Focusing Effect and the Poverty Trap*

Andrea Canidio†

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Abstract

I build a dynamic consumption-savings model in which agents' choices are distorted by the *focusing effect*: agents overweight the utility of goods in which their options differ more. I show that the consumption-savings choice depends both on the marginal return on savings and on the *total return* on savings, implying that the incentive to save may increase with the initial level of wealth. As a consequence, a salience-based poverty trap may exist when the marginal return on savings is sufficiently high and sufficiently flat. I also consider the case of a perfect credit market and show that a poverty trap may emerge when the salience of consumption is bounded above. I discuss policy implications. In particular, imposing upon an agent a punishment for decreasing savings below a threshold leads to a higher level of savings, even when the threshold triggering the punishment is not binding.

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†Economics and Political Science Area, INSEAD, Boulevard de Constance, 77300 Fontainebleau, France; email: andrea.canidio@insead.edu.

1 Introduction

The theoretical literature on poverty traps argued that poor households remain poor because they lack high marginal-return investment opportunities. If poor households can invest in high marginal-return projects, they will rapidly accumulate wealth and catch up with richer households over time.[1] If instead these projects are not available to poor people but only to richer people, then poor people will remain poor indefinitely.[2] However, convincing empirical evidence shows that poor households often have profitable investment opportunities available, but decide not to invest. Poor households routinely forgo investments such as buying a mosquito net, vaccinating their children (see Banerjee and Duflo, 2011), using fertilizers (see Duflo, Kremer, and Robinson, 2011), switching to more valuable crops (see Udry and Anagol, 2006), or keeping spare change for their small businesses (see Beaman, Magruder, and Robinson, 2013). Most of these investments yield returns of 50-100% and are divisible (e.g. fertilizer), allowing for arbitrarily small investment levels. This evidence raises the suspicion that, contrary to what argued by the theoretical literature, poor households remain poor not because they lack investment opportunities but because behavioral biases prevent them from exploiting these opportunities.

Two basic observations can help explain why poor people fail to exploit profitable investment opportunities. First, high marginal-return investments produce extremely low *total* returns when the scale of the investment is small. Hence, a poor person facing a very high marginal-return investment opportunity may still earn a very low total return on her investment. Second, the behavioral literature long argued that the *focusing effect* can cause people to overlook small gains. The focusing effect (or focusing illusion) occurs whenever an agent places too much or too little importance on certain aspects of her choice set or on certain pieces of information (i.e. certain elements are more *salient* than others). Several papers demonstrate that the focusing effect distorts people's choices. Most famously, Schkade and Kahneman (1998) show that when asked about comparing life in California and in the Midwest, most people report California as the best place to live, citing the weather - i.e. the dimension in which the two choices differ the most - as the main reason. Despite this, actual measures of life satisfaction in the two regions are similar, implying that the Midwest is better than California in some dimensions other than the weather. However, the difference between the two regions in these other dimensions is too small to be salient, causing people to ignore them when evaluating the two options.[3]

The goal of this paper is to evaluate whether the focusing effect can be a cause for poverty even in the presence of high marginal-return investment opportunities. To do so, I introduce the

[1] This mechanism is at the center of the neoclassical growth models, starting from the Solow model and the Ramsey–Cass–Koopmans model.

[2] See the literature on occupational choice started by Banerjee and Newman (1993) and Galor and Zeira (1993).

[3] See also Kahneman, Krueger, Schkade, Schwarz, and Stone (2006), who show that people place too much weight on differences in monetary compensation when comparing employment contracts. The interpretation is that the non-monetary components of each employment contract do not vary much across contracts and are therefore not salient.

focusing effect as modeled by Kőszegi and Szeidl (2013) into a dynamic consumption-savings model. I show that the salience of future consumption depends on the *total* return on savings. Hence, a \$1 investment with a 100% return is less salient than a \$100,000 investment with a 1% return.[4] As a consequence, the salience of future consumption is higher for wealthier agents than for poorer ones, because wealthier agents can invest at a higher scale and generate a higher total returns. The main result I derive is that the poverty trap may emerge when the marginal return on savings is very high, and may disappear when the marginal return on savings is low. If the marginal return on savings is very high, then the *total* return on savings increases rapidly with the scale of the investment. It follows that the salience of future consumption relative to current consumption increases with the agent's initial wealth, and a poverty trap may emerge. For example, if fertilizer delivers an approximately constant percentage return at different investment levels, then a poverty trap is possible. A poor agent and a relatively richer agent both invest in fertilizer, but the total return on investment is lower for the poor agent. It follows that the salience of investing—and the incentive to invest—is lower for the poor agent than for the richer one. These differences in the initial incentive to save may lead, in the long run, to different steady-state wealth levels. Instead, if the marginal return on savings is low, a poverty trap can emerge only under much more stringent conditions. In other words, the presence of high marginal-return investment opportunities can actually be a cause for poverty and inequality.

Formally, I build a model in which agents live for two periods. In the first period, they consume and save. In the second period, they consume and leave bequests. Agents' utility depends on their consumption and on the bequests left to the following generation. The utility from present and future consumption and the utility from bequests are discounted by *focus weights*. Following Kőszegi and Szeidl (2013), the focus weight on a specific component of the utility function increases in the maximum utility achievable in that component within the agent's choice set. As a consequence, if the total return on savings increases, then the future will be more salient because the maximum achievable level of future consumption—and the maximum achievable future utility—increases. In addition, the salience of present consumption, future consumption and bequests increase with the level of wealth available at the beginning of life, but the relative speed at which the different focus weights grow depends on the return on savings.

As a consequence, the focusing effect introduces a wealth-dependent discount factor in the form of a *focus wedge*, which is defined as the salience of future consumption relative to present consumption. I assume that the marginal return on investment is monotonically decreasing, arbitrarily large for a very small investment, and arbitrarily small for a very large investment, as in the neoclassical growth

[4] The fact that consumers' and firms' decisions depend on total (or average) costs or benefits is a very old observation. For example, Hall and Hitch (1939) claim that firms base their prices on total cost. See Machlup (1967) for a summary of the debate between proponents of the "marginalist" and proponent of the "behavioral" view of the firm. More recently, Ito (2014) shows that consumers react to average prices and not to marginal prices. Here, decisions will be distorted by the maximum total return achievable, i.e. the total return in case the agent saves all her initial wealth.

models. I derive sufficient conditions for the focus wedge to increase with an agent's initial wealth, involving the strength of the focusing effect at different utility levels, the utility function, and the return on investment. I show that multiple steady states are possible when the marginal return on investment is sufficiently flat and high. The incentive to save at different wealth levels is determined by the focus wedge—which may be increasing with initial wealth—and by the marginal return on savings—which is always decreasing in the level of savings. If the marginal return is almost flat and high, multiple steady states may exist because salience is the main determinant of the incentive to save at different wealth levels. The model has a unique per-period equilibrium, so that different steady states are reached depending on the initial wealth level. Hence, the observation that poor people have high marginal-return projects available is consistent with the existence of a poverty trap, provided that wealthier agents can access the same high-marginal return projects available to poorer agents.

I also consider the case of a perfect credit market by assuming that agents can freely borrow and lend at a given interest rate. This case is interesting because the marginal return on savings is now given by the market interest rate, which may be low. I show that a poverty trap is possible also in this case, but only under stronger conditions. The driving force is the shape of the utility function. If the utility function is bounded above, the salience of consumption (present or future) is bounded above. Hence, as wealth increases the difference between the salience of present consumption and future consumption decreases. The distortion in the consumption-saving decision introduced by the focusing effect is less relevant for rich agents than for poor agents, implying that rich agents save more than poor agents do.

The first obvious policy implication is that a one-off wealth transfer may have long-term effects because it may push the economy to a higher steady state. A second, more interesting, aspect of the model is that any policy that alters the agent's choice set even temporarily may have a permanent effect, irrespective of whether the agent chooses any of the new options introduced by the policy or whether the policy constrains the agent's choice. For example, a nonlinear subsidy to savings may increase the savings rate even if the agent does not benefit from the subsidy. A one-off tax on consumption may make savings relatively more salient, even if the agent does not bear the cost of the tax.

Finally, I allow agents to manipulate their choice set and future preferences using a commitment-saving device, which imposes a punishment if the stock of savings drops below a given threshold. This type of commitment devices are widespread and commonly used in the developing world. For example, in ROSCAS the penalties for not saving enough are harsh social sanctions and social exclusion. Similarly, poor households save by borrowing at very high interest rates from money lender and MFIs, who deliver a punishment if there is no repayment.[5] The typical explanation for the value of commitment relies on hyperbolic discounting (see Laibson, Repetto, and Tobacman,

[5] Ananth, Karlan, and Mullainathan (2007) document that in several parts of the world microentrepreneurs borrow at daily rates of around 10%, for several years in a row.

2003). Here instead, commitment decreases the salience of present consumption, increases the value of future consumption and allows for asset accumulation. Also, a model in which choices are distorted by the focusing effect delivers a specific empirical prediction: commitment-savings devices should have an impact on savings *even when they are non-binding*. This implies that, for example, agents who adopt commitment-savings devices may increase their savings above the minimum amount imposed by the commitment device.

The remainder of the paper is structured as follows. The next subsection describes the relevant literature. In Section 2 I introduce the Kőszegi and Szeidl (2013) model of focusing, and apply it to a simple two-period consumption-savings problem. I assume that technology is linear and derive conditions under which the savings function is convex in initial wealth.[6] In Section 3 I expand the simple two-period model to an infinite horizon, consider a generic concave technology, and show that a poverty trap is possible. In Section 4 I consider the case of perfect credit markets. In Section 5 I analyze the effect of imposing a commitment savings device on an agent with preferences distorted by the focusing effect. Section 6 concludes. All the proofs are in appendix.

1.1 Relevant Literature

In the economic literature, the focusing effect is called *salience*, *focusing*, and *attention* (or *inattention*.) Different strands of literature use different words, reflecting different ways of formalizing this concept.

