The value of public information in vertically differentiated markets*

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Abstract

Generating public information about vertically differentiated products increases expected vertical differentiation and softens competition. We show that this will induce firms to overinvest (underinvest) in information generation, if the deadweight loss in the subsequent market equilibrium is high (low). Moreover, information generation by one firm has a positive externality on the other firm. It follows that coordination (e.g. via industry associations) increases information generation. When product qualities are endogenous, information generation may prevent quality degradation and thus have an additional social benefit.

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1 Introduction

Do sellers generate the efficient level of public information regarding their goods? We contribute to answering this fundamental question by examining the case of a competitive market with vertically differentiated products. We do so by considering a canonical model of price competition with vertically differentiated products (as in Shaked and Sutton, 1982), in which consumers and firms have identical beliefs regarding the products qualities. Our innovation is to allow firms, before setting their prices, to generate costly, unbiased signals correlated with the quality of their products, observed publicly by all consumers and all firms.

For example, a product's technical specifications may be perfectly known to all consumers and all firms. However, the consumption utility generated by this product may depend on harder-to-measure attributes such as its aesthetic appeal, its ergonomics and ease of use, the presence of unexpected bugs or defects, both in absolute terms and relative to other products. These attributes can be (partially) learned via informative public signals: industry competitions, trade shows, industry classifications, reviews by experts in the media (examples are *Consumer Reports* in the US or *Which?* in the UK), quality tests and certification by professional agencies (e.g., rating agencies for financial products, TÜV for industrial goods). These signals are often commissioned by firms, either individually or via industry bodies tasked with organizing industry competitions or maintaining classification systems.[1]

As a preliminary result, we show that under mild assumptions (log concavity of the distribution of consumer's taste for quality and a sufficiently high minimum taste for quality) in Shaked and Sutton (1982)'s model a consumer's equilibrium choice of product quality, low or high, is independent of the distribution of qualities in the market. Equipped with this result, we then derive our main proposition: that, for given average taste for quality,

[1] A case in point are wine classification systems, usually maintained by associations of vintners. See, for example, the Bordeaux wine classification of 1855, and, more importantly, its more recent and regularly updated offshoot Cru Bourgeois. Similar systems in Burgundy, Champagne, Douro, and other regions. While observable variables such as soil quality of the vineyard and the weather of the vintage determine expected qualities, the true quality of a fine wine often only realizes after years of storage.

firms will under- (over-) invest in information generation when the market share of the leader is high (low). This means that inefficiency in information generation is linked to the deadweight loss in the pricing equilibrium.

For intuition, note that in the first best allocation all consumers consume the product of the quality leader. Hence, when the market share of the quality follower is large, there is a substantial deadweight loss in the pricing equilibrium. This loss strictly increases in the amount of information generated, because, when new information arrives, the expected quality distance between leader and follower grows. Firms fail to internalize the impact of information generation on the subsequent deadweight loss, which implies that when the deadweight loss is large in equilibrium, firms overinvest in information generation relative to the social optimum.

Conversely, when few consumers purchase from the quality follower, the deadweight loss is small and so is its sensitivity to new information. We show that the social benefit of information generation then exceeds its private benefit and firms will underinvest in information generation relative to the social optimum. This underinvestment is most severe in the limit case in which the quality leader captures the entire market. The reason is that the quality leader's profits—and his incentives to generate information—depend on the *marginal* valuation of its product, i.e. the willingness to pay of the least quality-sensitive consumer. The social value of information generation instead depends on the *average* consumer's valuation.[2]

We also show that public information generates a positive externality among firms, because drawing *any* signal increases a firm's expected profits, including signals about the opponent's product. Hence, even if prices are set competitively, firms can soften competition by cooperating in generating unbiased, publicly available information about product quality, for example, by introducing a classification system or an industry-wide competition. Such coordination decreases aggregate surplus whenever the level of information generation in the Nash equilibrium already exceeds the social optimum, which

[2]Interestingly, in the underinvestment case, consumers may benefit from generating additional information. This paper focuses on firm behavior, see Terstiege and Wasser (2019) for optimal information generation from the consumers' perspective.

provides a novel perspective on the regulation of industry cooperation.

Finally, we extend the model by allowing firms to achieve vertical differentiation also via costless quality degradation. It is well known that, absent information generation, in equilibrium firms will increase the expected quality distance in the product market by way of quality degradation. The possibility of generating public information mitigates this problem, because learning provides an alternative means to generate quality dispersion in the market. Hence, when quality degradation is a concern, encouraging firms to cooperate in information generation may be socially desirable, as it prevents quality degradation.

Related literature.

Our primary goal is to contribute to the literature studying information generation in competitive settings. We also contrast our results with those in the literature on information generation in monopoly settings and the literature on information disclosure.

Information generation. The existing literature studying information generation in competitive settings has mostly focused on horizontal competition (see, in particular, Anderson and Renault, 2000, 2009, Levin, Peck, and Ye, 2009). Similar to our paper, also with horizontal competition, increasing distance between quality levels via information generation increases firms market power. However, the welfare implications differ markedly between horizontal and vertical competition.[3]

The small literature on information generation with vertical differentiation tends to rely on very specific informational environments. For instance, Bouton and Kirchsteiger (2015) examine the role of reliable rankings of sellers and show that their presence can reduce consumers' welfare. Bergemann

[3]There is, of course, the classical paradox that by construction fully revealing market equilibrium prices will not provide incentives for costly information generation (Grossman and Stiglitz, 1980), which has been resolved by allowing agents to take into account the effect of their actions on prices and beliefs of other agents (Milgrom, 1981; Verrecchia, 1982).

and Välimäki (2000) consider a dynamic setting, in which information is generated through repeated purchases, and find that information generation increases firms' market power and may reduce social welfare. By contrast we consider a generic form of information generation (i.e., any unbiased signal correlated with the distribution of quality), examine incentives for over- or underinvestment in information, as well as the role of coordination in information generation.

Information generation has also been studied in monopoly settings. E.g., Ottaviani and Prat (2001) find that a monopolist always benefits from generating and disclosing signals that are affiliated to the buyer's valuation. Other studies have examined the incentives to generate public information in auctions (e.g. the seminal paper by Milgrom and Weber, 1982). Closer to our model, Ganuza and Penalva (2010) study an auction, in which one side of the market (the firm) generates information related to the *other* side of the market (the buyers' valuations). They argue that the amount of information generated by the firm will fall short of the social optimum, because information increases the dispersion in buyers' evaluations and information rents. Roesler and Szentes (2017) examine a related issue, identifying the optimal information environment from the buyers' point of the view. Out setup differs in the presence of a second firm/seller, which affects the incentive to generate information: both over- and under-provision of information can be outcome, depending on the market share of the quality leader.

Finally, the literature has studied models, in which firms generate private information, that is, signals that are informative relative to each consumer's idiosyncratic preferences (for example, Lewis and Sappington, 1994, Moscarini and Ottaviani, 2001, Johnson and Myatt, 2006). We study public information instead, which is about the quality of the goods sold in the market, where "quality" refers to the attributes of a product that are valued by all consumers. Hence, all consumers prefer higher quality to lower quality, but may trade off quality and price differently.[4]

[4]Because the taste for quality of each consumer is constant, but information generation affects the expected quality, in our setting new information always proportionally shifts the consumer's willingness to pay.

Information disclosure A related problem is that of a privately informed firm deciding how much information to disclose to consumers. To the best of our knowledge, Meurer and Stahl (1994) are the first to point out that information disclosure by one firm generates a positive externality on competing firms. However, they only consider horizontal competition and a very specific information structure (i.e., informative advertising).[5]

More closely related to our work is Board (2009) studying information disclosure by firms under vertical competition. In his setup there are equilibria in which disclosure of information by firms may be partial, and hence a policy forcing disclosure is socially desirable. By contrast, our setup yields over-provision of information when the market share of the quality follower is high.[6] Related is also Jovanovic (1982), showing that a monopolist will disclose too much information to consumers relative to the social optimum, whereas in our setup a monopolist will under-invest in information.

In the remainder of the paper we first present the model. Then in Section 3 we derive the equilibrium in the pricing game for given expected qualities. In Section 4 we solve the full game, in which firms can invest to generate information before setting prices. Section 5 adds a stage to the game, in which each firm can degrade its product at no cost. The last section concludes. All mathematical derivations missing from the text are in the appendix.

2 Model

Our starting point is the canonical model of a duopoly with vertically differentiated products (see Gabszewicz and Thisse, 1979, Shaked and Sutton, 1982, and Chapter 7 of Tirole, 1988's textbook). The market consists of 2 firms and a mass 1 of buyers. Each firm produces a good of quality $s_i \in [\underline{s}, \overline{s}]$

[5]See also Vives (1999), chapter 8, discussing incentives of firms to share (but not generate) private information in different models of oligopolistic competition. Related to our findings, both Meurer and Stahl (1994) and Vives (1999) argue that firms may benefit from organizing a trade association to gather information from their members and then share it.

[6]Clearly, an interesting extention is to consider both the choice of information generation and the subsequent choice of information disclosure. We briefly discuss such extension in our conclusions.

for $i \in \{1,2\}$. A buyer's utility is given by

$$U = \begin{cases} \theta s_i - p_i & \text{if good } i \text{ is purchased} \\ 0 & \text{in case of no purchase,} \end{cases}$$

where p_i is the price of the good produced and $\theta \in \mathbb{R}_+$ is an i.i.d. taste parameter with cumulative distribution function $F(x) = \mathrm{pr}(\theta \leq x)$ that is continuous, differentiable, and has a continuous first derivative. We assume that the support of θ has a minimum $\underline{\theta}$ (so that $F(x) = 0$ for all $x \leq \underline{\theta}$), but may or may not have a maximum. If a maximum exists, we call it $\overline{\theta} > \underline{\theta}$, otherwise we write $\overline{\theta} = \infty$.

Each firm has zero marginal cost of production, so that profit is given by price times quantity sold.

Information and Learning

We depart from the canonical model by assuming that the quality levels s_i are unknown to both buyers and firms, who have common ex-ante beliefs about s_i. Call $q_i = E[s_i]$ the initial expected quality of firm i's product, and assume, without loss of generality, that $q_1 \geq q_2$.

Firm i can generate information by paying a cost k and drawing a signal σ_i, which is informative with respect to s_i and may be informative with respect to s_{-i} as well.[7] We allow for any possible correlation between any σ_1 and σ_2. Information generated is public: all market participants receive the signal and update their belief about quality. We adopt the convention that $\sigma_i = \emptyset$, if firm i does not generate information. We therefore write $\sigma = (\emptyset, \emptyset)$ if no firm generates information, $\sigma = (\emptyset, \sigma_i)$ when firm $i \in \{1,2\}$ generates information but not firm $-i$, and $\sigma = (\sigma_1, \sigma_2)$ when both firms generate

[7] We abstract away from the choice of precision of the signal (as in the Bayesian persuasion literature, see in particular Gentzkow and Kamenica, 2016) as well as from the possibility of signal jamming. We will show below that both firms' expected profits increase in the precision of both signals. Hence, firms have no incentive to jam each other's signal, and, for given cost of drawing a signal, firms prefer the most precise signal available. Hence, our results carry over to such a case. However, if signals of different precision differ in their cost, firms will face a trade off. This trade off depends crucially on the details of the cost function, and we prefer to leave this extension for future research.

information.

