A micropolar model for the analysis of dispersive waves in chiral mass-in-mass lattices

A. Bacigalupo
University of Trento
andrea.bacigalupo@unitn.it

L. Gambarotta
University of Genoa
luigi.gambarotta@unige.it

ABSTRACT. The possibility of obtaining band gap structures in chiral auxetic lattices is here considered and applied to the case of inertial locally resonant structures. These periodic materials are modelled as beam-lattices made up of a periodic array of rigid rings, each one connected to the others through elastic slender ligaments. To obtain low-frequency stop bands, elastic circular resonating inclusions made up of masses located inside the rings and connected to them through an elastic surrounding interface are considered and modeled. The equations of motion are obtained for an equivalent homogenized micropolar continuum and the overall elastic moduli and the inertia terms are given for both the hexachiral and the tetrachiral lattice. The constitutive equation of the beam lattice given by the Authors [15] are then applied and a system of six equations of motion is obtained. The propagation of plane waves travelling along the direction of the lines connecting the ring centres of the lattice is analysed and the secular equation is derived, from which the dispersive functions may be obtained.

KEYWORDS. Auxetic materials; Chirality; Cellular materials; Mass-in-mass dynamic systems; Dispersive waves.

INTRODUCTION

In recent years a considerable interest in acoustic metamaterials was witnessed by several research (see for reference [1]), most of them focused to the design of artificial materials having periodic microstructure conceived to get complete sound attenuation for a certain frequency range, namely acoustic wave spectral gap. Sonic crystals with spectral gaps [2] have been developed on the realization that composites with locally resonant structural units may exhibit effective negative elastic constants at certain frequency ranges, as shown by Liu *et al.* [3] and by Huang *et al.* [4,5]. Recently, Bigoni *et al.* [6] proposed a periodic metamaterial with internal locally resonant structures that supports tunable low-frequency stop bands. This effect is associated with localized rotational modes obtained from a chiral microstructure of the periodic cell.

Tee *et al.* [7] have recently obtained a band gap structure in periodic tetrachiral materials. Spadoni *et al.* [8] have obtained analogous results with reference to hexachiral lattices. These kind of materials are auxetics [9] and their structure is

characterized by a periodic array of the rings connected by four or six elastic ligaments. Lakes [10], with reference to the planar isotropic case of hexachiral lattice, proposed this microstructure geometry firstly. Later Prall and Lakes [11] showed for this material a Poisson's ratio of -1 under the hypothesis of ignoring the axial strain of the ligaments. Further studies on the homogenization of hexachiral auxetic materials have been carried out by Spadoni and Ruzzene [12] and by Liu *et al.* [13]. Liu *et al.* [14] proposed a hexachiral metacomposite by integrating a two-dimensional hexachiral lattice with elastic resonating inclusions to obtain low-frequency band gap. This metacomposite has been analysed through a numerical model where the ligaments are modelled as multi-beam elements and the inclusions as a two dimensional FEM model.

In this paper, hexa- and tetra-chiral beam lattices are considered having local resonators at the nodes of the periodic array. The model is developed in closed-form and is based on a micropolar homogenization of the lattice. This approach partially relies on the results by Bacigalupo and Gambarotta [15], which developed and compared the results from the micropolar and a second displacement gradient homogenization for both the hexachiral and the tetrachiral periodic cells in order to evaluate the validity limits of the beam lattice model.

