Dynamic Adverse Selection and the Supply Size

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January 11, 2016

Abstract

In this paper we examine the problem of dynamic adverse selection in a stylized market where the quality of goods is a seller's private information while the realized distribution of qualities is public information. We obtain that full trade occurs in every dynamic competitive equilibrium. Moreover, we show that if prices can be conditioned on the supply size then a dynamic competitive equilibrium always exists, while it fails to exist if prices cannot be conditioned on the supply size and the frequency of exchanges is high enough. We conclude that the possibility to condition prices on the supply size allows to reach efficiency in the limit for exchanges becoming more and more frequent, while otherwise the welfare loss due to delays of exchanges remains bounded away from zero.

JEL classification code: D82, L15.

Keywords: dynamic adverse selection; supply size; frequency of exchanges; asymmetric information.

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1 Introduction

Since the publication of the seminal work by Akerlof (1970), the problem of adverse selection has been widely investigated by economic theorists. One quite recent development in this regard is the exploration of the dynamics of exchanges under asymmetric information, and in particular of the phenomenon of dynamic adverse selection (see e.g., Hendel and Lizzeri, 1999; Janssen and Roy, 2002; Hendel et al., 2005; Moreno and Wooders, 2010, 2015).[1] Although several important aspects of dynamic adverse selection have been investigated, so far no attention has been given to the consequences of the public access to information about the supply size, e.g., the number of goods or services still on the market. In the present paper we explore this case, identifying the potential benefits accruing from the public availability of such a piece of information.

In a market where trade can take place sequentially, the public access to the supply size can solve dynamic adverse selection problems. The intuition behind this result is actually very simple and can be explained with a short example. Suppose that the supply side is constituted by two sellers, one who wants to sell one unit of a high quality good and the other who wants to sell one unit of a low quality good. Qualities are private information of the sellers but it is public knowledge that there is one high quality good and one low quality good. The supply size tells agents how many goods – but not which qualities – are still circulating in the market. Upon arrival at the market, buyers are told that two goods are being supplied. On the first day of the market, buyers are available to trade only at a low price, certain that for a low price they can only buy the low quality good. This choice is reasonable since a high price would lead to expected losses because of information asymmetries. Given a low price, the seller of the high quality good will not accept as for her accepting would mean a certain loss. What about the seller of the low quality good? She

[1] A clear and formal account of dynamic adverse selection can be found in Bolton and Dewatripont (2005, Ch. 9) where the consequences of multi-stage contracting are analyzed.

could opt to wait in the hope of higher prices in the following days. However, if she also refuses to trade, then the supply size does not change – two goods still circulate – and, hence, the following days buyers would be available to trade only at a low price. Therefore, the seller of the low quality good finds it optimal to accept a low price, as the alternative would be indefinite waiting. As a result, the low quality good is sold and the supply size shrinks by one unit. The second day of the market, upon disclosure of the fact that the supply size has diminished by one unit, buyers still on the market find it reasonable to trade at a high price as they know that the day before there were one high quality good and one low quality good, and that the previous low price could be reasonably accepted only by the low quality seller. Finally, since there is no good reason to expect a higher price in the future, the high quality seller accepts trading immediately and the market clears.

We note that the knowledge of the supply size allows to infer the exact quality of goods that remain unsold only if the realized initial distribution of qualities is common knowledge. This idea is reminiscent of the mechanism underlying the so-called "pac-man" conjecture for durable goods monopoly (Bagnoli et al., 1989) which has been opposed to the Coase conjecture, giving rise to a lively discussion on what exactly is the most reasonable prediction in durable goods markets with a monopolist (see von der Fehr and Kühn, 1995; Cason and Sharma, 2001). In short, the pac-man conjecture says that when consumers are not individually negligible, then the monopolist can make them pay their reservation prices (possibly discounted, depending on the discount factor and the distribution of reservation prices), i.e., the monopolist can discriminate prices and eat a bit of consumers' surplus in each trading stage. The intuition underlying the pac-man conjecture is similar to the one presented here in that the monopolist can condition his price offers on the number of consumers still on the market. This allows him to induce consumers with high reservation prices to buy in the first periods at a higher price. Relevant similarities however end here since, differently from the model in Bagnoli et al. (1989), we focus on adverse selection in competitive markets, and hence we consider the case of many buyers and asymmetric information about qualities. In particular, in our model buyers know the initial distribution of qualities brought to the market by sellers, but cannot say which seller has what quality.

We remark that the knowledge of the *supply* size is relevant in our adverse selection model because sellers are the ones who are privately informed. If it is buyers to be privately informed (e.g., buyers' willingness to pay is private information while quality is observable, as in Bagnoli et al., 1989) then the relevant piece of information is not the supply size but the *demand* size. This is so because the piece of information that is relevant for mitigating the negative effects of asymmetric information is, in general, the size of the *informed side* of the market.

Even if we restrict to cases where the realized initial distribution of qualities is common knowledge, the practical relevance of our results as a solution to dynamic adverse selection problems rests on the possibility that prices can be conditioned on the supply size. At least in some circumstances, the supply size can be quite a simple piece of information to retrieve, e.g., when the market is small and all participants can directly observe goods. When markets are large, instead, we may think of an active role by a dedicated authority, that may gather all the relevant information and disclose it to the public. This suggests that our findings have perhaps a normative insight.

Another important issue is whether the public knowledge of the supply size is sufficient to let agents enjoy the whole potential surplus from exchanges, as would happen in the absence of asymmetric information on the quality of goods. This desirable outcome in general is not guaranteed because sequential trade may take a substantial amount of time and, hence, agents might be forced to wait long periods before enjoying their payoff which, reasonably, would be discounted accordingly. However, since the amount of time waited before exchanges take place has no special role in ensuring full trade, there is one straightforward way to increase total surplus: shortening the time between exchange opportunities. This entails that, if exchange opportunities are frequent enough, then agents roughly enjoy the whole potential surplus from exchanges.

The remaining of the paper can be summarized as follows. In section 2 we present the model; in section 3 we give our main results and we briefly comment on them; in section 4 we provide a discussion of crucial assumptions; in section 5 we briefly summarize our contribution, we stress its relevance for welfare objectives, and we sketch some directions for future research; in section 6 we relate the present paper to the existing literature on adverse selection.