One branch of literature proposes models in which the focusing effect is embedded directly into the agent's preferences. These are models in which the decision maker's attention is unconsciously and automatically drawn toward certain elements of her choice set, that are therefore overvalued when making choices. For my purposes, the most relevant works using this framework are Kőszegi and Szeidl (2013) (who call this distortion *focusing*), and Bordalo, Gennaioli, and Shleifer (2013) (who call this distortion *salience*). Kőszegi and Szeidl (2013) assume that the variation in utility levels achievable within the choice set affects the agent's choice, and pushes the agent to overvalue the goods (or goods attributes) that vary the most within the choice set. Bordalo, Gennaioli, and Shleifer (2013) assume that agents overvalue the goods that differ the most with respect to a reference point. Kőszegi and Szeidl (2013) is well suited to analyze how wealth accumulation affects the salience of present and future consumption, because wealth accumulation leads to an expansion of the choice set. Performing the same analysis using the Bordalo, Gennaioli, and Shleifer (2013) approach requires first to establish how wealth accumulation affects the agent's reference point.

Beside the different modeling choices, Bordalo, Gennaioli, and Shleifer (2013) depart from Kőszegi and Szeidl (2013) (and, therefore, from the model presented here) in the relevance given to the *Weber–Fechner law* of human perception and to *diminishing sensitivity*. The Weber–Fechner

[6] A convex savings function may have multiple fixed points and therefore generate multiple steady states when embedded in an infinite-horizon consumption savings model. See Moav (2002) for a treatment of how convex savings function can generate poverty traps.

law states that the perception of a stimulus grows less than proportionally with the intensity of a stimulus, and can be incorporated in the framework presented here by assuming that the focus function is concave (see page 10). Diminishing sensitivity is related to the observation that a given change in price is more salient when applied to an object having a low starting price than when applied to an object having a high starting price, and is one of the founding assumptions of Bordalo, Gennaioli, and Shleifer (2013). For example, a $2 discount on the price of a cookie box is more salient than a $2 discount on the price of a TV. In a consumption-savings setup, diminishing sensitivity determines how the salience of a given change in the total return on investment depends on the level of the investment, i.e. whether an extra return of $2 is more or less salient when the initial investment is $10 or $20. Here, instead, the starting point of the analysis is establishing how, for *given level of investment,* the salience of future consumption changes as total return on investment increases by $2, $3, $4 dollars and so on. The key observation is that, according to the Kőszegi and Szeidl (2013) model, everything else equal the salience of future consumption should increase with the return on investment. For this reason, I largely abstract away from diminishing sensitivity.[7]

A second strand of literature models the focusing effect as a rational choice given some information-absorption constrains. In other words, agents know that they can use only a given "amount of information", and they choose strategically what variables to consider when making a decision (hence the name *rational inattention*, see Sims, 2003). Recently, Gabaix (2014) develops a model of limited attention in which agents reduce the complexity of a decision problem by ignoring some signals. Similarly to my model, a signal is more likely to be ignored if the variance of this signal is low. However, here salience is defined over utility-relevant outcomes rather than signals. Also, dimensions are never fully neglected or fully considered, but rather change their salience as the set of possible outcomes expands or shrinks.

Within the rational-inattention literature, Banerjee and Mullainathan (2008) show that when attention is a scarce resource a poverty trap is possible. In their model, attention can be employed either in production (where it reduces mistakes) or at home (where it solves problems). Crucially, attention and home consumption are substitutes at home, while attention and human capital are complements at work. High-human capital agents devote attention at work and generate high income, while low-human capital agents devote attention at home generating low income. If income level and human capital of offspring are correlated (e.g. because of credit market imperfections), then this mechanism delivers an attention-based poverty trap. Contrary to Banerjee and Mullainathan (2008), in the model presented here attention is not an input in the production functions, but it enters directly into the utility function. However, I show in the Appendix that my model can be interpreted as the reduced form of a costly attention model. The key assumption is that the cost of attention depends on the size of the choice set, and that, as the choice set changes, the agent optimally puts more attention to the dimension that changed the most.

[7] See Section B of the Appendix for conditions under which diminishing sensitivity holds in this context as well.

Some authors have shown that behavioral biases are worsened by poverty and can generate behavioral poverty traps. Bernheim, Ray, and Yeltekin (2013) analyze a growth model with time-inconsistent agents. They consider all possible equilibrium consumption-savings paths, and show that the one with highest wealth accumulation increases with wealth. Agents can commit to a high level of savings by employing a *personal rule*: if any past self deviated from the equilibrium, then the present self consumes as much as possible (i.e. as much as allowed by an equilibrium). The punishment that follows a deviation therefore increases with wealth, meaning that equilibria with higher investment rates become sustainable as wealth increases.

Few authors argued that behavioral biases may lead to a convex savings function and poverty traps. Banerjee and Mullainathan (2010) analyze a consumption-savings problem in which different goods have a different discount factor. Some goods are *temptation goods*: the present discounted value of their future consumption is low (or even zero); nonetheless they will be consumed in the future. The presence of temptation goods creates a discount between present and future. Banerjee and Mullainathan (2010) show that if the share of temptation goods consumed as a fraction of wealth decreases with wealth, then poorer people discount the future more than rich people do. Thus a temptation-based poverty trap may exist. Moav and Neeman (2012) develop a theory in which conspicuous consumption and the concern for social status may lead to a poverty trap. In their model, people infer the social status of other people by observing their human capital investment and expenditure in conspicuous consumption. Hence, concerns for status acts as a regressive tax, affecting poor and low human-capital agents more than rich and high human-capital agents.[8]

My work is related in several ways to the papers just discussed. As in Bernheim, Ray, and Yeltekin (2013), I do not propose here a new behavioral bias, but rather show that a distortion already identified in the behavioral economic literature can be a cause for poverty. Similarly to Banerjee and Mullainathan (2010) and Moav and Neeman (2012), the behavioral distortion I introduce into the model generates a wealth-dependent discount factor, which may induce wealthier people to save proportionately more than poorer people. Also, there are no equilibria multiplicity here, and different steady states are reached depending on the starting wealth level. Differently from all papers mentioned above, here the poverty trap emerges not only because choices are distorted by a specific behavioral bias, but also because of the return on investment. Hence, my model has a specific empirical prediction: a poverty trap is more likely to emerge when the marginal return on investment is high and decreases slowly with the size of the investment.

2 Focusing Effect in a Two-Period Consumption-Savings Problem

Kőszegi and Szeidl (2013) introduce the focusing effect into a choice problem by assuming that the decision maker evaluates each consumption bundle available in her choice set C using a *focus-*

[8] See also Moav and Neeman (2010).

weighted utility

$$\tilde{U}(x_1, x_2, .., x_n) = g_1 u_1(x_1) + g_2 u_2(x_2) + ... + g_n u_n(x_n),$$

where $x_1, x_2, .., x_n$ are different goods in a consumption bundle $x \in C$. The weight g_s is the *focus weight* attached to good s, which represents the salience of good s and is defined as:

$$g_s = g\left(\max_{x\in C} u_s(x_s) - \min_{x\in C} u_s(x_s)\right),$$

where $g()$ is the *focus function,* assumed strictly increasing. In this formulation, the agent's attention is unconsciously and automatically drawn toward the goods in which the agent's options differ more, where these differences are given by the maximum and minimum utility that is possible to achieve by consuming a given good. In turns, the focus weight attached to a good (or, its salience) determines the sensitivity of the agent to the utility generated by consuming that good.

Kőszegi and Szeidl (2013) analyze how, for a given choice set, the focusing effect distorts the decision maker's choice relative to a rational benchmark. Here, instead, I'm interested in how the decision maker's preferences change as the choice set changes. In particular, if a new consumption bundle is introduced into the choice set, and this bundle is better or worse than the other bundles in a given dimension, then the salience of this dimension will increase. Therefore, the focusing effect is relevant in a consumption-savings setup because, as wealth increases, new consumption bundles become available, which are better than the ones previously affordable and therefore change the salience of all choices available.

Therefore, consider the simplest possible consumption-savings problem with focusing effect:

$$\max_{c_1, c_2} \{h_1(b)u_1(c_1) + h_2(f(b))u_2(c_2)\}$$

$$\begin{aligned} s.t\ f(s) &= c_2 \geq 0 \\ s &= b - c_1 \geq 0, \end{aligned}$$

where b is initial wealth, s is savings, c_1 and c_2 are consumption levels in the two periods of life.[9] The production function $f(.)$ determines the return on savings and represents the outer envelope of the return on all investment opportunities that are available to the agent. The utility functions $u_1(.)$ and $u_2(.)$ are strictly increasing and strictly concave. I assume away corner solutions by imposing that $\lim_{c_t \to 0} u_t'(c_t) = \infty$ for $t \in \{1, 2\}$.

The focusing effect enters the above problem via the *focus weights* $h_1(b)$ and $h_2(f(b))$, defined

[9] If the agent consumes multiple goods in each periods, then each good should enter in the utility function with its own focus weight. In this case, all the focus weights relative to present consumption goods increase with b, and all the focus weights relative to future consumption goods increase with $f(b)$ (see the next paragraph). Therefore, also in this more complex environment, the future becomes more salient when the return on savings increases.

as

$$h_1(x) \equiv g\left(u_1(x) - u_1(0)\right),$$

$$h_2(x) \equiv g\left(u_2(x) - u_2(0)\right),$$

where $g(.)$ is the *focus function* assumed strictly increasing. In other words, the salience of consumption in a given period depends on the maximum and on the minimum utility level achievable in that period. The minimum utility levels achievable are $u_1(0)$ and $u_2(0)$, both assumed to be well defined.[10] The maximum utility level achievable in a given period depends on the maximum consumption achievable in that period, which in period 1 is b (in case the agent does not save anything), and in period 2 is $f(b)$ (in case the agent saves everything). In turns, the salience of present and future consumption determines the sensitivity of the agent to the utility enjoyed in the present and in the future, and distorts the consumption-savings choice. Note that the salience of consumption in both periods increases with wealth. However, as wealth increases, the salience of period-1 consumption and the salience of period-2 consumption will, in general, grow at different rates.