Denote the *ex-post expected quality* by $\hat{q}_i$, that is, the expected quality of firm i given a specific realization of σ. Note that by iterating expectations $E[\hat{q}_i|\sigma] = q_i$ for any signal configuration σ (where the expectation is taken over the possible realizations of σ). This means that ex ante, before any signal is drawn, the expected ex-post quality is equal to the initial expected quality.

Timing

To summarize, the timing of the game is as follows.

1. Given the initial beliefs about qualities, firms simultaneously decide whether to acquire information at cost k, yielding a vector of signals σ.

2. Realizations of signals are publicly revealed, leading to a revision of the beliefs about the products' qualities and to $\hat{q}_1, \hat{q}_2$.

3. Firms announce prices simultaneously. Consumers decide if and from whom to buy and consume. Payoffs are realized.

Solution Concept

To derive the outcome of the game described above we employ a subgame perfect Nash equilibrium of signal generation choices σ_1 and σ_2 and price choices p_1 and p_2 depending on the signals.

Assumptions

We conclude the description of the model by introducing some restrictions on the distribution of the taste parameter θ. These restrictions guarantee the existence and uniqueness of a pure strategy Nash equilibrium in the pricing game (stage 4 in the timeline above), as we will show.

Assumption 1 (Log-concavity)**.** *The density $f(\theta)$ is log-concave.*

This assumption comes with only a very modest loss of generality, as log-concavity is satisfied by a host of widely used distributions. Nonetheless, it puts some useful structure on $F(\theta)$ and $f(\theta)$. For example, log concavity implies that $f(\theta)$ is unimodal (see e.g. Dharmadhikari and Joag-Dev, 1988). Hence, $f(\theta)$ is strictly positive for $\theta \in (\underline{\theta}, \overline{\theta})$ (remember that if there is no upper bound, we write $\overline{\theta} = \infty$). Furthermore, log concavity of $f(\theta)$ ensures that both $F(\theta)$ and $1 - F(\theta)$ are log-concave (see Prékopa, 1973 and Bagnoli and Bergstrom, 2005). This, in turns, implies that $F(\theta)/f(\theta)$ increases, $(1 - F(\theta))/f(\theta)$ decreases, and $(1 - 2F(\theta))/f(\theta)$ also decreases, all facts that we will use extensively in our derivations.

Finally, we assume that there is enough potential revenue in the left tail of the taste distribution, in the sense that $\underline{\theta}$ is sufficiently high and the taste distribution has enough mass at or near $\underline{\theta}$.

Assumption 2 (Covered Market). *Either* $\underline{\theta} \cdot f(\underline{\theta}) > 1$*, or*

$$\underline{\theta} \cdot m \geq \frac{\overline{s} - \underline{s}}{\overline{s}}, \tag{A2}$$

where $m \equiv \min_{\theta \in [\underline{\theta}, \theta^*]} f(\theta)$ *and* θ^* *is implicitly defined as* $\theta^* = \frac{1-F(\theta^*)}{f(\theta^*)}$.

Note that, because of log-concavity, $\frac{1-F(\theta)}{f(\theta)}$ is strictly decreasing and hence θ^* exists and is unique as long as $f(\underline{\theta})\underline{\theta} \leq 1$.

Condition (A2) is a generalization of the standard *covered market* condition.[8] As we will show, it guarantees that equilibrium prices are such that all consumers prefer to purchase from one of the firms to not purchasing.[9] Indeed, any distribution that is bounded below with $f(\underline{\theta}) > 0$ satisfies (A2), if appropriately scaled up. This is because increasing $\underline{\theta}$ decreases $\theta^* - \underline{\theta}$ and,

[8]For example, in Chapter 7 of Tirole (1988)'s textbook, the taste parameter is distributed uniformly with $\overline{\theta} - \underline{\theta} = 1$, and the model is solved assuming the covered market condition $\frac{|\hat{q}_1 - \hat{q}_2|}{\max\{\hat{q}_1, \hat{q}_2\}} \leq \underline{\theta}$. Condition (A2) is a generalization of this condition, because it applies to all possible distributions of the taste parameter, and to all possible quality levels (in Tirole, 1988 the quality levels are given exogenously).

[9]To the best of our knowledge, (1)-(2) are the weakest conditions existing in the literature guaranteeing existence, uniqueness and full analytical characterization of the pricing equilibrium with covered market. Studies examining the non covered-market case (Moorthy, 1988, Choi and Shin, 1992) or not imposing ex-ante whether the market will be covered (Wauthy, 1996) restrict their attention to uniform taste distributions.

therefore, (weakly) increases m. If the new $\underline{\theta}$ is sufficiently large relative to the maximum possible dispersion in quality $\overline{s} - \underline{s}$, then Condition (A2) will hold. Since a truncation of a log-concave distribution is also log-concave (Bagnoli and Bergstrom, 2005, Theorem 7), any log-concave distribution that is unbounded below or bounded below but with mass equal to zero at the lower bound satisfies Assumptions 1 and 2, if appropriately truncated.

3 The Pricing Game

Consider a given realization of the signal vector σ. If this realization is such that $\hat{q}_1 = \hat{q}_2$, then the two firms compete á la Bertrand and set equilibrium prices $p_1 = p_2 = 0$. All consumers purchase from one of the two firms, and are indifferent between purchasing from firm 1 or 2. If instead $\hat{q}_1 \neq \hat{q}_2$, the two firms may charge positive prices. Since information generation may reverse the initial quality ranking of firms, we will refer to the quality leader by L and the follower by F, so that $\hat{q}_L \equiv \max\{\hat{q}_1, \hat{q}_2\} > \hat{q}_F \equiv \min\{\hat{q}_1, \hat{q}_2\}$.

Denote a firm i's posted price by p_i. The demand for goods can be characterized by two thresholds. The first threshold X is given by the consumer who is indifferent between purchasing from either firm, if there is such a consumer, and by $\underline{\theta}$ ($\overline{\theta}$) if all consumers weakly prefer L (F):

$$X \equiv \begin{cases} \frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} & \text{if } \frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} \in (\underline{\theta}, \overline{\theta}) \\ \underline{\theta} & \text{if } \frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} \leq \underline{\theta} \\ \overline{\theta} & \text{if } \frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} \geq \overline{\theta}. \end{cases}$$

The second threshold Y is given by the consumer who is indifferent between the lower quality firm F and not consuming, if there is such a consumer, and by $\underline{\theta}$ ($\overline{\theta}$) if all consumers weakly prefer F (not to consume):

$$Y \equiv \begin{cases} \frac{p_F}{\hat{q}_F} & \text{if } \frac{p_F}{\hat{q}_F} \in (\underline{\theta}, \overline{\theta}) \\ \underline{\theta} & \text{if } \frac{p_F}{\hat{q}_F} \leq \underline{\theta} \\ \overline{\theta} & \text{if } \frac{p_F}{\hat{q}_F} \geq \overline{\theta}. \end{cases}$$

These thresholds can be shown to have some useful properties, using that the quality leader can always out-price the follower.

Lemma 1. *In any pure strategy Nash equilibrium:*

- *A positive measure of consumers purchase from the quality leader:* $Y \leq X < \overline{\theta}$.
- *If not all consumers purchase from the quality leader* ($X > \underline{\theta}$)*, then there is positive demand for the quality follower* ($X > Y \geq \underline{\theta}$).

Hence, in any pure strategy Nash equilibrium the demand for good L is $1 - F(X)$ and the demand for good F is $F(X) - F(Y)$.[10] Profits are given by:

$$\pi_L(p_L, p_F) = p_L \cdot (1 - F(X)) \text{ and } \pi_F(p_L, p_F) = p_F \cdot (F(X) - F(Y)).$$

Given this, we can derive the two best responses:

Lemma 2. *The quality leader's best response is*

$$p_L(p_F) = \max\left\{\frac{1 - F(X)}{f(X)}(\hat{q}_L - \hat{q}_F), \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F\right\},$$

which is a continuous function. The quality follower's best response is

$$p_F(p_L) = \begin{cases} [0, +\infty) & \text{if } p_L \leq \underline{\theta}(\hat{q}_L - \hat{q}_F) \\ \min\left\{\frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F), \underline{\theta}\hat{q}_F\right\} & \text{otherwise.} \end{cases}$$

which is a upper-hemicontinuous, compact valued, convex correspondence.

The proof of the Lemma relies on Assumption 1 to establish existence and uniqueness of the best responses. In addition, Assumption 2 implies that the market is covered: for every p^L the follower's optimal price is such that $Y = \underline{\theta}$.

[10] From now on we focus exclusively on pure strategy equilibria, even when we do not explicitly mention this.

A Nash equilibrium is a pair p^* such that $p_i(p_{-i}(p_i^*)) = p_i^*$ for $i = L, F$. Depending on the distribution of the taste parameter θ, this equilibrium can be effectively a *monopoly*, by which we mean that the market is supplied by a unique seller, or a *duopoly*, in which both sellers supply some customers. The following proposition gives sharp conditions on the type distribution for either case.

Proposition 1 (Market Equilibrium Outcome)**.**

(i) If $1 \leq \underline{\theta} \cdot f(\underline{\theta})$, then in the unique pure strategy Nash equilibrium $X^* = Y^* = \underline{\theta}$, i.e. the quality leader supplies the entire market, and prices are $p_F^* = 0$ and $p_L^* = \underline{\theta}(\hat{q}_L - \hat{q}_F)$.

(ii) If instead $1 > \underline{\theta} \cdot f(\underline{\theta})$, then in the unique pure strategy Nash equilibrium $Y^* = \underline{\theta}$ and $X^* > \underline{\theta}$ where

$$X^* = \frac{1 - 2F(X^*)}{f(X^*)} \tag{1}$$

Equilibrium prices are $p_L^* = \frac{1-F(X^*)}{f(X^*)}(\hat{q}_L - \hat{q}_F)$ and $p_F = \frac{F(X^*)}{f(X^*)}(\hat{q}_L - \hat{q}_F)$.

To establish point (ii) of the Proposition, we first prove that Assumption 2 implies that the follower's profit maximization problem must have an interior solution in equilibrium. Knowing this, we then use Assumption 1 to show the existence and uniqueness of the equilibrium.

Hence, $\underline{\theta} \cdot f(\underline{\theta})$ determines whether in the market outcome the quality leader supplies the whole market, or both firms share the market. If $\underline{\theta} \cdot f(\underline{\theta}) \geq 1$ (monopoly) profits are given by

$$\pi_L = \underline{\theta}(\hat{q}_L - \hat{q}_F) \text{ and } \pi_F = 0,$$

if instead $\underline{\theta} \cdot f(\underline{\theta}) < 1$ (duopoly) profits are given by

$$\pi_L = \frac{(1 - F(X^*))^2}{f(X^*)}(\hat{q}_L - \hat{q}_F) \text{ and } \pi_F = \frac{(F(X^*))^2}{f(X^*)}(\hat{q}_L - \hat{q}_F).$$

Note also that the cutoff X^*, separating consumers buying from L from those buying from F, cannot be greater than the median by construction. Hence, the quality leader's market share is greater than one half.