2 The model

We consider a competitive market where n sellers offer their goods to m buyers, with $m > n$. Each seller comes to the market with one good of quality $q \in \{L, H\}$, where L denotes low quality and H high quality. The quality of a good is a private information of its seller, while the initial number of goods of quality L, denoted with n_L, and the initial number of goods of quality H, denoted with $n_H = n - n_L$, are public information. Exchanges take place at consecutive trading stages, and buyers and sellers stay on the market until they complete a transaction.

Sellers are homogeneous apart from the quality of the good possessed. Goods are durable, and provide a stream of services over time, whose present values for a seller are, respectively, s_L for a low quality good and s_H for a high quality good. Buyers, too, are homogeneous, with b_L and b_H denoting present values for a buyer. We assume $s_H > s_L$ and $b_H > b_L$, so that a high quality good is more valuable than a low quality good for sellers and buyers. We also assume $b_L > s_L$ and $b_H > s_H$, which means that both qualities can potentially lead to a profitable exchange for the two trading parties. Furthermore, we assume that $s_H > (n_L b_L + n_H b_H)/n$, that is we restrict our analysis to cases of *proper adverse selection*: a price equal to the expected value for a buyer of a good in the market is not enough to let sellers trade high quality goods.

Goods are traded over time at trading stages that are of length Δ. We use $t \in \{0, \dots, T\}$ to denote a generic trading stage, with trading stage t being $t\Delta$ from the initial stage. If we denote with δ the discount factor for one unit of time, with $0 < \delta < 1$, then the discount factor between two consecutive stages is δ^{Δ}. We define the *frequency of trading stages* as $1/\Delta$.

When a transaction is completed so that a good of quality $q \in \{L, H\}$ is exchanged at stage t for a price p, the seller of such good earns a payoff equal to $\delta^{t\Delta}(p - s_q)$, and the buyer of the good earns a payoff equal to $\delta^{t\Delta}(b_q - p)$. In case no transaction occurs, zero payoffs are earned.

The definition of a dynamic competitive equilibrium is based on Moreno and Wooders (2015, section 3 of the supplementary appendix), in that we derive supplied and demanded quantities over time as the result of individual optimal behavior, and we then require market clearing at each stage and rational expectations over the relevant variables. In addition, reasonable constraints are imposed on expectations for stages at which no goods are traded. The main novelty that we introduce concerns the price setting. In particular, we consider a *price mechanism* that is a function $\pi : \{0, \dots, T\} \times \{0, \dots, n\} \to \mathbb{R}_+$, which takes as arguments a stage t and the number of goods g that are still on the market at t, and gives as output a price $\pi(t, g)$ at which exchanges can take place at time t. We say that the price mechanism is *unconditional on the supply size* if $\pi(t, g) = \pi(t, g')$ for every $0 \leq t \leq T$, $g \geq 1$ and $g' \geq 1$. Otherwise, we say that the price mechanism is *conditional on the supply size*. Conventionally, we set $\pi(t, g) = 0$ if $g = 0$.

Market supply and market demand are described by the triple $\mathcal{M} = (S_H, S_L, D)$, where $S_q = (S_q^0, \dots, S_q^T)$ for $q \in \{L, H\}$ is a sequence of quantities of goods of quality q supplied over time, and $D = (D^0, \dots, D^T)$ is a sequence of quantities demanded over time.

Expectations held by agents over the relevant variables are collected in $\mathcal{E} = (p_L, p_H, p_B, q_B)$, where $p_q = (p_q^0, \ldots, p_q^T)$ for $q \in \{L, H\}$ is a sequence of prices expected over time by a seller of a good of quality q, while $p_B = (p_B^0, \ldots, p_B^T)$ and $q_B = (q_B^0, \ldots, q_B^T)$ are the sequences of prices and qualities, respectively, expected over time by a buyer. We stress that, since these expectations are used to assess the optimality of an agent's choice, they must be considered as the price (and quality, in case of a buyer) that the agent expects over trading stages in the presupposition that she is still on the market. This remark will turn out to be relevant when interpreting minimal constraints on expectations (conditions ME1-2 described in the following).

We now list and discuss briefly the conditions for a triple $(\pi, \mathcal{M}, \mathcal{E})$ to be a dynamic competitive equilibrium. We start by considering conditions S1-3 on the optimality of sellers' supply. Condition S1 requires feasibility and symmetry of sellers' choices, i.e., all sellers of goods of the same quality supply at the same trading stages. Under such restriction, it is useful to denote with τ_L and τ_H the stage at which goods of low quality and high quality, respectively, are supplied; also, we conventionally set $\tau_q = \infty$ if goods of quality q are never supplied.[2] Condition S2 states that, if some positive quantity is supplied at stage t, then supplying at stage t must be optimal. Similarly, condition S3 states that if some good is not supplied, then not supplying at all must be optimal.

S1. $S_q^t \in \{0, n_q\}$ for every t, and $\sum_{t=1}^{T} S_q^t \leq n_q$ for $q \in \{L, H\}$;

S2. $S_q^t > 0$ implies $\delta^{t\Delta}(p_q^t - s_q) \geq \delta^{t'\Delta}(p_q^{t'} - s_q)$, for all t' and $\delta^{t\Delta}(p_q^t - s_q) \geq 0$;

S3. $\sum_{t=0}^{T} S_q^t < n_q$ implies $\delta^{t\Delta}(p_q^t - s_q) \leq 0$, for all t.

We now come to conditions concerning the demand side. Condition D1 is feasibility. Condition D2 requires that, if some positive quantity is demanded at stage t, then demanding

[2]Allowing sellers to supply at some $\tau_q < \infty$ with a positive probability (possibly smaller than one) would not sensibly affect our results (see the discussions on the price conditional mechanism) but would greatly complicate the analysis.

at stage t must be optimal. Similarly, condition D3 requires that, if some buyer's demand is null at every stage, then never demanding must be optimal.