Here I simplify the analysis by assuming that the production function is linear so that $f(x) = a{\cdot}x$, and I consider a more standard concave production function in the next section. The first order condition of the problem is

$$u_1'(b - s(b)) = a\left[\frac{h_2\left(a \cdot b\right)}{h_1(b)}\right] u_2'(a \cdot s(b)),$$

where $s(b)$ is the savings function. Note that the *total return on investment* affects the salience of future consumption and the consumption-savings decision through the *focus wedge* $\frac{h_2(a \cdot b)}{h_1(b)}$. In turn, the shape of the focus wedge determines how the incentive to save changes as initial wealth increases. Note also that whether the focus wedge is greater or smaller than one at a given b—and whether the agent over- or under-invests relative to the rational benchmark—depends on the shape of the functions $g()$, $u_1()$, $u_2()$, and on the value of a. However, I will show that under- or over-investment relative to the rational benchmark does not matter for the existence of a poverty trap. Even in situations where the focus wedge is everywhere larger than one, a poverty trap may exist if the focus wedge is somewhere increasing in initial wealth, so that wealthier agents may save a

[10] The model can accommodate for a minimum subsistence consumption level γ. The focus weights become

$$h_1(b - f^{-1}(\gamma)) \equiv g\left(u_1(b - f^{-1}(\gamma)) - u_1(\gamma)\right)$$

$$h_2(f(b - \gamma)) \equiv g\left(u_2(f(b - \gamma)) - u_2(\gamma)\right),$$

In this case, a higher total return on savings makes both future and present consumption more salient. The reason is that as the total return on investment increases, the level of savings required to meet the subsistence consumption level in period 2 decreases, and the maximum consumption level achievable in period 1 increases. This additional effect is second order and all derivations presented in the main text go through for the case $\gamma > 0$ as well. For example, Lemma 1 shows that $\frac{h_2()}{h_1()}$ may be increasing in b when the marginal return on savings is high. Clearly, if the marginal return on savings is high, then $f^{-1}(\gamma)$ is small, eventually becoming negligible.

larger fraction of their wealth than poorer agents. In this situation, some agents may over-invest relative to the rational benchmark, but *under-invest relative to the investment level required to exit poverty* and reach the high steady state. This second benchmark is the one that is most relevant in this context.[11]

The main assumption, which I maintain throughout the paper, is:

Assumption 1. *$g(x)$ is strictly increasing, strictly concave, continuous and differentiable, with $g(0) > 0$.*

The concavity of $g(x)$ can be related to the *Weber-Fechner law of human perception*, which states that the perception of a sensation is proportional to the logarithm of the intensity of the stimulus causing it. In this case, the focus weight measures the perception of the utility of consumption. Weber-Fechner law implies that the focus weight should be proportional to the logarithm of the utility function. Here I make a more general assumption and I simply impose that $g(x)$ is concave.

The specific value of $g(0)$ may seem arbitrary, because when no wealth is available the solution to the utility maximization problem is independent on the value of the focus weights. However, the value of $g(0)$ is important in determining how the salience of present and future consumption evolve as $b \to 0$. The condition $g(0) > 0$ implies that, when wealth approaches zero, present and future consumption become equally salient, independently on the shape of the functions $u_1(.)$, $u_2(.)$ and $f(.)$. Alternatively, if $g(0) = 0$, in some cases the ratio of the focus weights $\frac{h_2(a \cdot b)}{h_1(b_t)}$ may diverge to infinity or go to zero. Hence, if $g(0) = 0$, as $b \to 0$ the present may become infinitely more salient than the future or vice versa. Condition $g(0) > 0$ rules out these situations.

The main result I derive in this section is that whenever a is large, it is possible that $s'(b) > 1$ for some b. This implies that $s(b)$ is somewhere convex, so that the return on savings $as(b)$ may cross the 45 degree line multiple times and may have multiple fixed points. On the other hand whenever a is sufficiently small $s'(b) < 1$ always, and $as(b)$ can have at most one fixed point. Note that a fixed point in the function $as(b)$ corresponds to a situation in which the agent starts each period of life with the same amount of wealth.

Lemma 1. *If a is sufficiently small, then $\frac{h_2(a \cdot b)}{h_1(b)}$ is always decreasing in b and $s'(b) < 1$ for all b. It follows that $as(b)$ can have at most one fixed point. If a is sufficiently large, then $\frac{h_2(a \cdot b)}{h_1(b)}$ is increasing in b at some b. If, furthermore, $u_2''()$ is sufficiently small, then $s'(b) > 1$ somewhere. It follows that $as(b)$ may have multiple fixed points.*

If the marginal return on savings is low, then the focus wedge is everywhere decreasing. The reason is that, as initial wealth expands, present consumption possibilities expand faster than future

[11] Note also that, because the agent lives for two periods (both in this version of the model and in its infinite-horizon version), I'm effectively abstracting away from other behavioral distortions such as time inconsistency. Time inconsistency has been shown to distort the agent's consumption-savings decision, but it is relevant for the existence of a poverty trap only if the under-investment introduced by time inconsistency decreases with the initial wealth level. For a theoretical argument along this line, see Bernheim et al., 2013.

consumption possibilities, and the present becomes relatively more salient than the future. If instead the marginal return on savings is large, then the focus wedge is somewhere increasing because future consumption possibilities expand faster than present consumption possibilities.[12] As a consequence, provided that the utility function is not too curved, the agent increases the amount saved more-than-proportionally with wealth. Note that the curvature of the utility function determines the sensitivity of the savings function to changes in the focus wedge. If $u_2(.)$ is very curved at a specific point (say, because it has an upper bound and it is already close to it), increasing the amount saved has a very small impact on the utility level achieved in period 2. Hence, independently on the focus wedge, as wealth increases the agent does not allocate extra resources to future consumption. On the other hand, if $u_2(.)$ is close to a straight line, then the focus wedge represents the marginal benefit of increasing future consumption. As a consequence, when the focus wedge increases, the amount saved increases as well.

The key result of this section is that, when the marginal return on investment is sufficiently large, then $as(b)$ may have multiple fixed points. In the next section, I introduce the infinite-horizon version of the model presented here, in which the level of initial wealth of an agent is endogenous and depends on the bequests left by the previous generation. In that model, a steady state is a fixed point in the bequests function (i.e. the function determining, for each level of initial wealth, the level of bequests left to the next generation). The bequests function will inherits many of the properties of the savings function just described. Therefore multiple fixed points — and multiple steady states — will emerge under conditions similar to the ones described in Lemma 1.

3 Focusing Effect in an Infinite-Horizon Consumption-Savings Model

Consider the infinite-horizon version of the problem described above. An agent is born at time t and lives for two periods. In the first period she consumes and decides how much to save. In the second period she consumes and decides how much to leave as bequests to her offspring. The problem faced by this agent is:

$$\max_{c_{1,t}\geq 0,\, c_{2,t}\geq 0,\, b_{t+1}\geq 0} \{h_1(b_t)u_1(c_{1,t}) + h_2(c^\star(f(b_t)))u_2(c_{2,t}) + h_b(f(b_t) - c^\star(f(b_t)))\nu(b_{t+1})\}$$

$$\begin{aligned} s.t\ \ f(b_t - c_{1,t}) &= c_{2,t} + b_{t+1} \\ b_t - c_{1,t} &\geq 0. \end{aligned}$$

Bequests received are denoted by b_t, while bequests left are denoted by b_{t+1}. Consumption by the agent born in period t during the period of life i is denoted by $c_{i,t}$, while $c^\star(x)$ is the amount that

[12] The focus wedge is somewhere increasing but not everywhere increasing because of the curvature of the utility function. Even if future consumption possibilities expand faster than present consumption possibilities as b grows, at some point the curvature of the utility function may cause present utility to grow faster than future utility.

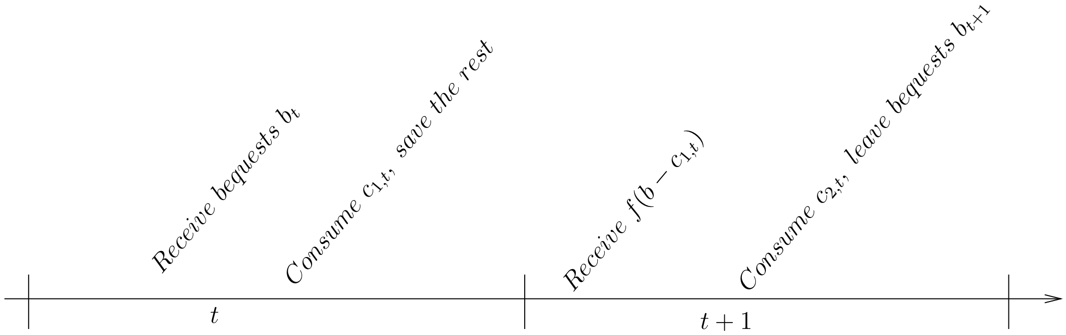


Fig. 1: Timeline

will be consumed in the second period of life if the agent saves x. The functions $u_1(.)$, $u_2(.)$ and $\nu(.)$ are assumed strictly concave, strictly increasing, continuous, differentiable with $u_1(0)$, $u_2(0)$, $\nu(0)$ finite. The production function $f(.)$ is assumed continuous, differentiable, strictly increasing, strictly concaves, unbounded above, with $f(0) = 0$, $\lim f'(x) = \infty$ and $\lim f'(x) = 0$.

The focus weights are

$$h_1(x) \equiv g\left(u_1(x) - u_1(0)\right)$$

$$h_2(x) \equiv g\left(u_2(x) - u_2(0)\right)$$

$$h_v(x) \equiv g\left(u_v(x) - \nu(0)\right).$$

Also here, the salience of period-1 consumption, period-2 consumption, and bequests depend on the maximum utility achievable in each element. However, here the agent cannot decide in period 1 to allocate all his wealth to bequests or to period-2 consumption. In period 1, the agent can only decided how much to consume and to save, and the agent's future self will then decide on how to split savings between bequests and period-2 consumption. Therefore, the salience of future consumption and future bequests depend on the consumption decision of period-2 self in case period-1 self saves all the available wealth.

