Benevolent mediation in the shadow of conflict*

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Abstract

Before the start of a negotiation, the negotiating parties may try to manipulate its outcome by making socially-wasteful investments, such as purchasing weapons or asking for legal opinions. We study the problem of a benevolent mediator who controls the bargaining protocol of the negotiation and wishes to minimize pre-negotiation wasteful investments. We derive the efficient bargaining protocol under different specifications of the information available to the mediator. Under some conditions the mediator will choose a protocol that benefits the strongest player. We therefore highlight a potential conflict between fairness and efficiency arising in negotiations.

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1 Introduction

Negotiating parties can influence the outcome of the negotiation by making various types of pre-negotiation investments. These investments are often a form of rent seeking, because they do not increase the total payoff to be shared during the negotiation, but only how this surplus is split. They are, however, quite common.

For instance, negotiations are often conducted under the shadow of conflict: in case an agreement is not reached, the negotiating parties will fight in a non-cooperative game. Because the outcome of the conflict defines the disagreement point of the negotiation, the bargaining parties may spend resources to prepare for conflict even if they expect to achieve a negotiated agreement. This is why, for example, a government and a rebel group may engage in military actions just before negotiating a peace agreement.[1] Similarly, before negotiating a settlement, two firms may ask for additional legal opinions or hire very expensive lawyers as a way to manipulate the outcome of the lawsuit that may follow the breakdown of the negotiation.[2]

Conflict is, however, only one possible alternative to achieving a negotiated agreement. The second alternative is further protracting the negotiation with the associated cost of waiting. These costs are an important elements in determining the players' negotiating outcomes, which is precisely the logic behind the seminal model by Rubinstein (1982).[3] Therefore, an additional type of wasteful, pre-negotiating investments are those aimed at reducing the cost protracting a negotiation. These investments have been so far overlooked by the literature, but examples abound. A particularly famous one is the 2-year lease for a house paid by the Vietnamese delegation at the onset of the Paris negotiations to end the Vietnam war (see Raiffa, 1982, page 16). In a similar spirit, prior to starting a wage negotiation, trade unions routinely create funds to support striking workers, so to reduce the cost of protract-

[1] There is ample evidence that conflicts are reactivated prior to the beginning of peace negotiations. For example, the mass killing of civilians (thus permanently weakening the opponents) is significantly more probable during the process of democratization of a country. See Esteban, Morelli, and Rohner (2015).

[2] For a review of the academic literature arguing that the bargaining parties may spend resources to prepare for conflict before the start of the negotiation see Jackson and Morelli (2011) and the literature review in Meirowitz, Morelli, Ramsay, and Squintani (forthcoming).

[3] See also Raiffa (1982), chapter 6, for several examples, including theoretical and experimental evidence.

ing their industrial action and the negotiation.

In this paper we study the problem of a benevolent mediator who wishes to maximize social welfare, that is, to efficiently share the "peace dividend" (i.e, the aggregate benefit of finding an agreement rather than triggering a conflict) in a way that minimizes wasteful pre-negotiation investments. This benevolent mediator could be a person, an institution, an international organization, or a country called in to mediate, for example, a civil conflict. In our model, the mediator controls the bargaining protocol: the probability that, in each period of the negotiation, each party can make an offer to the other party. The mediator announces the bargaining protocol at the beginning of the game,[4] and then the negotiating parties make their investments. Finally, the negotiation starts.

Absent the mediator, the possibility of making pre-negotiation investments may generate inefficient outcomes. To illustrate this point, we model the unmediated negotiation as a bargaining game in alternating offers with positive outside options, as in Binmore, Rubinstein, and Wolinsky (1986) and Binmore, Shaked, and Sutton (1989). In the equilibrium of this game, either the players' payoffs depend on their discount factors and are independent from the two outside options, or one of the two players is kept at his outside option. In the first case the players have an incentive to invest in increasing their discount factor by making what we call an *investment in patience*. In the second case the players have an incentive to manipulate the outside options. More precisely, the player that expects to be kept at his outside option benefits from increasing his own outside option by making what we call *defensive investments*. The other player instead benefits from decreasing the opponent's outside option by making what we call *offensive investments*.