D1. $\sum_{t=0}^{T} D^t \leq m$;

D2. $D^t > 0$ implies $\delta^{t\Delta}(q_B^t - p_B^t) \geq \delta^{t'\Delta}(q_B^{t'} - p_B^{t'})$, for all t' and $\delta^{t\Delta}(q_B^t - p_B^t) \geq 0$;

D3. $\sum_{t=0}^{T} D^t < m$ implies $\delta^{t\Delta}(q_B^t - p_B^t) \leq 0$, for all t.

Condition C1 is dynamic market clearing: at every stage, the quantity demanded must be equal to the quantity supplied.

C1. $S_L^t + S_H^t = D^t$, for all t.

We now turn to conditions on expectations. In doing so, we restrict attention to profiles $(\pi, \mathcal{M}, \mathcal{E})$ where conditions S1-3, D1-3, and C1 hold. For such profiles, we denote the price set at time t with $p^t = \pi(t, n - \mathbb{I}(t \geq \tau_L + 1)n_L - \mathbb{I}(t \geq \tau_H + 1)n_H)$, where $\mathbb{I}$ is the indicator function that takes value 1 if the condition in its argument is true and 0 otherwise. Given this, we can introduce rational expectations with three conditions. The first, RE1, states that whenever trading occurs buyers' expectations about the average quality sold are correct. Condition RE2 states that sellers' expectations about prices are correct up to the trading stage τ_q where a seller of quality q is supposed to supply her good. Condition RE3 states that buyers' expectations are always correct (see hereafter in this section for a brief remark).

RE1. $S_L^t + S_H^t > 0$ implies $q_B^t = \frac{S_L^t b_L + S_H^t b_H}{S_L^t + S_H^t}$;

RE2. $p_q^t = p^t$ for all $t \leq \tau_q$;

RE3. $p_B^t = p^t$ for all t.

Finally, we provide reasonable constraints to expectations on prices and quality for stages at which no goods are traded. Condition ME1 incorporates a lower bound on the market size into a seller's expected price at trading stages where such seller is supposed to have already sold her good: at such stages, if the seller imagines herself to be still on the market, she must expect the price mechanism π to take as input a supply size that considers at least herself and all other sellers who, reasonably, cannot have sold their goods at prior expected prices, since always lower than their reservation values. Condition ME2 requires that buyers never expect quality to fall below the lowest quality still on the market if all goods of quality t are sold at stage τ_q, which is either b_L or b_H. Conventionally, the expected quality is set equal to zero at stages where all goods should already be sold.

ME1. $p_q^t = \pi(t, g_q^t)$, with $g_q^t \geq 1+(n_q-1)\mathbb{I}(p_q^k < s_q,\ \text{for all } k < t)+n_{q'}\mathbb{I}(p_q^k < s_{q'} \text{ for all } k < t)$, where $q \neq q'$;[3]

ME2. $S_L^t + S_H^t = 0$ implies $q_B^t \geq b_L$ if $t < \tau_L$, $q_B^t \geq b_H$ if $\tau_L < t < \tau_H$, and $q_B^t = 0$ if $t > \tau_L$ and $t > \tau_H$.

A triple $(\pi, \mathcal{M}, \mathcal{E})$ is a seller-symmetric *dynamic competitive equilibrium with a price mechanism (DCE-PM)* if conditions S1-3, D1-3, C1, RE1-3, and ME1-2 are all satisfied.

A brief remark on expectations can be worth doing. Both sellers and buyers take the price mechanism as given when evaluating the optimality of their choices. Still, a seller's decision not to supply at the stage where she is supposed to do so alters the future supply size and, hence, may alter prices at later stages; therefore, in our setup sellers can be considered to some degree as price-makers. Moreover, the way in which a seller's decisions alter prices depends on which stage she is supposed to supply her good (i.e., τ_L or τ_H). If $\tau_L \neq \tau_H$, this creates the possibility that the expected price at some stage is different across sellers depending on the quality of the good possessed; by exploiting this possibility, in Proposition 3 we are able to keep the optimal supplying stages for low quality and high quality goods separated one from the other, thus allowing full trade. Finally, we stress that, differently

[3] We note that g_q^t can be interpreted as the expectation held by a seller of quality q about the supply size at time t.

from sellers, buyers' decisions have no impact on the expected sequence of prices; therefore, in our setup buyers can be considered as fully price-takers. This can be understood as a consequence of the fact that buyers outnumber sellers: the competition on the buyers' side implies that all buyers obtain zero payoffs, hence they are indifferent between buying at τ_L, or τ_H, or not buying at all; therefore, we can imagine that, if a buyer chooses of not demanding when she is supposed to do so, she will be replaced by another buyer, as if the two buyers switched their decisions. We stress that, even if we suppose that this kind of switch does not take place, so that buyers as well have an impact on the evolution of prices, our results would still hold true. Indeed, the decision by a buyer of not demanding a good would change future market sizes, and hence prices, but it cannot lead to a positive expected payoff since at any future stage buyers will outnumber sellers in any case. Therefore, buyers would gain no benefit from the strategic use of the ability to affect prices.

3 Dynamic competitive equilibria with a price mechanism

We begin our formal investigation of competitive equilibria with Proposition 1, which provides rather stringent necessary conditions. In particular, goods of low quality must be exchanged in the first trading stage, and goods of high quality must be exchanged in the second trading stage. Moreover, selling prices are b_L and b_H, for goods of low quality and high quality, respectively. The intuition is as follows. Competition on the demand side – i.e., the fact that buyers outnumber sellers – implies that prices must be equal to buyers' expected valuation. Given that low quality goods and high quality goods cannot be sold together due to the assumption of proper adverse selection, the price for a low quality good cannot exceed b_L; hence low quality goods are immediately traded, thus avoiding any reduction of value due to time discounting. Once low quality goods have exited the market, minimal expectations raise the price to b_H, and since the price can never exceed that value, high quality goods are traded at next stage to avoid time discounting. We note that this result, which hinges on ME1-2, rules out the possibility of having a competitive equilibrium where some goods are never sold.

