction), where the firm-level shock is aggregated using the lagged December 2007 credit between the firm and each bank as weights		0.150	0.050	0.121	0.152	0.180
Supplier (Bank) Shock (1st Order Effect)	Direct (bank credit supply) shock of the supplier of a firm		0.151	0.044	0.129	0.151	0.173
Dummy Direct (Bank) Shock	A binary variable that takes the vaule of one when the Direct Shock is above its median and zero otherwise	0/1	0.541	0.498	0.000	1.000	1.000
Dummy Supplier (Bank) Shock (1st Order Effect)	A binary variable that takes the vaule of one when the Supplier (Bank) Shock (1st Order Effect) is above its median and zero otherwise	0/1	0.505	0.500	0.000	1.000	1.000
Higher Order Supplier (Bank) Effect	A network aggregate of shocks hitting suppliers of the supplier of any order (see equations (8) and (8R) and Section 5 and 6)		0.108	0.041	0.081	0.106	0.133
Supplier Reduction of Bank Debt	The (negative) change in overall bank credit of the supplier between 2008 and 2009	%	10.071	58.433	-6.275	8.696	28.814
Supplier (Bank) Net Interbank Borrowing	The net interbank position (interbank deposits minus interbank loans) of the supplier's weighted average banks, where weights are based on the lagged credit	0.0x	0.027	0.019	0.015	0.024	0.036
Δlog(sales from supplier to customer/sales of customer)	The log of change of a supplier's sales to the firm between 2008 and 2009 minus the log of change of the firm (customer)'s sales	%	6.280	62.476	-31.823	2.272	36.550
	Node Level						
Δlog(sales)	The log of change of firm' sales to all its customers between 2008 and 2009	%	-19.970	40.779	-40.314	-16.800	-0.585
Δlog(employment)	The log of change of firm' employment between 2008 and 2009	%	-8.967	30.296	-20.743	-0.358	0.000
Direct (Bank) Shock	Bank (supply) Shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction), where the firm-level shock is aggregated using the lagged December 2007 credit between the firm and each bank as weights		0.148	0.061	0.109	0.149	0.185
Supplier (Bank) Shock (1st Order Effect)	Aggregate of all the firm's suppliers' Direct (Bank) Shock, weighted by the lagged sales from each supplier to the firm		0.063	0.043	0.030	0.058	0.088
Customer (Bank) Shock (1st Order Effect)	Aggregate of all the firm's customers' Direct (Bank) Shock, weighted by the lagged sales from the firm to each customer		0.026	0.032	0.002	0.015	0.039
Upstream (1st & Higher Order Effects)	A network aggregate of shocks hitting customers of any order (see equations (10) and (10R) and Section 5 and 6)		0.034	0.049	0.000	0.008	0.054
Bidirectional (Up & Down 1st & Higher Order Effects)	A network aggregate of shocks hitting suppliers and customers of any order (see equations (10) and (10R) and Section 5 and 6)		0.041	0.046	0.004	0.026	0.064

Notes: This table reports the definition, mean, standard deviation and first, second and third quartiles of the variables used in the analysis. See Section 3 to 6 of the paper for a more in depth explanation.

TABLE A2

DIFFERENCE IN MEAN TESTS DEPENDING ON EX-ANTE LINKS WITH BANKS WITH STRONG NEGATIVE CREDIT SUPPLY

	Firms Exposed to Unconstrained Banks		Firms Exposed to Constrained Banks		Difference in Means	Normalized Differences	Dependent Variable: Bank Credit Supply Shock	
	Mean	S.D.	Mean	S.D.	t test	test	Coefficient	S.E.
Firm Characteristics								
Short Term Debt	49.57	(15.36)	49.90	(15.36)	4.39	0.02	0.001	(0.001)
Log(Age)	2.63	(0.34)	2.63	(0.34)	-2.57	-0.01	-0.000	(0.001)
Own Funds/Total Assets	31.56	(14.93)	31.36	(14.93)	-2.63	-0.01	0.001	(0.001)
Log(Total Assets)	7.57	(0.97)	7.58	(0.97)	1.21	0.00	0.003	(0.003)
Liquidity Ratio	16.25	(13.72)	16.11	(13.72)	-2.08	-0.01	-0.000	(0.001)
Average Bank Characteristics								
Log(Total Assets)	18.32	(0.77)	17.82	(0.69)	-138.55	-0.48	-0.090**	(0.037)
Own Funds/Total Assets	0.05	(0.01)	0.05	(0.01)	29.97	0.10	0.006	(0.054)
Net Interbank Borrowing	0.02	(0.01)	0.03	(0.01)	82.81	0.29	0.055**	(0.026)
ROA	0.01	(0.00)	0.01	(0.00)	40.09	0.14	0.020	(0.031)
NPL	0.03	(0.01)	0.03	(0.01)	28.73	0.10	-0.007	(0.035)
Loans/Deposits	0.62	(0.09)	0.63	(0.09)	7.70	0.03	0.012	(0.035)
% Construction & Real Estate	0.47	(0.05)	0.48	(0.06)	36.33	0.13	0.046	(0.040)
Savings Bank	0.53	(0.50)	0.41	(0.49)	-48.33	-0.17	-0.061	(0.055)
Network variables								
Bonacich centrality	0.64	(0.43)	0.66	(0.44)	10.28	0.04	-0.001	(0.002)
Bonacich centrality (γ=1)	1.47	(0.38)	1.49	(0.38)	13.02	0.05	-0.003	(0.002)
Upstreamness	1.68	(0.42)	1.70	(0.42)	11.09	0.04	0.005	(0.003)
In degree	18.61	(8.97)	19.04	(9.09)	9.72	0.03	0.007	(0.004)
Out degree	12.36	(9.31)	12.97	(9.44)	13.25	0.05	0.002	(0.002)
Expenditure share (γ_i)	0.00	(0.00)	0.00	(0.00)	5.15	0.02	-0.001	(0.002)
R-squared							0.212	
No. of Observations	80,884		85,999				166,883	

Notes: This table (in the first four columns) reports means and standard deviations of firm characteristics as of December 2007. Firms are classified in two groups. The first two columns refer to firms that ex-ante worked with unconstrained banks (its bank credit supply is below the median of the bank supply factor estimated following Amiti & Weinstein (2018), see below and Section 3), while the third and fourth columns refer to firms that worked with constrained banks (above the median). Column (5) reports the t-statistic of the differences in mean and column (6) shows the normalized difference test (a scale-and-sample-size-free estimator) proposed by Imbens and Wooldridge (2009), for which Imbens and Rubin (2015) suggested a heuristic threshold of 0.25 in absolute value for significant differences. Bank characteristics at the firm level are computed as a weighted average of the bank variables at the firm-bank level, using as weights the credit amount of each relationship. Columns (7) and (8) shows the results of a OLS regressions where the dependent variable is the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction), where the firm-level shock is aggregated using the lagged credit between the firm and each bank as weights, and the rest of the variables are standardized. Industry*province dummies are included. Vector of Bonacich centralities is defined with $\mathbf{v} = \kappa(\mathbf{I} - \alpha\mathbf{G})^{-1}\boldsymbol{\gamma}$ where $\kappa > 0$ is a normalizing constant, upstreamness is the measure defined in Antras et al. (2012), In degree is the number of suppliers, and Out degree is the number of customers. Coefficients are listed in the first row, robust standard errors are reported in the adjacent column which are corrected for clustering at the four-digit NACE, province and main bank. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A3

FIRM-LEVEL EFFECTS OF BANK SUPPLY SHOCKS

Dependent Variable:	ΔCredit		
	(1)	(2)	(3)
	2009	2008	2007
Direct (Bank) Shock	-0.604**	-0.214	0.297
	(0.285)	(0.612)	(0.353)
Firm Controls	Yes	Yes	Yes
Spatial & Industry Fixed Effects	Yes	Yes	Yes
R-squared	0.057	0.086	0.102
Observations	196,171	99,257	87,085

Notes: This table reports estimates from OLS. See Section 4. Observations are at the level of the firm (node-level). The dependent variable is the change in bank credit. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we first use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction) and then we aggregate it using the lagged credit between the firm and each bank as weights. As we cannot control for firm fixed effects, we control for industry, zip code fixed effects and main bank fixed effects. All shocks are standardized. For the list of firm controls, see Section 4. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the level of the main bank). In each column, the word Yes indicates that the set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A4

LINK-LEVEL: PROPAGATION OF BANK CREDIT SUPPLY SHOCKS THROUGH THE NETWORK OF CUSTOMERS/SUPPLIERS. REDUCED FORM. DUMMY SHOCK

Panel A. Upstream propagation (indirect shocks via bank credit supply shocks to first-order customers)

			IV. Instrument: Bank Net Interbank Borrowing		IV. Instrument: Bank Shock	
			1st Stage	2º Stage	1st Stage	2º Stage
	(1)	(2)	(3)	(4)	(5)	(6)
Dummy Direct (Bank) Shock	-0.914*					
	(0.493)					
Dummy Customer (Bank) Shock (1st Order Effect)		-2.620**		-4.803**	3.366***	
		(1.173)		(1.984)	(0.602)	
Customer (Bank) Net Interbank Borrowing			0.413***			
			(0.017)			
Customer Reduction of Bank Debt						-0.558***
						(0.265)
Customer:						
Controls	-	Yes	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	Yes	Yes	Yes
Fixed Effects	Yes	No	No	No	No	No
Firm:						
Controls	Yes	-	-	-	-	-
Spatial*Industry Fixed Effects	Yes	-	-	-	-	-
Fixed Effects	No	Yes	Yes	Yes	Yes	Yes
Firm*Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Effective First Stage F statistic	-	-	91.77	-	31.88	-
R-squared	0.404	0.488	-	-	-	-
Observations	1,119,169	1,119,169	1,119,169	1,119,169	1,119,169	1,119,169

Panel B. Downstream propagation (indirect shocks via bank credit supply shocks to first-order suppliers)

			IV. Instrument: Bank Net Interbank Borrowing		IV. Instrument: Bank Shock	
			1st Stage	2º Stage	1st Stage	2º Stage
	(1)	(2)	(3)	(4)	(5)	(6)
Dummy Direct (Bank) Shock	-2.902**					
	(1.452)					
Dummy Supplier (Bank) Shock (1st Order Effect)		-1.180**		-3.335***	2.173***	
		(0.516)		(1.225)	(0.434)	
Supplier (Bank) Net Interbank Borrowing			0.848***			
			(0.037)			
Supplier Reduction of Bank Debt						-0.453**
						(0.094)
Supplier:						
Controls	-	Yes	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	Yes	Yes	Yes
Fixed Effects	Yes	No	No	No	No	No
Firm:						
Controls	Yes	-	-	-	-	-
Spatial*Industry Fixed Effects	Yes	-	-	-	-	-
Fixed Effects	No	Yes	Yes	Yes	Yes	Yes
Firm*Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Effective First Stage F statistic	-	-	1,121.63	-	11.07	-
R-squared	0.358	0.483	-	-	-	-
Observations	1,114,421	1,114,421	1,114,421	1,114,421	1,114,421	1,114,421