Assumption 2. *$u_2(x) = \nu(x)$ for all x.*

By assuming that $u_2(x) = \nu(x)$ the problem can be rewritten as

$$\max_{c_t \geq 0,\, b_{t+1} \geq 0} \left\{ h_1(b_t) u_1(c_t) + h_2\left(\frac{f(b_t)}{2}\right) 2 \cdot u_2(b_{t+1}) \right\}$$

$$\begin{aligned} s.t \;\; f(b_t - c_t) &= 2b_{t+1} \geq 0 \\ b_t - c_t &\geq 0, \end{aligned}$$

where I used the fact that, in period 2, savings will be equally split between consumption and bequests. Hence, under Assumption 2, b_{t+1} is a linear function of the return on savings. In addition, I can simplify the notation and call c_t the consumption in the first period of life, and call b_{t+1} both bequests left and consumption in the second period of life.

Assumptions 2 (together with assumption 1) will be maintained throughout the paper. In addition, I will often employ two additional assumptions, which are:

Assumption. *$g(x)$ is bounded above.*

Assumption (Functional forms)**.** *$f(x) = a \cdot x^\alpha$ for $\alpha \in (0,1)$ and $a > 0$; $u_1(x) = u_2(x) \equiv u(x) = \frac{(x+\epsilon)^\sigma}{\sigma}$ for $\epsilon \geq 0$ and $\sigma < 1$ (with the restriction $\epsilon > 0$ whenever $\sigma \leq 0$).*[13]

Assuming that $g(x)$ is bounded implies that $h(.)$ is always bounded. Going back to the Weber-Fechner law, $h(.)$ bounded implies that intensity of the utility of consumption decreases more rapidly than a logarithm for low utility levels, and less rapidly than a logarithm for high utility levels. Hence, $g(x)$ bounded above implies that the Weber-Fechner law holds in approximate terms, because it is always possible to approximate a logarithmic function with a bounded function. The second assumption (functional forms) is made for convenience. The use of the above two assumptions will be clearly stated when presenting the relevant propositions. Finally, I will abstract away from possible corner solutions in the utility maximization problem, by implicitly assuming that ϵ is sufficiently small (or zero, if $\sigma > 0$).

Under Assumptions 1 and 2 the first-order condition of the utility maximization problem is:

$$u_1'(c_t) = \Delta(b_t) f'(b_t - c_t) u_2'(b_{t+1}), \tag{1}$$

where $\Delta(b_t)$ is the focus wedge:

$$\Delta(b_t) \equiv \frac{h_2\left(\frac{f(b_t)}{2}\right)}{h_1(b_t)}.$$

The solution to the utility maximization problem is always unique for every b_t. Compared with the model described in the previous section, here initial wealth for generations $t > 1$ is determined endogenously and depends on the previous-generation's savings decision. I argued in the previous section that the savings function may be convex in initial wealth. Here I show that a similar argument implies that bequests left may be a convex function of initial wealth, and that multiple

[13] This utility function is a HARA (Hyperbolic Absolute Risk Aversion) utility function, because

$$-\frac{u''(x)}{u'(x)} = -\frac{\sigma - 1}{x + \epsilon}.$$

The utility function displays constant relative risk aversion if $\epsilon = 0$; constant absolute risk aversion if both $\epsilon \to \infty$ and $\sigma \to -\infty$; decreasing absolute risk aversion otherwise. The parameter σ measures the curvature of the utility function. In particular, if $\sigma < 0$ the utility function is bounded above; if $0 \leq \sigma < 1$ the utility function is unbounded above.

steady states may exist. When this happens, because the per-period equilibrium is always unique, different steady states are reached depending on the wealth level of generation 0, giving rise to a poverty trap.

The next three lemmas derive sufficient conditions under which $\Delta(b_t)$ is locally increasing for some b_t.

Lemma 2. *If $u_1'(0)$ is finite, the focus wedge is increasing in b_t for b_t sufficiently close to zero.*

When $u_1'(0)$, the utility of present consumption cannot grow arbitrarily fast. Therefore, for low b_t the driving force in the evolution of the focus wedge is the marginal return on investment. If the return on investment increases sufficiently fast with the size of the investment, then wealthier agents weight future consumption relatively more than poorer agents. Because the marginal return on investment is higher for low b_t, the focus wedge increases for low b_t. Finally, note that Lemma 2 does not impose any restriction on $u_2'(0)$, meaning that the statement is true when $u_2'(0)$ is finite as well.

Lemma 3. *Assume that $u_2(x)$ is unbounded above, and $g(x)$ is bounded above. The focus wedge is somewhere increasing and somewhere decreasing in b_t as long as $\exists b_t$ s.t. $u_2\left(\frac{f(b_t)}{2}\right) \neq u_1(b_t)$.*

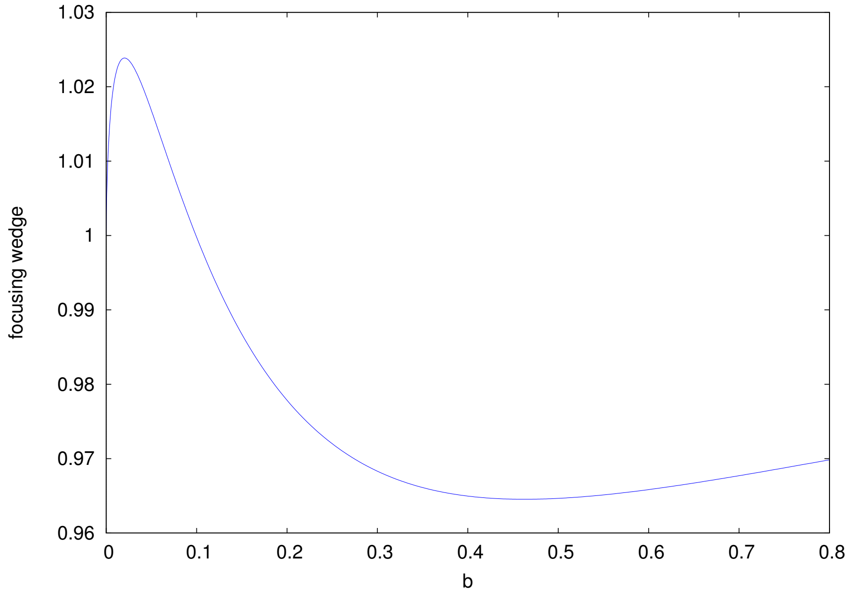
Boundedness of $g(x)$ implies that for very rich agents present and future consumption are approximately equally salient. When $g(0) > 0$, present and future are approximately equally salient also for very poor agents. Hence, unless the two focus weights are identical everywhere, the focus wedge must be increasing somewhere and decreasing somewhere. Note that Lemma 3 is true only if $u_2(x)$ is unbounded above. The reason is that if $u_2(x)$ is bounded above, the upper bound on $h_2(x)$ may be smaller than the upper bound on $h_1(x)$, implying that, by construction, for rich agents the future is less salient than the present. Finally, Lemma 3 does not impose any restriction on $u_1(x)$, that could be unbounded as well.

Lemma 4. *Suppose that $u_1(x) = u_2(x) \equiv u(x)$ for all x, and that $u(x)$ is bounded above. The focus wedge is somewhere increasing and somewhere decreasing in b.*

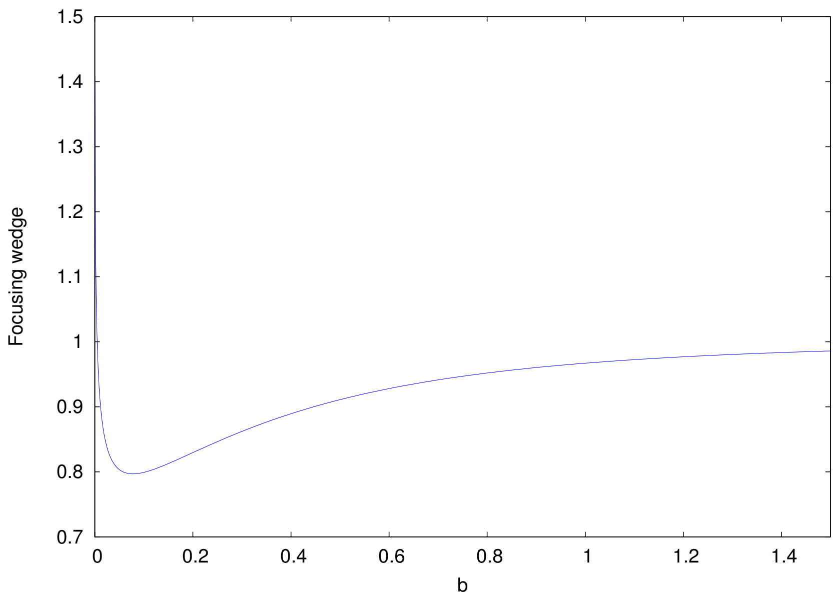
The intuition of the above lemma is similar to the one described in Lemma 3, because when the utility function is bounded also the salience of consumption is bounded.

Both Lemma 3 and Lemma 4 show that the focus wedge may be somewhere increasing and somewhere decreasing. The fact that the focus wedge is somewhere increasing will turn out to be relevant for the existence of multiple steady states. Intuitively, in the absence of salience (i.e. for $g(x)$ constant), the incentive to invest decreases with wealth, because the marginal return on investment decreases with the level of the investment. This effect is reinforced whenever the focus wedge is decreasing in wealth. However, if at *some* wealth level $\bar{b}$ the focus wedge is increasing, agents with initial wealth above $\bar{b}$ may leave a larger share of their initial wealth as bequests compared to agents with wealth below $\bar{b}$, and multiple steady states may exist.[14]

[14] The fact that the focus wedge may be decreasing with wealth is relevant in determining the exact mapping



(a) $u_1(x) = u_2(x) = \frac{1}{-5} \cdot (x+1)^{-5}$, $f(x) = x^{0.7}$ and $g(x) = (x+5)^{0.5}$, so that the conditions of Lemma 2 and Lemma 4 hold.



(b) $u_1(x) = u_2(x) = \frac{1}{0.7} \cdot x^{0.7}$, $f(x) = .5 \cdot x^{0.8}$ and $g(x) = 1 - e^{-5x}$, so that the conditions of Lemma 3 hold.

Fig. 2: The focus wedge

3.1 The Steady State

To derive the steady state of the economy, I impose the functional form assumptions described in the previous section:

- $f(x) = a \cdot x^{\alpha}$ for $\alpha \in (0,1)$ and $a > 0$,
- $u_1(x) = u_2(x) \equiv u(x) = \frac{(x+\epsilon)^{\sigma}}{\sigma}$ for $\epsilon \geq 0$ and $\sigma < 1$ (with the restriction $\epsilon > 0$ whenever $\sigma \leq 0$).