Proposition 1. *If $(\pi, \mathcal{M}, \mathcal{E})$ is a DCE-PM then low quality goods are sold at time 0 for a price equal to b_L, high quality goods are sold at time 1 for a price equal to b_H.*

Proposition 1 is silent on whether a competitive equilibrium actually exists. We answer this question by distinguishing the case where the price mechanism is unconditional and the case where it is conditional on the supply size. Proposition 2 shows that, when the price mechanism is unconditional on the supply size, a competitive equilibrium exists if and only if the frequency of trading stages is low enough. The reason for this result is the same as in Janssen and Roy (2002). Waiting for future prices is more costly for sellers of low quality goods, since low quality goods provide a flow of services per time that is lower than the flow provided by high quality goods. Therefore a kind of single crossing condition holds in our setting, potentially allowing separation between sellers of goods of different quality. However, and differently from Janssen and Roy (2002), in order for a separation outcome to constitute a competitive equilibrium, we must have that trading stages are sufficiently far in time. Indeed, once low quality goods have exited the market at stage 0, the condition of minimal expectations forces the expected quality to be high, so that in equilibrium high quality goods will be necessarily traded at stage 1 (as we know from Proposition 1). If stage 1 is close in time to stage 0, then sellers of low quality goods find it profitable to wait until stage 1, and no competitive equilibrium can exist. The failure to obtain a dynamic competitive equilibrium with full trade has already been noted by Moreno and Wooders (2015). However, while in their model the failure is originated by the market closing too early with respect to the amount of waiting time which is effective as a screening device, in our model there is an additional source of failure due to an excessively high frequency of trading stages. We stress that such a kind of failure only arises if minimal conditions are imposed to buyers' expectations on quality, determining that the price has to rise immediately after low quality goods have exited the market.

Proposition **2.** *Suppose the price mechanism is unconditional on the supply size. Then, a threshold* $1/\Delta^*$ *exists such that, if the frequency of trading stages is smaller than or equal to* $1/\Delta^*$*, a DCE-PM exists; otherwise, no DCE-PM exists.*

Proposition 2 is also a useful benchmark for the following Proposition 3, which shows that the existence of a competitive equilibrium can be obtained for any frequency of trading stages (hence, even when trading stages are very close in time one to the other), provided that we employ a price mechanism that is conditional on the supply size.

Proposition **3.** *If the price mechanism is conditional on the supply size, then a DCE-PM exists for any frequency of trading stages.*

A brief discussion contrasting Proposition 3 with Proposition 2 can help to appreciate the results. As we know from Proposition 1, a profile where low quality goods are sold in the first trading stage, and high quality goods are sold in the second trading stage, is the only candidate to be a dynamic competitive equilibrium. For such a profile to be actually a dynamic competitive equilibrium, we must have that sellers of low quality goods do not find profitable to wait until the second stage. This can be obtained, along the lines of Proposition 2, by setting the second stage sufficiently far in time from the first stage, and exploiting the waiting time as a screening device. However, if the price mechanism is conditional on the supply size, an additional possibility is given: the actual sequence of prices can depend on sellers' choices, so that waiting until the second stage for a seller of a low quality good prevents the price from rising. This is the case if the price mechanism is such that only if n_L goods are sold in the first stage, then the price will rise in the second stage to b_H.

To summarize, our results can be understood as the dynamic interplay of prices that are conditional on the supply size and expectations that satisfy minimal requirements for stages at which no goods are traded. Such requirements on expectations guarantee that full trade is obtained in any DCE-PM, but at the same time they may cause the inexistence of DCE-PM if the price mechanism is unconditional on the supply size: indeed, minimal expectations imply that future prices must be high, which makes strategic waiting particularly effective, and causes the inexistence of a DCE-PM in the case trading stages are too close in time (so that waiting does not allow separation of sellers' types). Instead, if the price mechanism is conditional on the supply size then existence of a DCE-PM is guaranteed: in such a case, a price mechanism can be constructed so that the price will not rise until n_L goods are left the market, which makes strategic waiting ineffective for a seller of a low quality good.

In addition to full trade, there are at least two other features of equilibria that are worth emphasizing. The first feature is the so-called *skimming property*: lower expected quality goods are sold earlier and at a lower price than higher quality goods. The skimming property is a quite common feature in dynamic models of trade under asymmetric information.[4] We note that the restriction to seller-symmetric profiles makes skimming very rapid: all low quality goods are sold at the first stage, all high quality goods are sold at the second stage. Indeed, if a profile is seller-symmetric, then the only possible case where goods of different qualities are sold at the same stage is when all goods are sold at the same stage; such a possibility is however not viable due to the assumption of proper adverse selection. If, instead, we consider profiles that are not seller-symmetric, then we cannot exclude cases in which a few units of low quality goods are sold together with a substantial number of high quality goods.[5]

[4] The property is usually stated in models of durable goods monopoly, see for instance Fudenberg et al. (1985) and Gul et al. (1986), but it can also be found in a literature closer to ours, see for instance Janssen and Roy (2002).

[5] We refer the interested reader to Bilancini and Boncinelli (2014), where we provide an algorithm that deals with such cases in a strategic price-setting, obtaining full trade equilibria by means of price offers that are conditional on the supply size. This suggests that, even if some minor results would be lost (for instance, exchanges can take place over more than two periods to allow the working of the mechanism), our

The second additional feature which characterizes equilibria is that all surplus goes to sellers. This depends crucially on the fact that there is competition among buyers whatever the number of goods still on market. If instead sellers outnumber buyers, then the competitive pressure on the supply size would lead buyers to earn positive payoffs, while full trade could reasonably be obtained in this case as well by means of prices that are conditional on the supply size; basically, a price mechanism like the one used in the proof of Proposition 3 would allow to punish deviations by preventing price increases, so that high quality goods cannot be (expected to be) traded at following stages.

4 Discussion

Our results rely on the knowledge of the initial distribution of qualities. This assumption is crucial for full trade to be obtained by means of prices that are conditional on the supply size: indeed, each seller must believe that from some point onward refusing to sell would not be followed by an increase in prices, so that her acceptance is crucial at some stage. Admittedly, in many real markets there is incomplete information about the initial distribution of qualities, and hence conditional pricing can fail to achieve full trade.

