The Technological Determinants of Long-Run Inequality

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Abstract

I explore the effect of skill-biased technological change and unbiased technological progress on long-run inequality using a theoretical model in which the supply of skilled and unskilled workers is endogenous. The main assumption of the model is that young agents can finance their education and become skilled workers by borrowing against their future income on an imperfect credit market. I show that whenever the rate of unbiased technological progress is sufficiently high there is no steady-state inequality, independently on the degree of skill bias. If instead the rate of unbiased technological progress is low, then the long-run skill premium increases with the technological skill bias. Therefore, similarly to the short run, in the long run higher technological skill bias may cause higher inequality. However, contrary to the short run, in the long run unbiased technological progress is more important than technological skill bias in determining inequality. I also discuss how the efficiency of the educational technology and the degree of financial development affect long-run inequality.
KEYWORDS: Inequality, Technology, Intergenerational Bequests, Skill-Biased Technological Change, Credit Rationing.
JEL NUMBERS: D91, J24, O11, O16, O33.

Acknowledgement. I'm thankful to Dilip Mookherjee for his precious help and support. I'm also indebted to Santanu Chatterjee, Andrew Newman, the participants to the microeconomic reading group at ECARES-ULB, three anonymous referees for helpful discussions and constructive comments.

1 Introduction

The evolution of the wage structure in the United States between the end of the 1970s and the beginning of the 1990s suggests that technology can increase short-run inequality. Following the introduction of the personal computer and the unfolding of the information technology era, the difference between the average wage of workers with a college degree and of workers with a high-school degree increased significantly. This wave of innovations was *skill biased*: it increased the productivity of skilled workers (workers with a college degree), leaving unchanged the productivity of unskilled workers.[1]

However, the long-run impact of technology on inequality is not well understood. The reason is that, in the long run, the supply of skilled workers may react to variations in wages. For example, parents may be willing to spend more on the education of their children when the return on education is higher. In addition, the short-run cost of education is fixed, but the long-run college tuitions are likely to be correlated with the skilled wage because college professors are skilled workers. Finally, a sufficiently well-developed financial system may allow students to borrow against their future income. When this income is higher, more people should be able to borrow and access education.

The goal of this paper is to explore theoretically the effect of technology on long-run inequality, by building an overlapping generation model in which the demand and, most importantly, the supply of different types of skills is endogenous. In the model, parents care about the future earnings of their offspring, and leave bequests that are used by young adults to access education either directly or by first borrowing on the credit market. In addition, the cost of education is endogenous and is proportional to the wage of skilled workers. Regarding the demand for skills, the model is fairly standard, in the sense that a competitive production sector pays workers their marginal product.

The central assumption of the model is that credit market is imperfect. Young agents can borrow against their future income. However, because of credit market frictions, only agents with a sufficient level of inherited wealth can access the credit market. It follows that changes in the wage structure affect the number of people who can access the credit market both directly via the credit market, and indirectly via the equilibrium level of bequests left by parents.

In the model, the degree of altruism of each parent is stochastic, so that there is always a positive probability that an unskilled parent has a skilled child (and vice versa). Nonetheless, if skilled and unskilled parents earn different amounts (i.e. there is inequality), the cost of leaving sufficient bequests so that a child can access education will be higher for an unskilled parent than for a skilled parent. The reason is that, in terms of forgone utility

[1] For empirical evidence, see, among many others, Juhn, Murphy, and Pierce (1993), Autor, Katz, and Kearney (2005), and Heckman, Lochner, and Taber (1998).

from consumption, the cost of educating a child is higher for poor parents than for rich parents. This observation implies that inequality is possible in steady state, because the higher is the difference in earnings between skilled and unskilled workers, the lower is social mobility, which in turn implies that skilled workers may be scarce and earn a wage premium over unskilled workers.

The above reasoning fails—and inequality is not possible in steady state—whenever all young individuals can borrow on the credit market and finance their education, independently on the inheritance received. Hence, steady-state inequality can exist only if credit rationing exists, so that only agents born with wealth above a certain threshold can become skilled workers. The main result of the paper is to show that the existence of credit rationing depends on the level of development of the financial sector, on the efficiency of the educational sector, and on the unbiased growth rate of the economy. Crucially, skill-biased technological change plays no role in the existence of credit rationing. In this sense, whereas the unbiased growth rate of the economy can be considered a first-order determinant of long-run inequality, skill-biased technological change is, at best, a second-order determinant.

Intuitively, the effect of an increase in skill bias on the functioning of the credit market has two components. Higher skill bias increases both the cost of education, and the future skilled wage, which can be used as collateral in the credit market. When the efficiency of the schooling technology (which determines the cost of education), the development of the financial sector (which determines the presence of credit market frictions) and the growth rate of the economy (which determines the future skilled wage) are high enough, the positive effect always dominates. As a consequence, for any level of skill bias a young agent who is born with zero wealth can access the credit market and become a skilled worker, because the future wage itself provides enough collateral to access the credit market. The converse is also true, because if a young agent who is born with no wealth can access the credit market for some level of skill bias, then this agent can access the credit market for any level of skill bias.

If credit market rationing is present because, for example, the growth rate of the economy is low, then higher technological skill bias increases long-run inequality. A higher skill bias increases the likelihood that the steady state of the economy is unequal, and increases the steady-state skill premium if the steady state is unequal. Intuitively, the cost of leaving bequests large enough so that a child can access education is always greater for an unskilled parent than for a skilled parent, the more so the higher is the skill bias. Hence, higher skill bias decreases intergenerational mobility. This effect, coupled with an increase in the skill premium, generates higher long-run inequality following an increase in the skill bias.

The same framework can be applied to understanding the long-run effects of other technological and policy changes. For example, the efficiency of the educational technology is a first-order determinant of long-run inequality, in the sense that when it is sufficiently high there is no inequality in steady state. Furthermore, in case steady-state inequality exists, an increase in the efficiency of the educational technology decreases the level of steady-state inequality. Therefore, the model suggests that innovation in the educational sector (i.e. the introduction of online learning or MOOC) should decrease long-run inequality, possibly to the point of bringing the economy to the equal steady state. An increase in the efficiency of the financial system delivers similar predictions; a more developed financial system decreases (and potentially eliminates) steady-state inequality. In addition, the effect of an improvement in the functioning of the financial system on long-run inequality is larger whenever the technological skill bias is large.

Relevant Literature

Within the economic literature, there is a growing recognition that the evolution of inequality should be studied by looking at variations of both the supply and the demand for skills. Piketty (2006) nicely summarizes:

> [...] the impact of technology on inequality depends on a large number of institutions, and these institutions vary a great deal over time and across countries. Chief among these are the institutions governing the supply and structure of skills, from formal schooling institutions to on-the-job training schemes. To a large extent, the dynamics of labor market inequality are determined by the race between the demand for skills and the supply of skills. New technologies tend to raise the demand for skills, but the impact on inequality depends as to whether the supply of skills is rising at a faster or lower rate. There is no general presumption that the race should go one way or the other.

Formally, my paper belongs to the literature on long-run wealth distribution started by Banerjee and Newman (1993) and Galor and Zeira (1993), who show that non-convexities in investment opportunities may generate permanent inequality.[2] Within this literature, my model is closely related to Mookherjee and Ray (2010). As in their model, I assume that agents care about their offspring's wealth (paternalistic altruism), and that wages are determined endogenously. However, contrary to Mookherjee and Ray (2010), in my model the cost of education and the existence of credit rationing are also endogenous. This difference is important because the cost of education and credit rationing are the channels through which technology affects the access to different professions and determines

[2] See also Piketty (1997); Matsuyama (2000); Mookherjee and Ray (2003); Mookherjee, Prina, and Ray (2012).

the supply of different types of skills. Also, similarly to Mookherjee and Napel (2007), the presence of shocks (in my case, to a parent's altruism, in their case to innate ability) generates steady states that are locally unique, which is relevant whenever the goal is to perform steady-state comparative-static analysis. Within this literature, Rigolini (2004) also considers a model derived from Mookherjee and Ray (2010) in which the cost of education is endogenous but agents are exogenously prevented from borrowing. He shows that, depending on the parameters of the utility function, the unbiased growth rate of the economy may increase or decrease the incentive for unskilled workers to acquire education. He also argues that the same result holds also with respect to skill-biased technological change.

Here I derive results that are substantially different from Rigolini's. In my model, if the unbiased growth rate of the economy is high enough there will be no long-run inequality, independently of the shape of the utility function. The reason is that, contrary to Rigolini's, here agents can borrow for education on an imperfect credit market, and the functioning of the credit market is affected by the growth rate of the economy. Furthermore, I show that an increase in the technological skill bias leads to higher long-run inequality. This result contradicts Rigolini's intuition, and is due to the fact that technological skill bias affects differentially the relative cost of education of skilled and unskilled agents. In steady state, an increase in the productivity of skilled workers increases the wage of skilled workers and the cost of education, leaving unchanged the wage of unskilled workers. As a consequence, following an episode of skill-biased technological change, skilled and unskilled parents' incentive to educate their children change in very different ways.