In steady state, $b_t = b_{t+1} = b_{ss}$ and $c_{ss} = b_{ss} - \left(\frac{2b_{ss}}{a}\right)^{\frac{1}{\alpha}}$, so that the steady-state level of bequests solves

$$\left(1 - \frac{\left(\frac{2b_{ss}}{a}\right)^{\frac{1}{\alpha}}}{b_{ss}+\epsilon}\right)^{\sigma-1} = \alpha \cdot 2^{1-\frac{1}{\alpha}} \cdot a^{\frac{1}{\alpha}} \cdot \Delta(b_{ss})\left(b_{ss}\right)^{\frac{\alpha-1}{\alpha}}, \tag{2}$$

for

$$\Delta(b_{ss}) = \frac{h\left(\frac{a \cdot b_{ss}^{\alpha}}{2}\right)}{h(b_{ss})}.$$

The left-hand side (LHS) of equation 2 is monotonically increasing in b_{ss}. On the right-hand side (RHS) of equation 2, $(b_{ss})^{\frac{\alpha-1}{\alpha}}$ is monotonically decreasing in b_{ss}, while $\Delta(b_{ss})$ is somewhere increasing. In other words, if $\Delta(b_{ss})$ were fixed, then LHS and RHS of equation 2 would cross only once and the model would have a unique steady state. However, because $\Delta(b_{ss})$ may be increasing for some b_{ss}, the RHS of equation 2 may be increasing for some b_{ss}, which implies that LHS and RHS of equation 2 may cross multiple times leading to multiple steady states.

Note the competing roles of the marginal return on investment and the focus wedge in determining the shape of the RHS of equation 2. The marginal return on investment always decreases with the size of the investment, making savings less appealing as wealth increases. At the same time, the focus wedge depends on the *total return* on investment and may increase with wealth, generating the opposite incentive. The relative importance of the marginal return on investment and the focus wedge in determining the shape of the RHS of equation 2 depends on the parameters α and a. If α is sufficiently large, then the marginal return on investment does not change much with the level of investment. If a is sufficiently large, then the total return increases fast with the size of the investment. If both α and a are sufficiently large, the shape of the RHS of equation 2 is mostly determined by the shape of the focus wedge.

Lemma 5. *If $a > 2$ and α arbitrarily close to 1 the RHS of equation 2 is increasing somewhere.*

A necessary condition for the existence of a steady state is that the RHS of equation 2 is increasing somewhere. The above lemma shows that this necessary condition is satisfied under

between initial conditions and steady states. When the focus wedge is decreasing for some b, the convergence to different steady states may be non-monotonic in the initial conditions, implying that, over some range, a higher initial conditions may lead to a lower steady state.

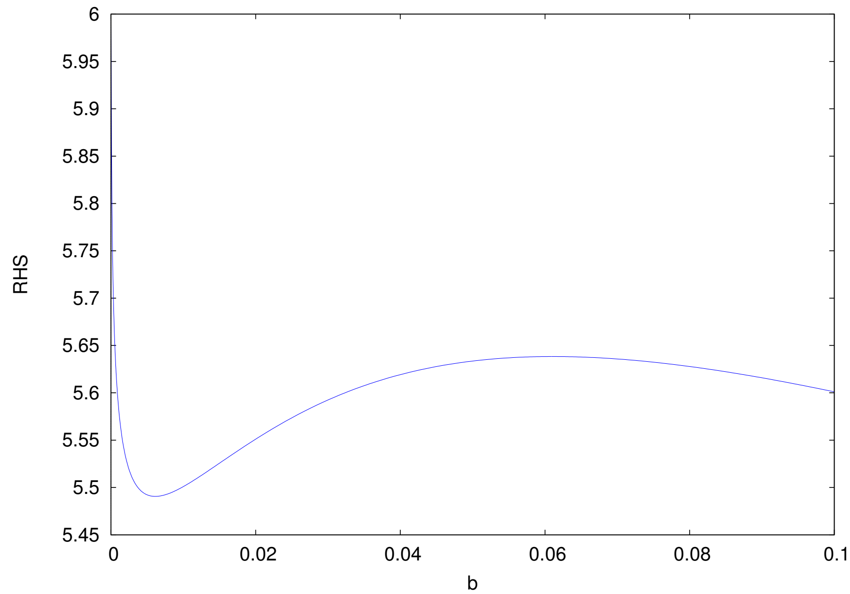


Fig. 3: RHS of equation 2 for $u(x) = \frac{1}{-5} \cdot (x+1)^{-5}$, $f(x) = 5 \cdot x^{0.98}$ and $g(x) = (x+5)^{0.5}$

some assumptions on the production function. To better understand these assumptions, assume that the agent can invest in several projects, each of them with a given minimal and maximal scale. The agent will engage first in the projects with higher return, and later in projects with lower returns as the size of the investment increases. The resulting production function is $f(x)$. For the sake of the argument, assume that each of these projects has a linear return. Consider a specific high-return project, for example purchasing fertilizer. Lemma 5 shows that a poverty trap is possible if fertilizer is the best investment available for agents starting their lives owning different levels of initial wealth. In this case, a poor agent who invests in fertilizer has a lower incentive to save than a richer agent who invests in fertilizer. The reason is that the salience of savings is greater for the agent who can invest more and rip a higher total return.

Different incentives to save at two specific initial wealth levels may translate into different steady states reached. The descendant of the agent who invested in fertilizer will be richer than her parent. For her, saving will be less costly than it was for her parent, and therefore she will continue to save and invest (in fertilizer, or some other investment opportunity). The opposite is true for the offspring of the agent who did not invest. In short, Lemma 5 shows that a poverty trap can exist if both poor agents and less-poor agents have access to the same high marginal-return projects. On the other hand, there is no steady state multiplicity whenever different projects are pursued by agents with different wealth levels, because the marginal return on investment decreases with wealth, dampening the incentive to invest.

Proposition 1. *There is always at least one steady state. For α arbitrarily close to 1, $\sigma > 0$, $\epsilon = 0$, there exist an $a > 2$ such that the economy has multiple steady states.*

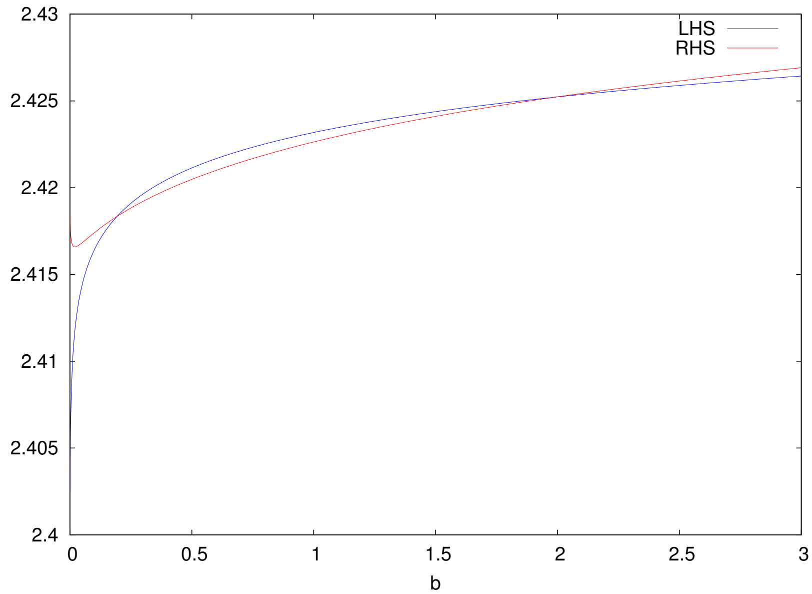


Fig. 4: LHS and RHS of equation 2 for $u(x) = \frac{1}{0.5} \cdot x^{0.5}$, $f(x) = 2.4103.x^{0.9995}$, $g(x) = (X+5)^{.2}$ (third steady state not shown)

In the previous section, I argue that $\Delta(b)$ can be increasing because of technology, because of the curvature of the focus function, or because of the curvature of the utility function. In this section I show that the existence of a steady state is determined by the shape of the production function. A natural question arises: is it possible to find different sufficient conditions for the existence of multiple steady states, weaker than the ones of Proposition 1, based on the shape of the utility function or the shape of the focus function?

To start with, note that if α is sufficiently low, then the marginal return on investment decreases rapidly with wealth. Regardless of the shape of the focus wedge, the economy has a unique steady state. It follows that multiple steady states can exist only if α is relatively high. In the limit case $\alpha \to 1$, one must assume that $a > 2$. The reason is that $c_{ss} = b_{ss} - \left(\frac{2b_{ss}}{a}\right)^{\frac{1}{\alpha}}$, meaning that if $\alpha \to 1$ and $a < 2$ then $c_{ss} \to 0$: consumption is zero in all steady states.[15] In other words, if $\alpha \to 1$, then $a > 2$ is a necessary condition for the existence of steady states with positive consumption.

Of course, the previous discussion does not imply that $\alpha \to 1$ and $a > 2$ are necessary conditions for steady-state multiplicity. It may be possible to find different necessary conditions, where α is high but bounded away from 1, and multiple steady states exist because of the curvature of the utility function or the curvature of the focus function. I do not explore this possibility here. However, in the next section I assume that there is a perfect credit market, so that the return on savings is linear. I show that a poverty trap emerges whenever the utility function is bounded above.

[15] It is possible to have steady states where consumption is zero but bequests are positive. However, I ruled them out by assuming that $\epsilon = 0$ and $\sigma \in (0,1)$, so that the marginal utility of consumption is infinity at zero.

4 Perfect Credit Market

So far I assumed that agents cannot borrow or lend but only invest in their own production function, and I showed that multiple steady states are possible. However, the reader may wonder whether the poverty trap arises because of the focusing effect, or because of the interaction between the focusing effect and the absence of a credit market. To address this question, here I introduce a perfect credit market into the model.