There are other real-world cases, however, where the assumption of public information of the initial distribution of qualities fits relatively well. Think for instance of artworks: there is one original piece of art, and a possibly large number of fake copies that are hardly distinguishable from the original. Outside the field of art, we can consider industrial limited series: a product is manufactured in a (known) limited number, but its market is flooded with a lot of counterfeited reproductions. Also, consider the case of faulty products: all the products that have been manufactured at a specific plant during a certain interval of time are known to be flawed; in the second-hand market, the exact number of such products is

main message remains true without the restriction to seller-symmetric profiles. The cost for eliminating this restriction would be paid in terms of a much more complicated analysis and exposition.

known by buyers.

Even when the information about the distribution of qualities is incomplete, there are some cases that can be handled with minor adjustments of our model. Suppose it is known that n_L goods are of low quality, but the actual quality of each of such goods is random: in particular, s_L is drawn with uniform probability in $[\underline{s}_L, \overline{s}_L]$, with buyers' valuation being βs_L, $\beta > 1$. Similarly, we have that s_H is drawn with uniform probability in $[\underline{s}_H, \overline{s}_H]$, and buyers' valuation is βs_H. It is easy to recognize that our results apply to this modified setting as well, provided that $\beta(\underline{s}_q + \overline{s}_q)/2 > \overline{s}_q$ for $q = L, H$, i.e., the expected value for buyers is larger than the maximum value for sellers.

Another case of incomplete information where conditional pricing allows full trade is when there are only two products for sale, and enough information is available to reconstruct the realized distribution of qualities. We denote the two products with 1 and 2, and we suppose that the sellers' value of product 1 is s_1, randomly drawn from the interval $[0, 1]$; similarly, the sellers' value of product 2 is s_2, randomly drawn from the interval $[0, 1]$. Crucially, we assume that there exists some function $f(s_1, s_2) = k$, that k is publicly observable, and that the previous equation determines an inverse relation between s_1 and s_2, as for instance in case the sum between s_1 and s_2 is known (i.e., $s_1 + s_2 = k$). Then, each seller is able to infer if her good is the lowest or highest quality and, under the assumption that the expected value for buyers is large enough, full trade can be reached with a price mechanism that is conditional on the supply size.

There are other assumptions in our model, besides the knowledge of the initial distribution of qualities, that are not easily relaxable. An important assumption is that when a good is sold to a buyer, such good is no longer traded in that market. This is crucial because the size of the supply size only allows to separate lowest qualities from the rest of the goods, implying that if the lowest quality goods do not leave the market once sold then higher quality goods will never be sold. Of course, this does not mean that re-sales cannot take place but requires, as for instance in Hendel et al. (2005), that buyers can at least distinguish goods already sold once from goods never sold before in that market.

Another relevant assumption is the perfect observability of the supply size. Indeed, as indicated by the analysis in Levine and Pesendorfer (1995), if the piece of information on which price offers are conditioned upon is not perfectly observable, then deviations from the full trade equilibrium can become strictly profitable as imperfect observability can hide opportunistic behavior.

There are many other details of our model which have a non-negligible role, but they can be regarded as less important for the gist of our results.

5 Conclusions

In this paper we have considered a simple model of a market where goods of different quality can be exchanged at consecutive trading stages, and quality is sellers' private information. In this setting we have shown that the public knowledge of the supply size can solve dynamic adverse selection problems. More precisely, we have employed the notion of dynamic competitive equilibrium, taken from Moreno and Wooders (2015), which we have conveniently adapted to our framework, and we have obtained the following results: full trade always occurs in dynamic competitive equilibria (Proposition 1); if prices cannot be conditioned on the supply size then a dynamic competitive equilibrium with full trade exists only if consecutive trading stages are sufficiently distant in time (Proposition 2); if instead prices can be conditioned on the supply size then a dynamic competitive equilibrium with full trade exists for any frequency of trading stages (Proposition 3).

By contrasting Proposition 2 and Proposition 3, we can derive potentially relevant implications for welfare objectives. Consider a public authority that regulates the functioning of the market under consideration. If the price mechanism is unconditional on the supply size, stage 1 must be sufficiently far in time from stage 0 to discourage low quality sellers from waiting. This however constraints the possibility of the public authority to achieve the maximum potential surplus from exchanges, which amounts to $n_L(b_L - s_L) + n_H(b_H - s_H)$, because waiting is costly. Despite full trade, we incur a surplus loss that cannot be reduced below a certain level. If instead the price mechanism is conditional on the supply size, stage 1 can be made as close as desired to stage 0, and full trade can be still achieved by virtue of Proposition 3. Therefore, a public authority can obtain a total surplus that is arbitrarily close to the maximum potential surplus from exchanges by setting stage 1 arbitrarily close in time to stage 0.

A part from some crucial assumptions that we have discussed in section 4, we believe that our results can be extended along different dimensions. One is about alternative market structures. While in this paper we have analyzed a price setting mechanism that is centralized, decentralized mechanisms can be considered as well. In the class of decentralized mechanisms, many different details might be considered: alternative bargaining schemes might be assumed (such as sellers making price offers or sellers and buyers alternating in making offers), price offers might not be a public information (we might use a matching model and only consider bilateral interactions), the relative number of buyers and sellers might be different or buyers might be willing to buy more than one good (for instance, we might consider a monopsonistic buyer). Each of these variants should be considered with care, some minor differences can emerge (like a different distribution of surplus), and some technical issues are likely to arise. Yet, we do not see any major obstacle preventing full trade to be achieved in such variants as long as the prices are conditional on the supply size. As a confirmation of this, we refer the interested reader to the working paper version of this article (Bilancini and Boncinelli, 2014), where a more decentralized mechanism is presented and analyzed: each buyer makes a take-it-or-leave-it offer, and sellers choose whether to sell or not at the highest price.[6]

[6] In Bilancini and Boncinelli (2014) an extension is also considered where the arrival of new sellers and buyers is allowed over time, and it is shown that if agents can be distinguished on the basis of their wave of

Another dimension that can be explored is agents' heterogeneity. Consider for instance the case in which buyers are heterogeneous in their valuations of goods. In such a case the competitive pressure on the buyers' side might decrease, possibly allowing some buyers to obtain a positive surplus. Moreover, as shown by Roy (2012), some issues regarding the optimal sorting of buyers over time can arise. However, full trade equilibria would not be ruled out unless we allow for some buyer type having a reservation price for some quality that is lower than a seller's reservation price. We might also let the discount factor vary across agents. This would have the intuitive consequence of creating differences in agents' valuation that evolve over time.