Finally, a large literature has studied the relationship between technology, occupational choice and inequality, addressing issues that are related to my paper. Owen and Weil (1998) show that a faster rate of unbiased technological progress relaxes credit constraints and reduces inequality, a mechanism that is present in my model too. With respect to this work, I consider skill-biased technological change and unbiased technological progress at the same time, and rank these two elements it terms of their impact on long-run inequality. Eicher (1996) studies the interaction between human capital accumulation and technological change, and how this interactions affects relative wages and economic growth. Similarly to my model, he assumes that agents borrow to access education and become skilled workers, that the cost of education depends on the skilled wage, and that future earnings can be used as collateral when borrowing. However, in his model every young agent is identical because there is neither heterogeneity in preferences nor intergenerational wealth transmission. Hence, by assumption, agents are indifferent between accessing either profession and there is no inequality. Several authors developed models in which economic inequality is caused by the interaction technology and heterogeneity in ex-ante ability (Galor and Moav, 2000, Guvenen and Kuruscu, 2012). Here I abstract away from innate ability and focus on the intergenerational transmission of wealth as a determinant of an agent's occupational choice.

Finally, a number of authors (Kuznets, 1955, Greenwood and Jovanovic, 1990, Townsend and Ueda, 2006, Galor and Moav, 2000, and many others) studied the effect of technology on inequality in the short run, while I'm concerned with the long run. Also, starting with Acemoglu (1998) a number of papers have argued that the technological skill bias reacts to the skill composition of the economy. In the body of the paper, instead, I assume that technology is exogenously given, which is equivalent to assuming that the country under consideration imports technology from abroad.[3] However, in appendix I show that the results of the model are robust to the introduction of an endogenous skill composition as long as the elasticity of the technological skill bias to the skill composition of the economy is not too large.

The paper proceeds as follows. In the next section, I illustrate the model. In the third section, I solve for the steady state of the economy, and derive all the relevant comparative statics. In the last section, I conclude with a brief summary of the main results and a discussion of their robustness. Unless otherwise stated, all proofs and mathematical derivations are in appendix.

2 The Model

A small open economy is composed of a measure one of agents, all identical but starting their lives with different levels of wealth. Each individual is alive for two periods. During the first period, she receives a bequest from her (only) parent and decides whether to go to school. Individuals who choose to go to school become skilled types, while those who do not go to school become unskilled types. During the second period of life, an individual works, earns a wage, consumes and bequeathes to her (only) child (see Figure 1).

Call w_t^s and w_t^u the wage of a skilled and of an unskilled type working in period t. Call $e_t^i \geq 0$ the bequest made by the member of household i active in period t to the member of the same household born in period t but active in period $t+1$. Call m_{t+1}^i the end-of-life resources of the agent active in period $t+1$, defined as the total resources available to this agent for consumption and bequests in period $t+1$:

$$m_{t+1}^i = \begin{cases} (e_t^i - \xi_t)(1+r) + w_{t+1}^s & \text{if } \{i,t\} \text{ is skilled,} \\ e_t^i(1+r) + w_{t+1}^u & \text{if } \{i,t\} \text{ is unskilled,} \end{cases}$$

where r is an exogenously given interest rate, and ξ_t is the cost of education in period t

[3] See Gancia and Zilibotti (2009), for a model in which an advanced country produces technology according to its own skill composition, and then exports this technology to other countries.

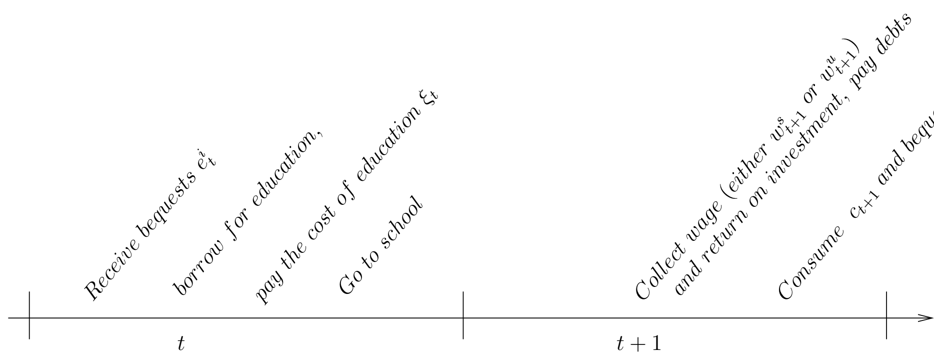


Fig. 1: Timeline, agent born in period t, active in period $t+1$.

(that will be derived endogenously). The utility function of an agent active in period t is given by:

$$V^i_t = u(c^i_t) + \beta^i_t v(m^i_{t+1}). \tag{1}$$

where the parameter $\beta^i_t \geq 0$ measures altruism, and is assumed to be an i.i.d. (both over time and across households) random variable, drawn at the beginning of life from a continuous distribution defined over $[0, \overline{\beta}]$ with $\overline{\beta} > 0$. In other words, agents are altruistic in the sense they care about the end-of-life resources available to their children, but the degree of altruism is heterogeneous across generations and households. The functions $u()$ and $v()$ are assumed increasing and concave.

This form of altruism is called *paternalistic altruism*.[4] It implies that parents care about their direct offspring but not about distant generations. It can be interpreted as an intermediate case between warm-glow altruism and dynastic altruism. Under warm-glow altruism, parents do not care about their offspring but about the bequests left. Hence, the level of bequests left does not respond, for example, to an increase in the return on education, which is a channel I'm interested in exploring and which is present under paternalistic altruism. Under dynastic altruism each parent cares about all subsequent generations. Bequests respond to an increase in the return on education even when only a remote descendant will be able to access it, which is a rather unrealistic feature of this form of altruism.[5]

Young individuals can use the bequests received to finance their education. If these bequests fall short of the cost of education, they can borrow the difference using their future wage as collateral. The budget constraint of an individual born in period $t-1$ and active

[4] See Mookherjee and Ray (2010).

[5] Because they generate different incentives to leave financial bequests, different forms of altruism are more or less likely to generate steady state inequality (see the discussion in Mookherjee and Ray, 2010). However, here I'm interested in studying how long-run inequality is affected by technology rather than the conditions for the existence of long-run inequality.

in period t who either goes to school without borrowing, or borrows and repays her loan is:

$$c_t^i + e_t^i = w_t^s + \left(e_{t-1}^i - \xi_{t-1}\right)(1+r). \tag{2}$$

However, agents may choose not to repay their loans. In this case, because of limited liability, when old they will loose only a part of their wealth. Thus, the budget constraint of an agent who finances part of the cost of education by borrowing but does not repay is:

$$c_t^i + e_t^i = \tau(w_t^s), \tag{3}$$

where $\tau(.)$ is a continuous and strictly increasing function, with $\tau(x) < x$ for all x. This function represents the efficiency of the financial sector of the economy, as it determines how much of an agent's future income can be pledge as collateral today. In countries with a well developed system of students' loans, future income can be easily sized by a lender in case a loan is not repaid, and therefore $\tau(x) \approx 0$. In other countries it is very difficult to seize future income, and therefore $\tau(x) \approx x$. In Section 3 I will impose a simple parametrization for the function $\tau(x)$ and explore how the ability to pledge future income affects long-run inequality.

Finally, the budget constraint of an agent who does not go to school is:

$$c_t^i + e_t^i = w_t^u + e_{t-1}^i(1+r). \tag{4}$$

Given this, banks will lend to agents only if the RHS of 2 is greater than the RHS of 3: access to the credit market and to school is determined by the bequests received at the beginning of life. More precisely, people can become skilled types if and only if:

$$e_{t-1}^i \geq \xi_{t-1} - \left(\frac{w_t^s - \tau(w_t^s)}{1+r}\right). \tag{5}$$

Although the function $\tau(.)$ is taken as given, the existence of credit rationing in the economy is determined endogenously and depends on w_t^s, and ξ_{t-1}. It follows that an economy with very severe credit constraints in some periods may evolve toward a perfect credit market in which everybody is able to borrow. Similarly, an economy with a perfect credit market may later on develop some credit rationing. Because inequality and credit market imperfections are strictly interconnected, an important part of the analysis that follows focuses on the long-run evolution of Equation 5.

I will show in Subsection 2.2 that, under fairly weak assumptions, both professions are always employed in production. This implies that becoming a skilled worker must always be at least as profitable as becoming an unskilled worker. In other words, in every period the RHS of 2 must be greater or equal than the RHS of 4:

$$w_t^s - (1+r)\xi_{t-1} \geq w_t^u. \tag{6}$$

It follows that the utility maximization problem for a skilled parent born in period $t-1$ is:

$$\max_{e_t^i \geq 0} \left\{ u\left(w_t^s + \left(e_{t-1}^i - \xi_{t-1}\right)(1+r) - e_t^i\right) + \beta_t^i v(m_{t+1}) \right\}$$

$$\text{s.t. } m_{t+1} = \begin{cases} w_{t+1}^s + \left(e_t^i - \xi_t\right)(1+r) & \text{if } e_t^i \geq \xi_t - \left(\frac{w_{t+1}^s - \tau(w_{t+1}^s)}{1+r}\right) \\ e_t^i(1+r) + w_{t+1}^u & \text{otherwise} \end{cases}$$

Similarly, for an unskilled parent:

$$\max_{e_t^i \geq 0} \left\{ u\left(w_t^u + e_{t-1}^i(1+r) - e_t^i\right) + \beta_t^i v(m_{t+1}) \right\}$$

$$\text{s.t. } m_{t+1} = \begin{cases} w_{t+1}^s + \left(e_t^i - \xi_t\right)(1+r) & \text{if } e_t^i \geq \xi_t - \left(\frac{w_{t+1}^s - \tau(w_{t+1}^s)}{1+r}\right) \\ e_t^i(1+r) + w_{t+1}^u & \text{otherwise.} \end{cases}$$

where, in both cases, the value of m_{t+1} depends on whether the bequests e_t are such that following generation can access education.

The following lemmas characterize the solution to the above problem.