Notes: This table reports estimates from WLS results. See Section 4. Observations are at the level of the firm-customer (Panel A) or firm-supplier (Panel B), i.e. link-level. The dependent variable is the change in the log of sales from supplier to customer between 2008 and 2009 for all columns but (3) and (5). In column (4) the firm bank shock is instrumented with the firm financial shock derived from the (weighted) average net interbank borrowing of the firm across all its banks before the crisis (column (3)) and corrected for the standardization of the instrumental variable. In column (6) the reduction in bank debt between 2008 and 2009 is instrumented with the firm financial shock (column (5)). The continuous bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). Here we discretize this variable based on the median of the distribution. All shocks are standardized. For the list of controls, see Section 4. First stage effective F statistic is based on Montial Olea and Pfleuger (2013) and it is robust to heteroskedasticity, serial correlation, and clustering. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels but for columns (4) and (6) that correct for firm and main bank). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A5

LINK-LEVEL: PROPAGATION OF BANK CREDIT SUPPLY SHOCKS THROUGH THE NETWORK. REDUCED FORM. BANK SHOCKS COMPUTED USING A FIRM VARYING DEMAND BY LOAN TYPE

Dependent Variable: Δlog(sales from supplier to customer)	Upstream propagation			Downstream propagation		
	Continuous Shock		Discrete Shock	Continuous Shock		Discrete Shock
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank) Shock	-1.094**			-3.072**		
	(0.442)			(1.470)		
Customer (Bank) Shock (1st Order Effect)		-2.445**	-2.923**			
		(1.233)	(0.098)			
Supplier (Bank) Shock (1st Order Effect)					-0.990***	-1.377***
					(0.372)	(0.508)
Customer/Supplier:						
Controls	-	Yes	Yes	-	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	-	Yes	Yes
Fixed Effects	Yes	No	No	Yes	No	No
Firm:						
Controls	Yes	-	-	Yes	-	-
Spatial*Industry Fixed Effects	Yes	-	-	Yes	-	-
Fixed Effects	No	Yes	Yes	No	Yes	Yes
Firm*Customer/Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.405	0.474	0.474	0.358	0.484	0.483
Observations	1,119,169	1,119,169	1,119,169	1,114,421	1,114,421	1,114,421

Notes: This table reports estimates from WLS results. See Section 4. Observations are at the level of firm-customer/supplier, i.e. link-level. The dependent variable is the change in the log of sales from supplier to customer between 2008 and 2009. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018), but allowing firm-loan type fixed effects (where loan types are asset-based loans, cash flow loans, trade finance agreements, and leases following Ivashina, Laeven, and Moral-Benito (2022)), as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. All shocks are standardized. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A6

LINK-LEVEL: PROPAGATION OF BANK CREDIT SUPPLY SHOCKS THROUGH THE NETWORK. REDUCED FORM.
BANK SHOCKS COMPUTED USING A FIRM VARYING DEMAND BY INDUSTRY AND PROVINCE OF THE FIRM VS BANK'S SPECIALIZATION

Dependent Variable: Δlog(sales from supplier to customer)	Upstream propagation			Downstream propagation		
	Continuous Shock		Discrete Shock	Continuous Shock		Discrete Shock
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank) Shock	-1.209**			-2.623**		
	(0.570)			(1.191)		
Customer (Bank) Shock (1st Order Effect)		-2.665**	-1.680*			
		(1.261)	(0.957)			
Supplier (Bank) Shock (1st Order Effect)					-0.896**	-0.937***
					(0.409)	(0.459)
Customer/Supplier:						
Controls	-	Yes	Yes	-	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	-	Yes	Yes
Fixed Effects	Yes	No	No	Yes	No	No
Firm:						
Controls	Yes	-	-	Yes	-	-
Spatial*Industry Fixed Effects	Yes	-	-	Yes	-	-
Fixed Effects	No	Yes	Yes	No	Yes	Yes
Firm*Customer/Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.404	0.474	0.474	0.358	0.483	0.484
Observations	1,119,169	1,119,169	1,119,169	1,114,421	1,114,421	1,114,421

Notes: This table reports estimates from WLS results. See Section 4. Observations are at the level of firm-customer/supplier, i.e. link-level. The dependent variable is the change in the log of sales from supplier to customer between 2008 and 2009. In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018), but allowing firm fixed effects to vary depending on whether the firm and the bank match in their industry and/or province (where the province or industry, NACE two digits, of the bank relates to its main province or industry computed in terms of total credit at December of 2007), as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. All shocks are standardized. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A7

LINK-LEVEL: PROPAGATION OF BANK CREDIT SUPPLY SHOCKS THROUGH THE NETWORK. REDUCED FORM.
BANK SHOCKS COMPUTED USING A FIRM VARYING DEMAND DEPENDING ON BANK'S SPECIALIZATION IN REAL ESTATE

Dependent Variable: Δlog(sales from supplier to customer)	Upstream propagation			Downstream propagation		
	Continuous Shock		Discrete Shock	Continuous Shock		Discrete Shock
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank) Shock	-0.992**			-3.017**		
	(0.471)			(1.417)		
Customer (Bank) Shock (1st Order Effect)		-2.146*	-2.453**			
		(1.196)	(1.104)			
Supplier (Bank) Shock (1st Order Effect)					-1.103**	-1.394***
					(0.556)	(0.497)
Customer/Supplier:						
Controls	-	Yes	Yes	-	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	-	Yes	Yes
Fixed Effects	Yes	No	No	Yes	No	No
Firm:						
Controls	Yes	-	-	Yes	-	-
Spatial*Industry Fixed Effects	Yes	-	-	Yes	-	-
Fixed Effects	No	Yes	Yes	No	Yes	Yes
Firm*Customer/Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.404	0.474	0.474	0.358	0.483	0.484
Observations	1,119,169	1,119,169	1,119,169	1,114,421	1,114,421	1,114,421

Notes: This table reports estimates from WLS results. See Section 4. Observations are at the level of firm-customer/supplier, i.e. link-level. The dependent variable is the change in the log of sales from supplier to customer between 2008 and 2009. In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use firm level shock estimated following Amiti & Weinstein (2018), but allowing firm fixed effects to vary depending on whether the bank is specialized in the real estate sector or not (in terms of total credit at December 2007), as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. All shocks are standardized. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A8. PANEL A

LINK-LEVEL: PROPAGATION OF BANK CREDIT SUPPLY SHOCKS THROUGH THE NETWORK. REDUCED FORM. BANK SHOCKS COMPUTED USING A FIRM VARYING DEMAND BY THE WEIGHT OF WEAK BANKS IN THE PROVINCE

Dependent Variable: Δlog(sales from supplier to customer)	Upstream propagation			Downstream propagation		
	Continuous Shock		Discrete Shock	Continuous Shock		Discrete Shock
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank) Shock	-0.913***			-2.738***		
	(0.497)			(1.041)		
Customer (Bank) Shock (1st Order Effect)		-2.464**	-2.654**			
		(1.202)	(1.118)			
Supplier (Bank) Shock (1st Order Effect)					-1.028*	-1.310***
					(0.524)	(0.426)
Customer/Supplier:						
Controls	-	Yes	Yes	-	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	-	Yes	Yes
Fixed Effects	Yes	No	No	Yes	No	No
Firm:						
Controls	Yes	-	-	Yes	-	-
Spatial*Industry Fixed Effects	Yes	-	-	Yes	-	-
Fixed Effects	No	Yes	Yes	No	Yes	Yes
Firm*Customer/Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.404	0.474	0.474	0.358	0.483	0.484
Observations	1,119,169	1,119,169	1,119,169	1,114,421	1,114,421	1,114,421

Notes: This table reports estimates from WLS results. Observations are at the level of firm-customer/supplier, i.e. link-level. The dependent variable is the change in the log of sales from supplier to customer between 2008 and 2009. In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. Bank shock is a variable capturing whether the firm borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018), but allowing firm-province fixed effects, where provinces are classified by quintiles based on the market share of the weak banks (following Bentolila et al., 2018), as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). All shocks are standardized. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A8. PANEL B

LINK-LEVEL: PROPAGATION OF BANK CREDIT SUPPLY SHOCKS THROUGH THE NETWORK. REDUCED FORM.
BANK SHOCKS COMPUTED USING A FIRM VARYING DEMAND BY WEAK BANKS

Dependent Variable: Δlog(sales from supplier to customer)	Upstream propagation			Downstream propagation		
	Continuous Shock		Discrete Shock	Continuous Shock		Discrete Shock
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank) Shock	-1.056**			-2.657**		
	(0.520)			(1.046)		
Customer (Bank) Shock (1st Order Effect)		-2.510**	-2.843**			
		(1.245)	(1.229)			
Supplier (Bank) Shock (1st Order Effect)					-0.949*	-1.288***
					(0.490)	(0.442)
Customer/Supplier:						
Controls	-	Yes	Yes	-	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	-	Yes	Yes
Fixed Effects	Yes	No	No	Yes	No	No
Firm:						
Controls	Yes	-	-	Yes	-	-
Spatial*Industry Fixed Effects	Yes	-	-	Yes	-	-
Fixed Effects	No	Yes	Yes	No	Yes	Yes
Firm*Customer/Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.404	0.474	0.474	0.358	0.483	0.484
Observations	1,119,169	1,119,169	1,119,169	1,114,421	1,114,421	1,114,421

Notes: This table reports estimates from WLS results. Observations are at the level of firm-customer/supplier, i.e. link-level. The dependent variable is the change in the log of sales from supplier to customer between 2008 and 2009. In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. Bank shock is a variable capturing whether the firm borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018), but allowing firm-weak bank fixed effects, where weak banks are defined following Bentolila et al. (2018), as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). All shocks are standardized. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A9

LINK-LEVEL: PROPAGATION OF A BANK SUPPLY SHOCK THROUGH THE NETWORK OF CUSTOMERS/SUPPLIERS: STRUCTURAL FORM. HETEROGENEITY WITH CUSTOMER/SUPPLIER

Dependent Variable: Δlog(sales from supplier to customer/sales of customer)	Upstream propagation			Downstream propagation		
	(1)	(2)	(3)	(4)	(5)	(6)
Customer (Bank) Shock (1st Order Effect)	-2.054**	-2.484**	-2.623**			
	(0.932)	(1.096)	(1.148)			
Customer (Bank) Shock (1st Order Effect)*Debt/Assets of the Customer	-0.740**	-0.945**	-1.445**			
	(0.354)	(0.399)	(0.583)			
Customer (Bank) Shock (1st Order Effect)*SME Customer		-0.971**	-0.967**			
		(0.438)	(0.439)			
Customer (Bank) Shock (1st Order Effect)*Debt/Assets of the Customer*SME Customer			-0.646**			
			(0.253)			
Supplier (Bank) Shock (1st Order Effect)				-1.217**	-1.316**	-1.335**
				(0.576)	(0.656)	(0.661)
Supplier (Bank) Shock (1st Order Effect)*Debt/Assets of the Supplier				-1.603	-1.608	-1.957
				(1.085)	(1.074)	(1.271)
Supplier (Bank) Shock (1st Order Effect)*SME Supplier					-0.248	-0.323
					(0.249)	(0.293)
Supplier (Bank) Shock (1st Order Effect)*Debt/Assets of the Supplier*SME Supplier						-0.703
						(0.558)
Debt/Assets of the Customer	0.155	0.121	0.213			
	(0.751)	(0.810)	(1.044)			
SME Customer		-0.317	-0.312			
		(0.357)	(0.373)			
Debt/Assets of the Customer*SME Customer			0.072			
			(0.530)			
Debt/Assets of the Supplier				0.065	0.067	-0.019
				(0.583)	(0.579)	(0.574)
SME Supplier					-1.075***	-1.132***
					(0.370)	(0.364)
Debt/Assets of the Supplier*SME Supplier						-0.321
						(0.321)
Higher Order Customer (Bank) Effect	1.770***	1.710***	1.709***			
	(0.631)	(0.621)	(0.626)			
Higher Order Supplier (Bank) Effect				-2.162*	-2.171*	-2.250*
				(1.168)	(1.162)	(1.179)
Supplier/Customer:						
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Firm*Supplier/Customer Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.478	0.478	0.478	0.493	0.493	0.493
Observations	1,119,169	1,119,169	1,119,169	1,114,420	1,114,420	1,114,420