Finally, a potentially interesting variant might consider the case where goods are non-durable. Indeed, if the goods brought to the market are exhausted upon consumption, then some of the arrangements which can reduce the loss due to adverse selection, such as leasing and secondary markets, become ineffective (see Waldman, 2003, for a comprehensive discussion of the functioning of real markets for durable goods). In particular, delays in exchanges are intrinsically useless as screening devices since the seller's cost of delaying a sale is the same for high quality and low quality goods, because goods do not provide a stream of services over time. It is worth pointing out that the effectiveness of conditioning prices on the supply size does not rest crucially on goods being durable, although the possibility for sellers to consume their own non-durable goods can generate a reduction of the supply size which should be carefully considered.

6 Related literature

Dynamic adverse selection has recently been considered with an emphasis on the efficiency-enhancing role of multiple-stage contracting. In particular, it has been shown that if traders discount future payoffs then the delay of exchanges can fruitfully work as a screening device,

arrival, then there exists a weak perfect Bayesian equilibrium that leads to full trade.

potentially allowing market transactions that would never be made in the traditional static framework of Akerlof (1970). Janssen and Roy (2002) show that delaying exchanges of high quality goods can effectively screen qualities and lead to full trade in a Walrasian dynamic setup (see also Janssen and Karamychev, 2002; Janssen and Roy, 2004, for an analysis of exchange cycles). The screening mechanism is based on the fact that sellers with higher-quality goods have greater incentive to wait for higher prices. Differently from Janssen and Roy (2002), in our model both the realized distribution of qualities and the supply size are public information. Moreover, the price mechanism allows conditioning prices on the supply size. The relevance of these differences can be appreciated by considering the welfare loss due to the delay of exchanges as the discounting becomes arbitrarily small. In Janssen and Roy (2002) the welfare loss does not vanish since, as the discounting diminishes, the delay required for efficient sorting becomes consequently larger. Instead, as we discuss in section 5, in our model the welfare loss tends to disappear when the discount factor tends to unity.

A related body of literature has focused on the role of contract types (e.g., leasing rather than selling) and the interaction between new and used good markets.[7] Of particular interest here is the contribution by Hendel et al. (2005) who show that, if consumers observe the number of times a durable good has changed hand, then the combination of multiple secondary markets and endogenous assignment of new goods can completely eliminate the inefficiencies caused by asymmetric information. A common element between our paper and Hendel et al. (2005) is that both provide a solution to dynamic adverse selection problems that does not exploit the delay in exchanges as a screening device but the availability of extra information.

In recent years there has been an upsurge of interest in the relation between the overall

[7] Hendel and Lizzeri (1999) investigate both the role and the existence of markets for used durables under dynamic adverse selection. Hendel and Lizzeri (2002) study the role of leasing contracts in durable goods markets.

informational structure and the likelihood of market failures due to adverse selection.[8] Many contributions in this stream of literature shed light on the role of some aspect of the informational structures in adverse selection problems, but dynamic adverse selection is rarely considered. As shown by Hörner and Vieille (2009), a dynamic environment can raise new and specific informational issues. They compare the effects of public versus private price offers in a dynamic adverse selection model with information and payoff structures as in Akerlof (1970). Differently from our model, they consider a situation where there is a unique seller who bargains sequentially with potential buyers until agreement is reached, if ever. Interestingly, Hörner and Vieille (2009) find that trade always eventually occurs when offers are private – i.e., buyers cannot observe past offers made by other buyers – while bargaining often ends at an impasse when offers are public. This happens because, with public offers, buyers can compete intertemporally deterring each other from making offers that lead to trade.[9]

Further evidence of the importance of the observability of price offers is provided by Moreno and Wooders (2010) who show that in a dynamic model of decentralized trade all goods entering the market are sold at some stage, notwithstanding asymmetric information on goods quality (see also Blouin, 2003, for a full trade result under decentralized trade).

[8] Some basic facts are now established. Kessler (2001) considers a lemons market where the seller can be uninformed with some probability and shows that welfare is non-monotonic in the amount of information on qualities. Levin (2001) shows that greater information asymmetries can reduce the gains from trade, although better information on the uninformed side unambiguously facilitates trade when demand is downward sloping. Creane (2008) proves that, in a pooling equilibrium where a monopolist sells a product of unknown quality to a group of consumers, welfare can be locally decreasing in the fraction of informed consumers. Sarath (1996) considers the issue of information disclosure to market participants and shows that entrusting the choice of (unverifiable) public information quality to traders who benefit from such information leads to inefficiencies, while delegating the choice of information quality to an independent agent who cannot share trade profits results in efficient implementation.

[9] We refer the interested reader to section 6 of Bilancini and Boncinelli (2014) for a discussion about how similar issues also affect our results in a non-competitive setting.

Decentralization of exchanges is modeled with a random matching mechanism where uninformed buyers make a take-it-or-leave-it offer to the informed seller they are matched with. If the seller accepts, then exchange takes place and both agents exit the market. If the seller rejects the offer, then both agents remain in the market and are randomly matched again. In this setting full trade emerges thanks to the cost associated with the delay of exchanges that works as a screening device.[10] Moreno and Wooders (2015) further investigate decentralized trade, showing that it can perform better than centralized trade if the trade horizon is finite and the cost associated with the delay of exchanges is low enough.[11] They also explore the role of taxes and subsidies. Camargo and Lester (2012) study the impact of policies aimed at mitigating the lemon problem in decentralized asset markets, showing that insuring buyers from the risk of buying a lemon can have ambiguous effects.[12]

Another paper that deals with the role of public information in markets plagued by dynamic adverse selection is Daley and Green (2012), which investigates the market for financial assets under asymmetric information with the public disclosure, at each trading stage, of an information that affects the future value of the traded asset. Daley and Green (2012) show that, depending on beliefs, in equilibrium there can be immediate trade, no trade at all, or partial trade. The most important difference between our model and theirs regards, again, which pieces of information are public knowledge.