Perhaps surprisingly, in equilibrium the demand faced by leader and follower does *not* depend on the expected qualities $\hat{q}_L$ and $\hat{q}_F$. This is because under log-concavity both firms' best responses are proportional to the expected quality distance $\hat{q}_L - \hat{q}_F$, and therefore the equilibrium distribution of market shares is independent of that distance. Thus the demand faced by leader and follower depends only on the taste distribution $F(\theta)$, which determines the cutoff X^*. This fact will be very convenient, since it implies that the signal realizations, and thus also the signal configurations, only affect the identity of quality leader and follower and their market prices, but not demand.

To provide some illustration for Proposition 1 note that if the taste θ follows a Pareto then $\underline{\theta} \cdot f(\underline{\theta}) \geq 1$ holds and one firm will capture the market if and only if $E(\theta)$ is finite. If the taste distribution is uniform instead, then the quality leader supplies the entire market if $\overline{\theta} \leq 2\underline{\theta}$, otherwise there is a duopoly with $X^* = \frac{\underline{\theta}+\overline{\theta}}{3}$. Which case will occur depends mainly on two intuitive effects. First, fixing either $\overline{\theta}$ or $\underline{\theta}$, as $\overline{\theta} - \underline{\theta}$ increases (and with it the variance of the distribution) the duopoly becomes more likely. This is because the quality leader will find it less profitable to attract the least quality sensitive consumers by lowering the price, thus leaving demand for the follower. By contrast, holding the range of the support $\overline{\theta} - \underline{\theta}$ constant, an equal increase in both $\overline{\theta}$ and $\underline{\theta}$ makes it more likely that the quality leader corners the market. This is because the incentive of the quality leader to sell to the least quality-sensitive consumer increases in her quality sensitivity.

4 Information generation

Equipped with the properties of the pricing equilibrium we are now in a position to examine the firms' choices of information generation. Depending on the properties of the type distribution, either the quality leader will corner the market (monopoly case) or both firms will supply some consumers. In the following we consider each case separately.

4.1 Case 1: Monopoly ($1 \leq \underline{\theta} f(\underline{\theta})$)

We start by considering the case $1 \leq \underline{\theta} f(\underline{\theta})$, in which the quality leader covers the entire market. Since all consumers consume the good that has higher expected quality, the pricing equilibrium is efficient.

Social value of information generation. Given the expected qualities $\hat{q}_1$ and $\hat{q}_2$ the social welfare is given by:[11]

$$S(\hat{q}_1, \hat{q}_2) = \max\{\hat{q}_1, \hat{q}_2\} E[\theta] \, .$$

Note that, by the law of iterated expectation, the two expected qualities are independent from σ. This implies that if no realization of the signal can reverse the quality ranking (so that firm 1 will be the leader for sure), then the expected social welfare is the same with or without information generation. If instead it is possible to reverse quality ranking, then by a straightforward application of the Jensen's inequality, social welfare is higher when new information is expected to arrive. For an intuition, suppose that a signal about the leader's quality is drawn. If the leader's quality is revealed to be better than expected, aggregate welfare will increase. If it is instead worse than expected, then aggregate welfare will decrease. The decrease in welfare is, however, bounded below by the quality of the ex-ante quality follower. This makes information generation about the quality leader socially beneficial in expectation whenever the quality follower may become the quality leader. The case of the follower is analogous.

The next lemma shows that this logic is remarkably general, extending for instance to the cases when each signal is informative of the other product as well, and when signals are correlated.

Lemma 3. *For any two signal configurations σ' and σ'', the social benefit generated from moving from σ' to σ'' is given by*

$$E[S(\hat{q}_1, \hat{q}_2)|\sigma''] - E[S(\hat{q}_1, \hat{q}_2)|\sigma'] = E[\theta] \left(\Delta(\sigma'', q_1, q_2) - \Delta(\sigma', q_1, q_2)\right),$$

[11] We assume throughout the paper that the investment in information generation k by itself is not socially valuable.

where

$$\Delta(\sigma, q_1, q_2) \equiv pr\{\hat{q}_2 \geq \hat{q}_1 | \sigma\} E[\hat{q}_2 - \hat{q}_1 | \hat{q}_2 \geq \hat{q}_1, \sigma], \tag{2}$$

is the expected quality gain.

The above lemma is more easily understood by considering the case $\sigma' = \emptyset$ but $\sigma'' \neq \emptyset$, that is, in case in which we move from no information generation to some information generation. As expected, from a social point of view, information generation is beneficial if and only if the quality ranking reverses for some realizations of the signal. If positive, the benefit of information generation increases with the expected quality gain $\Delta(\sigma, q_1, q_2)$, which depends both on the signal σ, but also on the quality distribution. Hence, more informative signals leading to more dispersed posteriors have higher expected quality gains. Similarly, for given $\sigma \neq (\emptyset, \emptyset)$ the expected quality gain increases as the priors q_1 and q_2 get closer. Note also that $\Delta(\sigma, q_1, q_2)$ depends only on the expected posterior quality distribution, but not on which firm draws a signal. Hence, if firms have access to the same signal technology, so that σ_1 and σ_2 induce the same posterior distribution given initial quality, then the social values of generating information on firm 1's and firm 2's products are identical.

The lemma also allows to compare social welfare under two different signal configurations $\sigma' \neq \emptyset$, $\sigma'' \neq \emptyset$. In this case, an important observation is that the expected quality gain increases with the number of signals, that is:

$$\Delta((\emptyset, \emptyset), q_1, q_2) \leq \Delta((\emptyset, \sigma_i), q_1, q_2) \leq \Delta((\sigma_1, \sigma_2), q_1, q_2). \text{ for } i \in \{1, 2\} \tag{3}$$

This is because more information (in the form of two signals rather than one) increases the dispersion in the distribution of the posterior.[12] Hence, expected social welfare is (weakly) monotone in the number of signals drawn.

[12] To see this, suppose that two signals are drawn sequentially starting with σ_1. The derivation above implies that given a realization of σ_1 drawing σ_2 is always weakly beneficial, strictly so if the ranking given the realization of σ_1 changes for some realizations of σ_2. Hence, in expectation (i.e., before drawing σ_1) information generation by both firms generates weakly greater social welfare (excluding the signal cost) than by only one firm.

The socially optimal investment in information generation then solves

$$\max_{\sigma} E[S(\hat{q}_1, \hat{q}_2)|\sigma] - \begin{cases} 2k \text{ if } \sigma = (\sigma_1, \sigma_2), \\ k \text{ if } \sigma = (\emptyset, \sigma_i) \text{ for } i \in \{1, 2\}, \\ 0 \text{ if } \ \sigma = (\emptyset, \emptyset). \end{cases}$$

Whether it is optimal to learn about only the quality leader, only the quality follower, or both will depend on the two expected qualities, on the two signals, and on the cost parameter k.

Private value of information generation. Given the outcome of the pricing game in Proposition 1 the firms' profits are

$$\pi_i(\hat{q}_i, \hat{q}_{-i}) = \begin{cases} \underline{\theta}|\hat{q}_i - \hat{q}_{-i}| & \text{if } \hat{q}_i > \hat{q}_{-i} \\ 0 & \text{otherwise,} \end{cases}$$

which increase in the distance between quality levels, strictly so for the quality leader. Firms' profits can be more easily compared with social welfare by rewriting them as:

$$\pi_i(\hat{q}_i, \hat{q}_{-i}) = \underline{\theta}(\max\{\hat{q}_i, \hat{q}_{-i}\} - \hat{q}_{-i}) = \frac{\underline{\theta}}{E[\theta]} S(\hat{q}_1, \hat{q}_2) - \underline{\theta}\hat{q}_{-i},$$

The above expression makes it clear that the monopolist fails to capture the entire social surplus for two reasons. The first one is that consumers could purchase from the quality follower, which implies that all consumers must enjoy positive surplus (captured by the second part of the above expression). The second reason is that the monopolist cannot price discriminate, and hence consumers with higher willingness to pay must enjoy higher surplus (captured by the first part of the above expression).

When it comes to information generation, however, the first reason is irrelevant because, as already discussed, unbiased signals do not affect the expected quality of the followers. The second reason instead plays a central role, because it implies that information generation increases the monopolist's profits, but by less than social welfare. The following proposition shows this formally (its proof is straightforward and is omitted).

Proposition 2. *Suppose there is a monopoly (i.e., $1 \leq \underline{\theta} f(\underline{\theta})$). Then for any two signal configurations σ' and σ'' a firm i's gain in payoffs from moving from σ' to σ'' is given by*

$$E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma''] - E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma'] = \underline{\theta}\left(\Delta(\sigma'', q_1, q_2) - \Delta(\sigma', q_1, q_2)\right), \quad (4)$$

and is proportional to the gain in social welfare with a factor $\underline{\theta}/E[\theta] < 1$.

Note that by (3) the benefit will be non-negative, if σ'' contains strictly more signals than under σ'. The proposition therefore confirms that the private returns to information generation are lower than the social returns, for any increase in the number of signals drawn and given any signal configuration. As a side comment, note that this implies that information generation increases consumer surplus, because social welfare is the sum of profits and consumer surplus.

Subgame Perfect Nash Equilibrium A firm's optimal choice of whether to acquire a signal, and thus the outcome of the two stage game, will depend on whether the expected increase in profits computed above outweighs the investment cost k. That is, the subgame perfect Nash equilibrium of the information generation cum pricing game will depend on the quality distribution q_1 and q_2, the signal technology, and the investment cost k. We list the pure strategy equilibria below:

- If $k > \underline{\theta}\left(\Delta(\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right)$ and $\underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2) \geq k$ then there is an equilibrium in which only firm $i \in \{1, 2\}$ generates information. There are two equilibria (each corresponding to a different firm generating information), if these inequalities hold for both $i = 1$ and $i = 2$.
- if $k \leq \underline{\theta}\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right)$ and $\underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2) \geq k$ for at least one $i \in \{1, 2\}$, then there is a unique equilibrium in which both firms generate information.

- if $k \leq \underline{\theta}\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right)$, but $\underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2) \leq k$ for both $i = 1, 2$, then there are multiple equilibria: one in which no firm generates information, and one in which both firms generate information.
- Otherwise, there is no information generation in equilibrium.

Therefore multiple equilibria are possible in two cases. First, if it is beneficial for each firm to generate information individually but not jointly, then there could be two equilibria depending on which firm generates information. Second, if the expected quality gain $\Delta(\sigma, q_1, q_2)$ increases in the number of signals drawn, then information generation choices will be strategic complements. An interesting case occurs when individual information generation is not profitable, but joint information generation is. This will be the case, for instance, when drawing one signal cannot perturb the posterior quality distribution sufficiently to reverse the quality ranking, but drawing two signals can (so that $\Delta((\emptyset, \sigma_1), q_1, q_2) = \Delta((\emptyset, \sigma_2), q_1, q_2) = 0$, but $\Delta((\sigma_1, \sigma_2), q_1, q_2) > 0$). If the cost k is sufficiently small, this case would produce a familiar coordination failure: an equilibrium without information generation, Pareto dominated by another equilibrium, in which both firms acquire information. In what follows, if there are multiple equilibria that can be Pareto ranked, we will always focus on the Pareto-preferred one.