This table reports estimates from WLS results. See Section 6. Observations are at the level of the firm-customer (columns (1) to (3)) or firm-supplier (columns (4) to (6)), i.e. link-level. The dependent variable is the change in the sales from supplier to customer, minus the change in the log of total sales of the customer between 2008 and 2009. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). For the definition of higher order bank shock effects, see Section 5 and 6 of the paper. SME is a dummy variable that takes the value 1 if the customer or the supplier of the firm is not a large firm, and 0 otherwise. Debt/Assets is the ratio between the debt with cost over total assets. All variables are standardized. For the list of controls, see Section 6. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A10

NODE-LEVEL: THE EFFECT OF BANK CREDIT SUPPLY SHOCK ON TRADE CREDIT

Dependent Variable:	Change commercial debtors: Customers		Δ(Commercial debtors: Customers/Sales)	
	(1)	(2)	(3)	(4)
Direct (Bank) Shock	0.609	0.626	0.181	0.171
	(0.595)	(0.598)	(0.135)	(0.133)
Customer (Bank) Shock (1st Order Effect)		-0.583**		0.353***
		(0.228)		(0.043)
Supplier (Bank) Shock (1st Order Effect)		0.102		-0.065**
		(0.264)		(0.026)
Firm Controls	Yes	Yes	Yes	Yes
Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes
R-squared	0.071	0.071	0.073	0.074
Observations	165,455	165,455	181,393	181,393

This table reports estimates from OLS. For columns (1) and (2), the dependent variable is the symmetric change between 2008 and 2009 for commercial debtors (customers) – trade credit is from suppliers to customers – in percentage. For columns (3) and (4), the dependent variable is the change between 2008 and 2009 for commercial debtors (customers) over total assets, in percentage. As we cannot control for firm fixed effects, we control for industry (NACE at two digits) times province, spatial (zip code), and main bank fixed effects. All shocks are standardized. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at main bank). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A11

LINK-LEVEL: PROPAGATION OF A BANK SUPPLY SHOCK THROUGH THE NETWORK OF CUSTOMERS/SUPPLIERS: STRUCTURAL FORM

Panel A: Aggregating suppliers at 2 or 3-digit NACE

Dependent Variable: Δlog(sales from supplier to customer/sales of customer)	2-digit NACE		3-digit NACE	
	(1)	(2)	(3)	(4)
Supplier (Bank) Shock (1st Order Effect)	-0.560**	-0.423*	-0.601***	-0.495**
	(0.256)	(0.250)	(0.221)	(0.221)
Higher Order Supplier (Bank) Shock		-1.122**		-0.960**
		(0.533)		(0.486)
Supplier/Customer:				
Controls	Yes	Yes	Yes	Yes
Customer Spatial*Industry Fixed Effects	Yes	Yes	Yes	Yes
Firm:				
Fixed Effects	Yes	Yes	Yes	Yes
Firm*Customer Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes
R-squared	0.481	0.483	0.410	0.412
Observations	526,429	526,429	697,019	697,019

Panel B: Firm fixed effects*supplier industry (2 or 3-digit NACE) fixed effects

Dependent Variable: Δlog(sales from supplier to customer/sales of customer)	2-digit NACE		3-digit NACE	
	(1)	(2)	(3)	(4)
Supplier (Bank) Shock (1st Order Effect)	-1.787*	-1.768*	-2.733**	-2.556**
	(0.950)	(0.920)	(1.340)	(1.165)
Higher Order Supplier (Bank) Shock		-1.574		-0.884
		(1.446)		(1.916)
Supplier/Customer:				
Controls	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	Yes	Yes	Yes	Yes
Firm:				
Firm Fixed Effects*Supplier Industry Fixed Effects	Yes	Yes	Yes	Yes
Firm*Supplier/Customer Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes
R-squared	0.595	0.595	0.595	0.646
Observations	757,067	757,067	577,859	577,859

Panel C: Estimated sigma from panel A and panel B

	Aggregating suppliers at:		Firm f.e.*supplier industry at:	
	2-digit NACE	3-digit NACE	2-digit NACE	3-digit NACE
Sigma	1.25***	1.28***	1.95**	2.37**
	(0.15)	(0.16)	(0.77)	(1.00)

This table reports estimates from WLS results. See Section 6. Observations are at the level of the firm-supplier, i.e. link-level. The dependent variable is the change in the sales from supplier to customer, minus the change in the log of total sales of the customer between 2008 and 2009. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). For the definition of higher order bank shock effects, see Section 5 and 6 of the paper. All variables are standardized. For the list of controls, see Section 6. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A12

LINK-LEVEL: PROPAGATION OF A BANK SUPPLY SHOCK THROUGH THE NETWORK OF CUSTOMERS/SUPPLIERS: STRUCTURAL FORM. LONG-RUN EFFECTS

Dependent Variable: Δlog(sales from supplier to customer/sales of customer)	Upstream	Downstream	Joint estimation Non-standardized
	(1)	(2)	(3)
Customer (Bank) Shock (1st Order Effect)	-2.008**		-42.541**
	(0.896)		(20.376)
Higher Order Customer (Bank) Shock	1.979*		56.372*
	(1.022)		(29.128)
Supplier (Bank) Shock (1st Order Effect)		-2.158**	-49.308**
		(0.903)	(20.709)
Higher Order Supplier (Bank) Shock		1.730	44.453
		(1.098)	(28.755)
Supplier/Customer:			
Controls	Yes	Yes	Yes
Spatial*Industry Fixed Effects	Yes	Yes	Yes
Firm:			
Fixed Effects	Yes	Yes	Yes
Firm*Supplier/Customer Spatial & Industry Fixed Effects	Yes	Yes	Yes
R-squared	0.525	0.538	0.531
Estimated Sigma			2.17** (0.88)
Observations	1,115,199	1,107,968	2,223,167

This table reports estimates from WLS results. See Section 6. Observations are at the level of the firm-customer (columns (1), (4) and (7)) or firm-supplier (columns (2), (5) and (8)), i.e. link-level. The dependent variable is the change in the sales from supplier to customer, minus the change in the log of total sales of the customer between 2008 and 2010. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). For the definition of higher order bank shock effects, see Section 5 and 6 of the paper. All variables are standardized but those of column (3), (6) and (9). For the list of controls, see Section 6. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A13

LINK-LEVEL: PROPAGATION OF A BANK SUPPLY SHOCK THROUGH THE NETWORK OF CUSTOMERS/SUPPLIERS: HETEROGENEOUS INPUT ELASTICITIES. STRUCTURAL FORM

Dependent Variable: Δlog(sales from supplier to customer/sales of customer)	Upstream propagation		Downstream propagation	
	(1)	(2)	(3)	(4)
Customer (Bank) Shock (1st Order Effect)	-2.006**	-1.960**		
	(0.921)	(0.886)		
Higher Order Customer (Bank) Shock	2.409**	1.598*		
	(1.000)	(0.857)		
Customer Capital Input Effect		2.713*		
		(1.555)		
Customer Labor Input Effect		-1.378**		
		(0.692)		
Supplier (Bank) Shock (1st Order Effect)			-1.077**	- 1.059**
			(0.547)	(0.515)
Higher Order Supplier (Bank) Shock			-3.326**	-2.713*
			(1.595)	(1.527)
Supplier Capital Input Effect				-4.062***
				(1.316
Supplier Labor Input Effect				-0.985
				(1.722)
Supplier/Customer:				
Controls	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	Yes	Yes	Yes	Yes
Firm:				
Fixed Effects	Yes	Yes	Yes	Yes
Firm*Supplier/Customer Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes
R-squared	0.478	0.478	0.493	0.494
No. of Observations	1,119,169	1,119,169	1,114,421	1,114,421

Notes: This table reports estimates from WLS. See online Appendix. Observations are at the level of the firm (node-level). The dependent variables is the change, between 2008 and 2009, in the log of firm-level aggregate sales to all customers over E, the (nominal) GDP. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). As we cannot control for firm fixed effects, we control for spatial and industry, and main bank fixed effects. All variables are standardized. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A14

NODE-LEVEL: FIRM-LEVEL EFFECTS OF BANK SUPPLY SHOCKS THROUGH THE PRODUCTION NETWORK: HETEROGENEOUS INPUT ELASTICITIES. STRUCTURAL FORM.

Dependent Variable: Δlog(sales/E)	(1)	(2)	(3)	(4)	(5)
Upstream (1st & Higher Order Effects)	-1.810***		-1.534***	-1.531***	-1.506***
	(0.326)		(0.321)	(0.351)	(0.321)
Bidirectional (Up & Down 1st & Higher Order Effects)		-1.542***	-0.715**	-0.725*	-0.498*
		(0.344)	(0.304)	(0.402)	(0.292)
Direct (Bank) Shock				0.033	
				(0.599)	
Bidirectional Capital Input Effect					-2.274***
					(0.283)
Bidirectional Labor Input Effect					-0.443
					(0.475)
Firm Controls	Yes	Yes	Yes	Yes	Yes
Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes
R-squared	0.358	0.357	0.358	0.358	0.359
Observations	196,171	196,171	196,171	196,171	196,171

Notes: This table reports estimates from WLS. See online Appendix. Observations are at the level of the firm (node-level). The dependent variables are the change, between 2008 and 2009, in the log of firm-level aggregate sales to all customers over E, the (nominal) GDP. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). See also Section 6 of the paper. As we cannot control for firm fixed effects, we control for spatial and industry, and main bank fixed effects. All variables are standardized. For the list of firm controls, see Section 4. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the level of the main bank). In each column, the word Yes indicates that the set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A15

SUMMARY STATISTICS: SUPPLIER-CUSTOMER DATASET

	Num. Obs.	Mean	S.D.	P25	Median	P75
			Links			
Links by Year:						
2008	13,810,158	78,039	5,281,888	4,922	9,453	25,396
2009	11,988,607	71,641	4,989,742	4,791	8,950	23,361
Links Appearing in Both Years:						
2008	7,655,815	115,145	7,035,309	6,353	13,424	38,043
2009	7,655,815	92,495	5,089,617	5,487	10,942	30,073
			Nodes			
2008						
Number of Customers	696,152	20	336	2	4	13
Number of Suppliers	808,973	17	55	3	7	17
2009						
Number of Customers	691,792	17	309	1	4	11
Number of Suppliers	789,118	15	52	2	6	15

Notes: This table reports means, standard deviations and first/second/third quartiles of annual bilateral transactions for 2008 and 2009 (Links), as well as the number of suppliers/customers for years 2008 and 2009 (Nodes). A firm is a supplier (customer) if it has at least one customer (supplier) in the network in a given year. Link ji between two firms appears in both years if supplier j reports a sale to customer i (or i reports a purchase from j) in both 2008 and 2009.