The possibility of obtaining band gap structures in chiral auxetic lattices is here considered and applied to the case of inertial locally resonant structures. These structures are obtained following Huang *et al.* [4] and Liu *et al.* [14], by the insertion of a circular mass connected through an elastic surrounding interface to each ring of the microstructure. The equations of motion are given within a micropolar continuum model and the overall elastic moduli and the inertia terms are obtained for both the hexachiral and the tetrachiral lattice. The constitutive equation of the beam lattice given in [15] are then applied and a system of six equations of motion is obtained. The propagation of plane waves travelling along the direction of the lines connecting the ring centres of the lattice is analysed and the secular equation is derived, from which the dispersive functions may be obtained.
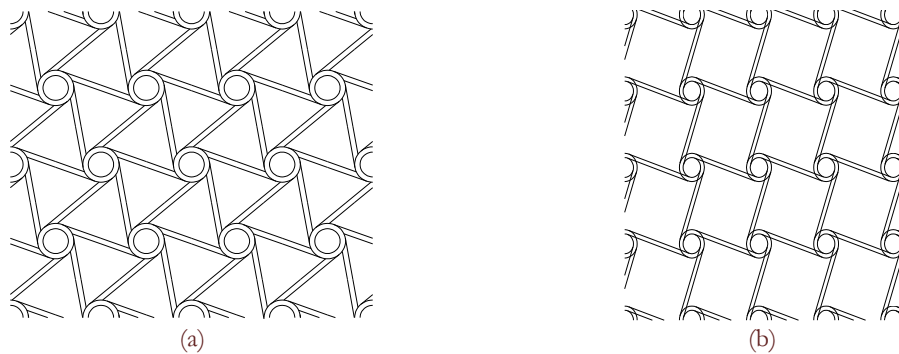

Figure 1: (a) Hexachiral lattice; (b) tetrachiral lattice.

CHIRAL MASS-IN-MASS PERIODIC MATERIAL: MICROPOLAR HOMOGENIZATION

The periodic materials shown in Figure 1 are considered as beam-lattices made up of a periodic array of rigid rings, each one connected to the others through *n* elastic slender ligaments rigidly connected to the rings. In Figure 2.a the periodic hexachiral cell (*n*=6) is shown, while in Figure 1.b is shown the tetrachiral cell (*n*=4). Each ligament is tangent to the joined rigid rings and has length *l* measured between the connection points, with section width t, thickness d and Young's modulus E_s. Both the ligaments and the rings have mass density ρ_s, so that the mass and the rotation inertia of the rings are $M_1 = 2\pi\rho_s r t$ and $J_1 = M_1 r^2$, respectively. The two dimensional composite materials are auxetic. The hexachiral lattice is isotropic and its Poisson's ratio becomes negative when increasing the chirality angle β. On the other hand, the tetrachiral material is strongly anisotropic and presents a directional dependency of the chirality on the direction of the applied stress, as shown in [15].