Which of the different cases will emerge depends not only on the signal structure, but also on the distance in ex-ante expected qualities $|q_1 - q_2|$. If, for given signals, this distance is sufficiently small, information generation by at least one firm is more likely in equilibrium. For intermediate $|q_1 - q_2|$, there may be multiple equilibria, in which either both firms generate information or neither does; the former equilibrium Pareto dominates the latter. If the distance is sufficiently large, neither firm will acquire any signal.

The characterization of the Nash Equilibrium and Proposition 2 imply the following proposition, derived in the appendix, stating that the equilibrium level of information generation is inefficiently low.

Proposition 3. *Suppose* $1 \leq \underline{\theta} f(\underline{\theta})$, *i.e. there is a monopoly.*

(i) There are values of k for which the number of signal drawn in equilibrium is strictly lower than socially optimal. For all other values of k the efficient number of signals is drawn.[13]

(ii) Consider any two distributions of the taste parameter $F(\theta)$ and $F'(\theta)$ such that either they have equal mean but different lower bounds $\underline{\theta} > \underline{\theta}'$, or they have equal lower bounds but $F(\theta)$ has lower mean than $F'(\theta)$. The set of k for which there is an inefficient equilibrium under $F'(\theta)$ contains the set of k for which there is an inefficient equilibrium under $F(\theta)$.

Hence, firms are more likely to draw fewer signals in equilibrium than efficient if the difference between private benefit (as measured by $\underline{\theta}$) and social benefit (as measured by $E(\theta)$) of information generation is large.

Coordination in information generation. An implication of Proposition 4 is that drawing a given signal benefits *both firms in the same way*, and hence there is a positive externality in information generation across firms. It follows that firms may coordinate their choice of information generation via, for example, an industry body.[14]

When firms can coordinate their information generation, they will choose a signal configuration that maximizes joint profits. By the previous derivations, the joint benefit of information generation by firm i is:

$$2\underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2),$$

and the joint benefit of information generation by both firms is

$$2\underline{\theta}\Delta((\sigma_1, \sigma_2), q_1, q_2).$$

Because information generation by one firm imposes a positive externality

[13]Note that for some k there could be coordination failure: there are multiple Nash equilibria, one with each firm drawing a signal, but not the other. One of these equilibria is inefficient, because the firm with the less informative signal generates information.

[14]As already noted, industry bodies are often responsible for creating and maintaining public information generation mechanisms such as classifications and competitions.

on the other firm, the firms' joint benefit from information generation is larger than each firm's individual benefit. Therefore there are cost parameters k, for which no firm generates information in any equilibrium described above, but information generation by one or both firms will occur when firms jointly decide on information generation and share its cost. Similarly, for some level of k only one firm generates information in equilibrium, and both firms generate information when they can coordinate.

This increase in information generation will be socially beneficial, if the joint benefit of information generation is less than its social benefit, that is, when $2\underline{\theta} \leq E[\theta]$. In this case there is underinvestment in information generation, both with and without coordination, but the underinvestment will be less severe when firms can coordinate.[15] These observations yield the following corollary to our results above.

Corollary 1. *Firms that coordinate their choice of information generation generate more information than is generated by individual choices in the Nash equilibrium. If $2\underline{\theta} \leq E[\theta]$, coordination increases social welfare and consumer surplus.*

4.2 Case 2: Duopoly ($1 > \underline{\theta} f(\underline{\theta})$)

Turn now to the case of a duopoly, i.e., $1 > \underline{\theta} f(\underline{\theta})$ and thus both firms sell to some consumers, jointly covering the market.

Social benefit of information generation. In contrast to above case, now the pricing game has an inefficient outcome. In the pricing equilibrium, the social welfare is given by:

$$\begin{aligned} S(\hat{q}_1, \hat{q}_2) &= \hat{q}_L \int_{X^*}^{\overline{\theta}} \theta dF(\theta) + \hat{q}_F \int_{\underline{\theta}}^{X^*} \theta dF(\theta) \\ &= \max\{\hat{q}_1, \hat{q}_2\}(1-F(X^*))E[\theta|\theta > X^*] + \min\{\hat{q}_1, \hat{q}_2\}F(X^*)E[\theta|\theta < X^*] \\ &= \max\{\hat{q}_1, \hat{q}_2\}E[\theta] - |\hat{q}_1 - \hat{q}_2|F(X^*)E[\theta|\theta < X^*]. \end{aligned} \quad (5)$$

[15] When $2\underline{\theta} > E[\theta]$, firms coordination may lead to overinvestment in information generation, and may reduce social welfare. We discuss more in details this possibility in the next subsection.

The first part of this expression is the first-best social welfare, resulting from all consumers consuming the high quality good. The second part is the deadweight loss generated by positive demand for the lower quality good.

Information generation therefore has two competing effects on social welfare. Similar to the monopoly case above, drawing a signal increases the expected highest quality in the market, which increases social welfare. In contrast to the monopoly case above, information generation also increases the expected quality distance, which in turn increases the deadweight loss, too. The strength of this second effect depends on the market share of the quality follower (i.e., on $F(X^*)$) and on the average taste for quality of the consumers purchasing the low-quality good (i.e., $E[\theta|\theta < X^*]$). Both quantities strictly increase in X^*, which is therefore a sufficient statistics for the social cost of information generation. The following lemma states the social benefit of information generation.

Lemma 4. *For any two signal configurations σ' and σ'', the social benefit generated from moving from σ' to σ'' is given by*

$$E[S(\hat{q}_1, \hat{q}_2)|\sigma''] - E[S(\hat{q}_1, \hat{q}_2)|\sigma'] = (E[\theta] - 2F(X^*)E[\theta|\theta < X^*]) \left(\Delta(\sigma'', q_1, q_2) - \Delta(\sigma', q_1, q_2)\right),$$

Recall that, by definition of X^*, the majority of consumers purchase the high quality good ($F(X^*) < 1/2$), which implies:[16]

$$E[\theta] - 2F(X^*)E[\theta|\theta < X^*] > E[\theta] - E[\theta|\theta < X^*] > 0.$$

Hence, the positive effect of information generation dominates: moving from fewer signals to more signals always increases social welfare, strictly so when increasing the number of signals strictly increases the expected quality gain.

[16] Recall also that in any Nash equilibrium of the pricing game the quality leader faces strictly positive demand and therefore $X^* < \overline{\theta}$.

Private benefit of information generation. To compare private and social returns, recall the firms' profits:

$$\pi_i(\hat{q}_i, \hat{q}_{-i}) = |\hat{q}_i - \hat{q}_{-i}| \begin{cases} \frac{(1-F(X^*))^2}{f(X^*)} & \text{if } \hat{q}_i \geq \hat{q}_{-i} \\ \frac{F(X^*)^2}{f(X^*)} & \text{if } \hat{q}_i \leq \hat{q}_{-i}. \end{cases} \tag{6}$$

Both firms' profits linearly increase in the distance between quality levels, but because $F(X^*) < \frac{1}{2}$ this increase is steeper for the quality leader.

Compared with the monopoly case, now the follower's profits are positive and increasing in the distance with the leader. The next proposition shows that, as a consequence, the private benefit of information generation is here larger than in the monopoly case. Together with the fact that the *social* value of information generation is here lower than in the monopoly case, this implies that the private value of information may exceed its social value.

Proposition 4. *Suppose there is a duopoly (i.e., $1 > \underline{\theta} f(\underline{\theta})$). Then for any two signal configurations σ' and σ'' a firm i's gain in payoffs from moving from σ' to σ'' is given by*

$$E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma''] - E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma'] = \left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)(\Delta(\sigma''_, q_1, q_2) - \Delta(\sigma'_, q_1, q_2)), \tag{7}$$

and is proportional to the gain in social welfare with a factor

$$\frac{X^* + 2\frac{F(X^*)^2}{f(X^*)}}{E[\theta] - 2F(X^*)E[\theta|\theta < X^*]}.$$

As in the case above the benefit will be non-negative, if σ'' contains strictly more signals than σ'. Indeed, Proposition 4 is the duopoly version of Proposition 2 and differs exclusively in that the factor of proportionality between private and social returns of information generation may be below or above unity, depending on X^*. The following corollary states the implied condition for overinvestment.

Corollary 2. *Each firm's private benefit from generating information is* *strictly higher than its social benefit if, and only if:*

$$\frac{F(X^*)^2}{f(X^*)} + F(X^*)E[\theta|\theta < X^*] > \frac{1}{2}(E[\theta] - X^*). \tag{8}$$

Note that condition (8) reduces to $\underline{\theta} > E[\theta]$, when there is a monopoly (i.e., $1 \leq \underline{\theta} f(\underline{\theta})$ and thus $X^* = \underline{\theta}$), so that no qualifier is needed. Indeed, the LHS of (8) strictly increases in the threshold consumer X^*, while the RHS of (8) is strictly decreasing in X^*. Hence, if X^* is sufficiently high (for example, close to the mean of the distribution of the taste parameter), then a firm's private benefit from acquiring a signal is higher than its social benefit. The reverse holds if X^* is sufficiently low (for example, close to the marginal consumer $\underline{\theta}$).

The next proposition derives conditions under which too few or too many signals are drawn in equilibrium (relative to the social optimum). Its proof contains the derivation of the subgame perfect equilibria of the information generation and pricing game (which is analogous to the monopoly case).

Proposition 5. *Suppose* $1 > \underline{\theta} f(\underline{\theta})$*, i.e. there is a duopoly*

(i) If Condition (8) holds there are values of k for which the number of signal drawn in equilibrium is strictly greater than socially optimal. For all other values of k the efficient number of signals is drawn. If condition (8) does not hold, then Proposition 3 applies.[17]

(ii) Consider any two distributions of the taste parameter $F(\theta)$ *and* $F'(\theta)$ *such that either they have equal mean but different* $X^{*\prime} > X^*$*, or they have equal* $X^{*\prime} = X^*$ *but* $F(\theta)$ *has lower mean than* $F'(\theta)$*. If Condition (8) holds at both* $F(\theta)$ *and* $F'(\theta)$*, then the set of k for which there is an inefficient equilibrium under* $F'(\theta)$ *contains the set of k for which there is an inefficient equilibrium under* $F(\theta)$*. If Condition (8) is violated at both* $F(\theta)$ *and* $F'(\theta)$*, then the set of k for which there is an inefficient equilibrium under* $F'(\theta)$ *is contained in the set of k for which there is an inefficient equilibrium under* $F(\theta)$*.*

[17] Also here, for some k there could be coordination failure: there are multiple Nash equilibria, one with each firm drawing a signal, but not the other. One of these equilibria is inefficient, because the firm with the less informative signal generates information.