A.2. Heterogeneous input elasticities

Equation (8) does not feature network propagation of effects coming from labor and capital. The intuition behind this is that $\mathbf{G}$ is a column stochastic matrix, ensuring that changes in wages and the price of physical capital affect all firms equally, given that they have identical labor and capital input shares in the production function.

We now explore the implications of relaxing the assumption that α, β and ρ are common to all firms, both for the link-level analysis from Section 5.4 and node-level analysis from Section 5.5.

As we show in Proposition 1, when input shares are firm specific (8) becomes:

$$\begin{aligned}\mathrm{d}\log\left(\frac{s_{ji}}{s_i}\right) =& -\theta_i-(\sigma-1)\theta_j \\ &-(\sigma-1)\boldsymbol{e}_j'\mathbf{AG}'\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}\boldsymbol{\theta}+(\sigma-1)\boldsymbol{e}_i'\mathbf{G}'\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}\boldsymbol{\theta} \\ &-(\sigma-1)\boldsymbol{e}_j'\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}(\boldsymbol{\beta}\mathrm{d}w+\boldsymbol{\rho}\mathrm{d}r)+(\sigma-1)\boldsymbol{e}_i'\mathbf{G}'\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}(\boldsymbol{\beta}\mathrm{d}w+\boldsymbol{\rho}\mathrm{d}r)\,.\end{aligned} \tag{17}$$

The first two lines of (17) are analogous to (8). The only difference is in that the scalar α is replaced with the diagonal matrix $\mathbf{A}\equiv diag(\alpha_i)$ in which i-th element is equal to α_i. The third line of the expression is new, and it captures how the change of the change in wage ($\mathrm{d}w$) and the change in the price of capital ($\mathrm{d}r$) affect the production costs of supplier j and customer i, respectively. Intuitively, the effects of a change in the wage and the capital price are proportional to how much a firm relies, directly and indirectly, on labor and capital in production, respectively.

In the empirical implementation, we estimate sector-specific parameters α, β and ρ using standard approach Wooldridge (2009). We assign these parameters to firms according to their respective sectors. While we do not observe $\mathrm{d}w$, $\mathrm{d}r$, we are able to calculate network measures $(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{\beta}$ and $(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{\rho}$. Therefore, when estimating equation (17) the theory implied effect of $\boldsymbol{e}_j'(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{\beta}$ is equal to $-(\sigma-1)\mathrm{d}w$, and analogously $(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{\rho}$. We report the estimation results in Table A13. We label estimated parameters of $\boldsymbol{e}_j'(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{\beta}$ and $\boldsymbol{e}_j'(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{\rho}$ as the *Supplier Labor Input Effect* and the *Supplier Capital Input Effect*, respectively. Similarly, the parameters of $\boldsymbol{e}_i'(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{\beta}$ and $\boldsymbol{e}_i'(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{\rho}$ are labeled as the *Customer Labor Input Effect* and the *Customer Capital Input Effect*, respectively. We find that our estimates of parameters reported in panel A of Table 3 are not affected.

By allowing firm specific values of parameters α, β and ρ equation (10) becomes:

$$\mathrm{d}\log\left(\frac{s_i}{E}\right)=-\boldsymbol{e}_i'\left(\mathbf{I}-\mathbf{H}\right)^{-1}\mathbf{H}\boldsymbol{\theta}+(1-\sigma)\boldsymbol{e}_i'\boldsymbol{\Lambda}\boldsymbol{\theta}+(1-\sigma)\boldsymbol{e}_i'\boldsymbol{\Lambda}(\boldsymbol{\beta}\mathrm{d}w+\boldsymbol{\rho}\mathrm{d}r), \tag{18}$$

where now, $\mathbf{H}=\mathbf{V}^{-1}\mathbf{GAMV}$ and $\boldsymbol{\Lambda}=(\mathbf{I}-\mathbf{H})\left(diag\left(\mathbf{H1}\right)-\mathbf{HG}'\right)\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}$.

Turning to the node-level analysis, relative to (10), equation (18) features bidirectional propagation coming from the changes in labor and capital prices due to bank shocks. Since, in this case, firms are different with respect to the intensity in which they use inputs, changes in labor and capital price affect firms differently. Note, also, that both upstream and bidirectional propagation of bank shocks in (18) depend on the firm-specific values of parameter α_i, which is captured by diagonal matrix $\mathbf{A}$. The intuition behind these two types of propagation remains the same, as explained Section 5.5.

We estimate equation (9) and report the results in Table A14 in online Appendix A. We label estimated parameters of $\boldsymbol{e}_i'\boldsymbol{\Lambda\beta}$ and $\boldsymbol{e}_i'\boldsymbol{\Lambda\rho}$ as the *Bidirectional Capital Input Effect* and the *Bidirectional Labor Input Effect*, respectively.

A.3. A more complete literature review

The fast-growing literature studying the phenomenon of shock propagation in large economies has mostly evolved by studying separately the real and the financial networks. In the first case, the main focus has been on the supply chains that underlie the production of the non-financial firms of the economy and the role of the network structure in the propagation and aggregation of, for the most part, productivity shocks.[48] In the second case, the analysis has mainly centered on the banks alone as the main actors, the links among them typically conceived as reflecting some form of financial flows.[49] In comparison with these two largely unconnected branches of the literature, our contribution considers both the real and financial sides of the economy and focuses the analysis on the interaction between them.

There is a rich literature that has explored whether credit-supply shocks may lead to significant real effects on the production side of the economy, but its analysis of the problem abstracts from the role played by the production network as a propagation structure of those shocks. As a representative sample of its more theoretical branch, we can refer to e.g. Holmstrom and Tirole (1997); Stein (1998); Gertler and Kiyotaki (2010), while for its empirical branch we can mention e.g. Khwaja and Mian (2008), Chodorow-Reich (2014), Greenstone et al. (2014), Jiménez et al. (2012, 2017), Amiti and Weinstein (2018), and Galaasen et al. (2020). There are only a few papers that are close to ours in that they also aim to understand the process by which bank shocks propagate through the real production network. To the best of our knowledge, the following two papers are the most related.[50]

The first paper is by Costello (2020), who studies the downstream propagation of shocks through their influence on the trade credit that firms extend to their customers. Relying on data obtained from a third-party trade credit information platform, this paper documents that firms with greater exposure to a large decline in finance reduce their trade credit to customers, and consequently induce negative effects on employment.[51] In contrast with this paper, we use

[48] See for instance Acemoglu et al. (2012); Barrot and Sauvagnat (2016); Baqaee (2018); Carvalho et al. (2020).

[49] See for instance Allen and Gale (2000); Freixas et al. (2000); Iyer and Peydro (2011); Niepmann and Schmidt-Eisenlohr (2013); Elliott et al. (2014); Cabrales et al. (2017).

[50] Another more distantly related paper is Alfaro et al. (2021), which investigates the propagation bank shocks through *industry-level input-output data*. We outline here three important differences. First, they analyze reduced-form estimates while we show that such reduced-form estimation may miss about half of the overall propagation effects, hence substantially underestimating the extent of shock propagation. Second, and relatedly, they do not investigate higher-order propagation effects, but our findings suggest that these high-order effects are as crucial as first-order effects. Third, their reliance on industry-aggregated data raises identification concerns that our transaction-level data at the firm and supplier-customer level can handle in a significantly more effective manner. In addition, we also refer to the paper by Dewachter et al. (2020), which complements our research by studying a dynamic Keynesian model that also displays an interplay of financial and production networks and is applied to Belgium data similar to ours. Their concern is quite different from ours in that their primary focus is on how bank concentration and its effect on bank competition bears on macroeconomic volatility.

[51] Related to this, Demir et al. (2024) show that a negative shock to the cost of import financing gets propagated

administrative registers and focus on the effects of bank shocks on sales at the firm-to-firm (link) and firm (node) levels. The advantage of our approach, relative to the one taken in Costello (2020), is that it enables us to account for the general equilibrium and higher-order network effects of bank shocks and interpret the estimates as structural parameters of the model. In contrast with Costello (2020), we show that: (a) besides downstream propagation, upstream propagation is also important, with even larger economic effects; (b) in addition to first-order effects, also higher-order effects matter; (c) complex bidirectional propagation matters as well. This type of propagation of bank shocks has not yet been studied in the literature.

The second paper is by Cortes et al. (2019), who uses firm-to-firm payment data across different banks from Brazil to approximate transaction data across firms and estimate the indirect effects of state-owned bank shocks. Methodologically, however, this paper differs from ours in several key respects. First, it only considers first-order propagation, while we also analyze the transmission of shocks through higher-order linkages. Second, it considers bank shocks by state-owned banks, while we consider bank shocks from all banks. Note that there is extensive literature showing that state-owned banks generate large inefficiencies (see e.g. La Porta et al. (2002)), and hence, changes in credit through such banks may not identify bank shocks appropriately. Third, due to data limitations, Cortes et al. (2019) only exploit transactions between firms working with different banks while we exploit all transactions. And fourth, in contrast to their paper, our approach is theory-based in that we propose and study a general equilibrium model of the problem and then use it for the estimation.

Our paper is also connected to studies that estimate the elasticity of substitution across intermediate inputs using various methods and sources of variation. Several papers use import data to estimate the elasticity of substitution between imported and domestic intermediate inputs. Halpern et al. (2015) uses annual Hungarian import data and estimates the elasticity of substitution between imported and domestic intermediate inputs to be between 4 and 7. In their estimation, they fix the elasticity of substitution between intermediate inputs and factors of production to be 1. Using monthly import data from Japan to USA in combination with the Tohoku earthquake shock, Boehm et al. (2019) estimates the elasticity of substitution between intermediate goods imported from Japan and other domestic (US) intermediate inputs to be between 0.201 and 0.624.

Another approach to estimating the elasticity is using sector-level data and plausible variation in input prices. Using government spending demand shocks and annual US sector level input-output tables Atalay (2017) estimates the elasticity of substitution between intermediate inputs between -0.13 and - 0.07. Using trade liberalization shock in India, Peter et al. (2022) estimates the long-run elasticity of substitution between material input categories to be 3.1.

The papers closest to ours use firm-level production network data to recover the elasticity of substitution across intermediate inputs. Even though they do not estimate the elasticity of substitution directly, Barrot and Sauvagnat (2016) argue that their results on the effects of supply chain disruption on firm-level outcomes are consistent with Leontief production function in the short run (quarterly data). Using partial annual production network data from Japan

from liquidity-constrained firms to their customers (see also Jacobson and von Schedvin (2015)).

and the shock caused by the Tohoku earthquake, Carvalho et al. (2020) estimate the elasticity of substitution across intermediate inputs to be 1.18. Finally, Fujiy et al. (2024), using monthly firm-level transaction data from India and disruptions caused by governmental responses to Covid-19 pandemic, find that the elasticity of substitution across intermediate inputs lies within a range of 0.50 to 0.66.