To obtain low-frequency stop bands, a metamaterial inclusion consisting of a softly heavy disk is located inside the ring as shown in Figure 3.a. This inclusion plays the role of low-frequency resonator with mass and rotation inertia denoted with M_2 and J_2, respectively. The motion of the rigid ring of the beam lattice is denoted by the displacement vector **u** and the rotation ϕ, respectively (see Figure 3.b), while the motion of the mass of the resonator is denoted by the displacement vector **v** and the rotation θ (see Figure 3.c).
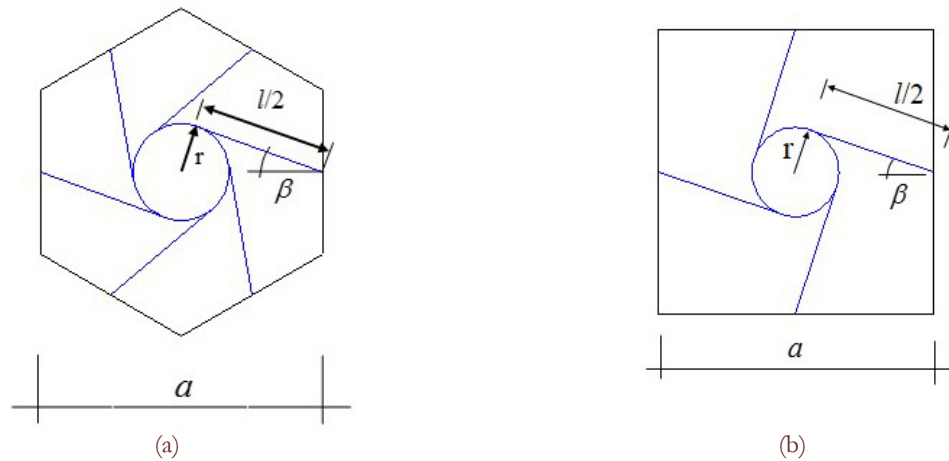
Figure 2: Periodic cell of the (a) hexachiral lattice; (b) tetrachiral lattice.
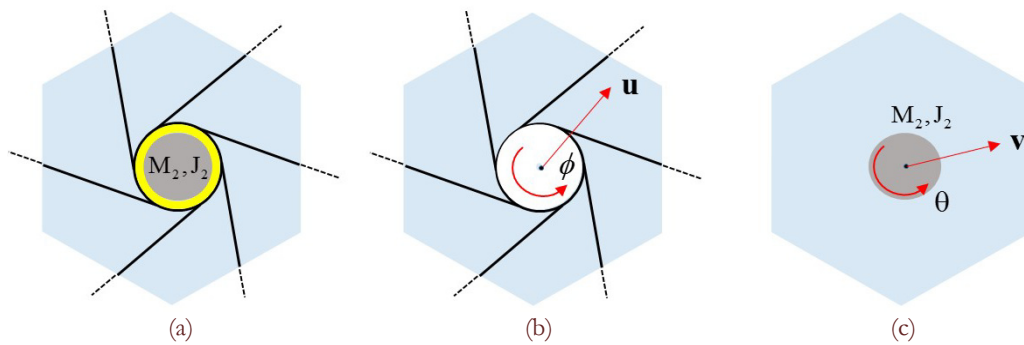

Figure 3: (a) Internal mass with external elastic thick layer; (b) rigid ring and related dofs; (c) internal mass and related dofs.

A soft elastic interface connects the internal mass to the rigid ring. To get a simplified formulation, the constitutive equation of the interface is assumed in the form

$$\mathbf{f} = -k_d(\mathbf{v} - \mathbf{u}), \quad c = -k_\theta(\theta - \varphi) \tag{1}$$

$\mathbf{f}$ being the force exerted by the rigid ring on the internal mass and c the corresponding couple (see Figure 4). The parameters k_d and k_θ are the isotropic translational stiffness and the rotational stiffness, respectively.
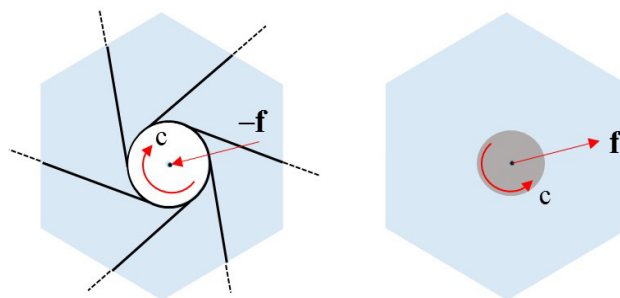

Figure 4: Contact force and couple between the rigid ring and the internal mass.

The lattice model is here approximated as a micropolar continuum model resulting from a homogenization process based on the macro-homogeneity criterion, involving both the total potential energy and the kinetic energy through the Hamilton principle (see [13, 16] for reference). The equations of motion for the homogenized beam-lattice are

$$
\begin{cases}
\sigma_{11,1} + \sigma_{12,2} + \hat{k}_d (v_1 - u_1) = \rho_1 \ddot{u}_1 \\
\sigma_{21,1} + \sigma_{22,2} + \hat{k}_d (v_2 - u_2) = \rho_1 \ddot{u}_2 \\
m_{1,1} + m_{2,2} + \sigma_{21} - \sigma_{12} + \hat{k}_\theta (\theta - \phi) = I_1 \ddot{\phi} \\
\hat{k}_d (u_1 - v_1) = \rho_2 \ddot{v}_1 \\
\hat{k}_d (u_2 - v_2) = \rho_2 \ddot{v}_2 \\
\hat{k}_\theta (\phi - \theta) = I_2 \ddot{\theta}
\end{cases}
\tag{2}
$$