Lemma 1. *Everything else equal, wealthier parents leave larger bequests, i.e., the optimal* e_t^i *is weakly increasing in* e_{t-1}^i.

Proof. Since the function $u()$ is concave, then e_t^i and e_{t-1}^i are complements in both objective functions. Hence, by Topkis (1998) Theorem 2.8.1, the optimal e_t^i increases in e_{t-1}^i. □

The above lemma relies on the fact that the marginal utility of own consumption is decreasing. Hence, everything else equal, as wealth increases, parents consume only part of the additional resources, and allocate the rest to their offspring.

Lemma 2. *Suppose the return on the skilled profession is at least as large as the return on the unskilled profession. Everything else equal (including* w_{t+1}^s*), if* $\tau(w_{t+1}^s)$ *decreases the probability of having a skilled child weakly increases.*

Proof. It follows from the fact that, for both skilled and unskilled parents, as $\tau(w_{t+1}^s)$ decreases the bequest level which is necessary to leave so that a child can access education also decreases. Hence, having a skilled child becomes less costly. □

Hence, everything else equal, if the credit market becomes more efficient, then the threshold required to access education decreases, and each parent becomes more likely to have a skilled child.

2.1 The Educational Technology.

Call S_t the number of skilled agents in the economy in period t. In every period, the educational sector produces skilled agents by hiring teachers $T_t \geq 0$ and educational capital $i_t \in \mathbb{R}$ according to the following production function:[6]

$$S_{t+1} = 2(A_t T_t)^{\frac{1}{2}} + i_t,$$

where A_t is a productivity parameter and is taken as given by the educational sector. However, due to positive spillovers in human capital accumulation, teacher's productivity increases with the number of skilled agents in the economy, so that

$$A_t = S_t.$$

Because teachers are skilled agents, they must receive the skilled wage, which is determined in the production sector (see Section 2.2). It follows that the educational sector solves:

$$\max_{T_t, i_t} \left\{ \xi_t \left(2(A_t T_t)^{\frac{1}{2}} + i_t \right) - w_t^s T_t - \tilde{c}_i i_t \right\},$$

where $\tilde{c}_t$ is the market price for educational capital. The above problem has a solution whenever $\xi_t \leq \tilde{c}_t$. The optimal number of teachers is

$$T_t = A_t \left(\frac{\xi_t}{w_t^s} \right)^2.$$

If the educational capital is supplied inelastically, then there is always an equilibrium in which $\tilde{c}_t = \xi_t$, and any supply of skilled workers can be an equilibrium supply. Under this assumption, the equilibrium S_{t+1} is determined on the labor market (see Section 2.3 for more details).

Finally, skilled agents are indifferent between joining the educational sector and working in the production sector. I assume that this indifference is broken in the following way: each

[6] The education capital represents all inputs other than teachers in the production of skilled agents (i.e., books, computers, school labs, ...). The fact that educational capital can be negative is simply for ease of exposition. The results in this section are unchanged if $i_t \geq 0$, and the production function for skilled agents is

$$S_{t+1} = \max \left\{ 2(A_t T_t)^{\frac{1}{2}} + i_t - \bar{i}, 0 \right\}.$$

for $\bar{i}$ sufficiently large.

skilled agent joins the educational sector with probability $\lambda^2 \in [0,1]$. It follows that in every period the supply of teachers is fixed at $\lambda^2 S_t$, with the remaining skilled agents employed in the production sector. It follows that, the cost of education must be:

$$\xi_t = w_t^s \lambda \left(\frac{S_t}{A_t} \right)^{\frac{1}{2}} = \lambda w_t^s, \tag{7}$$

which is the same as in Rigolini (2004). That is, the cost of education must be such that the number of teachers employed is exactly $\lambda^2 S_t$.

Note that the equilibrium cost of education (7) is weakly increasing in λ. Hence, λ can also be interpreted as a measure of the efficiency of the educational sector. For given S_t and S_{t+1}, as the educational technology becomes more efficient (i.e. λ decreases) fewer teachers are required to train the same amount of skilled workers. As a consequence, the equilibrium cost of education decreases, leading to the following lemma.

Lemma 3. *Suppose the return on the skilled profession is at least as large as the return on the unskilled profession. Everything else equal, if λ decreases the probability of having a skilled child weakly increases.*

Proof. A decrease in λ decreases ξ_t and causes three effects. First, it increases the return on education, increasing the benefit of having a skilled child. Second, a decrease in λ makes education more affordable, in the sense of decreasing the bequest level required to access education. Third, a decrease in λ makes a skilled parent willing to leave larger bequests. This last point follows again from Topkis (1998) Theorem 2.8.1, and the fact that λ and e_t^i are substitutes in a skilled parent objective function. All three effects work in the same direction and increase the probability of having a skilled child. □

The above lemma follows from the fact that as the efficiency of the educational technology increases, the threshold required to access education decreases. In addition, a more efficient educational technology increases the return on education, making parents more willing to finance it, and simultaneously making skilled parents richer and able to finance it.[7]

[7] Note that, in every period, the educational sector generates profits equal to 2λ. For simplicity, I assume that the educational sector is foreign owned so that these profits do not affect the workings of the economy. If these profits are equally distributed among all agents in the economy, then everyone's wealth at the beginning of life is $2\lambda + e_t$. In this case, a more efficient educational technology also reduces the profits earned, and makes people poorer. Modulo this effect, all results derived below continue to hold, but with different thresholds.

2.2 The Production Function.

The economy produces a consumption good according to the following production function:

$$Y_t = K_t^{\alpha}\left(\left(a_t(1-\lambda^2)S_t\right)^{\epsilon} + (b_tU_t)^{\epsilon}\right)^{\frac{1-\alpha}{\epsilon}}, \tag{8}$$

where K is capital, $(1-\lambda^2)S$ and U are skilled and unskilled types employed in production, and a_t and b_t represent the productivity of the two types of workers. Note that, if both a_t and b_t change by the same amount, this translates into a change in the overall productivity of the economy (i.e. a *Hicks-neutral* productivity increase). Instead, variations to $\frac{a_t}{b_t}$ reflect variations in the skill bias of the economy.

Markets are competitive and all inputs are paid their marginal product:

$$r = \frac{\partial Y_t}{\partial k_t} = \alpha\left(\frac{\left(\left(a_t(1-\lambda^2)S_t\right)^{\epsilon} + (b_tU_t)^{\epsilon}\right)^{\frac{1}{\epsilon}}}{K_t}\right)^{1-\alpha}, \tag{9}$$

$$w_t^s = \frac{\partial Y_t}{\partial S_t} = K_t^{\alpha}(1-\alpha)\left(\left(a_t(1-\lambda^2)S_t\right)^{\epsilon} + (b_tU_t)^{\epsilon}\right)^{\frac{1-\alpha}{\epsilon}-1} a_t^{\epsilon}(1-\lambda^2)^{\epsilon}S_t^{\epsilon-1}, \tag{10}$$

$$w_t^u = \frac{\partial Y_t}{\partial U_t} = K_t^{\alpha}(1-\alpha)\left(\left(a_t(1-\lambda^2)S_t\right)^{\epsilon} + (b_tU_t)^{\epsilon}\right)^{\frac{1-\alpha}{\epsilon}-1} b_t^{\epsilon}U_t^{\epsilon-1}. \tag{11}$$

Rearranging, we get

$$w_t^s = a_t(1-\lambda^2)\left(1 + \frac{b_t}{a_t}\left(\frac{U_t}{(1-\lambda^2)S_t}\right)^{\epsilon}\right)^{\frac{1}{\epsilon}-1}\cdot\chi,$$

$$w_t^u = b_t\left(\frac{a_t}{b_t}\left(\frac{(1-\lambda^2)S_t}{U_t}\right)^{\epsilon} + 1\right)^{\frac{1}{\epsilon}-1}\cdot\chi,$$

where

$$\chi = \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha),$$

so that the wage received by each type of worker in a given period is determined by the ratio between the number of skilled and unskilled agents employed in production and by their productivity parameters $\{a_t, b_t\}$. Consistent with the literature, the *skill premium* of the economy is defined as the ratio of the two wages:

$$\eta_t \equiv \frac{w_t^s}{w_t^u} = \left(\frac{a_t(1-\lambda^2)}{b_t}\right)^{\epsilon}\left(\frac{U_t}{S_t}\right)^{1-\epsilon}. \tag{12}$$

I assume that $0 \leq \epsilon < 1$. When $\epsilon < 1$ the marginal product of labor at zero is infinity: no matter how high wages are, firms will always demand a strictly positive amount of each type of worker. Furthermore, if $\epsilon \geq 0$, for given S_t and U_t an increase skill bias causes an increase in the skill premium, which is the relevant empirical case (see the literature on skill-biased technological change and short-run inequality discussed in the introduction). Finally, the two types of workers are complements whenever $1 - \alpha > \epsilon$, while they are substitutes whenever $1 - \alpha < \epsilon$.

2.3 Market Clearing Conditions.

Because the economy can freely borrow and lend on the international capital market, the domestic market clearing interest rate is equal to the international one, and the capital market is always in equilibrium. We already argued that the educational sector can supply any number of skilled workers at ξ_t, and therefore the educational market is always in equilibrium. In addition, by Walras' Law I can ignore the consumption good's market. Hence, the economy is in a competitive equilibrium if the two labor markets clear.

The demand for skilled and unskilled workers is given by equations 10 and 11. The supply depends on whether the returns on the two professions are equal. If all agents prefer to be skilled, the supply of skilled workers is given by the number of workers whose wealth satisfies condition 5. If instead agents are indifferent between the two professions, any agent whose wealth satisfies condition 5 can be either skilled or unskilled. The following definition formalizes this intuition.