Assume that agents can borrow and lend at an interest rate r. For any technology $f(x)$ satisfying the standard Inada conditions, a perfect credit market implies two things:

1. In every period, the agent can borrow at rate r and invest in $f(x)$ until $f'(x) = 1 + r$. This is equivalent to assuming that the agent receives a lump-sum payment $y(r)$, increasing in r and equal to the infra-marginal benefit of borrowing at rate r and investing in $f(x)$.

2. After receiving $y(r)$, the agent saves linearly at the interest rate r.

The consumption-savings problem is now

$$\max_{c_t, b_{t+1}} \left\{ h_1\left(b_t + y(r)\right) u_1(c_t) + h_2\left(\frac{\left(b_t + y(r)\right)(1+r)}{2}\right) 2 \cdot u_2(b_{t+1}) \right\}$$

$$s.t \quad \left(b_t + y(r) - c_t\right)(1+r) = 2b_{t+1}.$$

When analyzing the case of no credit market, I assumed that $a > 2$, implying a return on savings above 100% for *some* savings levels. With a perfect credit market, the logic behind Proposition 1 continues to apply, and it is possible to show that for some $r > 1$ a poverty trap exists. However, assuming a return on savings above 100% for *all* savings levels is quite unreasonable. I will therefore limit the analysis to the case $r < 1$. Therefore, this section provides conditions under which multiple steady states emerge when the return on savings is low.

In a steady state, $b_t = b_{t+1} = b_{ss}$ and $c_{ss} = y(r) - \frac{b_{ss}(1-r)}{(1+r)}$. Assuming again that $u_1(x) = u_2(x) = \frac{1}{\sigma}(x+\epsilon)^\sigma$ for $\sigma < 1$ and $\epsilon \geq 0$ (with $\epsilon > 0$ if $\sigma \leq 0$), the steady-state level of bequests solves

$$\left(\frac{y(r) - b_{ss}\frac{(1-r)}{(1+r)} + \epsilon}{b_{ss} + \epsilon}\right)^{\sigma - 1} = (1+r)\Delta(b_{ss}). \tag{3}$$

The LHS of equation 3 is increasing in b_{ss}. The shape of the RHS instead depends on the shape of $\Delta(b_{ss})$.

Lemma 6. *Assume that $r < 1$, and that $\sigma < 0$ (so that the utility function is bounded above). $\Delta(b_{ss})$ is increasing somewhere.*

For an agent, the present is more or less salient than the future depending on the return on investment at a specific b, which here is determined by the value of r. Because $r < 1$, then the

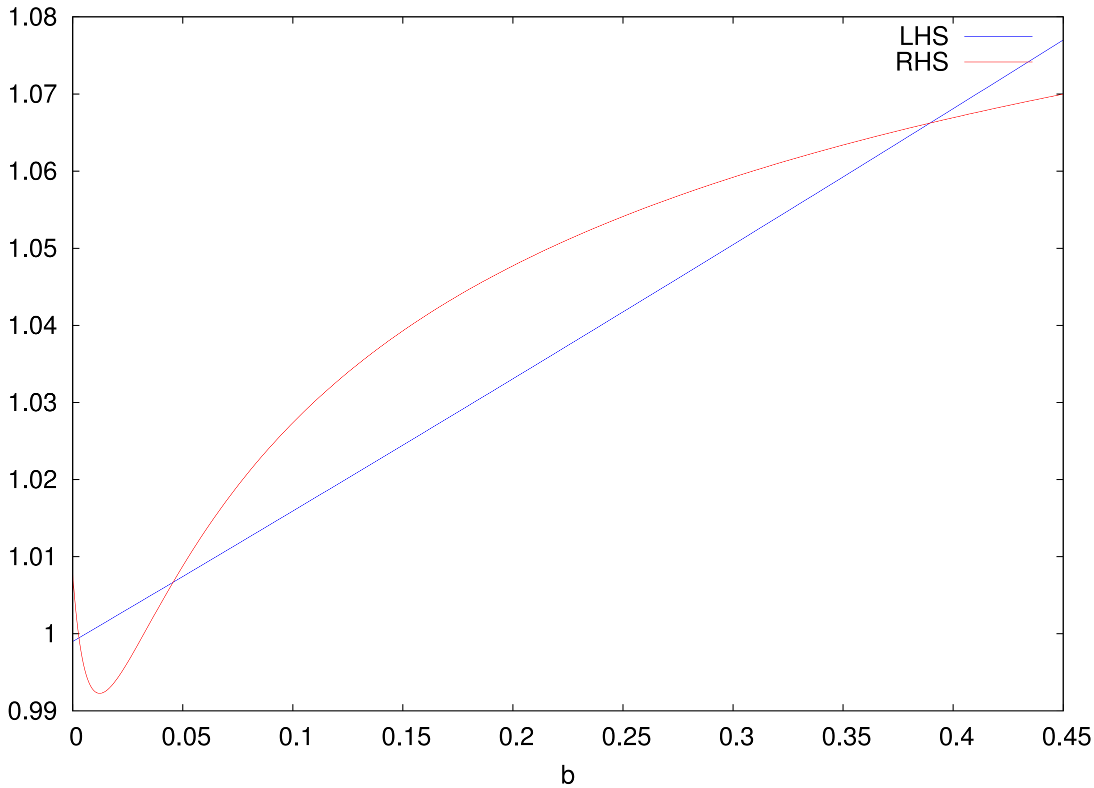


Fig. 5: LHS and RHS of equation 3 for $g(x) = \log(x/0.0005 + 2)$, $u(x) = -\frac{1}{0.01}(x + 10)^{-0.01}$, $r = .2$, $y(r) = 0.01$.

future is always less salient than the present. However, if the utility function is bounded above, then the difference in salience between present and future becomes smaller as b_{ss} increases. The future is discounted less and less, meaning that wealthier agents save a larger fraction of their initial wealth compared to poorer agents.[16] Figure 5 illustrates a numerical examples, in which under the conditions assumed in Lemma 6 multiple steady states exist.

Finally, it is also possible to show that multiple steady states emerge when the utility function is unbounded but the function $g()$ is bounded above. The reason is that, also in this case, the salience of consumption is bounded above and the distortion introduced by the focusing effect decreases as wealth increases.

[16] Bounded utility functions have been frequently reported in the literature. See, for example, Havránek, Horvath, Iršová, and Rusnak (2013) for a meta-analysis or the cross-country estimates. The utility function used here does not display constant elasticity of substitution (CES), but can approximate a CES function by choosing ϵ small.

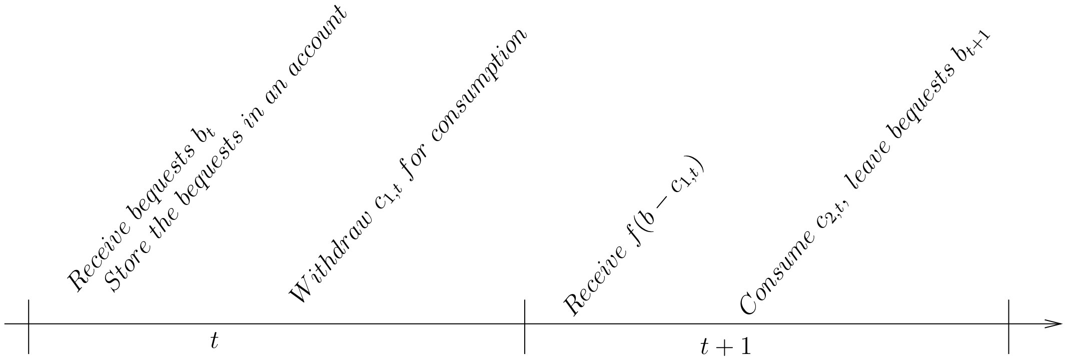


Fig. 6: Timeline with choice of accounts.

5 Commitment Savings

When preferences are distorted by the focusing effect, agents' choices depend on the set of available choices in a way that may lead to a poverty trap. It is therefore interesting to note that, in a consumption-savings set up, people often strategically manipulate their choice set by mean of various commitment-saving devices. The typical explanation for the use of commitment devices relies on time inconsistency. In this section, I show that commitment increases savings also when the relevant behavioral bias is the focusing effect, in a way that is empirically distinguishable from other behavioral bisases.

Assume that the first period of an agent's life is divided into two sub-periods. In the first sub-period, an agent receives her bequests and stores them into an account. In the second sub-period, she withdraws some money for consumption and saves the rest. There are two types of accounts, a normal account for simple wealth storage, and a commitment account that is subject to a withdrawal tax if the account balance drops below a given threshold during period 1.

From the agent's point of view, adopting the commitment account has one obvious implication: if the threshold that triggers the punishment is above what the agent would have saved without commitment, then adopting the commitment account may cause the agent to increase her savings in order to avoid the punishment. However, in this context, the commitment account increases savings through a second channel. Remember that the focus weight on present consumption is a function of the utility that can be achieved today if savings are set to zero. If the agent adopts the normal account, the focus weight on present consumption is $h(b_t)$. If the agent adopts the commitment account, the focus weight on present consumption is $h(b_t - \tau\kappa)$ where τ is the tax and κ is the threshold (the focus weight on future consumption is unchanged). In other words, the commitment mechanism affects the choice set and decreases the maximum utility level that is achievable today, which in turns decreases the salience of present consumption relative to future consumption.

It follows that the focusing effect has one distinctive empirical implication. Without the focusing effect, whenever an agent anticipates that she will not save enough she may adopt a commitment account similar to the one previously discussed. The commitment savings account may push the agent from savings below the threshold to savings exactly at the threshold, but never above the threshold. The focusing effect introduces an additional element: the commitment threshold makes present consumption less salient. This implies that some people may save below the threshold, adopt the commitment technology, and start saving above the threshold. Alternatively, they may already save above the threshold, but nonetheless adopt the commitment device to save even more.

When comparing the distribution of savings between a control group and a treatment group to which the commitment account was offered, the presence of the focusing effect can be detected by comparing the two distributions of savings. Without the focusing effect, offering the account causes all savings levels below κ to (weakly) loose mass, causes the savings level κ to gain mass, and causes no changes in the proportion of agents saving strictly above κ. The presence of the focusing effect instead implies that savings levels above κ may also gain mass.