where the stiffnesses $\hat{k}_d = k_d / A_{cell}$ and $\hat{k}_\theta = k_\theta / A_{cell}$ are introduced, A_{cell} being the area of the periodic cell, together with the mass densities $\rho_1 = \dfrac{M_1}{A_{cell}}$ and $\rho_2 = \dfrac{M_2}{A_{cell}}$, and the micro-inertia terms $I_1 = \dfrac{J_1}{A_{cell}}$ and $I_2 = \dfrac{J_2}{A_{cell}}$, respectively. In Eq. (2) σ_{ij} and m_i are the overall stress components, namely the asymmetric stress components and the micro-couples, respectively. In case of hexachiral lattice, one obtains $A_{cell} = 2\sqrt{3} l^2 / \cos^2 \beta$ and the following inertial parameters are obtained

$$
\rho_1 = \frac{\sqrt{3}\pi}{24} \rho_s \delta \sin(2\beta)
$$

$$
I_1 = \frac{\sqrt{3}\pi}{24} \rho_s \delta l^2 \tan\beta \sin^2\beta
$$

$$
\rho_2 = \frac{M_2}{A_{cell}} = \frac{\sqrt{3}\pi}{24} \rho_i \sin^2\beta
$$

$$
I_2 = \frac{J_2}{A_{cell}} = \frac{\sqrt{3}\pi}{192} \rho_i l^2 \sin^2\beta \tan^2\beta
$$

For the tetrachiral lattice one obtains $A_{cell} = l^2 / \cos^2 \beta$ and the resulting inertia parameters are $\rho_1 = \dfrac{\pi}{24} \rho_s \delta \sin(2\beta)$, $I_1 = \dfrac{\pi}{48} \rho_s \delta l^2 \sin^2\beta$, $\rho_2 = \dfrac{M_2}{A_{cell}} = \dfrac{\pi}{48} \rho_i \sin^2\beta$, $I_2 = \dfrac{J_2}{A_{cell}} = \dfrac{\pi}{192} \rho_i l^2 \sin^2\beta \tan^2\beta$.

To obtain the displacement formulation of the equations of motion, the compatibility equations involving the macrostrain components $\gamma_{11} = u_{1,1}$, $\gamma_{22} = u_{2,2}$, $\gamma_{12} = u_{1,2} + \phi$, $\gamma_{21} = u_{2,1} - \phi$ and the curvatures $\chi_1 = \phi_{,1}$ and $\chi_2 = \phi_{,2}$ have to be considered together with the constitutive equation.

MICROPOLAR CONSTITUTIVE EQUATION FOR HEXA- AND TETRA-CHIRAL LATTICES

The constitutive equation of hexachiral honeycomb corresponds to that obtained in [13, 15] and is written as follows

$$
\begin{Bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{12} \\
\sigma_{21} \\
m_1 \\
m_2
\end{Bmatrix}
=
\begin{bmatrix}
2\mu+\lambda & \lambda & -A & A & 0 & 0 \\
\lambda & 2\mu+\lambda & -A & A & 0 & 0 \\
-A & -A & \mu+k & \mu-k & 0 & 0 \\
A & A & \mu-k & \mu+k & 0 & 0 \\
0 & 0 & 0 & 0 & S & 0 \\
0 & 0 & 0 & 0 & 0 & S
\end{bmatrix}
\begin{Bmatrix}
\gamma_{11} \\
\gamma_{22} \\
\gamma_{12} \\
\gamma_{21} \\
\chi_1 \\
\chi_2
\end{Bmatrix}
\tag{3}
$$

in which the five elastic moduli

$$\mu = \frac{\sqrt{3}}{4} E_s \delta \left(1 + \delta^2\right)$$

$$\lambda = \frac{\sqrt{3}}{4} E_s \delta \left(1 - \delta^2\right)\left(\cos^2\beta - \sin^2\beta\right)$$