Definition 1. Define $F_{t-1}(e)$ as the c.d.f of the wealth distribution across the generation born in period $t-1$. For given $F_{t-1}(e)$ and $\lambda^2 S_{t-1}$ (number of teachers available in period $t-1$), a competitive equilibrium in period t is a $\{U_t, S_t\}$ that satisfies two conditions:

- *Feasibility*:
$$S_t = 1 - U_t, \tag{13}$$

- *Market clearing*: if agents strictly prefer being skilled types:
$$w_t^s - \lambda w_{t-1}^s (1 + r) > w_t^u, \tag{14}$$
then:
$$U_t = F_{t-1}\left(\lambda w_{t-1}^s - \frac{(w_t^s - \tau(w_t^s))}{(1 + r)}\right), \tag{15}$$
and equations 10 and 11 hold. Alternatively, if agents are indifferent between the two occupations:
$$w_t^s - \lambda w_{t-1}^s (1 + r) = w_t^u, \tag{16}$$

there exist a measure μ such that

$$U_t = F_{t-1}\left(\lambda w_{t-1}^s - \frac{(w_t^s - \tau(w_t^s))}{(1+r)}\right) + \mu, \tag{17}$$

where $0 \le \mu \le 1 - F_{t-1}\left(\lambda w_{t-1}^s - \frac{(w_t^s - \tau(w_t^s))}{(1+r)}\right)$ and, again, 10 and 11 are satisfied.

Note that, by varying S_t it is possible to make the return on education either arbitrarily high or arbitrarily low. For example, if $S_t \to 0$ there are arbitrary few skilled workers employed in production in period t, and w_t^s is arbitrarily large. If instead $S_t \to 1$, the unskilled wage is arbitrarily large, so education is not profitable. Hence, equations (13)-(17) always have a unique solution, which imply the following proposition (missing details are in Appendix).

Proposition 1. *A unique competitive equilibrium exists for every $F_{t-1}(e)$ and S_{t-1}.*

3 Steady State

Let us assume that the productivity parameters a_t and b_t grow at a common constant rate, so that $\frac{a_t}{b_t} \equiv \frac{a}{b}$ and $\frac{a_t}{a_{t-1}} = \frac{b_t}{b_{t-1}} = \gamma$. It follows that an increase in γ represents an increase in the rate of unbiased (i.e., Hicks-neutral) technological progress, while an increase in $\frac{a}{b}$ represents an increase in the technological skill bias.

Definition 2. The economy is in a *steady state* if the fraction of skilled and unskilled workers is constant over time. A steady state is *equal* if the returns on the two professions are equal. A steady state is *unequal* if the two professions yield different returns.

Call S_{ss} and $U_{ss} = 1 - S_{ss}$ the number of skilled and unskilled agents in steady state, and note that $\{U_{ss}, S_{ss}\}$ together with the value of a_t and b_t are sufficient to determine, in every period, the two wages, the cost of education, and the minimum initial bequests required to access the credit market. Also, simple algebra shows that, in steady state, wages grow at a rate γ, so that $w_{t-1,ss}^s\gamma = w_{t,ss}^s$ and $w_{t-1,ss}^u\gamma = w_{t,ss}^u$. I call $\eta_{ss} = \frac{w_{t,ss}^s}{w_{t,ss}^u} = \left(\frac{(1-\lambda^2)a_t}{b_t}\right)^\epsilon \left(\frac{U_{ss}}{S_{ss}}\right)^{1-\epsilon}$ the steady-state skill premium.

Assumption 1. *The net return on the skilled profession is positive:*

$$\gamma > \lambda(1+r) \tag{18}$$

Assumption 1 is equivalent to

$$\frac{w_{t,ss}^s}{1+r} > \frac{\lambda w_{t,ss}^s}{\gamma} = \lambda w_{t-1,ss}^s$$

where the RHS is the cost of education in a steady state, and the LHS is the skilled wage discounted by one period. This condition implies that the cost of education grows relatively slowly with the skilled wage, so that an increase in the skilled wage always makes the skilled profession more attractive. Assumption 1 is necessary to guarantee the existence of a steady state. Without it, in a sequence of competitive equilibria the number of skilled workers must shrink over time, so that in every period the skilled wage is greater than the cost of education and some agents join the skilled profession.

Te remainder of this section proceeds as follows. In order to characterize the steady state, I first assume that the number of skilled agents in the economy is fixed over time at an arbitrary level $\overline{S}$. This implies that wages grow at rate γ and that the skill premium is also fixed at $\overline{\eta}$. I show that, for any $\overline{\eta}$ there is a unique stationary normalized wealth distribution (Lemma 4). Finally, I solve for the steady state of the economy by finding the $\overline{\eta}$ generating a wealth distribution such that the number of agents who become skilled in each period is $\overline{S}$ (Proposition 2). In general, the economy may have multiple steady states, but these steady states are locally unique. This allows me to perform some comparative static analyses.

Distribution of wealth for time-invariant $\overline{S}$. For given time-invariant $\overline{S}$, the utility maximization problem for a skilled parent is:

$$\max_{e_t^i \geq 0} \left\{ u\left(w_t^s \left(1 - \frac{\lambda(1+r)}{\gamma}\right) + e_{t-1}^i(1+r) - e_t^i \right) + \beta_t^i v(m_{t+1}) \right\}$$

$$\text{s.t.}\, m_{t+1} = \begin{cases} e_t^i(1+r) + w_t^s\left(\gamma - \lambda(1+r)\right) & \text{if } e_t^i \geq \lambda w_t^s - \left(\frac{\gamma w_t^s - \tau(\gamma w_t^s)}{1+r}\right) \\ e_t^i(1+r) + w_t^u \gamma & \text{otherwise} \end{cases}$$

and for an unskilled parent

$$\max_{e_t^i \geq 0} \left\{ u\left(w_t^u + e_{t-1}^i(1+r) - e_t^i\right) + \beta_t^i v(m_{t+1}) \right\}$$

$$\text{s.t.}\, m_{t+1} = \begin{cases} e_t^i(1+r) + w_t^s\left(\gamma - \lambda(1+r)\right) & \text{if } e_t^i \geq \lambda w_t^s - \left(\frac{\gamma w_t^s - \tau(\gamma w_t^s)}{1+r}\right) \\ e_t^i(1+r) + w_t^u \gamma & \text{otherwise} \end{cases}$$

Note that the constraints can be made stationary by redefining the problem in terms of *normalized bequests* $\tilde{e}_t^i = \frac{e_t^i}{w_t^s}$ and normalized end-of-life wealth $\tilde{m}_t^i = \frac{m_t^i}{w_t^s}$.[8] In addition, it is easy to see that the entire problem is stationary in $\tilde{e}_{t-1}^i$ if and only if $u()$ and $v()$ are identical quadratic functions and the limited-liability constraint is linear. This consideration

[8] Alternatively, it is possible to normalize by w_t^u or γ^t, with equivalent results.

motivates the following assumption.

Assumption 2. $u(x) = v(x) = \frac{x^{1-\sigma}}{1-\sigma}$ *for* $\sigma > 0$, $\tau(w^s_{t,ss}) = \theta w^s_{t,ss}$ *for* $\theta \in [0,1]$.

Hence, the agent can pledge only a fraction θ of future earnings against a loan, so that the parameter θ measures the degree of imperfection in the credit market (with high θ corresponding to lower frictions), and can be thought of as an index of the financial development of the economy. The parameter σ measures the elasticity of substitution between own consumption and offspring's wealth.

Expressed in terms of normalized bequests, the utility maximization problem for time-invariant $\bar{S}$ becomes, for a skilled parent:

$$\max_{\tilde{e}^i_t \geq 0} \left\{ \frac{1}{1-\sigma} \left(\left(1 - \frac{\lambda(1+r)}{\gamma} \right) + \tilde{e}^i_{t-1} \frac{(1+r)}{\gamma} - \tilde{e}^i_t \right)^{1-\sigma} + \frac{\beta^i_t}{1-\sigma} (\tilde{m}_{t+1})^{1-\sigma} \right\}$$

$$\text{s.t. } \tilde{m}^i_{t+1} = \begin{cases} \tilde{e}^i_t(1+r) + (\gamma - \lambda(1+r)) & \text{if } \tilde{e}^i_t \geq \left(\lambda - \gamma \left(\frac{1-\theta}{1+r} \right) \right) \\ \tilde{e}^i_t(1+r) + \frac{\gamma}{\bar{\eta}} & \text{otherwise} \end{cases}$$

and for an unskilled parent

$$\max_{\tilde{e}^i_t \geq 0} \left\{ \frac{1}{1-\sigma} \left(\frac{1}{\bar{\eta}} + \tilde{e}^i_{t-1} \frac{(1+r)}{\gamma} - \tilde{e}^i_t \right)^{1-\sigma} + \frac{\beta^i_t}{1-\sigma} (\tilde{m}_{t+1})^{1-\sigma} \right\}$$

$$\text{s.t. } \tilde{m}^i_{t+1} = \begin{cases} \tilde{e}^i_t(1+r) + (\gamma - \lambda(1+r)) & \text{if } \tilde{e}^i_t \geq \left(\lambda - \gamma \left(\frac{1-\theta}{1+r} \right) \right) \\ \tilde{e}^i_t(1+r) + \frac{\gamma}{\bar{\eta}} & \text{otherwise} \end{cases}$$