We conclude by reiterating the important point made in Ruhl et al. (2008) and Boehm et al. (2019): caution is required when comparing results across different studies, as the elasticity of substitution is inherently linked to the time horizon and the nature of the shocks considered.

Appendix B: Proofs — For online publication

Lemma 1. *The marginal cost of firm i is given by*

$$mc_i = (1+\theta_i)\frac{1}{\kappa_i} r^{\rho_i} w^{\beta_i} P_i^{\alpha_i}, \tag{19}$$

where $\kappa_i \equiv \zeta_i \rho_i^{\rho_i} \alpha_i^{\alpha_i} \beta_i^{\beta_i}$ *and* $P_i \equiv \left[\sum_{k\in N_i^+} g_{ki} p_k^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$.

Proof of Lemma 1. Given any feasible production plan $[\ell_i, k_i, (z_{ij})_{j=1}^n]$ and shock θ_i, firm i minimizes:

$$(1+\theta_i)\left(w\ell_i + rk_i + \sum_{j\in N_i^+} p_j z_{ji}\right), \tag{20}$$

subject to the technological constraint:

$$y_i \le \zeta_i k_i^{\rho_i} \ell_i^{\beta_i} \left[\left(\sum_{j\in N_i^+} g_{ji}^{\frac{1}{\sigma}} z_{ji}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\alpha_i}.$$

The above constraint holds with equality. Hence the Lagrangian of this problem is:

$$\mathscr{L} = (1+\theta_i)\left(w\ell_i + rk_i + \sum_{j\in N_i^+} p_j z_{ji}\right) - \varphi_i\left[\zeta_i k_i^{\rho_i} \ell_i^{\beta_i} M_i^{\alpha_i} - y_i\right].$$

The first-order necessary conditions (FONC) which are also sufficient, given the postulated convexity conditions, read:

$$\begin{aligned}
&(1+\theta_i)p_j = \varphi_i\zeta_i k_i^{\rho_i}\ell_i^{\beta_i}\alpha_i M_i^{\alpha_i-1}\frac{\partial M_i}{\partial z_{ji}} = \varphi_i\alpha_i y_i \frac{1}{M_i}\frac{\partial M_i}{\partial z_{ji}} = \varphi_i\alpha_i y_i\frac{1}{M_i}\left[\sum_k g_{ki}^{\frac{1}{\sigma}} z_{ki}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} g_{ji}^{\frac{1}{\sigma}} z_{ji}^{-\frac{1}{\sigma}},\\
&(1+\theta_i)w = \varphi_i\beta_i\frac{1}{\ell_i}y_i,\\
&(1+\theta_i)r = \varphi_i\rho_i\frac{1}{k_i}y_i.
\end{aligned} \tag{21}$$

From (21) follows directly that for any two interemediate inputs j and k used by firm i, we have:

$$\frac{p_j}{p_k} = \left[\frac{g_{ji}}{g_{ki}}\right]^{\frac{1}{\sigma}}\left[\frac{z_{ji}}{z_{ki}}\right]^{-\frac{1}{\sigma}} \Rightarrow z_{ki} = \left[\frac{p_j}{p_k}\right]^{\sigma}\frac{g_{ki}}{g_{ji}}z_{ji}.$$

Substituting z_{ji} from above in (5) we get:

$$M_i = \left[\sum_{k\in N_i^+, k\neq j} g_{ki}^{\frac{1}{\sigma}}\left[\left[\frac{p_j}{p_k}\right]^{\sigma}\frac{g_{ki}}{g_{ji}}z_{ji}\right]^{\frac{\sigma-1}{\sigma}} + g_{ji}^{\frac{1}{\sigma}} z_{ji}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \Rightarrow z_{ji} = g_{ji} p_j^{-\sigma} P_i^{\sigma} M_i, \tag{22}$$

where

$$P_i \equiv \left[\sum_{k\in N_i^+} g_{ki} p_k^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

is the price index of intermediate inputs firm i uses in the production. Using (21), the definition of P_i and (22) we can write the conditional demand for intermediate inputs, labor and capital:

$$\begin{aligned}
\ell_i(y_i; w, r, \boldsymbol{p}, \theta_i) &= \varphi_i \frac{1}{1+\theta_i} \beta_i \frac{y_i}{w}, \\
k_i(y_i; w, r, \boldsymbol{p}, \theta_i) &= \varphi_i \frac{1}{1+\theta_i} \rho_i \frac{y_i}{r}, \\
M_i(y_i; w, r, \boldsymbol{p}, \theta_i) &= \varphi_i \frac{1}{1+\theta_i} \alpha_i \frac{y_i}{P_i}, \\
z_{ji}(y_i; w, r, \boldsymbol{p}, \theta_i) &= \varphi_i \frac{1}{1+\theta_i} \alpha_i g_{ji} p_j^{-\sigma} P_i^{\sigma-1} y_i.
\end{aligned} \tag{23}$$

Substituting (23) in (20) we get that $mc_i = \varphi_i$. Then, to derive the expression for φ_i, substitute (23) in (4) to obtain:

$$y_i = \zeta_i \left(\frac{\varphi_i \rho_i y_i}{(1+\theta_i)r}\right)^{\rho_i} \left(\frac{\varphi_i \beta_i y_i}{(1+\theta_i)w}\right)^{\beta_i} \left(\frac{\varphi_i \alpha_i y_i}{(1+\theta_i)P_i}\right)^{\alpha_i} = \frac{\zeta_i}{1+\theta_i} \varphi_i y_i \left(\frac{\rho_i}{r}\right)^{\rho_i} \left(\frac{\beta_i}{w}\right)^{\beta_i} \left(\frac{\alpha_i}{P_i}\right)^{\alpha_i},$$

which gives:

$$mc_i = \varphi_i = \frac{1+\theta_i}{\zeta_i} \rho_i^{-\rho_i} \beta_i^{-\beta_i} \alpha_i^{-\alpha_i} r^{\rho_i} w^{\beta_i} P_i^{\alpha_i} = \frac{1+\theta_i}{\kappa_i} r^{\rho_i} w^{\beta_i} P_i^{\alpha_i},$$

as desired. $\square$

Steady state and normalization

To facilitate the calibration, following Baqaee (2018); Baqaee and Farhi (2019), we define the steady state as a contingency in which there are no financial shocks $\theta_i = 0, \forall i$, and $\kappa_i = \mu_i$, $\forall i$.

From the Lemma 1 and the pricing rule, we can write:

$$\log p_i = \log \left[\frac{1+\theta_i}{\kappa_i} \mu_i\right] + \rho_i \log r + \beta_i \log w + \frac{\alpha_i}{1-\sigma} \log \left[\sum_k g_{ki} p_k^{1-\sigma}\right],$$

which in the steady state reduces to:

$$\log p_i = \rho_i \log r + \beta_i \log w + \frac{\alpha_i}{1-\sigma} \log \left[\sum_k g_{ki} p_k^{1-\sigma}\right].$$

Clearly, $p_i = 1 \forall (i \in N)$, $w = 1$ and $r = 1$ satisfies this equation for every i.

The consumer's problem

Lemma 2. *Let* $\bar{p} = \prod_i \left(\frac{p_i}{\gamma_i}\right)^{\gamma_i}$ *denote the price index of consumption goods. The consumer chooses consumption plan such that*

$$\frac{p_i c_i}{\bar{p} c} = \gamma_j \text{ and } \frac{c^{-\gamma}}{L^\eta} = \frac{\bar{p}}{w}.$$

Proof of Lemma 2. The consumer solves the following problem:

$$\max_{\boldsymbol{c},L} \frac{1}{1-\delta}\left[\overbrace{\prod_i c_i^{\gamma_i}}^{c}\right]^{1-\delta} - \frac{L^{1+\eta}}{1+\eta}$$

$$s.t. \sum_i p_i c_i \leq E.$$

The monotonicity of preferences implies $\bar{p}c = E$. The fact that $\frac{p_i c_i}{\bar{p}c} = \gamma_j$ is directly obtained by solving for the expenditure minimization problem, where $\bar{p}c$ is the resulting expenditure. $\frac{c^{-\gamma}}{L^\eta} = \frac{\bar{p}}{w}$ follows directly from the utility maximization problem with respect to c and L. $\square$

Cost share, revenue share, sales share

We now introduce some additional notation. Let $s_{ji} \equiv p_j z_{ji}$ and $s_i = p_i y_i$. Furthermore, let $\tilde{\omega}_{ji}$ denote cost share on intermediate input j among intermediate inputs firm i uses in production, that is $\tilde{\omega}_{ji} \equiv \frac{s_{ji}}{\sum_\ell s_{\ell i}}$. Let $\omega_{ji} \equiv \frac{s_{ji}}{s_i}$ denote the expenditure share in sales on input j (we'll refer to it as *revenue share*). Finally, let $h_{ji} \equiv \frac{s_{ji}}{s_j}$ denote the sales share.

Let us now relate the technological parameters $(g_{ji})_{ij}$ with cost shares $\tilde{\omega}_{ji}$, revenue shares ω_{ji}, and sales shares h_{ji}.

Lemma 3. *The following holds in the equilibrium:*

$$\tilde{\omega}_{ji} = g_{ji} p_j^{1-\sigma} P_i^{\sigma-1} = \frac{g_{ji} p_j^{1-\sigma}}{\sum_k g_{ki} p_k^{1-\sigma}}, \text{ or in terms of quantities, } \tilde{\omega}_{ji} = g_{ji}^{\frac{1}{\sigma}} \left[\frac{z_{ji}}{M_i}\right]^{\frac{\sigma-1}{\sigma}}, \tag{24}$$

$$\omega_{ji} = \frac{\alpha_i}{(1+\theta_i)\mu_i}\tilde{\omega}_{ji}, \tag{25}$$

and

$$h_{ji} = \omega_{ji}\frac{s_i}{s_j} = \frac{\alpha_i}{(1+\theta_i)\mu_i}\tilde{\omega}_{ji}\frac{s_i}{s_j}. \tag{26}$$

Proof of Lemma 3. The first equation in (24) follows directly from the fact that $s_{ji} = g_{ji} p_j^{1-\sigma} P_i^{\sigma-1} P_i M_i$. To prove that the second equation in (24) we note that the FONC of the cost minimization problem with respect to z_{ji} in (21) gives:

$$(1+\theta_i)p_j = mc_i \alpha_i y_i M_i^{\frac{1-\sigma}{\sigma}} g_{ji}^{\frac{1}{\sigma}} z_{ji}^{-\frac{1}{\sigma}} \Rightarrow \frac{(1+\theta_i)p_j z_{ji}}{mc_i \alpha_i y_i} = M_i^{\frac{1-\sigma}{\sigma}} g_{ji}^{\frac{1}{\sigma}} z_{ji}^{\frac{\sigma-1}{\sigma}} \overbrace{\Rightarrow}^{\text{by (23)}} \tilde{\omega}_{ji} = g_{ji}^{\frac{1}{\sigma}} M_i^{\frac{1-\sigma}{\sigma}} z_{ji}^{\frac{\sigma-1}{\sigma}}.$$