$$k = \frac{\sqrt{3}}{2} E_s \delta \left(\sin^2\beta + \delta^2 \cos^2\beta\right) \tag{4}$$

$$S = \frac{\sqrt{3}}{12} E_s \delta a^2 \left(3\sin^2\beta + 4\delta^2 \cos^2\beta\right)$$

$$A = -\frac{\sqrt{3}}{2} E_s \delta \left(1 - \delta^2\right)\sin\beta\cos\beta$$

depend on the Young's modulus, the slenderness ratio $\delta = t/l$ and the angle β of inclination of the ligaments. The constitutive Eq. (3) show the coupling between the extensional strains γ_{11} and γ_{22} and the asymmetric strains γ_{12} and γ_{21} through the elastic constant A. The elastic moduli, with the exception of parameter μ, depend on the parameter of chirality β, but only the constant A is an odd function of this parameter, i.e. it reverses its sign when the handedness of the material pattern is flipped over. In case of symmetric macro-strain fields, the fourth order elastic tensor for the hexagonal system corresponds to that of the transversely isotropic system whose elastic moduli in the plane of the lattice are:

$$E_{\text{hom}} = \frac{2\sqrt{3}E_s\delta^3\left(1 + \delta^2\right)}{\left(3\delta^2 + \delta^4\cos^2\beta + \sin^2\beta\right)}$$

$$\nu_{\text{hom}} = \frac{\left(\delta^2\cos^2\beta - \sin^2\beta\right)\left(1 - \delta^2\right)}{3\delta^2 + \delta^4\cos^2\beta + \sin^2\beta} \tag{5}$$

$$G_{\text{hom}} = \frac{\sqrt{3}}{4} E_s\delta\left(1 + \delta^2\right)$$

For the tetrachiral lattice, the constitutive equation is written [15]:

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \\ \sigma_{21} \\ m_1 \\ m_2 \end{Bmatrix} = \begin{bmatrix} 2\mu & 0 & 0 & B & 0 & 0 \\ 0 & 2\mu & -B & 0 & 0 & 0 \\ 0 & -B & \kappa & 0 & 0 & 0 \\ B & 0 & 0 & \kappa & 0 & 0 \\ 0 & 0 & 0 & 0 & S & 0 \\ 0 & 0 & 0 & 0 & 0 & S \end{bmatrix} \begin{Bmatrix} \gamma_{11} \\ \gamma_{22} \\ \gamma_{12} \\ \gamma_{21} \\ \chi_1 \\ \chi_2 \end{Bmatrix} \tag{6}$$

where four elastic moduli are related to the lattice parameters as follows

$$\mu = \frac{E_s\delta}{2}\left(\cos^2\beta + \delta^2\sin^2\beta\right)$$

$$k = E_s\delta\left(\sin^2\beta + \delta^2\cos^2\beta\right)$$

$$B = -E_s\delta\left(1 - \delta^2\right)\sin\beta\cos\beta \tag{7}$$

$$S = \frac{1}{12}E_s\delta a^2\left(3\sin^2\beta + 4\delta^2\cos^2\beta\right)$$

Similarly to the hexachiral honeycomb, a coupling is obtained between the extensional strains and the asymmetric strains through the elastic modulus B which is an odd function of the parameter of chirality β, while the other elastic moduli are even functions. In case of symmetric macro-strain fields, the resulting classical fourth order elasticity tensor has the elastic

moduli of the tetragonal system. The elasticity tensor depends on the chirality parameter, but unlike the hexachiral honeycomb, some elasticities are odd functions of β [15].