I call this problem the *normalized utility maximization problem.* It is quite straightforward to see that lemmas 1, 2 and 3 apply here as well: wealthier parents leave larger normalized bequests (i.e. the optimal $\tilde{e}^i_t$ increases with $\tilde{e}^i_{t-1}$), and the probability of having a skilled child increases with the efficiency of the educational technology and the credit market (i.e. decreases with λ and θ). Instead, other parameters have an ambiguous effect on the normalized utility maximization problem. For example, an increase in γ makes the return on both professions higher, relaxes the credit constraint, and reduces the value of bequests received by a parent. There is no presumption that for every possible $\tilde{e}_{t-1}$ and β^t_i the overall effect is positive.[9]

More interestingly, changes in the skill premium $\bar{\eta}$ affect skilled and unskilled parents differently. In the normalized utility maximization problem, the benefit of having a skilled

[9] This result is related to Rigolini (2004), who finds an ambiguous effect of hicksian growth rate on inequality. In a model where there is no credit market, he shows that an increase in future wages can either lead to a higher investment in education or a higher consumption, depending on the parameters of the utility function.

child is independent on the profession of the parent, but the cost of having a skilled child depends on the parent's profession. For given skill premium $\bar{\eta}$, in case the return on the two professions is unequal the cost of having a skilled child is greater for an unskilled parent than for a skilled parent even when both parents have the same level of initial wealth (but different wages). In addition, the cost of having a skilled child is decreasing in initial wealth $\tilde{e}^i_{t-1}$, which also tends to depress the incentive for unskilled parents to have skilled children.

More importantly, the transition probability between professions depends on the steady-state skill premium $\bar{\eta}$. For skilled parents, a higher $\bar{\eta}$ increases the chances to have a skilled child because it increases the benefit of a child's education leaving unchanged the costs. The reason is that a higher $\bar{\eta}$ increases the cost of education but also the relative wage of a skilled parent, who is therefore better able to provide a large bequest to her child. For an unskilled parent, instead, the effect is ambiguous. The benefit of having a skilled child increases, but the relative wage of an unskilled parent decreases. Hence, the cost of a child's education as measured by the marginal benefit of consuming instead of leaving bequests increases with $\bar{\eta}$.

It is easy to see that the evolution of normalized bequests for given $\bar{\eta}$ is a Markov process, because the normalized bequests left in every period depend only on the starting normalized bequests and on the realization of β^i_t. Furthermore, for given $\bar{\eta}$, the skill bias $\frac{a}{b}$ has no impact on the evolution of normalized bequests, which implies the following proposition.

Lemma 4. *Assume that there is a finite upper bound to the level of normalized bequests that can be accumulated. For every $\bar{\eta}$ there exist a unique stationary distribution of normalized bequests. We call its CDF $\tilde{F}(\tilde{e}, \bar{\eta})$.*

First of all, note that the above lemma is true only if the distribution of normalized bequests has an exogenously given upper bound. This assumption is made for technical reasons and will be maintained throughout the paper, but should not be considered as restrictive because this upper bound can be arbitrarily high. Note also that, because the steady state distribution of normalized bequests is stationary, then the distribution of bequests grows at the same rate as the skilled wage.

Characterization of the steady state After establishing that for every time-invariant $\overline{S}$ (and $\overline{\eta}$) there is a unique stationary distribution of normalized wealth, I can now characterize the steady state of the economy. The following lemma derives necessary conditions for equal and unequal steady states to exist.

Lemma 5. *In an equal steady state, the following condition must hold:*

$$\left(1 - \frac{\lambda(1+r)}{\gamma}\right)^{-1} = \eta_{ss}. \tag{19}$$

In an unequal steady state, the following conditions must hold:

$$\left(1 - \frac{\lambda(1+r)}{\gamma}\right)^{-1} < \eta_{ss}, \tag{20}$$

$$\theta > 1 - \frac{\lambda(1+r)}{\gamma}. \tag{21}$$

Proof. Immediate from the definition of equal and unequal steady states, 5, 6, 12, and using the fact that, in steady state, $w^s_{t-1,ss}\gamma = w^s_{t,ss}$. □

When condition 19 holds, the steady-state return on the skilled profession is equal to the steady-state return on the unskilled profession. Instead, when condition 20 holds, the steady-state return on the skilled profession is strictly greater than the steady-state return on the unskilled profession. In this case, there can be a steady state only if condition 21 also holds, because under this condition credit market frictions are sufficiently severe and an agent born with zero wealth cannot access education.

Remark 1. There is no unequal steady state whenever:

$$\gamma > \frac{\lambda(1+r)}{1-\theta}.$$

i.e. the Hicks-neutral growth rate is sufficiently high, the efficiency of the educational technology is high (low λ), the interest rate is low, and credit market frictions are low (high θ).

The above remark follows directly from condition 21. If the growth rate is high, then the credit market works perfectly, and everybody is able to access education. The reason is that future wages are very high (relative to current wages) and can be used as collateral for borrowing. Similarly, γ, r and θ determine whether credit rationing is possible in steady state, and hence whether inequality is possible. Therefore, γ, λ, r and θ are first-order determinants of steady-state inequality, because they determine whether inequality can exist. As we will see, skill bias a/b is a second order determinant, because it plays a role only if condition 21 is satisfied.

In order to fully characterize the steady state of the economy, I need to find a time-invariant measure of skilled agents $\overline{S}$ (and a time-invariant skill premium $\overline{\eta}$) generating a wealth distribution such that the number of skilled agent is $\overline{S}$.

Proposition 2. *Define*

$$\hat{\eta} \equiv \bar{\eta} \textit{ such that } \left(\frac{\tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), \bar{\eta}\right)}{1 - \tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), \bar{\eta}\right)}\right)^{1-\epsilon} = \left(\frac{b}{a(1-\lambda^2)}\right)^{\epsilon} \cdot \bar{\eta}. \tag{22}$$

If $\hat{\eta} > \left(1 - \frac{\lambda(1+r)}{\gamma}\right)^{-1}$*, then there exists an unequal steady state* $\{S_{ss}, U_{ss}\}$*, with*

$$U_{ss} = 1 - S_{ss},$$

$$\left(\frac{a(1-\lambda^2)}{b}\right)^{\epsilon}\left(\frac{U_{ss}}{S_{ss}}\right)^{1-\epsilon} = \hat{\eta}.$$

Instead, if $\hat{\eta} \leq \left(1 - \frac{\lambda(1+r)}{\gamma}\right)^{-1}$ *and*

$$\left(\frac{a(1-\lambda^2)}{b}\right)^{\epsilon}\left(\frac{(1+\lambda)\tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), \bar{\eta}\right)}{1 - \tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), \bar{\eta}\right)}\right)^{1-\epsilon}\Bigg|_{\bar{\eta}=1-\frac{\lambda(1+r)}{\gamma}} < \left(1 - \frac{\lambda(1+r)}{\gamma}\right)^{-1},$$

there exists an equal steady state $\{S_{ss}, U_{ss}\}$*, with*

$$U_{ss} = 1 - S_{ss},$$

$$\left(\frac{a(1-\lambda^2)}{b}\right)^{\epsilon}\left(\frac{U_{ss}}{S_{ss}}\right)^{1-\epsilon} = \left(1 - \frac{\lambda(1+r)}{\gamma}\right)^{-1}.$$

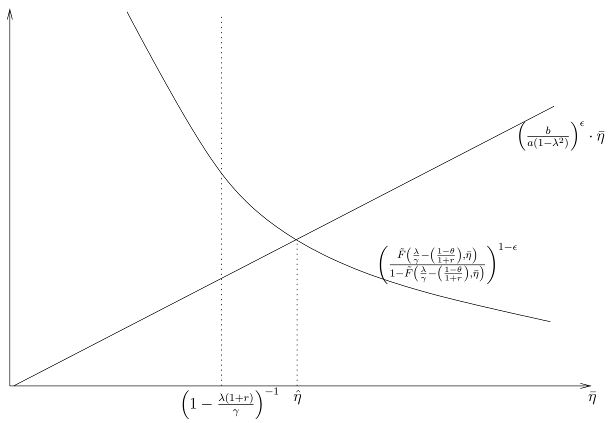
The above proposition shows that the existence of a steady state as well as the number of steady states of the economy depends on the number of solutions to Equation 22. Furthermore, the value of the solutions to 22 determines whether this steady state is equal or unequal, and, if the steady state is unequal, the steady-state skill premium. Finally, note that whenever Equation 22 has multiple solutions and multiple steady states exist, these steady states will be locally unique. See Figure 2 for an illustration of the possible cases.