Why would the agent choose a commitment-savings account? A natural normative benchmark is a situation in which, at the beginning of life, the agent has full control over her lifetime consumption path. In this benchmark, when young the agent can decide that in period 2 she will consume all her wealth, or that she will leave all her wealth as bequests. The benchmark focus wedge is

$$\Delta^{\star}(b) \equiv \frac{h(f(b))}{h(b)} > \Delta(b) \equiv \frac{h\left(\frac{f(b)}{2}\right)}{h(b)},$$

which is always greater than the focus wedge used by the agent when deciding how much to save. In other words, in this normative benchmark the agent places more weight on the future than in equilibrium. If the agent uses $\Delta^{\star}(b)$ to discount the future when the type of account is chosen, then some level of commitment is valuable for every wealth level.[17] By adopting the commitment account, the agent can make the present less salient, and therefore push the focus wedge toward $\Delta^{\star}(b)$.

Lemma 7. *Suppose that a commitment account can be purchased at a cost and that, once the account is purchased, the agent can set the punishment $\tau\kappa$ optimally. There exists a $\underline{b}$ such that all agents with wealth level below $\underline{b}$ do not purchase the account. Furthermore, if the focus function $g(x)$ is bounded above, there is also a $\bar{b}$ such that all agents with wealth level above $\bar{b}$ do not purchase the account.*

When $g(x)$ is bounded above, the value of commitment goes to zero for $b \to \infty$ and for $b \to 0$. If the commitment device can be adopted at a cost, then the poorest and the richest agents do not purchase commitment, but agents with intermediate wealth levels might purchase it. A similar

[17] A second possibility is that multiple behavioral biases are at play. The agent chooses the commitment account because she anticipates to be dynamically inconsistent. Once the account is chosen, the focusing effect determines how savings respond to the commitment device.

result holds if the account is free, but the punishment $\tau\kappa$ is given: very rich and very poor agent do not adopt the account, but other agents might.

6 Conclusions

I develop a consumption-savings model where agents' choices are distorted by the focusing effect: when choosing from a choice set, a decision maker overweights the goods in which her options differ the most. It follows that, as wealth increases, the salience of consumption today relative to consumption tomorrow changes. In particular, if the marginal return on investment is sufficiently high, then the salience of future consumption relative to present consumption increases with wealth because, as wealth grows, future consumption possibilities expand faster than present consumption possibilities.

I show that, if the marginal return on investment is high and sufficienlty flat, then a poverty trap may emerge. In this case, the *percentage* return on investment at different wealth levels is approximately constant, but the *total* return on investment increases rapidly with wealth. Because the salience of future consumption depends on the total return, wealthier agents place more importance on future consumption and save more than poorer agents do. Wealth inequality and poverty are transmitted from generation to generation and a poverty trap may emerge.

I also consider the case of a perfect credit market. If the utility function is bounded above, then a poverty trap may exist also in this case. In utility terms, the difference between consuming in the first period of life and in the second period of life becomes smaller as wealth increases. Therefore, the distortion introduced by the focusing effect becomes less severe as wealth increases, so that rich agents have higher incentives to save than poor agents.

Finally, I argue that when preferences are distorted by the focusing effect, commitment-saving devices increase savings in a way that is empirically distinct from other behavioral biases. With the focusing effect, when a punishment for dropping savings below a given threshold is imposed on the agent, the agent increases the level of savings even when this threshold is not binding. The reason is that the punishment reduces the maximum utility achievable in the current period of life and the salience of present consumption. I propose an empirical test to detect the presence of the focusing effect. This test is based on the comparison between the distribution of savings within a treatment group to which a commitment savings account was proposed, and a control group.

A Appendix: A Rational Inattention Interpretation

The consumption-savings problem I analyzed in this paper can also be interpreted as the reduced form of a rational-inattention model. Assume that, in every period, after the agent decides on the amount to save and to consume, two types of mistakes can occur. In one type of mistake, consumption is destroyed but savings are left untouched. In the other type of mistake savings are destroyed but consumption is left untouched. I assume that the occurrence of each mistake is independent on the realization of the other mistake.

The agent can monitor the two consumption sets in search of potential mistakes and correct them. However, monitoring requires effort, and effort is costly. Call p_c the probability of a mistake in which consumption is destroyed, and p_s the probability of a mistake in which savings are destroyed. I assume that

$$1 - p_c = e_c$$

$$1 - p_s = e_s.$$

where e_c and e_s represent the effort exerted in preventing mistakes. The cost of monitoring depends on the size of the two choice sets (present and future) available to the agent, and is assumed:

$$C(e_c + e_s) = \mu_c\left(b, f(b)\right)\frac{e_c^2}{2} + \mu_s\left(b, f(b)\right)\frac{e_s^2}{2},$$

where $\mu_c\left(b, f(b)\right)$ and $\mu_s\left(b, f(b)\right)$ are increasing in both arguments. The optimal-effort problem is

$$\max_{e_c, e_s}\left\{e_c u_1(c_t) + 2e_s u_2(b_{t+1}) - \left[\mu_c\left(b, f(b)\right)\frac{e_c^2}{2} + \mu_s\left(b, f(b)\right)\frac{e_s^2}{2}\right]\right\},$$

with solution

$$e_c^\star = \frac{u_1(c)}{\mu_c\left(b, f(b)\right)}$$

$$e_s^\star = \frac{u_2\left(b_{t+1}\right)}{2 \cdot \mu_s\left(b, f(b)\right)}.$$

I make the following assumption

Assumption.

$$\frac{\mu_s\left(b, f(b)\right)}{\mu_c\left(b, f(b)\right)} = \frac{\tilde{h}_c(b)}{\tilde{h}_s(f(b))},$$

where $\tilde{h}_c(x)$ *and* $\tilde{h}_s(x)$ *are increasing functions.*

Under the above assumption, the optimal $e_c^\star/e_s^\star$ increases with the size of the current consumption choice, and decreases with the size of the future consumption choice. Intuitively, if the two choice sets increase, the impact of mistakes is minimized by monitoring relatively more the choice set that increased the most. Under this assumption, this costly-attention model replicates the main feature of the model discussed in the body of the paper: as wealth increases, the agent changes her relative valuation of future consumption as a function of how rapidly the future consumption possibilities expand relative to present consumption possibilities.

By assuming that $u_1(x) = u_2(x) = x^\sigma$ for $\sigma \in (0, \frac{1}{2})$, the consumption-savings problem becomes:

$$\frac{\mu_c(b, f(b))}{\tilde{h}_c(b)} \cdot \max_{c_t, b_{t+1}} \left\{ \tilde{h}_c(b) \cdot c_1^{2\sigma} + \tilde{h}_s(f(b)) b_{t+1}^{2\sigma} - \left[\mu_c(b, f(b)) \frac{(e_c^\star)^2}{2} + \mu_s(b, f(b)) \frac{(e_s^\star)^2}{2} \right] \right\}$$

$$s.t \;\; f(b_t - c_1) = 2b_{t+1},$$

which is almost equivalent to the consumption-savings problem discussed previously. The two problems become identical by setting

$$\tilde{h}_c(b_t) = g(b_t^{2\sigma}); \; \tilde{h}_s(f(b_t)) = 2 \cdot g\left(\left(\frac{f(b_t)}{2} \right)^{2\sigma} \right).$$

B Decreasing sensitivity

One of the most widely reported empirical regularities relative to the focusing effect is *diminishing sensitivity:* the fact that a given monetary gain is less salient when considered at a higher initial reference point. In a typical experiment, agents are presented two choices: buying an object at a given price now, or traveling to another store and purchase the same object at a cheaper price. The experiment is then repeated by asking the same question, but using a more expensive object. Crucially, the net gain of traveling to the other store (i.e. the difference in the two prices) is kept constant. Usually, agents are less willing to travel to a second store if the good under consideration is expensive, and more willing to travel to a second store if the good under consideration is cheap, implying that the salience of a given change in price is decreasing with the price level (see, for example, Kahneman and Tversky, 1984).

The model considered here differs from the set up of these experiments in one important way. Here, agents never consider binary choices but rather face a compact choice set. As a consequence, it is not possible to define salience with respect to a given change in present or future consumption. In other words, salience is defined over consumption, not over changes in consumption. Nonetheless, within the framework presented it is possible to look at a question related to diminishing sensitivity: how the salience of present (or future) is affected by a lump-sum transfer, and how this effect changes when considered at different wealth levels.

Suppose that a given agent is allocated an extra T dollars for future consumption. The relative salience of future consumption after receiving the transfer is $\frac{h_2(f(b)+T)}{h_1(b)}$, which is decreasing with b if

$$\frac{h_2'(f(b)+T)}{h_2(f(b)+T)} f'(b) < \frac{h_1'(b)}{h_1(b)},$$

and increasing otherwise. Because $h_2()$ is concave, the above expression is always satisfied for T sufficiently large, and is satisfied for every $T \geq 0$ if the focus wedge $\frac{h_2(f(b))}{h_1(b)}$ is decreasing. In other words, at every b there is always some T for which the agent displays diminishing sensitivity toward future consumption. Furthermore, under either Lemma 3 or 4, there is some b for which the agent displays diminishing sensitivity toward future consumption for all T. Similarly, suppose an agent is allocated an extra T dollars that can be used exclusively for present consumption. The relative salience of this extra payment is $\frac{h_1(b+T)}{h_2(f(b))}$,which is decreasing with b if

$$\frac{h_2'(f(b))}{h_2(f(b))} f'(b) > \frac{h_1'(b+T)}{h_1(b+T)},$$

and increasing otherwise. Again, for every b, there T is sufficiently large such that diminishing sensitivity toward present consumption holds. Furthermore, if the focus wedge $\frac{h_2(f(b))}{h_1(b)}$ is increasing then diminishing sensitivity toward present consumption holds for all b. Again, under either Lemma 2, 3 or 4, there is some b and some T for which the agent displays diminishing sensitivity toward present consumption.

C Appendix: Mathematical Derivations

Proof of Lemma 1.