PLANE WAVES PROPAGATION

The free wave motion in the hexachiral material is written in terms of the components of the generalized displacement field $\mathbf{U}(\mathbf{x}) = \{u_1 \quad u_2 \quad \phi \quad v_1 \quad v_2 \quad \theta\}^T$ in the following form

$$
\begin{cases}
(2\mu + \lambda)u_{1,11} - 2Au_{1,12} + (\mu + k)u_{1,22} + Au_{2,11} + (\mu - k + \lambda)u_{2,12} - Au_{2,22} - 2A\phi_{,1} + 2k\phi_{,2} + \hat{k}_d(v_1 - u_1) = \rho_1\ddot{u}_1 \\
Au_{1,11} + (\mu - k + \lambda)u_{1,12} - Au_{1,22} + (\mu + k)u_{2,11} + 2Au_{2,12} + (2\mu + \lambda)u_{2,22} - 2k\phi_{,1} - 2A\phi_{,2} + \hat{k}_d(v_2 - u_2) = \rho_1\ddot{u}_2 \\
S\phi_{,11} + S\phi_{,22} + 2Au_{1,1} + 2Au_{2,2} - 2ku_{1,2} + 2ku_{2,1} - 4k\phi + \hat{k}_\theta(\theta - \phi) = I_1\ddot{\phi} \\
\hat{k}_d(u_1 - v_1) = \rho_2\ddot{v}_1 \quad , \\
\hat{k}_d(u_2 - v_2) = \rho_2\ddot{v}_2 \quad , \\
\hat{k}_\theta(\phi - \theta) = I_2\ddot{\theta} \quad .
\end{cases}
\tag{8}
$$

If a harmonic plane wave propagating along axis x_1 in an infinite planar micropolar medium is admitted, the generalized displacement field at a point is assumed in the following form $\mathbf{U} = \hat{\mathbf{U}}^T \exp[i(qx_1 - \omega t)]$, where q and ω denote the wave number and the circular frequency, respectively, and $\hat{\mathbf{U}}^T = \{\hat{u}_1 \quad \hat{u}_2 \quad \hat{\phi} \quad \hat{v}_1 \quad \hat{v}_2 \quad \hat{\theta}\}$ is the vector of the amplitudes $(i = \sqrt{-1})$. Substituting the assumed generalized displacement field in the equation of motion (8) one obtains the secular equation system for the equivalent continuum model

$$
\begin{bmatrix}
\begin{bmatrix} (2\mu + \lambda)q^2 + \\ + \hat{k}_d - \rho_1\omega^2 \end{bmatrix} & Aq^2 & 2iAq & -\hat{k}_d & 0 & 0 \\[2ex]
Aq^2 & \begin{bmatrix} (\mu + k)q^2 + \\ + \hat{k}_d - \rho_1\omega^2 \end{bmatrix} & 2ikq & 0 & -\hat{k}_d & 0 \\[2ex]
-2iAq & -2ikq & \begin{bmatrix} Sq^2 + 4k + \\ + \hat{k}_\theta - I_1\omega^2 \end{bmatrix} & 0 & 0 & -\hat{k}_\theta \\[2ex]
-\hat{k}_d & 0 & 0 & \begin{bmatrix} \hat{k}_d - \rho_2\omega^2 \end{bmatrix} & 0 & 0 \\[2ex]
0 & -\hat{k}_d & 0 & 0 & \begin{bmatrix} \hat{k}_d - \rho_2\omega^2 \end{bmatrix} & 0 \\[2ex]
0 & 0 & -\hat{k}_\theta & 0 & 0 & \begin{bmatrix} \hat{k}_\theta - I_2\omega^2 \end{bmatrix}
\end{bmatrix}
\begin{Bmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \hat{\phi} \\ \hat{v}_1 \\ \hat{v}_2 \\ \hat{\theta} \end{Bmatrix} = \mathbf{0}
\tag{9}
$$

The solution of the eigenproblem (9), characterized by a hermitian matrix, provides dispersion functions $\omega(q)$ defined in the domain $q \in \left[-\dfrac{\pi}{a}, \dfrac{\pi}{a}\right]$, a being the characteristic size of the cell as shown in Figure 2. Since the model has six degrees of freedom, it follows that in the general case six plane waves may propagate, each characterized by a dispersion function. It follows that this model exhibits plane waves more complex than those obtained and discussed by Parfitt and Eringen [17] for isotropic achiral two dimensional micropolar solids and more recently by Khurana and Tomar [18] for 3D chiral

micropolar media. Since the matrix of the coefficient in the eigenproblem (9) is hermitian, the eigenvalues are real, but not necessarily positive. Therefore, for some values of the model parameters is expected a reduced number of dispersive functions.