Call $\text{pr}\{s \to u\}$ the probability that a skilled parent has an unskilled child, and call $\text{pr}\{u \to s\}$ the probability that an unskilled parent has a skilled child. Suppose that $\hat{\eta} > \left(1 - \frac{\lambda(1+r)}{\gamma}\right)^{-1}$, so that all agents who can become skilled do so. Because the steady state distribution of normalized wealth is stationary, it must be the case that

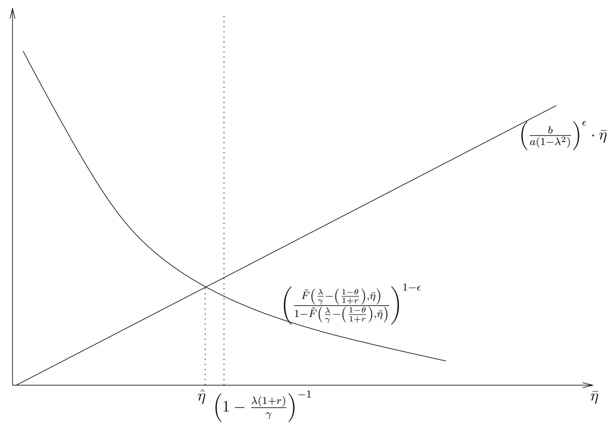
$$\tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), \bar{\eta}\right)\text{pr}\{u \to s\} = \left(1 - \tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), \bar{\eta}\right)\right)\text{pr}\{s \to u\}$$

i.e. the flow of people in and out the skilled profession must equalize. Rearranging:

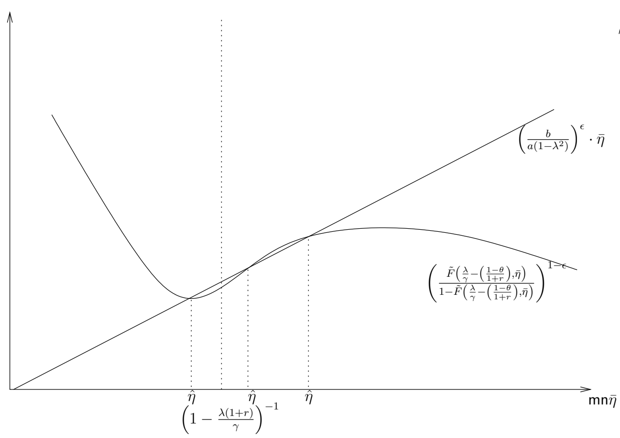
$$\frac{\tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), \bar{\eta}\right)}{1 - \tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), \bar{\eta}\right)} = \frac{\text{pr}\{s \to u\}}{\text{pr}\{u \to s\}}.$$



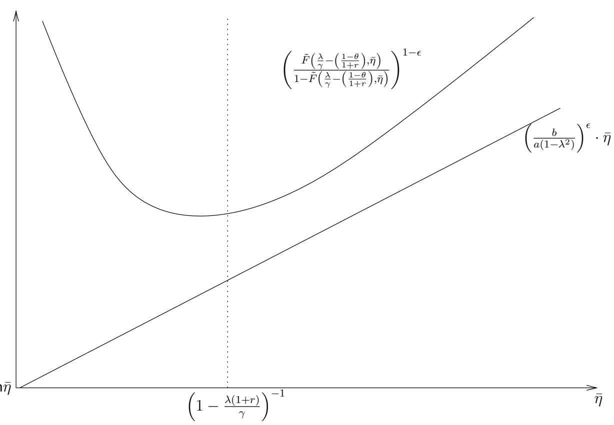
(a) Unique, unequal steady state.



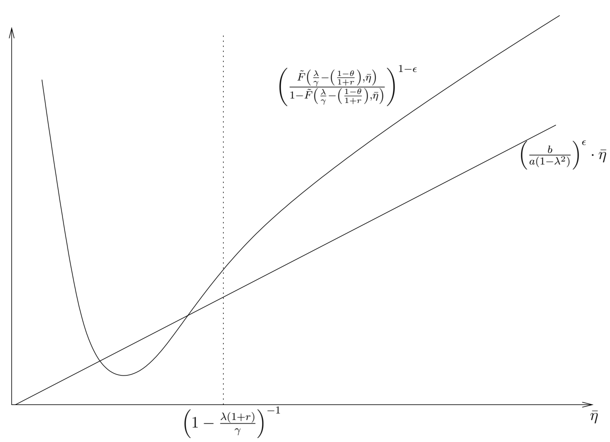
(b) Unique, equal steady state.



(c) Multiple steady states (one equal, two unequal).



(d) No steady state.



(e) No steady state.

Fig. 2: Equation 22 determines the existence, uniqueness, and the type of steady state.

Hence, Equation 22 can be rewritten as

$$\left(\frac{\text{pr}\{s \to u\}}{\text{pr}\{u \to s\}}\right)^{1-\epsilon} = \left(\frac{b}{a(1-\lambda^2)}\right)^{\epsilon} \cdot \bar{\eta},$$

and we can study the existence, uniqueness and the type of steady state in the economy by looking at the ratio of the transition probabilities across professions. In particular, we know that $\text{pr}\{s \to u\}$ decreases monotonically and goes to zero as $\bar{\eta}$ increases (i.e. the probability that a skilled parent has an unskilled child decreases monotonically to zero with $\bar{\eta}$). However, we do not know how $\text{pr}\{u \to s\}$ changes with $\bar{\eta}$, because for an unskilled parent, $\bar{\eta}$ increases both the costs and the return on her child's education. Whenever $\text{pr}\{u \to s\}$ decreases with $\bar{\eta}$, the expectation of a higher skill premium (higher $\bar{\eta}$) may lead to fewer skilled workers in the economy, creating a positive feedback that may generate multiple equilibria. On the other hand, whenever $\text{pr}\{u \to s\}$ increases with $\bar{\eta}$, the expectation of a higher skill premium always leads to an increase in the number of skilled workers, and a unique steady state always exists. More formally, whenever $\text{pr}\{u \to s\}$ increases with $\bar{\eta}$, the LHS of Equation 22 decreases monotonically to zero, and the economy has a unique steady state. This steady state can be either equal or unequal, corresponding to either panel (a) or (b) of Figure 2.

Remark 2. For $\epsilon \to 1$, there exists a unique steady state.

Remember that when $\epsilon = 1$ the two types of labor enter linearly into the production function. For ϵ approximately close to 1, the LHS of equation 22 is approximately an horizontal straight line, and a steady state (either equal or unequal) always exists.

Comparative statics. Because the steady states are generically unique, it is possible to perform some comparative statics exercises. I choose to focus on equilibria in which the LHS of equation 22 crosses the RHS of equation 22 from above, because this is how the two curves cross whenever $\epsilon \to 1$ (see the previous remark). Of course, all comparative statics are reversed if we consider equilibria in which the LHS of equation 22 crosses the RHS of equation 22 from below.

Corollary 1. *Suppose that the LHS of equation 22 crosses the RHS of equation 22 from above (which is the case whenever $\epsilon \to 1$). In case the steady state of the economy is unequal, the steady state skill premium increases with the skill bias a/b.*

This corollary follows by simple inspection of Equation 22. Therefore, despite the fact that the supply of skilled workers reacts to the skill premium, a higher skill bias leads to more inequality in the long run. The key observation is that the supply of skilled workers from skilled parents behaves very differently from the supply of skilled workers from unskilled parents. As a/b increases, a skilled worker becomes relative more likely to have a skilled parent rather than an unskilled parent. Hence, an increase in the skill bias simultaneously decreases social mobility and increases the return on the skilled profession, causing an increase in inequality. Finally, note that the skill bias has an effect on the steady state skill premium only if the economy is already unequal. The reason is that, in an equal steady state, the skill premium is independent on the skill bias and is equal to $\left(1 - \frac{\lambda(1+r)}{\gamma}\right)^{-1}$. Hence, an increase in the degree of technological skill bias should lead to an increase in inequality (as measured by the skill premium) if and only if the economy is unequal to start with. Otherwise, the skill bias will not change. This consideration is relevant because it implies that there is a connection between the initial degree of inequality and the increase in inequality due to skill biased technological change.

Lemma 6. *Suppose that the LHS of equation 22 crosses the RHS of equation 22 from above (which is the case whenever $\epsilon \to 1$). Furthermore, suppose the steady state of the economy is unequal. The steady-state skill premium decreases with θ and increases with λ. Furthermore, the impact of a given change in θ on the steady-state skill premium is increasing in $\frac{a}{b}$.*

As already discussed, inequality exists because it is easier for a skilled parent than for an unskilled parent to have a skilled child. As the above lemma shows, this same logic implies that when the educational technology is inefficient or credit market frictions are large inequality (as measured by the skill premium) will be greater. Finally, the effect of improving the efficiency of the credit market on long-run inequality will be larger in economies with high skill bias than on those with low skill bias.

4 Conclusion

The main message of the model is that the unbiased rate of technological progress is much more important for long-run inequality than the degree of technological skill bias. This result is actually the opposite of what happens in the short-run, when the supply of skilled and unskilled workers is fixed. In the short run, a change in the rate of unbiased technological progress affects the earnings of all workers in the same way, leaving inequality unchanged, while an episode of skill-biased technological change increases inequality. In the long run, unbiased technological progress affects the credit market while skill bias does not, which makes unbiased technological progress much more relevant for the existence of inequality. The model also allows for the study of how the different determinants of long-run inequality interact with each other. I show that an increase in the efficiency of the educational technology or an improvement in the functioning of the credit market decrease long-run inequality. The latter effect is stronger the higher the degree of technological skill bias in the economy.

The main result of the model is Lemma 5 and, more precisely, remark 1. This result depends exclusively on the workings of the credit market in steady state, i.e. whether someone born with zero wealth can borrow from a zero-profits risk-neutral bank. It is therefore robust to, for example, other types of utility functions, forms of altruisms, or the introduction of innate ability. Finally, whether technological skill bias should be seen as endogenous or exogenous depends on whether the economy is a large advanced country that produces its own technology, or rather a country that imports technology. In the body of the paper, I implicitly assumed that technology is not produced endogenously but rather imported from abroad. In Appendix A I show that, as long as the elasticity of the skill bias to the skill composition is not too high, all results presented in the body of the paper continue to hold, with the exception of those relative to how inequality depends on the skill bias.