The equation (25) follows directly from (23) and the pricing rule $p_i = \mu_i mc_i$. Finally, (26) follows from equality $\frac{s_{ji}}{s_j} = \frac{s_{ji}}{s_i}\frac{s_i}{s_j}$, which concludes the proof. $\square$

Let $\boldsymbol{\Omega}$, $\tilde{\boldsymbol{\Omega}}$ and $\mathbf{H}$ be matrices with elements ω_{ij}, $\tilde{\omega}_{ij}$ and h_{ij} respectively. Recall that, $\mathbf{A}$, $\mathbf{M}$ and $\mathbf{T}$ stand for diagonal matrices with elements α_i, $\frac{1}{\mu_i}$ and $\frac{1}{1+\theta_i}$ on the main diagonal, respectively. Lemma 3 implies that in steady state:

$$\tilde{\mathbf{\Omega}} \overbrace{=}^{\text{steady state}} \mathbf{G},$$

$$\mathbf{\Omega} = \tilde{\mathbf{\Omega}}\mathbf{AMT} \overbrace{=}^{\text{steady state}} \mathbf{GAM},$$

$$\mathbf{H} = \mathbf{V}^{-1}\mathbf{\Omega}\mathbf{V} \overbrace{=}^{\text{steady state}} \mathbf{V}^{-1}\mathbf{GAMV},$$

where $\mathbf{V}$ are diagonal matrices with elements diagonal elements equal to entries of the vector $\boldsymbol{v}(\boldsymbol{\theta}) = (v_i(\boldsymbol{\theta}))_{i=1}^n = (\mathbf{I} - \mathbf{GAMT})^{-1}\boldsymbol{\gamma}$, which, in steady state, is equal to $\boldsymbol{v}(\mathbf{0}) = (\mathbf{I} - \mathbf{GAM})^{-1}\boldsymbol{\gamma}$.

Effect of shocks on prices

Lemma 4. *In the steady state:*

$$\frac{\partial \log \boldsymbol{p}}{\partial \theta_k} = \left[\mathbf{I} - \mathbf{AG}'\right]^{-1}\left[\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta} + \frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho} + \boldsymbol{e}_k\right].$$

In the special case when $\alpha_i = \alpha$, $\beta_i = \beta$:

$$\frac{\partial \log \boldsymbol{p}}{\partial \theta_k} = \left[\mathbf{I} - \alpha\mathbf{G}'\right]^{-1}\boldsymbol{e}_k + \frac{\beta}{1-\alpha}\frac{\partial \log w}{\partial \theta_k}\mathbf{1} + \frac{\rho}{1-\alpha}\frac{\partial \log r}{\partial \theta_k}\mathbf{1}.$$

Proof. Consider $\frac{\partial \log p_i}{\partial \theta_k}$. From the expression for marginal cost of firm i (Lemma 1), we have:

$$\log p_i = \log(1+\theta_i) + \log \mu_i - \log \kappa_i + \beta_i \log w + \rho_i \log r_i + \alpha_i \log P_i.$$

Differentiating with respect to θ_k we get:

$$\frac{\partial \log p_i}{\partial \theta_k} = \frac{\partial \log(1+\theta_i)}{\partial \theta_k} + \beta_i \frac{\partial \log w}{\partial \theta} + \rho_i \frac{\partial \log r}{\partial \theta_k} + \alpha_i \frac{\partial \log P_i}{\partial \theta_k}.$$

To get the expression for $\frac{\partial \log P_i}{\partial \theta_k}$ we note that from the definition of P_i it follows that:

$$\frac{\partial \log P_i}{\partial \theta_k} = \frac{1}{\sum_\ell g_{\ell i} p_\ell^{1-\sigma}} \sum_j g_{ji} p_j^{1-\sigma} \frac{\partial \log p_j}{\partial \theta_k}.$$

At the steady state:

$$\left.\frac{\partial \log P_i}{\partial \theta_k}\right|_{\boldsymbol{\theta}=0} = \sum_j g_{ji} \left.\frac{\partial \log p_j}{\partial \theta_k}\right|_{\boldsymbol{\theta}=0} = \boldsymbol{e}_i'\mathbf{G}'\frac{\partial \log \boldsymbol{p}}{\partial \theta_k}.$$

In what follows we omit $|_{\boldsymbol{\theta}=0}$ whenever it is clear that the derivatives are evaluated at the steady state. Finally, evaluating at the steady state:

$$\frac{\partial \log p_i}{\partial \theta_k} = -\delta_{ki} + \beta_i \frac{\partial \log w}{\partial \theta_k} + \rho_i \frac{\partial \log r}{\partial \theta_k} + \alpha_i \sum_j g_{ji} \frac{\partial \log p_j}{\partial \theta_k},$$

where δ_{ki} denotes the Kroenecker's delta. Writing this expression for each price in vector notation gives:

$$\frac{\partial \log \boldsymbol{p}}{\partial \theta_k} = \left[\mathbf{I} - \mathbf{AG}'\right]^{-1}\left[\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta} + \frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho} + \boldsymbol{e}_k\right].$$

In the special case when $\alpha_i = \alpha$, $\beta_i = \beta$ for all i the above expression becomes:

$$\frac{\partial \log \boldsymbol{p}}{\partial \theta_k} = \left[\mathbf{I} - \alpha \mathbf{G}'\right]^{-1} \boldsymbol{e}_k + \frac{\beta}{1-\alpha}\frac{\partial \log w}{\partial \theta_k}\mathbf{1} + \frac{\rho}{1-\alpha}\frac{\partial \log r}{\partial \theta_k}\mathbf{1}.$$

□

The following corollary is implied by Lemma 4.

Corollary 1. *At the steady state:*

$$\frac{\partial \log \boldsymbol{P}}{\partial \theta_k} = \mathbf{G}'\left[\mathbf{I} - \mathbf{A}\mathbf{G}'\right]^{-1}\left[\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta} + \frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho} + \boldsymbol{e}_k\right],$$

and when $\alpha_i = \alpha$, $\beta_i = \beta$ for all i:

$$\frac{\partial \log \boldsymbol{P}}{\partial \theta_k} = \frac{\beta}{1-\alpha}\frac{\partial \log w}{\partial \theta_k}\mathbf{1} + \frac{\rho}{1-\alpha}\frac{\partial \log r}{\partial \theta_k}\mathbf{1} + \mathbf{G}'\left[\mathbf{I} - \alpha \mathbf{G}'\right]^{-1}\boldsymbol{e}_k.$$

Link-level outcomes

Proposition 1. *The first order approximation of the change $\log \frac{s_{ji}}{s_i} = \log w_{ji}$ at the steady state is given with* (17)*. In the special case when all firms have equal input shares α, β and ρ* (17) *becomes* (8)*.*

Proof of Proposition 1. We use the following approximation:

$$d\log s_{ji} = \sum_{k\in N}\frac{\partial \log s_{ji}}{\partial \theta_k}\theta_k, \tag{27}$$

where derivatives are evaluated at point $\boldsymbol{\theta} = \mathbf{0}$.

From firms' pricing rule ($p_i = \mu_i mc_i$), Lemma 1, and (22) it directly follows that:

$$p_j z_{ji} = g_{ji}(\mu_j mc_j)^{1-\sigma}P_i^{\sigma}M_i \Rightarrow p_j z_{ji} = g_{ji}(\mu_j(1+\theta_j)\kappa_j^{-1}r_j^{\rho_j}w^{\beta_j}P_j^{\alpha_j})^{1-\sigma}P_i^{\sigma}M_i,$$

which together with (23) implies:

$$p_j z_{ji} = g_{ji}(\mu_j(1+\theta_j)\kappa_j^{-1}r_j^{\rho_j}w^{\beta_j}P_j^{\alpha_j})^{1-\sigma}P_i^{\sigma-1}\mu_i^{-1}(1+\theta_i)^{-1}\alpha_i s_i.$$

Taking logs, and using $s_{ji} \equiv p_j z_{ji}$ we get:

$\log s_{ji} = (1-\sigma)\log(1+\theta_j) - \log(1+\theta_i) + \log g_{ji} + (1-\sigma)(\log \mu_j - \log \kappa_j + \beta_j \log w + \rho_j \log r) + \log \alpha_i +$
$\log s_i - \log \mu_i + (1-\sigma)\alpha_j \log P_j + (\sigma - 1)\log P_i.$

Differentiating with respect to θ_k we get:

$$\begin{aligned}\frac{\partial \log s_{ji}}{\partial \theta_k} = &- \frac{1}{1+\theta_i}\delta_{ki} - (\sigma-1)\frac{1}{1+\theta_j}\delta_{kj} + \frac{\partial \log s_i}{\theta_k} - \\ &(\sigma-1)\left[\beta_j\frac{\partial \log w}{\partial \theta_k} + \rho_j \frac{\partial \log r}{\partial \theta_k} + \alpha_j \frac{\partial \log P_j}{\partial \theta_k}\right] + (\sigma-1)\frac{\partial \log P_i}{\partial \theta_k},\end{aligned} \tag{28}$$

where δ_{jk} is Kroeneker's delta.

Corollary 1 implies that we can write (28) as:

$$\frac{\partial \log s_{ji}}{\partial \theta_k} = -\frac{1}{1+\theta_i}\delta_{ki} - (\sigma-1)\frac{1}{1+\theta_j}\delta_{kj} + \frac{\partial \log s_i}{\theta_k} -$$
$$(\sigma-1)\boldsymbol{e}_j'\left[\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta} + \frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho} + \mathbf{AG}'\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}\left(\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta} + \frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho} + \boldsymbol{e}_k\right)\right] +$$
$$(\sigma-1)\boldsymbol{e}_i'\left[\mathbf{G}'\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}\left(\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta} + \frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho} + \boldsymbol{e}_k\right)\right], \tag{29}$$

which simplifies to:

$$\frac{\partial \log s_{ji}}{\partial \theta_k} = -\frac{1}{1+\theta_i}\delta_{ki} - (\sigma-1)\frac{1}{1+\theta_j}\delta_{kj} + \frac{\partial \log s_i}{\partial \theta_k} -$$
$$(\sigma-1)\boldsymbol{e}_j'\left[\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}\left(\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta} + \frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho}\right) + \mathbf{AG}'\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}\boldsymbol{e}_k\right] + \tag{30}$$
$$(\sigma-1)\boldsymbol{e}_i'\left[\mathbf{G}'\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}\left(\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta} + \frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho}\right) + \mathbf{G}'(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{e}_k\right].$$

Using (30) in (27) and evaluating derivatives at $\boldsymbol{\theta}=\mathbf{0}$ gives (17), where we use the fact that $\mathbf{I}+\mathbf{AG}'(\mathbf{I}-\mathbf{AG}')^{-1} = (\mathbf{I}-\mathbf{AG}')^{-1}$.