In case of a tetrachiral lattice with internal resonant masses the free wave equation of motion takes the form:

$$\begin{cases} 2\mu u_{1,11} + k u_{1,22} + B u_{2,11} - B u_{2,22} - B\phi_{,1} + k\phi_{,2} + \hat{k}_d\left(v_1 - u_1\right) = \rho_1 \ddot{u}_1 \\ B u_{1,11} - B u_{1,22} + k u_{2,11} + 2\mu u_{2,22} - k\phi_{,1} - B\phi_{,2} + \hat{k}_d\left(v_2 - u_2\right) = \rho_1 \ddot{u}_2 \\ S\phi_{,11} + S\phi_{,22} + B u_{1,1} + B u_{2,2} - k u_{1,2} + k u_{2,1} - 2k\phi + \hat{k}_\theta\left(\theta - \phi\right) = I_1 \ddot{\phi} \\ \hat{k}_d\left(u_1 - v_1\right) = \rho_2 \ddot{v}_1 \\ \hat{k}_d\left(u_2 - v_2\right) = \rho_2 \ddot{v}_2 \\ \hat{k}_\theta\left(\phi - \theta\right) = I_2 \ddot{\theta} \end{cases} \tag{10}$$

In this case the secular equation system is written as follows:

$$\begin{bmatrix} \begin{bmatrix} 2\mu q^2 + \hat{k}_d + \\ -\rho_1\omega^2 \end{bmatrix} & Bq^2 & iBq & -\hat{k}_d & 0 & 0 \\ Bq^2 & \begin{bmatrix} kq^2 + \hat{k}_d + \\ -\rho_1\omega^2 \end{bmatrix} & ikq & 0 & -\hat{k}_d & 0 \\ -iBq & -ikq & \begin{bmatrix} Sq^2 + 2k + \\ +\hat{k}_\theta - I_1\omega^2 \end{bmatrix} & 0 & 0 & -\hat{k}_\theta \\ -\hat{k}_d & 0 & 0 & \begin{bmatrix} \hat{k}_d - \rho_2\omega^2 \end{bmatrix} & 0 & 0 \\ 0 & -\hat{k}_d & 0 & 0 & \begin{bmatrix} \hat{k}_d - \rho_2\omega^2 \end{bmatrix} & 0 \\ 0 & 0 & -\hat{k}_\theta & 0 & 0 & \begin{bmatrix} \hat{k}_\theta - I_2\omega^2 \end{bmatrix} \end{bmatrix} \begin{Bmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \hat{\phi} \\ \hat{v}_1 \\ \hat{v}_2 \\ \hat{\theta} \end{Bmatrix} = \mathbf{0} \tag{11}$$

Again the solution of the eigenproblem (11) with hermitian matrix provides dispersion functions $\omega(q)$, defined in the domain $q \in \left[-\dfrac{\pi}{a}, \dfrac{\pi}{a}\right]$.

The analytical formulation here derived, which is based on a micropolar continuum model enriched with additional degrees of freedom of the resonating masses located inside the rings, will be developed in future research to analyze the wave propagation and to appreciate the influence of the geometrical and mechanical parameters of the microstructure and of the resonant devices on the frequency spectra and on the conditions of wave attenuation. The reliability of the results has to be evaluated from the comparison with the rigorous solutions obtained from a Floquet-Bloch analysis of the generalized beam lattice proposed in this paper. Finally, it should be noted that the validity of this model relies on the rigidity assumption of the rings [15], a condition that may be easily implemented in the physical model.

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