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A Appendix: endogenous skill bias

Several authors argued that the degree of technological skill bias is endogenous. For example, Acemoglu (1998) assumes that technological change is *directed*, in the sense that the technological skill bias is a function of the skill composition of the economy. He identifies two effects: a *price effect* which encourages the development of technologies complementing the most expensive input, and a *market size effect* which encourages the development of technologies complementing the input with the largest market. The combination of the two effects imply that the technological skill bias can be an increasing or a decreasing function of the ratio of skilled-to-unskilled agents in the economy.

The model can easily accommodate the case of endogenous skill bias. For the sake of simplicity, I do not model here the process by which the skill composition determines the skill bias, but simply assume that the steady state skill bias is a function of the steady state skill composition of the economy, i.e.,

$$\frac{a}{b} \equiv \nu\left(\frac{S_{ss}}{U_{ss}}\right),$$

which I can use to rewrite the steady state skill-premium as:

$$\eta_{ss} = \left(\frac{(1-\lambda^2)a_t}{b_t}\right)^{\epsilon}\left(\frac{U_{ss}}{S_{ss}}\right)^{1-\epsilon} = \left((1-\lambda^2)\cdot\nu\left(\frac{S_{ss}}{U_{ss}}\right)\right)^{\epsilon}\left(\frac{U_{ss}}{S_{ss}}\right)^{1-\epsilon} \equiv (1-\lambda^2)^{\epsilon}\psi\left(\frac{S_{ss}}{U_{ss}}\right).$$

As long as the skill bias is taken as given by agents and firms, we can follow the same steps already described. It is quite immediate to see that Lemma 5 and Remark 1 hold here as well: there is no unequal steady state whenever:

$$\gamma > \frac{\lambda(1+r)}{1-\theta}.$$

Hence, also here, the Hicks-neutral growth rate is a first order determinant of the existence of long run inequality, because when it is sufficiently large everybody has access to the credit market and no inequality is possible.

Following the same steps described above, it is possible to characterize the unique time-invariant wealth distribution for given time-invariant skill premium $\bar{\eta}$. After that, it is possible to look for the time-invariant skill premium $\bar{\eta}$ generating a wealth distribution and a number of skilled and unskilled agents that is consistent with the starting time-invariant skill premium $\bar{\eta}$, as expressed in equation 22. The only difference is that, here $\frac{a}{b}$ depends on $\bar{\eta}$. Assuming that the function $\psi()$ is monotonic, I can write

$$\frac{a}{b} = \nu\left(\psi^{-1}\left(\frac{\bar{\eta}}{(1-\lambda^2)^{\epsilon}}\right)\right) \equiv \tilde{\nu}\left(\frac{\bar{\eta}}{(1-\lambda^2)^{\epsilon}}\right)$$

Leading the the following corollary, which is an adaptation of Proposition 2.

Corollary 2. *Define*

$$\hat{\eta} \equiv \bar{\eta} \text{ such that } \left(\frac{\tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), \bar{\eta}\right)}{1-\tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), \bar{\eta}\right)}\right)^{1-\epsilon} = \left(\tilde{\nu}\left(\frac{\bar{\eta}}{(1-\lambda^2)^{\epsilon}}\right)(1-\lambda^2)\right)^{-\epsilon} \cdot \bar{\eta}. \quad (23)$$

If $\hat{\eta} > \left(1 - \frac{\lambda(1+r)}{\gamma}\right)^{-1}$*, then there exists an unequal steady state* $\{S_{ss}, U_{ss}\}$*, with*

$$U_{ss} = 1 - S_{ss},$$

$$(1-\lambda)^{\epsilon} \cdot \psi\left(\frac{S_{ss}}{U_{ss}}\right) = \hat{\eta}.$$

Instead, if $\hat{\eta} \leq \left(1 - \frac{\lambda(1+r)}{\gamma}\right)^{-1}$ *and*

$$\left(\tilde{\nu}\left(\frac{\bar{\eta}}{(1-\lambda^2)^{\epsilon}}\right)(1-\lambda^2)\right)^{\epsilon}\left(\frac{(1+\lambda)\tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), \bar{\eta}\right)}{1-\tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), \bar{\eta}\right)}\right)^{1-\epsilon}\Bigg|_{\bar{\eta}=1-\frac{\lambda(1+r)}{\gamma}} < \left(1 - \frac{\lambda(1+r)}{\gamma}\right)^{-1},$$

there exists an equal steady state $\{S_{ss}, U_{ss}\}$*, with*

$$U_{ss} = 1 - S_{ss},$$

$$(1-\lambda^2)^{\epsilon} \cdot \psi\left(\frac{S_{ss}}{U_{ss}}\right) = \left(1 - \frac{\lambda(1+r)}{\gamma}\right)^{-1}.$$

The difference between this model and the model discussed in the body of the paper are evident by comparing the RHS of equation (23) with the RHS of equation (22). First of all, with exogenous skill bias, a unique steady state exists whenever $\epsilon \to 1$. This is not true anymore when skill bias is endogenous. Due to the interaction of price effect and market size effect, the RHS of equation (23) does not have to be monotonic or display any type of regularity.

However, it is quite evident that if the function $\nu()$ is sufficiently inelastic, then the LHS of equation (23) is sufficiently close to a straight line, and the results derived in the main text applies here as well: when $\epsilon \to 1$ there is a unique steady state in which the LHS of equation (23) crosses its RHS from above. Hence, if $\nu()$ is sufficiently inelastic, then Lemma 6 applies here as well.

Overall, with endogenous skill bias, when the Hicks-neutral growth rate is high enough, no long-run inequality is possible. Furthermore, when the elasticity of the skill bias to the skill composition of the economy is sufficiently low, the steady state comparative statics with respect to λ and θ derived in the body of the paper continue to apply here as well.

B Appendix: mathematical derivations

Proof of Proposition 1.

Proof. At the beginning of period $t-1$, the state of the economy is given by the wealth distribution $F_{t-1}(e)$ and the number of teachers $\lambda^2 S_{t-1}$. In turn, $\lambda^2 S_{t-1}$ is sufficient to determine the equilibrium wages in period $t-1$ and the cost of education λw_{t-1}^s.

In order to show that a competitive equilibrium exists, I need to show that there exists a S_t with $U_t = 1 - S_t$ such that either

$$w_t^s(S_t, U_t) - \lambda(1+r)w_{t-1}^s \geq w_t^u(S_t, U_t) \tag{24}$$

$$F_{t-1}\left(\lambda w_{t-1}^s - \frac{(w_t^s(S_t, U_t) - \tau(w_t^s(S_t, U_t)))}{1+r}\right) = U_t \tag{25}$$

or

$$w_t^s(S_t, U_t) - \lambda(1+r)w_{t-1}^s = w_t^u(S_t, U_t) \tag{26}$$

$$F_{t-1}\left(\lambda w_{t-1}^s - \frac{(w_t^s(S_t, U_t) - \tau(w_t^s(S_t, U_t)))}{1+r}\right) \leq U_t \tag{27}$$

First of all, note that the returns on the two professions change continuously and monotonically with S_t. It follows that, for S_t sufficiently low equation 24 holds (think of $S_t = 0$), for S_t sufficiently high equation 24 is violated (think of $S_t = 1$, so that there are no unskilled workers in the economy), and there is a unique value of S_t at which 26 holds.

Call S_t' the value of S_t such that 26 holds. If condition 27 holds at S_t', then S_t' is the unique equilibrium S_t, where uniqueness follows from the fact that any other $S_t < S_t'$ would necessary violate 25 and any other $S_t > S_t'$ would necessary violate 24.

Suppose instead that condition 27 is violated at S_t'. Note that any $S_t < S_t'$ satisfies condition 24. To find an equilibrium, we need to find a $S_t < S_t'$ that satisfies equation 25. Note that $\lambda w_{t-1}^s - \frac{(w_t^s(S_t,U_t) - \tau(w_t^s(S_t,U_t)))}{1+r}$ increases continuously with S_t, and U_t decreases continuously with S_t. Hence, if $F_{t-1}(e)$ is continuous, then there exists a unique S_t such that equation 25 is satisfied. If the distribution $F_{t-1}(e)$ is instead discontinuous, the supply of unskilled workers will change discontinuously at some value of $S_t = S''_t$ whenever there is a positive mass of agents with wealth equal to:

$$e'' = \lambda w^s_{t-1} - \frac{(w^s_t(S''_t, 1 - S''_t) - \tau\left(w^s_t(S''_t, 1 - S''_t)\right))}{1 + r}$$

When this is the case, for any $S_t < S''_t$ the number of agents without access to credit is U, but at $S_t = S''_t$ this number jumps discontinuously at $U''_t > U$. However, note that at S''_t banks are indifferent between lending to agents with wealth e'' or not. We can assume that the banking sector lends to any arbitrary fraction of agents with wealth equal to e''. Exploiting this indifference, when $S_t = S''_t$ the supply of unskilled workers can take any value between U and U''. Also in this case, there is a unique S_t such that equation 25 is satisfied. □

Proof of Lemma 4.