We then explore whether and how wasteful investment can be minimized or even eliminated by a mediator via an appropriate choice of the bargaining protocol. In our model, the mediator cannot commit to destroying surplus, and therefore can only announce bargaining protocols that are efficient both on- and off-equilibrium. Despite this, we show that the mediator can achieve full efficiency when he either observes offensive and defensive investments, or he observes the investments in patience. When offensive and defensive investments are observed, the mediator will implement a *permanent proposer* bargaining protocol, in which, in every period,

[4] Instead of an explicit announcement, we can equivalently think of established norms or protocols followed by a mediator in case of intervention. In this case, we want to know which norms or protocols minimize wasteful pre-negotiation investments.

the same player proposes with probability 1 to the other player. This bargaining protocol eliminates all incentives to invest in patience. Furthermore, by choosing the probability that each player will be the permanent proposer, the mediator can determine the share of peace dividend accruing to each player in the equilibrium of the bargaining game. This share can be manipulated so to compensate for any offensive or defensive investments, hence eliminating the incentives to invest. Similarity, if the mediator observes the players' investments in patience, he can implement a *constant random proposer* bargaining protocol in which a given player proposes to the other with constant probability in every period. By varying the probability that each player makes a proposal in response to the players' investment in patience, the mediator can maintain the players on the same utility level independently from their investments (offensive, defensive or in patience).[5]

When the mediator cannot observe neither the investments in patience, nor offensive and defensive investments, he will choose randomly which player is the permanent proposer so to eliminate investments in patience. The probability that each player is the permanent proposer determines the share of the peace dividend accruing to each player. We characterize the waste minimizing sharing rule and show that it is asymmetric, giving a larger share to the strongest player and inducing the weakest player not to invest at all, where "weak" and "strong" are defined by the outcome of the potential conflict in absence of investments.

The intuition is that by choosing the share of surplus accruing to each player, the mediator determines the sensitivity of each player's payoff to the two outside options, and therefore the incentives to manipulate these outside options via offensive and defensive investments. More precisely, allocating a larger share of surplus to a player makes both players' payoffs more strongly dependent on the *other* player's outside option. By favoring the strongest player, the mediator increases the "fight" over the weak player's outside option—with the weak player making defensive investments and the strong player making offensive investments—and decreases the fight over the strong player's outside option—with the strong player making defensive investments and the weak player making offensive investments. In our specification, the mediator minimizes total waste by inducing the players to fight over the lowest of the two outside options (that of the weak player), and away from the highest outside option (that of the strong player). Therefore, there may be a trade off between equity and

[5] Interestingly, there is no benefit for the mediator to observing both the investments in patience, and offensive and defensive investments.

efficiency: to minimize wasteful, pre-negotiation investment the mediator should be biased toward the strongest player.

A final point we make is that the mediator may benefit from organizing a *concession game*. This game is akin to a tournament in which each player tries to influence the bargaining protocol by making concessions to the other player—where concessions are visible, costly actions that benefit the opponent. This tournament can help the mediator because the equilibrium level of concessions is a function of the size of the peace dividend, which is itself a function of offensive and defensive investments. Even if the mediator does not observe these investments, he can nonetheless affect them by designing the concession game appropriately. In particular, we show that the mediator can construct the concession game so that offensive investments (those that increase the peace dividend and hence the incentive to make concessions) are fully eliminated.