One recent contribution which is very much related to our paper is that by Boukouras and

[10] Moreno and Wooders (2010) refer to "frictions" as both the discounting of future gains and the possible delay in matching with a trading partner. In line with Janssen and Roy (2002), as frictions go to zero payoffs tend to the ones obtained in the single-period competitive equilibrium because, although traders become more patient, delay increases even more.

[11] The definition of competitive equilibrium in section 2 the present paper builds on the one given by Moreno and Wooders (2015) for *centrialized* trade.

[12] Decentralized trade in the form of random matching is also considered by Kultti et al. (2012) who study a dynamic adverse selection model where matching between sellers and buyers randomly generates a competitive situation that varies across different matchings.

Koufopoulos (2015), which focuses on the design of a static direct revelation mechanism in an economy where agents have private information about their types and where the realized distribution of types is public information. Importantly, their mechanism obtains both full trade and full surplus. Compared to their contribution, our analysis focuses on a more specific problem, i.e., adverse selection, and considers exchanges that take place over time, allowing for prices to be conditioned on the number of goods that are left on the market.

Acknowledgements

We are very much indebted to two anonymous referees and an associate editor, whose useful suggestions allowed us to make the paper more readable, delivering its message in a clearer and more concise way. We also thank Alessandro Lizzeri, Antonio Nicolò and Gabor Virag for their useful comments, which helped us to improve the paper. We also want to thank people participating in the session "Markets with Adverse Selection" of the 13th SAET conference at MINES ParisTech, as well as people attending the seminars held at the Department of Economics, University of Bologna, at the Department of Economics, University of Pisa, at the Department of Economics, University of Padova, and those providing insightful comments during the annual BO.MO.PA. and G.R.A.S.S. meetings hosted by RECent and the University of Modena and Reggio Emilia, and the EARIE 2012 hosted by L.U.I.S.S. University.

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A Proofs

A.1 Proof of Proposition 1

Preliminarily, we observe that, since buyers outnumber sellers, never buying must be optimal for buyers in a DCE-PM, which means that buyers' expected payoffs of buying are non-positive at any stage. Furthermore, if $\tau_q \neq \infty$, then buyers' expected payoffs of buying at τ_q must be equal to zero.

Suppose that $(\pi, \mathcal{M}, \mathcal{E})$ is a DCE-PM. We start by noting that low quality goods and high quality goods cannot be sold at the same trading stage. Indeed, if ad absurdum $\tau_L = \tau_H = t \neq \infty$, then buyers' rational expectations on quality imply $q_B^t = (n_L b_L + n_H b_H)/n$ (by RE1), and buyers' rational expectations on prices imply $p_B^t = p^t$ (by RE3); moreover, since buyers must obtain zero expected payoffs, we have that $q_B^t = p_B^t$, and hence $p^t = (n_L b_L + n_H b_H)/n$. This, together with the assumption of proper adverse selection (i.e., $s_H > (n_L b_L + n_H b_H)/n$), implies that $p^t < s_H$, which turns into $p_H^t < s_H$ by sellers' rational expectations on prices (RE2), against the optimality of selling at τ_H for a seller H (S2).

We now observe that we cannot have $p^0 = \pi(0, n) < b_L$, because otherwise buyers would obtain positive expected payoffs, $q_B^0 > p_B^0$. Indeed, $p_B^0 = p^0$ by buyers' rational expectations on prices (RE3), and $q_B^0 \geq b_L$ by buyers' rational and minimal expectations on quality (RE1 if $\tau_L = 0$ or $\tau_H = 0$, and ME2 if $\tau_L \neq 0 \neq \tau_H$).

Therefore we have $p^0 \geq b_L$. But then $\tau_L = 0$. In fact, if ad absurdum $\tau_L = t \geq 1$, then the conditions of buyers' rational expectations on prices and quality, together with the fact that $\tau_H \neq \tau_L$ (established in a previous paragraph), give $q_B^t = b_L$ (by RE1) and $p_B^t = p^t$ (by RE3); since buyers must obtain zero-expected payoffs, this implies $p^t = b_L$, which means that $p^0 - s_L > \delta^{t\Delta}(p^t - s_L)$ for any δ since $0 < \delta < 1$; by virtue of sellers' rational expectations on prices (RE2), this translates into $p_L^0 - s_L > \delta^{t\Delta}(p_L^t - s_L)$, meaning that selling at time t would be worse than selling at time 0 (against S2). If instead, again ad absurdum, $\tau_L = \infty$, we notice that $p^0 \geq b_L > s_L$; by virtue of sellers' rational expectations (RE2), this translates into $p_L^0 - s_L > 0$, violating the optimality of never accepting to sell (against S3). We also notice that $\tau_L = 0$ implies that $p^0 = b_L$, due to buyers' zero-expected payoffs and rational expectations on quality and prices (RE1 and RE3), and the fact that $\tau_H \neq \tau_L$.

We then observe that we cannot have $p^1 = \pi(1, n_H) < b_H$, because otherwise buyers would obtain positive expected payoffs, $q_B^1 > p_B^1$. Indeed, $p_B^1 = p^1$ by buyers' rational expectations on prices (RE3), and $q_B^1 \geq b_H$ by buyers' rational and minimal expectations on quality (RE1 if $\tau_H = 1$, and ME2 if $\tau_H \neq 1$).

Therefore we have $p^1 \geq b_H$. But then $\tau_H = 1$. In fact, if ad absurdum $\tau_H = t \geq 2$, then the conditions of buyers' rational expectations on prices and quality, together with the fact (established in a previous paragraph) that $\tau_H \neq \tau_L$, give $q_B^t = b_H$ (by RE1) and $p_B^t = p^t$ by (RE3); since buyers must obtain zero-expected payoffs, this implies $p^t = b_H$, which means that $\delta^\Delta(p^1 - s_H) > \delta^{t\Delta}(p^t - s_H)$ for any δ since $0 < \delta < 1$; by virtue of sellers' rational expectations on prices (RE2), this translates into $\delta^\Delta(p_L^1 - s_H) > \delta^{t\Delta}(p_L^t - s_H)$, meaning that selling at time t would be worse than selling at time 1 (against S2). If instead, again ad absurdum, $\tau_H = \infty$, we notice that $p^1 \geq b_H > s_H$; by virtue of sellers' rational expectations on prices (RE2), this translates into $p_H^1 - s_H > 0$, violating the optimality of never accepting to sell (against S3). We finally notice that $\tau_H = 1$ implies that $p^1 = b_H$, due to buyers' zero-expected payoffs and buyers' rational expectations on quality and prices (RE1 and RE3), and the fact that $\tau_H \neq \tau_L$.