Proof. The lemma follows from Theorem 2 in Hopenhayn and Prescott (1992). The existence of the stationary distribution is guaranteed by the fact that the transition function describing the Markov process is *increasing*. Consider two households with initial normalized bequests level $\tilde{e}^1_{t-1}$ and $\tilde{e}^2_{t-1}$ with $\tilde{e}^1_{t-1} \geq \tilde{e}^2_{t-1}$. Assuming that the altruism parameter β^i is the same between these two households, then by lemma 1 for every β^i we have $\tilde{e}^1_t \geq \tilde{e}^2_t$. This implies that the normalized wealth distribution in period t is increasing (in a stochastic dominance sense) in the level of normalized wealth in period $t-1$. The uniqueness of the the stationary distribution is guaranteed by the fact that the Markov process satisfies the Monotone Mixing Condition (MMC), i.e. there exists a level of normalized bequests $\tilde{e}^\star$ and an integer $m > 0$, such that:

- starting from the largest possible level of normalized bequests (the upper bound to the level of normalized bequests that can be accumulated), there is a positive probability of falling below $\tilde{e}^\star$ in m periods. This is always true for any $\tilde{e}^\star$ and m because if the altruism parameter is sufficiently low a parent will leave no bequest.
- starting form the zero wealth, there is a positive probability of being above $\tilde{e}^\star$ in m periods. Again, if $\tilde{e}^\star$ is sufficiently low, this is true for any m because sufficiently altruistic parent will leave positive bequests.

□

Proof of Proposition 2.

Proof. For a given $\bar{\eta}$, the mass of people who cannot access education is $\tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), \bar{\eta}\right)$. Assuming that all agents who can access education become skilled types, we can characterize the steady-state skill premium $\hat{\eta}$ as

$$\hat{\eta} = \left(\frac{a(1-\lambda^2)}{b}\right)^{\epsilon}\left(\frac{U_{ss}}{S_{ss}}\right)^{1-\epsilon} = \left(\frac{a(1-\lambda^2)}{b}\right)^{\epsilon}\left(\frac{\tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), \hat{\eta}\right)}{1-\tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), \hat{\eta}\right)}\right)^{1-\epsilon}$$

Clearly, if $\hat{\eta} > \left(1 - \frac{\lambda(1+r)}{\gamma}\right)^{-1}$ the return on the skilled profession is greater than the return on the unskilled profession, and all agents who can access education will access education. Hence, $\hat{\eta}$ is the steady-state skill premium corresponding to an unequal steady state. Whenever $\hat{\eta} < \left(1 - \frac{\lambda(1+r)}{\gamma}\right)^{-1}$, the return on the skilled profession is lower than the return on the skilled profession at $\hat{\eta}$, nobody will want to access education, and an unequal steady state cannot exist.

However, note that if

$$\left(\frac{a(1-\lambda^2)}{b}\right)^{\epsilon}\left(\frac{\tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), 1-\frac{\lambda(1+r)}{\gamma}\right)}{1-\tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), 1-\frac{\lambda(1+r)}{\gamma}\right)}\right)^{1-\epsilon} < \left(1 - \frac{\lambda(1+r)}{\gamma}\right)^{-1}$$

It is always possible to find a $\phi \in [0,1]$ such that

$$\left(\frac{a(1-\lambda^2)}{b}\right)^{\epsilon}\left(\frac{\tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), 1-\frac{\lambda(1+r)}{\gamma}\right) + \phi\left(1-\tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), 1-\frac{\lambda(1+r)}{\gamma}\right)\right)}{\left(1-\tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), 1-\frac{\lambda(1+r)}{\gamma}\right)\right)(1-\phi)}\right)^{1-\epsilon} = \left(1 - \frac{\lambda(1+r)}{\gamma}\right)^{-1}$$

where ϕ is the fraction of agents with access to the credit market who remain unskilled, because they are indifferent between the two professions. If instead

$$\left(\frac{a(1-\lambda^2)}{b}\right)^{\epsilon}\left(\frac{\tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), 1-\frac{\lambda(1+r)}{\gamma}\right)}{1-\tilde{F}\left(\frac{\lambda}{\gamma} - \left(\frac{1-\theta}{1+r}\right), 1-\frac{\lambda(1+r)}{\gamma}\right)}\right)^{1-\epsilon} > \left(1 - \frac{\lambda(1+r)}{\gamma}\right)^{-1}$$

no equal steady state can exist. □

Proof of Proposition 6.

Proof. The steady-state skill premium is defined as:

$$\left(\frac{\tilde{F}\left(\frac{\lambda}{\gamma}-\left(\frac{1-\theta}{1+r}\right),\hat{\eta}\right)}{1-\tilde{F}\left(\frac{\lambda}{\gamma}-\left(\frac{1-\theta}{1+r}\right),\hat{\eta}\right)}\right)^{1-\epsilon}=\left(\frac{b}{a(1-\lambda^2)}\right)^{\epsilon}\hat{\eta}.$$

By the implicit function theorem, whenever there is an increase in λ (i.e. a decrease in the efficiency of the educational technology), the change in the steady-state skill premium is:

$$\frac{\partial\left(\frac{\tilde{F}\left(\frac{\lambda}{\gamma}-\left(\frac{1-\theta}{1+r}\right),\hat{\eta}\right)}{1-\tilde{F}\left(\frac{\lambda}{\gamma}-\left(\frac{1-\theta}{1+r}\right),\hat{\eta}\right)}\right)^{1-\epsilon}}{\partial\hat{\eta}}\cdot\frac{\partial\hat{\eta}}{\partial\lambda}+\frac{\partial\left(\frac{\tilde{F}\left(\frac{\lambda}{\gamma}-\left(\frac{1-\theta}{1+r}\right),\hat{\eta}\right)}{1-\tilde{F}\left(\frac{\lambda}{\gamma}-\left(\frac{1-\theta}{1+r}\right),\hat{\eta}\right)}\right)^{1-\epsilon}}{\partial\lambda}=\left(\frac{b}{a(1-\lambda^2)}\right)^{\epsilon}\frac{\partial\hat{\eta}}{\partial\lambda}-\epsilon\frac{2\lambda}{(1-\lambda^2)}\left(\frac{b}{a(1-\lambda^2)}\right)^{\epsilon}$$

$$\frac{\partial\hat{\eta}}{\partial\lambda}=\left[\left(\frac{b}{a(1-\lambda^2)}\right)^{\epsilon}-\frac{\partial\left(\frac{\tilde{F}\left(\frac{\lambda}{\gamma}-\left(\frac{1-\theta}{1+r}\right),\hat{\eta}\right)}{1-\tilde{F}\left(\frac{\lambda}{\gamma}-\left(\frac{1-\theta}{1+r}\right),\hat{\eta}\right)}\right)^{1-\epsilon}}{\partial\hat{\eta}}\right]^{-1}\left[\frac{\partial\left(\frac{\tilde{F}\left(\frac{\lambda}{\gamma}-\left(\frac{1-\theta}{1+r}\right),\hat{\eta}\right)}{1-\tilde{F}\left(\frac{\lambda}{\gamma}-\left(\frac{1-\theta}{1+r}\right),\hat{\eta}\right)}\right)^{1-\epsilon}}{\partial\lambda}+\epsilon\frac{2\lambda}{(1-\lambda^2)}\left(\frac{b}{a(1-\lambda^2)}\right)^{\epsilon}\right]$$

As discussed in the text, here I'm considering steady states in which the LHS of equation 22 crosses the RHS of equation 22 from above. Hence, $\left(\frac{b}{a(1-\lambda^2)}\right)^{\epsilon}>\frac{\partial\left(\frac{\tilde{F}\left(\frac{\lambda}{\gamma}-\left(\frac{1-\theta}{1+r}\right),\hat{\eta}\right)}{1-\tilde{F}\left(\frac{\lambda}{\gamma}-\left(\frac{1-\theta}{1+r}\right),\hat{\eta}\right)}\right)^{1-\epsilon}}{\partial\hat{\eta}}$. Furthermore, because of Lemma 3, everything else equal the probability of becoming a skilled agent decreases with λ, so that $\frac{\partial\left(\frac{\tilde{F}\left(\frac{\lambda}{\gamma^{\frac{1}{\epsilon}}}-\left(\frac{1-\theta}{1+r}\right),\hat{\eta}\right)}{1-\tilde{F}\left(\frac{\lambda}{\gamma^{\frac{1}{\epsilon}}}-\left(\frac{1-\theta}{1+r}\right),\hat{\eta}\right)}\right)^{1-\epsilon}}{\partial\lambda}>0$. Therefore the steady-state skill premium increases with λ.

Similarly, we have

$$\frac{\partial\hat{\eta}}{\partial\theta}=\left[\left(\frac{b}{a(1-\lambda^2)}\right)^{\epsilon}-\frac{\partial\left(\frac{\tilde{F}\left(\frac{\lambda}{\gamma}-\left(\frac{1-\theta}{1+r}\right),\hat{\eta}\right)}{1-\tilde{F}\left(\frac{\lambda}{\gamma}-\left(\frac{1-\theta}{1+r}\right),\hat{\eta}\right)}\right)^{1-\epsilon}}{\partial\hat{\eta}}\right]^{-1}\frac{\partial\left(\frac{\tilde{F}\left(\frac{\lambda}{\gamma}-\left(\frac{1-\theta}{1+r}\right),\hat{\eta}\right)}{1-\tilde{F}\left(\frac{\lambda}{\gamma}-\left(\frac{1-\theta}{1+r}\right),\hat{\eta}\right)}\right)^{1-\epsilon}}{\partial\theta}$$

Therefore, the steady-state skill premium decreases with θ because by Lemma 2

$$\frac{\partial\left(\frac{(1+\lambda)\tilde{F}\left(\frac{\lambda}{\gamma^{\frac{1}{\epsilon}}}-\left(\frac{1-\theta}{1+r}\right),\eta_{ss}\right)}{1-\tilde{F}\left(\frac{\lambda}{\gamma^{\frac{1}{\epsilon}}}-\left(\frac{1-\theta}{1+r}\right),\eta_{ss}\right)}\right)^{1-\epsilon}}{\partial\theta}<0.$$

Furthermore, this effect is stronger the larger is the skill bias $\frac{a}{b}$. □