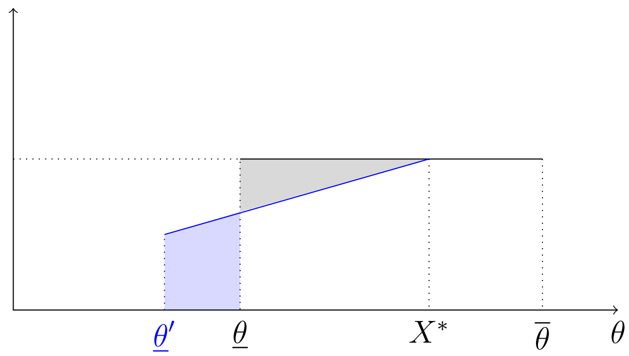

Figure 1: Left-tail spread from a uniform distribution. Note that the two shaded areas are equal.

Note that, again, if Condition (8) holds the difference between private benefit (as measured by $\left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)$) and social benefit (as measured by $(E[\theta] - 2F(X^*)E[\theta|\theta < X^*]))$ of information generation increases in X^* and decreases in $E[\theta]$. Hence, if Condition (8) holds then inefficiencies are more likely in equilibrium the higher is X^* and the lower is $E[\theta]$. Similarly, if Condition (8) is violated, the difference between private benefit and social benefit of information generation decreases in X^* and increases in $E[\theta]$, which implies that inefficiencies are more likely in equilibrium the lower is X^* and the higher is $E[\theta]$.

With respect to specific distributions, for a uniform distribution both $E[\theta]$ and X^* increase with $\underline{\theta}$ and decrease with $\overline{\theta}$. Hence, point (ii) of the above proposition is mute when $\underline{\theta}$ or $\overline{\theta}$ change. For other distributions, such as the truncated normal distribution, point (ii) has a bite: one can change $E(\theta)$ independently from X^* by modifying the parameters of the distribution.

In general, starting from any log-concave distribution for which (A2) holds strictly, one can decrease its lower bound $\underline{\theta}$, while holding constant $E[\theta|\theta > X^*]$, $f(X^*)$ and $F(X^*)$ and maintaining log-concavity. We call this modification a *left-tail spread*. The new distribution will still satisfy (A2). Figure 1 shows an example.

Clearly, performing such spread does not affect X^*. It does however decrease $E(\theta|\theta < X^*)$, and, as a consequence, $E(\theta)$. It follows that a left-tail spread increases the difference between private and social benefit of information generation. By point (ii) of the above proposition, if Condition (8) holds such spread makes inefficiencies in the information generation stage more likely. If Condition (8) is violated such spread makes inefficiencies in the information generation stage less likely.

A final observation is that making such spread always reduces the deadweight loss in the pricing equilibrium. There is therefore an interesting connection between the inefficiencies in the pricing equilibrium and the inefficiency in the information generation stage. First, the size of the deadweight loss determines whether firms under- or over-invest in information generation. This is because reducing the deadweight loss via a left-tail spread makes if more likely that (8) is violated and therefore firms underinvest in information generation. On the other hand, reducing the deadweight loss via a (negative) left-tail spread makes if more likely that (8) holds and firms overinvest in information generation.

Second, this implies that the relationship between inefficiencies in the pricing stage and inefficiencies in the information-generation stage is non-monotonic. Starting from low deadweight loss (so that (8) is violated), as the deadweight loss increases the set of k for which there is under investment in information generation progressively shrinks to zero. As the deadweight loss increases further, condition (8) holds and the set of k for which there over investment in information generation expands.

We summarize the above observations in the following corollary.

Corollary 3. *Take any two distributions $F(\theta)$ and $F'(\theta)$ that satisfy Assumptions 1 and 2 and have the same $E[\theta|\theta > X^*]$ and the same X^*. Suppose $F(\theta)$ has a higher deaweight loss than $F'(\theta)$.*

- *If there is overinvestment in information generation under $F'(\theta)$, then there is overinvestment in information generation under $F(\theta)$ as well. The inefficiencies in the information generation stage are more severe under $F(\theta)$ than under $F'(\theta)$.*

- *If there is underinvestment in information generation under $F(\theta)$, there is underinvestment in information generation under $F'(\theta)$ as well. The* *inefficiencies in the information generation stage are more severe under $F'(\theta)$ than under $F(\theta)$.*

Coordination in information generation. As in the case of a monopoly, information generation by one firm imposes a positive externality on the other firm, because of (7). Hence, the firms' joint benefit of information generation exceeds each firm's private benefit. Allowing firms to coordinate in information generation can thus only increase the number of signals drawn, which will increase firms' joint profits. If condition (8) holds, the number of signals drawn in a Nash equilibrium is already higher than socially optimal, and thus coordination decreases social surplus. This observation implies the following corollary.

Corollary 4. *If (8) holds, then coordination in information generation decreases aggregate welfare and consumer surplus.*

If instead (8) does not hold, then similarly to the monopoly case examined above coordination in information generation may increase welfare.

5 Extension: endogenous quality

In our model, information generation is the only way in which firms can achieve vertical differentiation. A relevant question is therefore whether and how the possibility of achieving vertical differentiation via other means changes our results.

In this extension, we address this possibility by allowing firms to costlessly degrade the quality of their product before generating information. A standard result in the literature (see, for example, Tirole, 1988) is that, absent information generation, the quality follower will always degrade its quality as much as possible in order to achieve maximum distance from the quality leader. Quality degradation is observed in some instances,[18] but is far from ubiquitous. In this section we show that information generation may act as a

[18]For example, several producers of electronic devices are known to intentionally reduce the performance and functionality of their products; e.g. the case of IBM printers.

strategic substitute to quality degradation, and therefore explain why quality degradation is rarely observed. This also implies that, when quality is endogenous, information generation has an additional social benefit because it may prevent harmful quality degradation.

Denote by $q_i^0 \in [\underline{s}, \overline{s}]$ firm i's initial quality with the convention that $q_1^0 > q_2^0$. Before the market opens, both firms simultaneously can decrease their expected quality at zero cost to any $q_i \in [\underline{s}, q_i^0]$.[19] Quality degradation is publicly observable. Recall that firms' profits increase in the distance between their expected quality levels. Hence, absent information generation, in the pure strategy Nash equilibrium of a quality degradation game the quality leader will maintain the initial quality $q_1 = q_1^0$, but the follower will degrade as much as possible to $q_2 = \underline{s}$.[20]

Turn now to a quality degradation and information generation game: after deciding whether to degrade its quality each firm can acquire information at a cost k. Introducing information generation may affect the choice quality degradation, because information generation provides an alternative means to increase the quality distance between firms. However, in contrast to degradation, information generation allows for upward revisions of the expected quality as well as downward revisions, increasing the expected highest quality and thus aggregate surplus.

Given that at least one firm generates information, the quality follower's profit can be written as:

$$E[\pi_2(\hat{q}_1, \hat{q}_2)|\sigma] = \left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)} \right) \Delta(\sigma, q_1, q_2) + \pi_2(q_1, q_2). \tag{9}$$

[19] More precisely, firm i can shift downward the quality distribution $F_i(s)$ to achieve any expected quality $q_i \in [\underline{s}, q_i^0]$. As discussed above the "consumption utility" generated by consuming a product s_i is unknown, but the product's technical specifications are publicly known and determine the expectation of s_i. With this interpretation in mind, quality degradation can be achieved by designing a product with worse technical specifications.

[20] The pure-strategy Nash equilibrium in which the quality follower degrades always exists. A second pure-strategy Nash equilibrium, in which the quality leader fully degrades its quality, but the quality follower does not, exists for some q_1^0, q_2^0. In case both equilibria exist, they can be ranked in terms of efficiency, because the welfare loss is smaller when the quality follower degrades than when the quality leader degrades. For ease of exposition, we only discuss the Nash equilibrium in which the quality follower degrades.

Note that $\pi_2(q_1, q_2)$ decreases in q_2, but $\Delta(\sigma, q_1, q_2)$ increases in q_2, and strictly so if $\Delta(\sigma, q_1, q_2) > 0$. Hence, if $\Delta(\sigma, q_1, q_2) = 0$ and information generation has no value, the above result carries over and the quality follower is better off by degrading as much as possible to maximize the distance to the quality leader. When $\Delta(\sigma, q_1, q_2) > 0$, however, it is possible that $E[\pi_2(\hat{q}_1, \hat{q}_2)|\sigma]$ increases in q_2, and hence that there is no incentive to degrade quality. That is, quality degradation and information generation can be alternative ways to achieve vertical differentiation.

To provide a sufficient condition for this case to occur we note that

$$\pi_2(q_1, q_2) = \frac{F(X^*)^2}{f(X^*)}(q_1^0 - q_2),$$

is arbitrarily close to zero whenever X^* is close to $\underline{\theta}$, because in this case the demand for the good sold by the quality follower is arbitrarily small. It is also arbitrarily close to zero if q_1^0 is close to $\underline{s}$, because the maximum distance that can be achieved between quality leader and follower is also arbitrarily small. In either of these cases, we have that

$$E[\pi_2(\hat{q}_1, \hat{q}_2)|\sigma] \approx \left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)}\right)\Delta(\sigma, q_1, q_2),$$

so that the quality follower's profit is strictly positive and strictly increases in q_2, if $\Delta(\sigma, q_1^0, q_2^0)$ is strictly positive, that is, if q_1^0 and q_2^0 are sufficiently close or if σ is sufficiently informative (in the sense of dispersion in the posterior expected qualities). This observation implies the following lemma.

Lemma 5. *For any X^*, there are q_1^0, q_2^0 such that the quality follower will fully degrade quality if no information is generated, but neither firm will degrade quality if information is generated by at least one firm.*

The lemma states that information generation can prevent harmful quality degradation. Indeed, a sufficiently low cost k will guarantee some information generation in equilibrium, which implies the following corollary.

Corollary 5. *There are q_1^0, q_2^0 and k such that information generation and no quality degradation constitute a subgame perfect Nash equilibrium of the* *quality degradation and information generation game.*

Note that, if the case described in Lemma 5 and the following corollary do not apply, the quality follower may partially degrade, even though information is generated in equilibrium. For example, if the signals σ_1 and σ_2 are discrete, then the probability of a quality ranking reversal may be discontinuous in the amount of quality degradation by the quality follower. That is, this probability may be very small when the quality follower degrades by a small amount, but jump discontinuously if the amount of quality degradation passes a given threshold. If, at the same time, the benefit of increasing vertical distance is large, the quality follower may prefer to partially degrade quality. We will not consider this possibility here.

Turning to social welfare, assume the case described in Lemma 5, i.e. information generation prevents quality degradation. The social benefit is:

$$\begin{aligned} &E[S(\hat{q}_1,\hat{q}_2)]-S(q_1^0,\underline{s}) = E[S(\hat{q}_1,\hat{q}_2)]-S(q_1^0,q_2^0)+S(q_1^0,q_2^0)-S(q_1^0,\underline{s}) \\ &= (E[\theta]-2F(X^*)E[\theta|\theta<X^*])\,\Delta((\emptyset,\sigma_i),q_1^0,q_2^0)+(q_2^0-\underline{s})F(X^*)E[\theta|\theta<X^*]. \end{aligned} \tag{10}$$

The first term of this expression is the benefit of information generation given initial quality levels. The second term stems from (5) and is the benefit from preventing quality degradation. It increases in $q_2^0-\underline{s}$ (the amount of quality degradation prevented by generating information), in the market share of the quality follower and in the average valuation of these consumers. The following proposition summarizes these observations.