In the special case when $\alpha_i=\alpha$ and $\beta_i=\beta$ for all firms i (30) simplifies to:

$$\frac{\partial \log s_{ji}}{\partial \theta_k} = -\frac{1}{1+\theta_i}\delta_{ki} - (\sigma-1)\frac{1}{1+\theta_j}\delta_{kj} + \frac{\partial \log s_i}{\partial \theta_k} -$$
$$(\sigma-1)\left[\frac{\beta}{1-\alpha}\frac{\partial \log w}{\partial \theta_k} + \frac{\rho}{1-\alpha}\frac{\partial \log r}{\partial \theta_k} + \alpha \boldsymbol{e}_j'\mathbf{G}'\left(\mathbf{I}-\alpha\mathbf{G}\right)^{-1}\boldsymbol{e}_k\right] +$$
$$(\sigma-1)\left[\frac{\beta}{1-\alpha}\frac{\partial \log w}{\partial \theta_k} + \frac{\rho}{1-\alpha}\frac{\partial \log r}{\partial \theta_k} + \boldsymbol{e}_i'\mathbf{G}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{e}_k\right]$$
$$= -\frac{1}{1+\theta_i}\delta_{ki} - (\sigma-1)\frac{1}{1+\theta_j}\delta_{kj} + \frac{\partial \log s_i}{\theta_k} - (\sigma-1)\left[\alpha \boldsymbol{e}_j'\mathbf{G}'\left(\mathbf{I}-\alpha\mathbf{G}\right)^{-1}\boldsymbol{e}_k - \boldsymbol{e}_i'\mathbf{G}'\left(\mathbf{I}-\alpha\mathbf{G}'\right)^{-1}\boldsymbol{e}_k\right].$$

This concludes the proof. □

Node-level outcomes

Proposition 2. *A first-order approximation of the change in* $\log\left(\frac{s_i}{E}\right)$ *at the steady state is given with* (18)*. In the special case when all firms have equal input shares* α*,* β *and* ρ (18) *becomes* (9)*.*

Proof of Proposition 2. We first provide expression for $\frac{\partial \log \boldsymbol{s}}{\partial \theta_k} - \frac{\partial \log E}{\partial \theta_k}$. Equations (18) and (9) then follow directly.

Market clearing condition for intermediate inputs read:

$$y_i = c_i + \sum_j z_{ij},$$

which be written as:

$$p_i y_i = p_i c_i + \sum_j p_i z_{ij} \Rightarrow s_i = \gamma_i E + \sum_j \omega_{ij} s_j,$$

and therefore:

$$\boldsymbol{s} = E(\mathbf{I} - \boldsymbol{\Omega})^{-1}\boldsymbol{\gamma} = E\boldsymbol{v}.$$

Taking derivatives, we get:

$$\begin{aligned}\frac{\partial \boldsymbol{s}}{\partial \theta_k} &= \frac{\partial E}{\partial \theta_k}(\mathbf{I}-\boldsymbol{\Omega})^{-1}\boldsymbol{\gamma} - (\mathbf{I}-\boldsymbol{\Omega})^{-1}\frac{\partial(\mathbf{I}-\boldsymbol{\Omega})}{\partial \theta_k}(\mathbf{I}-\boldsymbol{\Omega})^{-1}E\boldsymbol{\gamma} \\ &= \frac{\partial \log E}{\partial \theta_k}\boldsymbol{s} + (\mathbf{I}-\boldsymbol{\Omega})^{-1}\frac{\partial \boldsymbol{\Omega}}{\partial \theta_k}\boldsymbol{s} = \left[\frac{\partial \log E}{\partial \theta_k}\mathbf{I} + \overbrace{(\mathbf{I}-\boldsymbol{\Omega})^{-1}}^{\boldsymbol{\Psi}}\frac{\partial \boldsymbol{\Omega}}{\partial \theta_k}\right]\boldsymbol{s}.\end{aligned}$$

For a given firm i, we have:

$$\frac{\partial s_i}{\partial \theta_k} = \frac{\partial \log E}{\partial \theta_k}s_i + \sum_\ell \sum_j \psi_{i\ell}\frac{\partial \omega_{\ell j}}{\partial \theta_k}s_j,$$

and consequently

$$\frac{\partial \log s_i}{\partial \theta_k} = \frac{\partial \log E}{\partial \theta_k} + \frac{1}{s_i}\sum_\ell \sum_j \psi_{i\ell}\frac{\partial \omega_{\ell j}}{\partial \theta_k}s_j = \frac{\partial \log E}{\partial \theta_k} + \frac{1}{s_i}\sum_\ell \sum_j \psi_{i\ell}\omega_{\ell j}\frac{\partial \log \omega_{\ell j}}{\partial \theta_k}s_j.$$

Substituting the expression for $\frac{\partial \log \omega_{\ell j}}{\partial \theta_k}$ (from Proposition 1), the previous expression becomes:

$$\frac{\partial \log s_i}{\partial \log \theta_k} = \frac{\partial \log E}{\partial \theta_k} + \frac{1}{s_i}\left[\sum_\ell \sum_j \psi_{i\ell}\omega_{\ell j}s_j\left(-\frac{\partial \log(1+\theta_j)}{\partial \theta_k} + (1-\sigma)\left(\frac{\partial \log p_\ell}{\partial \theta_k} - \sum_r \tilde{\omega}_{rj}\frac{\partial \log p_r}{\partial \theta_k}\right)\right)\right].$$

To write the expression for $\frac{\partial \log \boldsymbol{s}}{\partial \theta_k}$, we consider the parts of the right-hand side of the previous expression in the brackets separately.

First, we note that:

$$-\frac{1}{s_i}\left[\sum_\ell \sum_j \psi_{i\ell}\omega_{\ell j}s_j\frac{\partial \log(1+\theta_j)}{\partial \theta_k}\right] = -\frac{1}{v_i}\left[\sum_\ell \sum_j \psi_{i\ell}\omega_{\ell j}v_j\frac{\partial \log(1+\theta_j)}{\partial \theta_k}\right],$$

where we used $s_i = Ev_i$. For each i this can be written in the matrix notation as:

$$-\frac{1}{1+\theta_k}\mathbf{V}^{-1}[\mathbf{I}-\boldsymbol{\Omega}]^{-1}\boldsymbol{\Omega}\mathbf{V}\boldsymbol{e}_k = -\frac{1}{1+\theta_k}(\mathbf{I}-\mathbf{H})^{-1}\mathbf{H}\boldsymbol{e}_k,$$

since:

$$\mathbf{V}^{-1}(\mathbf{I}-\boldsymbol{\Omega})^{-1}\boldsymbol{\Omega}\mathbf{V}\boldsymbol{e}_k = \left(\mathbf{V}^{-1}\boldsymbol{\Omega}\mathbf{V} + \mathbf{V}^{-1}\boldsymbol{\Omega}\mathbf{V}\mathbf{V}^{-1}\boldsymbol{\Omega}\mathbf{V} + \ldots\right)\boldsymbol{e}_k = \left(\sum_{i=1}^{\infty}\mathbf{H}^i\right)\boldsymbol{e}_k = (\mathbf{I}-\mathbf{H})^{-1}\mathbf{H}\boldsymbol{e}_k.$$

The expression:

$$(1-\sigma)\frac{1}{s_i}\left[\sum_\ell \sum_j \psi_{i\ell}\omega_{\ell j}s_j\frac{\partial \log p_\ell}{\partial \theta_k}\right],$$

for each $i \in N$ can be written as:

$$(1-\sigma)\mathbf{V}^{-1}\left[\mathbf{I}-\mathbf{\Omega}\right]^{-1} diag(\mathbf{\Omega V 1})\frac{\partial \log \boldsymbol{p}}{\partial \theta_k}.$$

Finally, consider:

$$-(1-\sigma)\frac{1}{s_i}\left[\sum_{\ell}\sum_{j}\psi_{i\ell}\omega_{\ell j}s_j\sum_{r}\tilde{\omega}_{rj}\frac{\partial \log p_r}{\partial \theta_k}\right],$$

and note that we can write this expression (for each i) in matrix notation as:

$$-(1-\sigma)\mathbf{V}^{-1}\left[\mathbf{I}-\mathbf{\Omega}\right]^{-1}\mathbf{\Omega V}\tilde{\mathbf{\Omega}}'\frac{\partial \log \boldsymbol{p}}{\partial \theta_k}.$$

Putting everything together, we get:

$$\frac{\partial \log \boldsymbol{s}}{\partial \theta_k}=\frac{\partial \log E}{\partial \theta_k}-\frac{1}{1+\theta_k}[\mathbf{I}-\mathbf{H}]^{-1}\mathbf{H}\boldsymbol{e}_k+(1-\sigma)\mathbf{V}^{-1}\left[\mathbf{I}-\mathbf{\Omega}\right]^{-1}\left[diag(\mathbf{\Omega V 1})-\mathbf{\Omega V}\tilde{\mathbf{\Omega}}'\right]\frac{\partial \log \boldsymbol{p}}{\partial \theta_k}. \tag{31}$$

Plugging in the expression for $\frac{\partial \log \boldsymbol{p}}{\partial \theta_k}$ (from Lemma 4) in (31) gives:

$$\begin{aligned}\frac{\partial \log \boldsymbol{s}}{\partial \theta_k}&=\frac{\partial \log E}{\partial \theta_k}-\frac{1}{1+\theta_k}[\mathbf{I}-\mathbf{H}]^{-1}\mathbf{H}\boldsymbol{e}_k+\\(1-\sigma)\mathbf{V}^{-1}\left[\mathbf{I}-\mathbf{\Omega}\right]^{-1}&\left[diag(\mathbf{\Omega V 1})-\mathbf{\Omega V}\tilde{\mathbf{\Omega}}'\right]\left[\mathbf{I}-\mathbf{A}\tilde{\mathbf{\Omega}}'\right]^{-1}\left[\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta}+\frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho}+\frac{1}{1+\theta_k}\boldsymbol{e}_k\right].\end{aligned} \tag{32}$$

Evaluating in the steady state ($\tilde{\omega}_{ij}=g_{ij}$, $\omega_{ij}=\frac{\alpha_j}{\mu_j}g_{ij}$, and $\boldsymbol{\theta}=\mathbf{0}$) the previous expression becomes:

$$\begin{aligned}\frac{\partial \log \boldsymbol{s}}{\partial \theta_k}-\frac{\partial \log E}{\partial \theta_k}&=-[\mathbf{I}-\mathbf{V}^{-1}\mathbf{GAMV}]^{-1}\mathbf{V}^{-1}\mathbf{GAMV}\boldsymbol{e}_k+\\(1-\sigma)\mathbf{V}^{-1}\left[\mathbf{I}-\mathbf{GAM}\right]^{-1}&\left[diag(\mathbf{GAMV1})-\mathbf{GAMVG}'\right]\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}\left[\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta}+\frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho}+\boldsymbol{e}_k\right].\end{aligned}$$

Let us now define $\mathbf{\Lambda}\equiv\mathbf{V}^{-1}\left[\mathbf{I}-\mathbf{GAM}\right]^{-1}\left[diag(\mathbf{GAMV1})-\mathbf{GAMVG}'\right]\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}$.