Note that if $a = 0$, then $\frac{h_2(a \cdot b)}{h_1(b)} \equiv \frac{g(0)}{h_1(b)}$, which is strictly decreasing because $g(0) > 0$. In addition,

$$\frac{\partial \left[\frac{h_2(a \cdot b)}{h_1(b)}\right]}{\partial b} = \frac{h_2(a \cdot b)}{h_1(b)} \left[\frac{a h_2'(ab)}{h_2(ab)} - \frac{h_1'(b)}{h_1(b)}\right].$$

These two observations together imply that at every b, $\lim_{a \to 0} \left[\frac{a h_2'(ab)}{h_2(ab)}\right] = 0$ and, by continuity, for a sufficiently close to zero $\frac{a h_2'(ab)}{h_2(ab)} < \frac{h_1'(b)}{h_1(b)}$. It follows that for a small $\frac{h_2(a \cdot b)}{h_1(b)}$ is decreasing in b. On the other hand, take any $b > 0$. If a is sufficiently large, then $h_2(ab) = g(u_2(ab) - u_2(0)) > h_1(b) = g(u_1(b) - u_1(0))$, which implies $\frac{h_2(a \cdot b)}{h_1(b)} > 1$. We also know that $\frac{h_2(a \cdot b)}{h_1(b)}|_{b=0} = \frac{g(0)}{g(0)} = 1$. Therefore, for a sufficiently large, $\frac{h_2(a \cdot b)}{h_1(b)}$ must be increasing in b somewhere.

By applying the implicit function theorem to the first order condition, we have:

$$u_1''(b - s(b))(1 - s'(b)) = a^2 u_2''(as(b)) s'(b) \frac{h_2(a \cdot b)}{h_1(b)} + a u_2'(as(b)) \frac{\partial \left[\frac{h_2(a \cdot b)}{h_1(b)}\right]}{\partial b}. \tag{4}$$

If $\frac{\partial \left[\frac{h_2(a \cdot b)}{h_1(b)}\right]}{\partial b} < 0$, then the RHS of the above expression is negative (and therefore $s'(b) < 1$). If, instead, at a given b, $\frac{\partial \left[\frac{h_2(a \cdot b)}{h_1(b)}\right]}{\partial b} > 0$, then the RHS of the above expression is positive whenever $u_2''()$ is close to zero. If the RHS of the above expression is positive, then $s'(b) > 1$.

Proof of Lemma 2.

Because $g(0) > 0$, $\Delta(0) = \frac{g(0)}{g(0)} = 1$. On the other hand, we have

$$\lim_{b\to 0}\left[\frac{u_2\left(\frac{f(b)}{2}\right) - u_2(0)}{u_1(b) - u_1(0)}\right] = \lim_{b\to 0}\left[\frac{u_2'\left(\frac{f(b)}{2}\right)\frac{f'(b)}{2}}{u_1'(b)}\right] = \infty,$$

because, $\lim_{b\to 0}\left\{\frac{f'(b)}{2}\right\} = \infty$ and $u_2'(0)$ is either a positive number or diverges to infinity as well. It follows that, for b sufficiently small

$$u_1(b) - u_1(0) < u_2\left(\frac{f(b)}{2}\right) - u_2(0),$$

and

$$\Delta(b) = \frac{g(\frac{f(b)}{2})}{g(b)} > 1.$$

Therefore $\Delta(b)$ must be increasing for b small.

Proof of Lemma 3.

Because $g(0) > 0$, $\Delta(0) = 1$. In addition, $\lim_{b\to\infty}\Delta(b) = \lim_{b\to\infty}\left(\frac{g\left(u_2\left(\frac{f(b)}{2}\right) - u_2(0)\right)}{g(u_1(b) - u_1(0))}\right) \geq 1$, with equality if $u_1(x)$ is unbounded above, and strict inequality if $u_1(x)$ is bounded above. Because $\Delta(b)$ is not everywhere identical to one, the focus wedge must be increasing somewhere.

Proof of Lemma 4.

We have $\Delta(0) = 1$, and $\lim_{b\to\infty}\left(\frac{g\left(u\left(\frac{f(b)}{2}\right) - u(0)\right)}{g(u(b) - u(0))}\right) = 1$. Because $\Delta(b)$ is not everywhere identical to one, the focus wedge must be increasing somewhere.

Proof of Lemma 5.

Define

$$\lim_{\alpha\to 1} a^{\frac{1}{\alpha}} \cdot 2^{1-\frac{1}{\alpha}}\alpha \frac{g\left(\frac{1}{\sigma}\left(\left(\frac{a\cdot b_{ss}^{\alpha}}{2} + \epsilon\right)^{\sigma} - \epsilon^{\sigma}\right)\right)}{g\left(\frac{1}{\sigma}\left((b_{ss} + \epsilon)^{\sigma} - \epsilon^{\sigma}\right)\right)} = a \cdot \frac{g\left(\frac{1}{\sigma}\left(\left(\frac{a\cdot b_{ss}}{2} + \epsilon\right)^{\sigma} - \epsilon^{\sigma}\right)\right)}{g\left(\frac{1}{\sigma}\left((b_{ss} + \epsilon)^{\sigma} - \epsilon^{\sigma}\right)\right)} \equiv \kappa(b).$$

We know that $\kappa(0) = a$. In addition, if $a > 2$, $\frac{1}{\sigma}\left(\left(\frac{a\cdot b_{ss}}{2} + \epsilon\right)^{\sigma} - \epsilon^{\sigma}\right) > \frac{1}{\sigma}\left((b_{ss} + \epsilon)^{\sigma} - \epsilon^{\sigma}\right)$ for all b_{ss}. Hence $\kappa(b_{ss}) > a$ for all $b_{ss} > 0$ meaning that $\kappa(b_{ss})$ is somewhere increasing. Finally, note that the RHS of equation 2 tends to infinity for $b_{ss} \to 0$, and to zero for $b_{ss} \to \infty$. At the same time, for every $b_{ss} > 0$, it is possible to find an α arbitrarily close to one, such that the distance between the RHS of equation 2 and $\kappa(b_{ss})$ is arbitrarily small. Hence, for every $b_{ss} > 0$ such that $\kappa(b_{ss})$ is increasing, it is possible to find an α sufficiently large such that the RHS of equation 2 is also increasing.

Proof of Proposition 1.

Simple algebra shows that the RHS of equation 2:

$$\left(1 - \frac{\left(\frac{2b_{ss}}{a}\right)^{\frac{1}{\alpha}}}{b_{ss} + \epsilon}\right)^{\sigma - 1} \tag{5}$$

is strictly increasing for $b_{ss} \in \left(0, \left(\frac{a}{2}\right)^{\frac{1}{1-\alpha}}\right)$. The LHS of equation 2

$$2^{1-\frac{1}{\alpha}} a\Delta(b_{ss})\alpha \left(b_{ss}\right)^{\frac{\alpha-1}{\alpha}} = 2^{1-\frac{1}{\alpha}} a^{\frac{1}{\alpha}} \frac{g\left(\frac{1}{\sigma}\left(\left(\frac{a \cdot b_{ss}^{\alpha}}{2} + \epsilon\right)^{\sigma} - \epsilon^{\sigma}\right)\right)}{g\left(\frac{1}{\sigma}\left(\left(b_{ss} + \epsilon\right)^{\sigma} - \epsilon^{\sigma}\right)\right)} \alpha b_{ss}^{1-\frac{1}{\alpha}} \tag{6}$$

is going to zero for $b_{ss} \to \infty$, and to infinity for $b_{ss} \to 0$. Hence equation 2 has at least one solution in $\left(0, \left(\frac{a}{2}\right)^{\frac{1}{1-\alpha}}\right)$.

Because $a > 2$, if $\epsilon = 0$ as $\alpha \to 1$ expression 5 becomes a straight line at $\left(1 - \frac{2}{a}\right)^{\sigma-1}$. In addition, if $a > 2$ and $\alpha \to 1$ expression 6 is arbitrarily close to $a\Delta(b_{ss})$ for all $b_{ss} > 0$, where

$$a\Delta(b_{ss}) = a\frac{g\left(\frac{1}{\sigma}\left(\frac{a \cdot b_{ss}}{2}\right)^{\sigma}\right)}{g\left(\frac{1}{\sigma}\left(b_{ss}\right)^{\sigma}\right)}$$

starts at a and is always above a. If $a < \left(1 - \frac{2}{a}\right)^{\sigma-1}$ but $a \approx \left(1 - \frac{2}{a}\right)^{\sigma-1}$ equation 2 has three solutions: at $b_{ss} = 0$ the LHS of 2 diverges to infinity while the RHS of 2 is finite; for $b_{ss} > 0$ but arbitrarily small the LHS of 2 is approximately equal to $a\Delta(b_{ss})$, which is approximately equal to a and is below the RHS of 2; for $b_{ss} > 0$ larger the LHS of 2 is approximately equal to $a\Delta(b_{ss})$, which is above a and is above the RHS of 2; for b_{ss} sufficiently large the LHS of 2 goes to zero while the RHS of 2 is positive.

Proof of Lemma 6.

Write

$$\Delta(b_{ss}) = \frac{g\left(\frac{1}{\sigma}\left(\frac{(b_{ss}+y(r))(1+r)}{2} + \epsilon\right)^{\sigma} - \frac{1}{\sigma}\left(\epsilon\right)^{\sigma}\right)}{g\left(\frac{1}{\sigma}\left(b_{ss} + y(r) + \epsilon\right)^{\sigma} - \frac{1}{\sigma}\left(\epsilon\right)^{\sigma}\right)},$$

which is always below one if $r < 1$. Also, if $\sigma < 0$ utility is bounded, and both numerator and denominator converge to $g(-\frac{1}{\sigma}\left(\epsilon\right)^{\sigma})$ as $b_{ss} \to \infty$. It follows that $\lim_{b\to\infty} \Delta(b_{ss}) = 1$. Hence $\Delta(b_{ss})$ must be increasing somewhere.

Proof of Lemma 7.

It follows simply because $\lim_{b\to 0}\left[\frac{h\left(\frac{f(b)}{2}\right)}{h(b)}\right] = \lim_{b\to 0}\left[\frac{h(f(b))}{h(b)}\right] = 1$, and because because $\lim_{b\to\infty}\left[\frac{h\left(\frac{f(b)}{2}\right)}{h(b)}\right] = \lim_{b\to\infty}\left[\frac{h(f(b))}{h(b)}\right] = 1$.

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