Proposition 6. *There are q_1^0, q_2^0 and k such that the social benefit of information generation with endogenous quality is strictly greater than the one with exogenous qualities.*

Proof. Immediate from Lemma 5, (10) and the following discussion. □

Since information generation can have an additional social benefit when quality is endogenous rather than exogenous, Proposition 5 may no longer apply. That is, when quality is exogenous, potentially the efficient number of signals is zero, but in equilibrium at least one firm generates information. Under endogenous quality choice the fact that a firm is expected to generate information prevents quality degradation. If the social benefit of preventing quality degradation (given by the second part of 10) is larger than the net social cost of an additional signal (given by the second part of 10 minus k), then one firm generating information is the socially optimal outcome with endogenous quality levels. Similarly, with exogenous quality levels there are situations in which the efficient number of signals is zero, which is also the equilibrium outcome. With endogenous quality levels, however, the absence of information generation leads to quality degradation and may, therefore, be inefficient. The following corollary summarizes these observations.

Corollary 6. *Suppose the case described in Lemma 5 holds. There are cases in which there is over-investment in information generation with exogenous quality, but the efficient level of information generation when quality levels are endogenous. Similarly, there are cases in which there is the efficient level of information generation with exogenous quality, but under-investment in information generation when quality levels are endogenous.*

6 Conclusion

We consider a standard duopoly with vertically differentiated products, and study firms' incentives to generate information. Our main result is that firms will under- or overinvest in information generation, depending on the inefficiencies in the pricing equilibrium. Taste distributions that generate a low deadweight loss in the pricing equilibrium are associated with underprovision of information. Conversely, taste distributions that generate a large deadweight loss are associated with overprovision of information. We also show that information generation has a positive externality on the other firm's profit and thus firms benefit from coordinating their information generation activities. Finally, we introduce the possibility of quality degradation and show that quality degradation and information generation are substitutes for increasing vertical product differentiation. Therefore the possibility of information generation may reduce harmful quality degradation.

This last result implies that there are situations in which information generation should be discouraged if quality levels are exogenous—possibly via a tax—but information generation should be encouraged if quality levels are endogenous—possibly via a subsidy. This insight carries over to whether cooperation and coordination of competing firms is allowed. There are situations in which coordination in information generation should be prevented if quality levels are exogenous, but should be allowed or even encouraged if quality levels are endogenous. This, however, implies that the optimal policy may be time inconsistent, because the policymaker may want to revise the policy after quality levels are set; this is an intriguing question for future research.

Our analysis assumed a covered market: in equilibrium all consumers purchase some product. Removing our Assumption 2 would potentially allow for equilibria in which some consumers do not purchase at all. If firms' quality levels are sufficiently close, however, their profits are close to zero and the market is covered. The logic laid out above continues to apply: information generation by firms is privately valuable, because it increases expected vertical distance and profits. From the social point of view, information generation may cause some consumers to stop consuming, which generates an additional source of inefficiency relative to the case of a covered market considered above. If the initial quality distance between firms is large, so that not all consumers purchase, our results may no longer apply: e.g. firms' benefits from information generation may become negative. A thorough analysis of this case is deferred to future work.

Finally as a natural extension, one could combine Board (2009)'s framework (which we discuss in the Introduction) with ours, that is, consider a model in which firms first generate information and then decide whether to make it public. Although we do not develop such extension here, an insight from our model can nonetheless be helpful. We show that a firm's profits increase with the informativeness of the signal drawn (or with the number of signals drawn). This implies that if a firm could commit to a policy of full disclosure, it will strictly prefer to do so because the possibility to hide some realizations of the signal effectively reduces the informativeness of the signal (from the consumers' viewpoint). We also note that some of the mechanism for information generation discussed earlier have this element of commitment (for example, industry competition or classifications by industry bodies). This full disclosure policy makes the equilibrium of such model identical to the equilibrium in ours. Of course, absent this commitment, the intuition from Board (2009) will be relevant: firms may hide intermediate realizations of the signal so to be considered "bad" and increase vertical differentiation. This in turn reduces the value of information generation. Exploring how this affects the choice of information generation, and the efficiency of the equilibrium, is left for future work.

Appendix

Proof of Lemma 1

Note first that $X = Y = \overline{\theta}$ quickly leads to a contradiction: if both prices are so high that no consumers purchase, then each one of the firms will earn strictly positive payoff by deviating to a small, but positive price, which will attract a positive measure of consumers because $F(\theta)$ is continuous and $\overline{\theta} > 0$.

Suppose that $X = \overline{\theta}$, i.e. the quality leader faces zero demand. Then, by the argument above, $Y < \overline{\theta}$ and the quality follower faces positive demand. This cannot be an equilibrium because the quality leader can set its price equal that of the quality follower, generate positive demand and earn positive profits.

Finally, suppose that $\underline{\theta} < X < \overline{\theta}$, i.e. the quality leader faces positive demand, but does not capture the entire market. Then $Y < X$. To see this suppose the contrary, i.e. $Y = X$. This cannot be an equilibrium because the quality follower will earn strictly positive profits by setting a small, but positive price, which will attract a positive measure of consumers because $F(\theta)$ is continuous and $\overline{\theta} > 0$.

Proof of Lemma 2

The best responses are defined as:

$$
\begin{aligned}
p_L(p_F) &= \text{argmax}_{p_L} \{\pi_L(p_L, p_F)\} \\
p_F(p_L) &= \text{argmax}_{p_F} \{\pi_F(p_L, p_F)\}.
\end{aligned}
$$

We first compute the leader's best response and then move to the follower's best response. As we will see, computing the leader's best response is quite straightforward, while computing the follower's best response is complicated by a kink in the profit function.

Quality leader's best response. Consider first the quality leader's problem. For given p_F the leader can out-price the follower and set $p_L \leq \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F$, so that $X = \underline{\theta}$. In this case the leader serves the entire market and its profit equals p_L. Hence, conditional on $X = \underline{\theta}$ the quality leader maximizes profits by setting $p_L = \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F$. If instead $p_L > \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F$, the leader serves only a fraction of the total market, and $X > Y \geq \underline{\theta}$. Using the definition of X the quality leader's problem becomes:

$$
\max_{p_L \geq \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F} \left\{ p_L \left(1 - F\left(\frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} \right) \right) \right\}.
$$

The first derivative of the objective function is

$$
1 - F(X) - \frac{p_L f(X)}{\hat{q}_L - \hat{q}_F}
$$

or

$$
\left(\frac{1 - F(X)}{f(X)} - \frac{p_L}{\hat{q}_L - \hat{q}_F} \right) f(X),
$$

and equals zero at

$$
p_L = \frac{1 - F(X)}{f(X)} (\hat{q}_L - \hat{q}_F), \tag{11}
$$

which is unique due to log-concavity. Log-concavity also implies that the second derivative of the objective function is negative at $p_L = \frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$. The quality leader's objective function therefore strictly increases for $p_L <$

$\frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$ and strictly decreases for $p_L > \frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$.

Quality follower's best response. We now turn to the quality follower F's best response. We first deal with the trivial case where the leader corners the market. Suppose the quality leader chooses $p_L \leq \underline{\theta}(\hat{q}_L - \hat{q}_F)$. Then, for any p_F, the quality leader covers the entire market and the quality follower's profit is zero for any p_L. Thus the quality follower's best response is

$$p_F(p_L) = [0, \infty) \;\text{ if } p_L \leq \underline{\theta}(\hat{q}_L - \hat{q}_F).$$

This establishes the first part of $p_F(p_L)$ in the lemma.

Suppose now that $p_L > \underline{\theta}(\hat{q}_L - \hat{q}_F)$ instead. Then there are $p_F > 0$ such that the follower has positive demand and profit. Note that the follower's profit function has a kink at price $p_F = \underline{\theta}\hat{q}_F$, but is well-behaved above and below, which allows us to characterize the follower's best response distinguishing the cases of $p_F \leq \underline{\theta}\hat{q}_F$ (in which case $Y = \overline{\theta}$ and the market is covered) and $p_F > \underline{\theta}\hat{q}_F$ (in which case $Y > \overline{\theta}$ and the market is not covered).

Covered market. If $p_F \leq \underline{\theta}\hat{q}_F$, then all consumers purchase one of the goods and $Y = \underline{\theta}$, so that a change in p_F only affects X. Conditional on $Y = \underline{\theta}$, the follower's profit function is

$$\max_{p_F \leq \underline{\theta}\hat{q}_F} \{p_F F(X)\}.$$

The objective function's first derivative is

$$\left(\frac{F(X)}{f(X)} - \frac{p_F}{\hat{q}_L - \hat{q}_F}\right) f(X).$$

The first derivative equals zero at $p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$, which is unique by log-concavity. Again, conditional on $Y = \underline{\theta}$, the follower's profit function is strictly concave at $p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$, which in turns imply that profits conditional on $Y = \underline{\theta}$ are first increasing then decreasing in p_F, reaching a maximum at $p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$.

Non-covered market. If instead $p_F > \underline{\theta}\hat{q}_F$ some consumers will not purchase, so that a change in p_F will affect both X and Y. Conditional on $\underline{\theta} \leq Y < X$, the follower's profit function is now

$$\max_{p_F \geq \underline{\theta}\hat{q}_F} \{p_F(F(X) - F(Y))\}.$$

The objective function's first derivative is

$$F(X) - F(Y) - p_F\left(\frac{f(X)}{\hat{q}_L - \hat{q}_F} + \frac{f(Y)}{\hat{q}_F}\right).$$

Now Condition (A2) becomes useful: it implies that the above expression is always negative, which implies that the quality follower always sets a price so that $Y = \underline{\theta}$ and the market is covered.

To see why, note that 11 implies that $X < \frac{1-F(X)}{f(X)}$ so that $X \leq \theta^*$. Hence, by the definition of m (see Assumption 2) $f(X) < m$ and $f(Y) < m$. Recall that the first order condition for the case $Y > \underline{\theta}$ is

$$F(X) - F(Y) - p_F\left(\frac{f(X)}{\hat{q}_L - \hat{q}_F} + \frac{f(Y)}{\hat{q}_F}\right).$$

Because $F(X) - F(Y) \leq 1$, $p_F > \underline{\theta}\hat{q}_F$ (whenever $Y > \underline{\theta}$), $\frac{\hat{q}_F}{\hat{q}_L} \geq \frac{\underline{s}}{\overline{s}}$, and $f(X), f(Y) \geq m$, the above expression is always smaller than

$$1 - m\underline{\theta}\left(1 - \frac{\underline{s}}{\overline{s}}\right)^{-1},$$

which is negative under (A2). Hence the first order condition for the case $Y > \underline{\theta}$ is always negative, and the quality follower is always better off by setting p_F such that $Y = \underline{\theta}$.

Hence, $p_F > \underline{\theta}\hat{q}_F$ cannot occur, implying the second part of $p_F(p_L)$ in the lemma.