A.2 Proof of Proposition 2

If the price mechanism is unconditional on the supply size, then Proposition 1 leaves as candidates for being a DCE-PM only profiles $(\pi, \mathcal{M}, \mathcal{E})$ such that $\pi(0, g) = b_L$ for every g, $\pi(1, g) = b_H$ for every g, $S_L = (n_L, 0, \ldots, 0)$, $S_H = (0, n_H, 0, \ldots, 0)$, and $D = (n_L, n_H, 0, \ldots, 0)$. We will now show that, among such candidate profiles, only a triple where $\pi(t, g) \leq b_H/\delta^{t\Delta}$ for all $g \geq 1$ and $t \geq 2$ (to discourage any seller to sell at $t \geq 2$) can be an equilibrium,[13] and it is actually an equilibrium if and only if Δ is smaller than or equal to some threshold $1/\Delta^*$ (to discourage L types to sell at $t = 1$). More precisely, we check conditions S1-3, D1-3 and

[13] We note that this condition is redundant if the price mechanism is constrained to set prices not greater than b_H.

C1, taking care that agents' expectations satisfy conditions RE1-3 and ME1-2.

We initially observe that $p^0 = b_L$, $p^1 = b_H$ and the market closes at the end of stage 1.

We first check the conditions about supply side. Condition S1 is clearly satisfied, and condition S3 is satisfied by false antecedent. We consider condition S2. We start from a seller of a good of quality H. By sellers' rational expectations (RE2) we have that $p^0_H = p^0$ and $p^1_H = p^1$. So, the optimal selling stage is either $t = 1$ or $t \geq 2$, since $\delta^\Delta(b_H - s_H) > 0 > b_L - s_H$, where the last inequality is implied by the assumption of proper adverse selection (i.e., $s_H > (n_L b_L + n_H b_H)/n$). We are free to set $\pi(t, g) \leq b_H/\delta^{t\Delta}$, for all $g \geq 1$ and $t \geq 2$, to obtain by ME1 that $\delta^\Delta(b_H - s_H) > \delta^{t\Delta}(\pi(t, g^t_H) - s_H)$, which implies that the optimal selling stage is $\tau_H = 1$.

We now deal with a seller of a good of quality L. By sellers' rational expectations (RE2) we have that $p^0_L = p_0$ and, in addition, we have that $p^1_L = p^1$ by sellers' minimal expectations (ME1). Since $b_L - s_L > 0$, never selling is not optimal, and selling at time $\tau_L = 0$ is better than selling at time 1 if and only if $\delta^\Delta \leq (b_L - s_L)/(b_H - s_L)$, which in terms of frequency of trading stages amounts to $1/\Delta \leq 1/\log_\delta((b_L - s_L)/(b_H - s_L)) = 1/\Delta^*$. Selling at $\tau_L = 0$ is also better than selling at time $t \geq 2$, by ME1, because we already set $\pi(t, g) \leq b_H/\delta^{t\Delta}$, for all $g \geq 1$ and $t \geq 2$.

We turn attention to conditions about demand side. Condition D1 is clearly satisfied. Since some buyers must find optimal not to buy, conditions D2 and D3 require that (i) buyers' expected payoff of buying at stage 0 and 1 must be both equal to zero, and (ii) buying at a later stages gives a non-positive payoff. To check (i), we have to show that $p^0_B = b^0_q$ and $p^1_B = b^1_q$; by virtue of buyers' rational expectations on quality and price (RE1 and RE3), this translates into $p^0 = b_L$ and $p^1 = b_H$, which is actually the case in the profile under consideration. To check (ii), we notice that we have $p^t_B = q^t_B$ also when $t > 1$; indeed, $p^t_B = p^t = \pi(t, 0) = 0$ due to RE3 and the definition of the price mechanism when $g = 0$, and $q^t_B = 0$ by ME2.

Finally, we simply observe that condition C1 is clearly satisfied.

A.3 Proof of Proposition 3

Consider a profile such that, for all t, $\pi(t,g) = b_L$ if $n_H + 1 \leq g \leq n$, and $\pi(t,g) = b_H$ if $1 \leq g \leq n_H$, and $S_L = (n_L, 0, \dots, 0)$, $S_H = (0, n_H, 0, \dots, 0)$, and $D = (n_L, n_H, 0, \dots, 0)$. We will show that conditions S1-3, D1-3, and C1 hold for such a profile, taking care that agents' expectations satisfy conditions RE1-3 and ME1-2.

We first check the conditions about supply side. Condition S1 is clearly satisfied, and condition S3 is satisfied by false antecedent. We consider condition S2. We start from a seller of a good of quality H. By sellers' rational expectations (RE2) we have that $p_H^0 = b_L$ and $p_H^1 = b_H$, while at any following stage $t \geq 2$ we have that $p_H^t \leq b_H$ (because of ME1). So, the optimal selling stage is $\tau_H = 1$, since $\delta^{\Delta}(b_H - s_H) > \delta^{t\Delta}(b_H - s_H) > 0 > b_L - s_H$, where the last inequality is implied by the assumption of proper adverse selection (i.e., $s_H > (n_L b_L + n_H b_H)/n$).

We now deal with a seller of a good of quality L. By sellers' rational expectations (RE2) we have that $p_L^0 = b_L$ and, in addition, we have that $p_L^t \leq b_L$ for any following stage $t \geq 1$ (because of ME1, considering that $g_L^t \geq n_H + 1$ for all $t \geq 1$). Since $b_L - s_L > \delta^{t\Delta}(b_L - s_L) > 0$, the optimal selling stage is $\tau_L = 0$.

The check for conditions D1-3 and C1 is exactly the same as in the proof of Proposition 2, to which we refer.