Our paper belongs to the literature studying hold-up problems, that is, how to achieve achieve efficiency (or reduce inefficiencies) when investments are not contractible. The vast majority of papers in this literature, however, assume that the outside options of the ex-post negotiation can be decided contractually ex-ante in order to induce the efficient level of investments.[6] Here instead we are interested in situations in which the outside options are determined by the players' investments and therefore cannot be specified contractually ex-ante. To the best of our knowledge, this question has not received much attention before, especially in the contest of mediation.[7] Whereas many papers have noted that the negotiating parties may want to invest before the start of the negotiation, existing economic models of mediation assume that the mediator's sole role is to maximize surplus *within the negotiation*.[8] Introducing pre-negotiation actions changes the role of the mediator,

[6] For example, in the seminal work of Grossman and Hart (1986) the allocation of ownership indirectly determines the players' outside options in the ex-post negotiation. In Aghion, Dewatripont, and Rey (1994) the players can directly specify ex-ante the outside options of the ex-post negotiation (together with the surplus share accruing to each player in the ex-post negotiation).

[7] There are papers in which the outside options of the ex-post negotiation depend both on the ex-ante allocation of ownership and on subsequent, non contractible investments (see Edlin and Reichelstein, 1996, Che and Hausch, 1999, and Chatterjee and Chiu, 2013). A few papers addressed our exact research question but in a specific environment: a network of buyers and sellers, in which each player can spend resources to link with an additional buyer/seller and therefore increase his bargaining power (see Kranton and Minehart, 2000, Kranton and Minehart, 2001 and Elliott, 2015).

[8] See the review by Jackson and Morelli (2011) on the different reasons why bargaining failures

because maximizing surplus requires considering also how the mediator can affect pre-negotiation wasteful investments.[9]

A second point of departure from the economic literature on mediation is that, in our model, the mediator can influence the negotiating outcome but may not be able to fully determine it.[10] The existing models of mediation instead typically assumes that the mediator can only make incentive compatible recommendations to the players (as in the seminal work by Myerson, 1986) and contrasts the role of the mediator with that of an arbitrator who has the power to impose an outcome on the players. Scholars in political science and international relationship instead propose a more nuanced view on the ability of third parties to determine the outcome of a negotiation. Our modelling choice is close to Fisher (2012)'s *power mediation* in which the mediator has some power over the outcome of the mediation but cannot fully determine it (as opposed to an arbitrator).[11]

The paper that is closest to ours is Meirowitz, Morelli, Ramsay, and Squintani (forthcoming), who also study the role of the mediator in reducing pre-negotiation wasteful investments. They compare mediated and unmediated negotiation and argue that mediated negotiation generates lower pre-negotiation wasteful investment in arms. In their framework, due to informational asymmetries inefficient negotiation breakdowns may occur. The mediator's role is to regulate the flow of information among parties, so to maximize the probability that an efficient settlement is reached. By doing so, he also determines the precision of each player's belief relative to the other player's strength and the incentives to modify this strength via

and an inefficient war may occur. Recent papers that explore the role of third party intervention in reducing the probability of an inefficient breakdown of the negotiations are Goltsman, Hörner, Pavlov, and Squintani (2009), Hörner, Morelli, and Squintani (2015), Balzer and Schneider (2015). Recent papers that explore the role of third party intervention in reducing the time required to reach an agreement are Fanning (2016) and Basak (2017), who model the negotiation as a war of attrition.

[9] An exception is Baliga and Sjöström (2004), who study how a negotiation (modeled as a cheap-talk stage) can reduce *subsequent* inefficient investments.

[10] This will happen whenever the mediator is less informed than the two players.

[11] Because, in our model, the mediator regulates the flow of offers between players, we establish an interesting connection between what Fisher (2012) calls *conciliation*, that is, the role of the mediator in establishing a communication link between the players, and power mediation. See also a similar taxonomy by Bercovitch (1997), who distinguishes between a mediator's communication strategies (i.e., transmitting messages) and manipulative strategies (i.e., influencing the outcome of the negotiation).

a pre-bargaining investment. His effect on the players pre-bargaining investment is, therefore, indirect and unintentional. Hence, our model differs from Meirowitz et al. (forthcoming) in one fundamental aspect: the mediator's goal is to achieve efficiency, which explicitly includes reducing wasteful pre-negotiation investments.[12]