Proof of Proposition 1

As a preliminary step, note that (A2) is equivalent to

$$\frac{1}{m\underline{\theta}} - 1 \leq \left(\frac{\overline{s}}{\overline{s} - \underline{s}}\right) - 1 \Leftrightarrow \left(\frac{1}{m\underline{\theta}} - 1\right)\left(\frac{\overline{s}}{\underline{s}} - 1\right) \leq 1. \tag{12}$$

We prove each part of the proposition separately.

(i) Recall the quality leader's best reply as derived above:

$$p_L(p_F) = \max\left\{\frac{1 - F(X)}{f(X)}(\hat{q}_L - \hat{q}_F), \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F\right\}.$$

By log-concavity $\frac{1-F(X)}{f(X)}$ decreases and is thus maximal for $x = \underline{\theta}$. Hence, if

$$\left(\underline{\theta} - \frac{1}{f(\underline{\theta})}\right)(\hat{q}_L - \hat{q}_F) + p_F \geq 0$$

the quality leader's captures the entire market. If $p_F > 0$, this cannot be an equilibrium because the quality follower should lower its price and earn positive profits. If instead $p_F = 0$ and $p_L = \underline{\theta}(\hat{q}_L - \hat{q}_F)$ then no firm can make a profitable deviation, and these prices constitute a Nash equilibrium. If $\underline{\theta} f(\underline{\theta}) \geq 1$, therefore, in equilibrium the leader captures the entire market.

(ii) Suppose instead $1 > \underline{\theta} f(\underline{\theta})$ from now on. The observations made in the text above imply that in this case the quality leader's best reply to $p_F = 0$ is $p_L = \frac{1-F(p_L/(\hat{q}_L-\hat{q}_F))}{f(p_L/(\hat{q}_L-\hat{q}_F))}(\hat{q}_L - \hat{q}_F)$. Hence, by Lemma 1 the Nash equilibrium will necessarily have $X > \underline{\theta}$ (implying $f(X) > 0$) and $p_F = \min\left\{\frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F), \underline{\theta}\hat{q}_F\right\} > 0$. Therefore there are two possible cases, depending on whether the quality follower's best response is a corner solution ($p_F = \underline{\theta}\hat{q}_F$) or an interior solution ($p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$).

Suppose first that the quality follower's best response has an interior solution:

$$\frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F) \leq \underline{\theta}\hat{q}_F, \tag{13}$$

so that $p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$. In this case, by definition of X, the equilibrium cutoff X solves

$$X = \frac{1 - 2F(X)}{f(X)}. \tag{14}$$

This equation has a unique solution because, by log concavity, its RHS is decreasing in X and we have assumed $1 > \underline{\theta} f(\underline{\theta})$. This constitutes a Nash equilibrium if indeed the solution X of equation (14) satisfies condition (13). Condition (13) can be rewritten as

$$\frac{F(X)}{f(X)\underline{\theta}} \left(\frac{\hat{q}_L}{\hat{q}_F} - 1 \right) \leq 1.$$

Note that $\frac{\hat{q}_L}{\hat{q}_F}$ is at most $\frac{\overline{s}}{\underline{s}}$, and that by (14) $\frac{F(X)}{f(X)} = \frac{1}{2}\left(\frac{1}{f(X)} - X \right)$, which is at most $\frac{1}{2}\left(\frac{1}{m} - \underline{\theta} \right)$.[21] Therefore,

$$\frac{F(X)}{f(X)\underline{\theta}} \left(\frac{\hat{q}_L}{\hat{q}_F} - 1 \right) < \frac{1}{2} \left(\frac{1}{m\underline{\theta}} - 1 \right) \left(\frac{\overline{s}}{\underline{s}} - 1 \right) < 1,$$

where the last inequality follows by (12). Hence, (13) holds and thus $p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$ and $p_L = \frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$, with X defined implicitly by (14) is a Nash equilibrium.

To conclude the proof, we show that there is no equilibrium in which $1 > \underline{\theta} f(\underline{\theta})$, and hence the quality leader's best response has an interior solution:

$$p_L = \frac{1 - F(X)}{f(X)}(\hat{q}_L - \hat{q}_F),$$

but at the same time (13) is violated, and hence the quality follower's best response has a corner solution:

$$p_F = \underline{\theta} \hat{q}_F.$$

[21] Here we make use again of a fact established in the proof of Lemma 2 (see its last paragraph): that $f(X) \geq m$.

If such equilibrium exists, then by definition

$$X = \frac{1 - F(X)}{f(X)} - \underline{\theta}\left(\frac{\hat{q}_L}{\hat{q}_F} - 1\right)^{-1} \tag{15}$$

This is consistent with a Nash equilibrium if indeed for this X (13) is violated.

Note that by (15) $\frac{F(X)}{f(X)}$ is smaller than $\frac{1}{f(X)} - X$ which, in turn, is smaller than $\frac{1}{m} - \underline{\theta}$. Also, $\frac{\hat{q}_L - \hat{q}_F}{\hat{q}_F}$ must be smaller than $\left(\frac{\overline{s}}{\underline{s}} - 1\right)$. It follows that

$$\frac{F(X)}{f(X)\underline{\theta}}\left(\frac{\hat{q}_L - \hat{q}_F}{\hat{q}_F}\right) \leq \left(\frac{1}{m\underline{\theta}} - 1\right)\left(\frac{\overline{s}}{\underline{s}} - 1\right) \leq 1,$$

where the last inequality follows by (12). Hence, (13) must hold and there cannot be a Nash equilibrium with $p_F = \underline{\theta}\hat{q}_F$.

Proof of Lemma 3

Suppose that no firm acquires information; then $\hat{q}_i = q_i$ for $i = 1, 2$ and (by assumption) firm 1 is the quality leader. The ex ante expected social welfare is then $S(q_1, q_2) = E[\theta]\, q_1 = E[\theta]\, E[\hat{q}_1|\sigma]$, where the last equality follows from the law of iterated expectation and holds for any σ. The social benefit of acquiring information is therefore given by the difference between expected social welfare given a chosen signal configuration σ and expected social welfare when no information is acquired:

$$\begin{aligned}
E[S(\hat{q}_1, \hat{q}_2)|\sigma] - S(q_1, q_2) &= E[\theta]\, E[\max\{\hat{q}_1, \hat{q}_2\}|\sigma] - E[\theta]\, E[\hat{q}_1|\sigma] \\
&= E[\theta]\{E[\hat{q}_1|\hat{q}_1 \geq \hat{q}_2, \sigma]\mathrm{pr}\{\hat{q}_1 \geq \hat{q}_2|\sigma\} + E[\hat{q}_2|\hat{q}_2 \geq \hat{q}_1, \sigma]\mathrm{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma\} \\
&\quad - E[\hat{q}_1|\hat{q}_1 \geq \hat{q}_2, \sigma]\mathrm{pr}\{\hat{q}_1 \geq \hat{q}_2|\sigma\} - E[\hat{q}_1|\hat{q}_2 \geq \hat{q}_1, \sigma]\mathrm{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma\}\} \\
&= E[\theta]\, E[\hat{q}_2 - \hat{q}_1|\hat{q}_2 \geq \hat{q}_1, \sigma]\mathrm{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma\} \equiv E[\theta]\, \Delta(\sigma, q_1, q_2).
\end{aligned}$$

The proposition follows by writing

$$E[S(\hat{q}_1, \hat{q}_2)|\sigma''] - E[S(\hat{q}_1, \hat{q}_2)|\sigma'] = (E[S(\hat{q}_1, \hat{q}_2)|\sigma''] - S(q_1, q_2)) + (S(q_1, q_2) - E[S(\hat{q}_1, \hat{q}_2)|\sigma'])$$

Proof of Proposition 3

We distinguish three cases:

1. It is socially optimal to generate no information, that is

   $$2k > E[\theta]\Delta((\sigma_1, \sigma_2), q_1, q_2) \text{ and } k > E[\theta]\Delta((\emptyset, \sigma_i), q_1, q_2) \quad i \in \{1, 2\}.$$

   By Proposition 2 each firm's best reply to the other firm not generating information is to not generate information either. Likewise, each firm i's best reply to the other firm $-i$ generating information is not to generate information, if

   $$k \geq \underline{\theta} \left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_{-i}), q_1, q_2) \right),$$

   which is always true, because

   $$\begin{aligned} &\underline{\theta}(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)) \\ &\leq E[\theta]\Delta((\sigma_1, \sigma_2), q_1, q_2) - \underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2) \leq 2k - \underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2) \leq k. \end{aligned}$$

   Hence, in the case when no information generation is socially optimal there is a unique Nash equilibrium, in which neither firm generates any information.

2. It is socially optimal for firm i to generate information, but not firm $-i$, that is

   $$\begin{aligned} &E[\theta]\Delta((\emptyset, \sigma_{-i}), q_1, q_2) \leq E[\theta]\Delta((\emptyset, \sigma_i), q_1, q_2) \equiv \hat{k}_1 \text{ and} \\ &\hat{k}_4 \equiv E[\theta](\Delta((\sigma_1, \sigma_2) q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)) < k < E[\theta]\Delta((\emptyset, \sigma_i), q_1, q_2). \end{aligned} \tag{16}$$

   The second inequality immediately implies

   $$k > \underline{\theta} \left[\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2) \right].$$

   This means that if firm i generates information, then firm $-i$'s best reply is to not generate information. Hence, there is no Nash equilibrium, in which both firms generate information.

Suppose that

$$k > \underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2) \equiv \hat{k}_0. \tag{17}$$

Then neither firm finds it profitable to generate information if the other firm does not. Hence, in the unique Nash equilibrium there is no information generation.

If instead

$$\underline{\theta}\Delta((\emptyset, \sigma_{-i}), q_1, q_2) < k \leq \underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2),$$

then there is a unique equilibrium in which firm i generates information. Finally, if

$$k \leq \underline{\theta}\Delta((\emptyset, \sigma_{-i}), q_1, q_2) \equiv \hat{k}_5,$$

then there are multiple equilibria, in which each firm may generate information, while the other one does not. In one of these equilibria firm $-i$ generates information, but not firm i. This is inefficient, because by assumption $\Delta((\emptyset, \sigma_i), q_1, q_2) < \Delta((\emptyset, \sigma_{-i}), q_1, q_2)$, i.e. firm i's signal generates more information (as measured by the dispersion of the posteriors) and higher social welfare than firm $-i$'s signal.

3. It is socially optimal for both firms to generate information, that is

$2k \leq E[\theta]\Delta((\sigma_1, \sigma_2), q_1, q_2)$ and
$E[\theta]\Delta((\sigma_1, \sigma_2), q_1, q_2) - 2k \geq E[\theta]\max\{\Delta((\emptyset, \sigma_1), q_1, q_2), \Delta((\sigma_1, \sigma_2), q_1, q_2)\} - k.$

That is, the net social benefit of drawing both signals is positive, and exceeds the net social benefit of drawing either individual signal. The above inequalities can be rewritten as

$$k \leq E[\theta] \left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \right.$$
$$\left. \max\left\{\frac{1}{2}\Delta((\sigma_1, \sigma_2), q_1, q_2), \Delta((\emptyset, \sigma_1), q_1, q_2), \Delta((\emptyset, \sigma_2), q_1, q_2)\right\}\right) \equiv \hat{k}_3.$$

A necessary condition for both firms to generate information in a Nash equilibrium (including the case of multiple equilibria) is

$$k < \underline{\theta} \left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right),$$

for both firms $i = 1, 2$, or

$$k \leq \underline{\theta} \left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \max\left\{\Delta((\emptyset, \sigma_1), q_1, q_2), \Delta((\emptyset, \sigma_2), q_1, q_2)\right\}\right) \equiv \hat{k}_2.$$

Therefore, if

$$k > \hat{k}_2 \text{ and } k \leq \hat{k}_3, \tag{18}$$

then the number of signals drawn in a Nash equilibrium is strictly less than in the social optimum. Otherwise, there will be a (possibly unique) Nash equilibrium that is efficient.