We can write (evaluating at the steady state):

$$\frac{\partial \log \boldsymbol{s}}{\partial \theta_k}-\frac{\partial \log E}{\partial \theta_k}=-[\mathbf{I}-\mathbf{V}^{-1}\mathbf{GAMV}]^{-1}\mathbf{V}^{-1}\mathbf{GAMV}\boldsymbol{e}_k+(1-\sigma)\mathbf{\Lambda}\left[\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta}+\frac{\partial \log r}{\partial \theta_k}+\boldsymbol{e}_k\right].$$

In the special case $\alpha_i=\alpha$ and $\beta_i=\beta$ for all $i\in N$ we get:

$$\frac{\partial \log \boldsymbol{s}}{\partial \theta_k}-\frac{\partial \log E}{\partial \theta_k}=-\alpha[\mathbf{I}-\alpha\mathbf{V}^{-1}\mathbf{GMV}]^{-1}\mathbf{V}^{-1}\mathbf{GMV}\boldsymbol{e}_k+(1-\sigma)\mathbf{\Lambda}\boldsymbol{e}_k,$$

where we used the fact that in the symmetric case $(\mathbf{I}-\alpha\mathbf{G}')^{-1}\mathbf{1}=\frac{1}{1-\alpha}\mathbf{1}$, and $\left[diag(\mathbf{GMV1})-\mathbf{GMVG}'\right]\mathbf{1}=\mathbf{0}$.

□

Auxiliary result

To obtain (10) from (9) we use the following result.

Lemma 5. *In steady state:*

$$\mathbf{V}^{-1}[\mathbf{I}-\mathbf{GAM}]^{-1}\left[diag(\mathbf{GAMV1})-\mathbf{GAMVG}'\right]\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}=[\mathbf{I}-\mathbf{H}]^{-1}\left[diag(\mathbf{H1})-\mathbf{HG}'\right]\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}.$$

Proof.

$$\begin{aligned}
&\mathbf{V}^{-1}[\mathbf{I}-\mathbf{GAM}]^{-1}\left[diag(\mathbf{GAMV1})-\mathbf{GAMVG}'\right]\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}=\\
&\mathbf{V}^{-1}[\mathbf{I}-\mathbf{GAM}]^{-1}\mathbf{VV}^{-1}\left[\mathbf{V}diag(\mathbf{V}^{-1}\mathbf{GAMV1})-\mathbf{VV}^{-1}\mathbf{GAMVG}'\right]\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}=\\
&[\mathbf{I}-\mathbf{H}]^{-1}\left[diag(\mathbf{H1})-\mathbf{HG}'\right]\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}.
\end{aligned}$$

□

Aggregation

We now examine the effect of the shocks on the real GDP. We consider the case with homogeneous α, β and ρ. We follow Baqaee and Farhi (2019) and choose as a numeraire the nominal GDP, therefore normalizing $E=\bar{p}c=\sum p_i c_i=1$, where $\bar{p}\equiv\sum_{i\in N}\left(\frac{p_i}{\gamma_i}\right)^{\gamma_i}$ is the consumer price index, and c is the aggregate production of the consumption good as defined in Section 5.3, which is equal to the real GDP in our model. Given the chosen normalization, we can write:

$$\frac{\partial\log c}{\partial\theta_k}=-\frac{\partial\log\bar{p}}{\partial\theta_k}=-\sum_{i\in N}\gamma_i\frac{\partial\log p_i}{\partial\theta_k}, \tag{33}$$

and consequently:

$$d\log c=-\sum_{i=1}^{n}\gamma_i d\log p_i. \tag{34}$$

Proposition 3. *The first-order approximation of the effects of financial shocks on GDP is given by:*

$$\mathrm{d}\log c=-\boldsymbol{\gamma}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{\theta}-\frac{\beta}{1-\alpha}\mathrm{d}\log w-\frac{\rho}{1-\alpha}\mathrm{d}\log r, \tag{35}$$

where

$$\mathrm{d}\log w=\frac{\eta}{1+\eta}\frac{\beta}{wL}\boldsymbol{s}'\mathbf{M}\mathrm{d}\log\boldsymbol{s}-\frac{1-\delta}{1+\eta}\mathrm{d}\log c, \tag{36}$$

$$\mathrm{d}\log r=\frac{\rho}{rK}\boldsymbol{s}'\mathbf{M}\mathrm{d}\log\boldsymbol{s}, \tag{37}$$

and

$$\mathrm{d}\log\boldsymbol{s}=-[\mathbf{I}-\mathbf{H}]^{-1}\mathbf{H}\boldsymbol{\theta}+(1-\sigma)\boldsymbol{\Lambda\theta}.$$

Proof of Proposition 3. We first find the expression for $\frac{\partial\log c}{\partial\theta_k}$. From Lemma 4:

$$\frac{\partial\log\boldsymbol{p}}{\partial\theta_k}=\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{e}_k+\frac{\beta}{1-\alpha}\frac{\partial\log w}{\partial\theta_k}\mathbf{1}+\frac{\rho}{1-\alpha}\frac{\partial\log r}{\partial\theta_k}\mathbf{1},$$

from where we can write:

$$\frac{\partial\log c}{\partial\theta_k}=-\boldsymbol{\gamma}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{e}_k-\left(\frac{\beta}{1-\alpha}\frac{\partial\log w}{\partial\theta_k}+\frac{\rho}{1-\alpha}\frac{\partial\log r}{\partial\theta_k}\right). \tag{38}$$

Therefore,

$$d\log c = -\boldsymbol{\gamma}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{\theta} - \frac{\beta}{1-\alpha}\sum_{k\in N}\frac{\partial \log w}{\partial \theta_k}\theta_k - \frac{\rho}{1-\alpha}\sum_{k\in N}\frac{\partial \log r}{\partial \theta_k}\theta_k =$$
$$-\boldsymbol{\gamma}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{\theta} - \frac{\beta}{1-\alpha}\mathrm{d}\log w - \frac{\rho}{1-\alpha}\mathrm{d}\log r,$$

which delivers equation (35).[52]

We now show that equations (37) and (36) hold. Combining the firm's optimal demand for capital (23) and the market clearing condition for capital, we get:

$$rK = \rho\sum_{i=1}^{n}\frac{s_i}{\mu_i} = \rho\boldsymbol{s}'\mathbf{M}\mathbf{1},$$

where we recall that K is the aggregate (inelastic) supply of the physical capital. Similarly, for labor, we get:

$$wL = \beta\sum_{i=1}^{n}\frac{s_i}{\mu_i} = \beta\boldsymbol{s}'\mathbf{M}\mathbf{1}.$$

Therefore, we can write:

$$\frac{\partial \log r}{\partial \theta_k} = \frac{\partial \log(rK)}{\partial \theta_k} = \frac{\partial \log(rK)}{\partial (rK)}\frac{\partial (rK)}{\partial \theta_k} = \frac{\rho}{rK}\left[\sum_{i=1}^{n}\frac{s_i}{\mu_i}\frac{\partial \log s_i}{\partial \theta_k}\right] = \frac{\rho}{rK}\boldsymbol{s}'\mathbf{M}\frac{\partial \log \boldsymbol{s}}{\partial \theta_k},$$

from where we get (37). We recall that from Proposition 2 $\mathrm{d}\log \boldsymbol{s} = -\left[\mathbf{I}-\mathbf{H}\right]^{-1}\mathbf{H}\boldsymbol{\theta} + (1-\sigma)\boldsymbol{\Lambda}\boldsymbol{\theta}$.

As for $\frac{\partial \log w}{\partial \theta_k}$, from the market clearing condition for labor we have:

$$\frac{\partial \log w}{\partial \theta_k} = \frac{\beta}{wL}\boldsymbol{s}'\mathbf{M}\frac{\partial \log \boldsymbol{s}}{\partial \theta_k} - \frac{\partial \log L}{\partial \theta_k}.$$

To calculate $\frac{\partial \log L}{\partial \theta_k}$, we note that from the representative household's problem, we have:

$$\log L = \frac{1-\delta}{\eta}\log c + \frac{1}{\eta}\log w,$$

and therefore:

$$\frac{\partial \log L}{\partial \theta_k} = \frac{1-\delta}{\eta}\frac{\partial \log c}{\partial \log \theta_k} + \frac{1}{\eta}\frac{\partial \log w}{\partial \theta_k},$$

which in turn implies:

$$\frac{\partial \log w}{\partial \theta_k} = \frac{\eta}{1+\eta}\frac{\beta}{wL}\boldsymbol{s}'\mathbf{M}\frac{\partial \log \boldsymbol{s}}{\partial \theta_k} - \frac{1-\delta}{1+\eta}\frac{\partial \log c}{\partial \theta_k},$$

and therefore:

$$\mathrm{d}\log w = \frac{\eta}{1+\eta}\frac{\beta}{wL}\boldsymbol{s}'\mathbf{M}\mathrm{d}\log \boldsymbol{s} - \frac{1-\delta}{1+\eta}\mathrm{d}\log c,$$

which is exactly (36). This completes the proof.

□

[52] Equation (35) corresponds to a more general expression derived in equation (4) in Baqaee and Farhi (2019), labeled as *ex-ante effect of distortions.* While Baqaee and Farhi (2019) do not provide an explicit expression for the ex-ante effect of distortions on the GDP, we are able to do it (see Corollary 2) thanks to our parametric assumptions.

Corollary 2.

$$
\begin{aligned}
\mathrm{d}\log c = & -\left(1-\frac{1-\delta}{1+\eta}\frac{\beta}{1-\alpha}\right)^{-1}\boldsymbol{\gamma}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{\theta} \\
& -\frac{1}{1-\alpha}\left(1-\frac{1-\delta}{1+\eta}\frac{\beta}{1-\alpha}\right)^{-1}\left(\frac{\eta\beta^2}{(1+\eta)wL}+\frac{\rho^2}{rK}\right)\boldsymbol{s}'\mathbf{M}\left(-\left[\mathbf{I}-\mathbf{H}\right]^{-1}\mathbf{H}\boldsymbol{\theta}+(1-\sigma)\boldsymbol{\Lambda}\boldsymbol{\theta}\right)
\end{aligned} \tag{39}
$$

Proof. Follows directly from Proposition 3.

□

Corollary 3. *In the absence of the production network*

$$
\mathrm{d}\log c = -\left(1-\frac{1-\delta}{1+\eta}\beta\right)^{-1}\boldsymbol{\gamma}'\boldsymbol{\theta} \tag{40}
$$

Proof. In counterfactual without network, we set $\alpha=0$, which implies $[\mathbf{I}-\alpha\mathbf{G}']^{-1}=\mathbf{I}$. Moreover, $\alpha=0$ implies $\mathbf{H}=0$, which basically captures the idea that firms are selling only to the final consumer, and therefore $\mathrm{d}\log\boldsymbol{s}=0$. Hence, (39) becomes (40). □

Recovering markups

From Lemma 3, it follows that at the steady state: $\mu_i=\alpha_i\frac{\tilde{\omega}_{ji}}{\omega_{ji}}$. We observe $\tilde{\omega}_{ji}$ and ω_{ji} in our VAT data. For firms that have more than one supplier, we calculate μ_i as the average across all suppliers, as follows:

$$
\mu_i=\alpha_i\frac{1}{d_i^-}\sum_j\frac{\tilde{\omega}_{ji}}{\omega_{ji}},
$$

where d_i^- is the in-degree of firm i.