We therefore established that the number of signals drawn in equilibrium is always smaller than the socially optimal number of signals, strictly so if either both conditions (16) and (17) hold, or both conditions in (18) hold. Note also that, in both cases, the set of such k for which fewer signals than optimal are drawn expands with $E[\theta] - \underline{\theta}$ and with the first difference of $\Delta(.)$.

We also established the possibility of a coordination failure: when the efficient number of signals is 1, either firm generating one signal may be a Nash equilibrium, and in particular only the firm with the less informative signal generating information may be an equilibrium, which is inefficient.

Proof of Lemma 4

Simply note that social welfare can be written as

$$\begin{aligned}
S(\hat{q}_1, \hat{q}_2) &= \max\{\hat{q}_1, \hat{q}_2\}E[\theta] - |\hat{q}_1 - \hat{q}_2|F(X^*)E[\theta|\theta < X^*] \\
&= \max\{\hat{q}_1, \hat{q}_2\}E[\theta] - (2\max\{\hat{q}_1, \hat{q}_2\} - \hat{q}_1 - \hat{q}_2)\, F(X^*)E[\theta|\theta < X^*] \\
&= \max\{\hat{q}_1, \hat{q}_2\}\left(E[\theta] - 2F(X^*)E[\theta|\theta < X^*]\right) + (\hat{q}_1 + \hat{q}_2)\, F(X^*)E[\theta|\theta < X^*].
\end{aligned}$$

The statement follows from the same derivations detailed in the proof of Lemma 3.

Proof of Proposition 4

For given σ, write

$$\begin{aligned}
E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma] = &\frac{(1 - F(X^*))^2}{f(X^*)}\mathrm{pr}\{\hat{q}_i \geq \hat{q}_{-i}|\sigma\}E[\hat{q}_i - \hat{q}_{-i}|\hat{q}_i \geq \hat{q}_{-i}, \sigma''] + \\
&\frac{F(X^*)^2}{f(X^*)}\mathrm{pr}\{\hat{q}_{-i} \geq \hat{q}_i|\sigma\}E[\hat{q}_{-i} - \hat{q}_i|\hat{q}_{-i} \geq \hat{q}_i, \sigma]
\end{aligned}$$

Suppose $\hat{q}_i \geq \hat{q}_{-i}$. By using the law of iterated expectation, write

$$\begin{aligned}
&\pi_i(q_i, q_{-i}) = E\left[\hat{q}_i - \hat{q}_{-i}|\sigma\right]\frac{(1 - F(X^*))^2}{f(X^*)} = \\
&\frac{(1 - F(X^*))^2}{f(X^*)}\left(\mathrm{pr}\{\hat{q}_i \geq \hat{q}_{-i}|\sigma\}E[\hat{q}_i - \hat{q}_{-i}|\hat{q}_i \geq \hat{q}_{-i}, \sigma] + \mathrm{pr}\{\hat{q}_{-i} \geq \hat{q}_i|\sigma\}E[\hat{q}_i - \hat{q}_{-i}|\hat{q}_{-i} \geq \hat{q}_i, \sigma]\right)
\end{aligned}$$

Similarly, if $\hat{q}_i \leq \hat{q}_{-i}$ write

$$\begin{aligned}
&\pi_i(q_i, q_{-i}) = E\left[\hat{q}_i - \hat{q}_{-i}|\sigma\right]\frac{F(X^*)^2}{f(X^*)} = \\
&\frac{F(X^*)^2}{f(X^*)}\left(\mathrm{pr}\{\hat{q}_i \geq \hat{q}_{-i}|\sigma\}E[\hat{q}_i - \hat{q}_{-i}|\hat{q}_i \geq \hat{q}_{-i}, \sigma] + \mathrm{pr}\{\hat{q}_{-i} \geq \hat{q}_i|\sigma\}E[\hat{q}_i - \hat{q}_{-i}|\hat{q}_{-i} \geq \hat{q}_i, \sigma]\right)
\end{aligned}$$

Both when $\hat{q}_i \geq \hat{q}_{-i}$ and when $\hat{q}_i \leq \hat{q}_{-i}$ we can then write

$$E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma] - \pi_i(q_i, q_{-i}) = \left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)\Delta(\sigma, q_1, q_2)$$

Writing

$$
\begin{aligned}
&E[\pi_i(\hat{q}_i,\hat{q}_{-i})|\sigma''] - E[\pi_i(\hat{q}_i,\hat{q}_{-i})|\sigma'] = \\
&\quad (E[\pi_i(\hat{q}_i,\hat{q}_{-i})|\sigma''] - \pi_i(q_i,q_{-i})) - (E[\pi_i(\hat{q}_i,\hat{q}_{-i})|\sigma'] - \pi_i(q_i,q_{-i})) = \\
&\quad \left(X^*+2\frac{F(X^*)^2}{f(X^*)}\right)(\Delta(\sigma'', q_1,q_2)-\Delta(\sigma', q_1,q_2)) \,,
\end{aligned}
$$

Concludes the proof.

Proof of Proposition 5

The pure strategy Nash equilibria of the information generation game for the case of a duopoly are similar to the ones derived for the case of a monopoly, modulo the different expression for the private benefit of information generation. We have:

- If $k > \left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)(\Delta(\sigma_1,\sigma_2), q_1, q_2) - \Delta((\emptyset,\sigma_i), q_1, q_2))$ and $\left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)\Delta((\emptyset,\sigma_i), q_1, q_2) \geq k$ for at least one $i \in \{1,2\}$, then there is an equilibrium in which only firm i generates information.
- if $k \leq \left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)(\Delta((\sigma_1,\sigma_2), q_1, q_2) - \Delta((\emptyset,\sigma_i), q_1, q_2))$ and $\left(X^* + 2\cdot\frac{F(X^*)^2}{f(X^*)}\right)\Delta((\emptyset,\sigma_i), q_1, q_2) \geq k$ for at least one $i \in \{1,2\}$, then there is a unique equilibrium in which both firms generate information.
- if $k \leq \left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)(\Delta((\sigma_1,\sigma_2), q_1, q_2) - \Delta((\emptyset,\sigma_i), q_1, q_2))$, but $\left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)\Delta((\emptyset,\sigma_i), q_1, q_2) \leq k$ for both $i = 1,2$, then there are multiple equilibria: one in which no firm generates information, and one in which both firms generate information.
- Otherwise there is no information generation in equilibrium.

We follow the structure of the proof of Proposition 3 and consider different cases. For ease of notation let us define the social value of information generation as

$$S \cdot \Delta(\sigma, q_1, q_2) \equiv (E[\theta] - 2F(X^*)E[\theta|\theta < X^*])\,\Delta(\sigma, q_1, q_2),$$

and the private value of information generation as

$$P \cdot \Delta(\sigma, q_1, q_2) \equiv \left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)\Delta(\sigma, q_1, q_2).$$

Condition (8) implies that $P > S$, so that the private benefit of information generation is higher than the social benefit. We distinguish three cases.

1. It is socially optimal to have no information generation, that is

   $$k > S\Delta((\emptyset, \sigma_i), q_1, q_2) \text{ and } 2k > S\Delta((\sigma_1, \sigma_2), q_1, q_2).$$

   At least one firm i will invest if $k < P\Delta((\emptyset, \sigma_i), q_1, q_2)$, and thus the number of signals generated in equilibrium is higher than socially optimal if

   $$\begin{aligned}\hat{k}_0 \equiv S\max\{\Delta((\emptyset, \sigma_i), q_1, q_2), \Delta((\sigma_1, \sigma_2), q_1, q_2)\} &< k \\ &< P\Delta((\emptyset, \sigma_i), q_1, q_2) \equiv \hat{k}_1.\end{aligned}$$

   Otherwise, if the above condition does not hold, there may be an equilibrium, in which both firms invest, if

   $$k \leq P\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right) \quad \forall i \in \{1, 2\}.$$

   In this case there are, however, multiple equilibria: one with both firms investing and one with neither firm investing.

2. It is socially optimal for firm i to generate information but not firm $-i$, that is

   $$S\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right) < k < S\Delta((\emptyset, \sigma_i), q_1, q_2) \text{ and}$$
   $$\Delta((\emptyset, \sigma_{-i}), q_1, q_2) < \Delta((\emptyset, \sigma_i), q_1, q_2).$$

Note that this case can only occur if $\Delta(.)$ has strictly decreasing differences in σ. Since $P > S$, at least one firm will invest in any Nash equilibrium, so the number of signals is at least the socially optimal one. For both firms to invest to be the unique Nash equilibrium it is necessary that

$$k < P\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right) \text{ and } k < P\Delta((\emptyset, \sigma_i), q_1, q_2),$$

Since $S < P$ the second condition holds. Hence, both firms will invest and there will be overinvestment if

$$\begin{aligned}\hat{k}_2 \equiv S\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right) < k \\ < P\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right) \equiv \hat{k}_3.\end{aligned}$$

If the above condition is violated, but

$$\hat{k}_4 \equiv P\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right) < k < P\Delta((\emptyset, \sigma_{-i}), q_1, q_2) \equiv \hat{k}_5,$$

then in equilibrium only one firm invests. If firm i invests then the equilibrium is efficient. If firm $-i$ invests, then the equilibrium is inefficient. In this last case, in equilibrium the information generated in equilibrium is *less* then the social optimum, because the firm with the least informative signal generates information in equilibrium.

3. It is socially optimal for both firms to generate information, that is

$$2k < E[\theta]\Delta((\sigma_1, \sigma_2), q_1, q_2) \text{ and}$$
$$E[\theta]\Delta((\sigma_1, \sigma_2), q_1, q_2) - 2k > E[\theta]\max\{\Delta((\emptyset, \sigma_1), q_1, q_2), \Delta((\emptyset, \sigma_2), q_1, q_2)\} - k.$$

A necessary and sufficient condition for a Nash equilibrium, in which both firms generate information, is:

$$k < P\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right),$$

for both firms $i = 1, 2$. Because $P > S$, there is always an equilibrium in which both firms generate information. Of course, there may also be another equilibrium, in which no firm generates information. But, as discussed in the text, when both equilibria are present the one in which both firms generate information Pareto dominates the other.

By restricting our attention to equilibria that are not Pareto dominated, we established that the number of signals drawn in equilibrium is always above the efficient one, strictly so in some cases. Also here, there is the possibility that the efficient number of signals is one, which is also the equilibrium one, but the "wrong" firm generates information in equilibrium.

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