A number of other authors also noted that the way the negotiation is conducted can affect pre-negotiation actions by the players. Both Esteban, Morelli, and Rohner (2015) and Garfinkel, McBride, and Skaperdas (2012) show that the surplus share accruing to each player can have an effect on decisions made prior to the beginning of the negotiation. In Esteban et al. (2015) the surplus share obtained by each party in a negotiation may affect the intensity of the pre-negotiation conflict. They show that an equal surplus-split rule may be welfare decreasing relative to an asymmetric surplus-split rule. Garfinkel et al. (2012) notice that by investing in arms players influence the probability of winning in a conflict—and hence the disagreement point—and the share of surplus in the case of a peaceful agreement. Their main result is that when fighting is not sufficiently destructive, arming will be unavoidable within the class of distribution rules they consider. Also related is the model in Anbarci, Skaperdas, and Syropoulos (2002), where each party starts by making wasteful investments in armaments. The paper compares the waste produced by three cooperative bargaining solutions: equal sacrifice, equal benefit, and Kalai-Smorodinski. The main result is that if players are symmetric equal sacrifice is the solution generating the lowest waste. Our contribution with respect to these papers is to study the full problem of a mediator who wishes to minimize pre-negotiation wasteful investments, including the informational constraints he may face.

Finally, our paper contributes to an important debate in political science and international relations regarding the merits of biased mediation (see Svensson, 2014 for a review). This literature argues that a mediator who is biased may be more likely to achieve an agreement, where "bias" typically refers to a distortion in the mediator's preferences. For example, in Kydd (2003) a bargaining party is more likely to believe the information transmitted by a mediator if the mediator is biased in favor of this party. In our paper, the mediator is, in principle, unbiased because

[12] Our models are complementary in two additional aspects. One is that they assume that the bargaining players are ex-ante identical, while our paper is mostly concerned with how ex-ante differences affect the optimal negotiation procedure. The second difference is in the treatment of information. We assume that players have complete information while they assume asymmetry of information among players.

his goal is to maximize welfare. However, if the players are asymmetric in the cost or the benefit of making pre-negotiation investments, he may choose to be *strategically* biased in order to decrease total waste. Hence, here bias is an equilibrium outcome.

The remainder of the paper is organized as follows. The next section describes the model. Section 3 solves the model for the case in which the mediator is absent. Section 4 introduces the mediator. Section 5 considers some extensions to the model. In particular, Section 5.3 allows the mediator to organize a pre-negotiation contest over the bargaining protocol. The last section concludes. Unless otherwise noted, all proofs are in Appendix.

2 The model

A total payoff S is to be shared between two players, 1 and 2, initially characterized by their ex-ante power levels ϕ_1 and ϕ_2. The players negotiate on how to split S, but if they fail to find an agreement a conflict will arise.

The timing is as follows. At the beginning of the game each player can make investments aimed at shifting the payoffs in case of conflict. We consider two kinds of such investments: offensive and defensive. Next, and just before the negotiation starts, the players make investments in patience: actions aimed at reducing the cost of extending the length of the negotiation. Finally, the negotiation starts. If there is no mediator, the negotiation is a bargaining game in alternating offers. If there is a mediator, the mediator will choose the bargaining protocol: the probability that each player makes an offer to the other player in every period of the negotiation. The mediator announces the bargaining protocol at the beginning of the game, and this announcement is fully incorporated into the players' investment choices. Figure 1 summarizes the timing of the game.

The outside option: conflict

The payoff in the Nash equilibrium of the conflict game are determined by the players' initial power levels ϕ_1 and ϕ_2, and by the players' offensive and defensive investments.[13]

A player's initial power level ϕ_i is the payoff achieved by this player in the conflict

[13] We call it "conflict game" because, in case conflict is triggered, the players may take additional costly actions, such as effort, giving rise to a game.

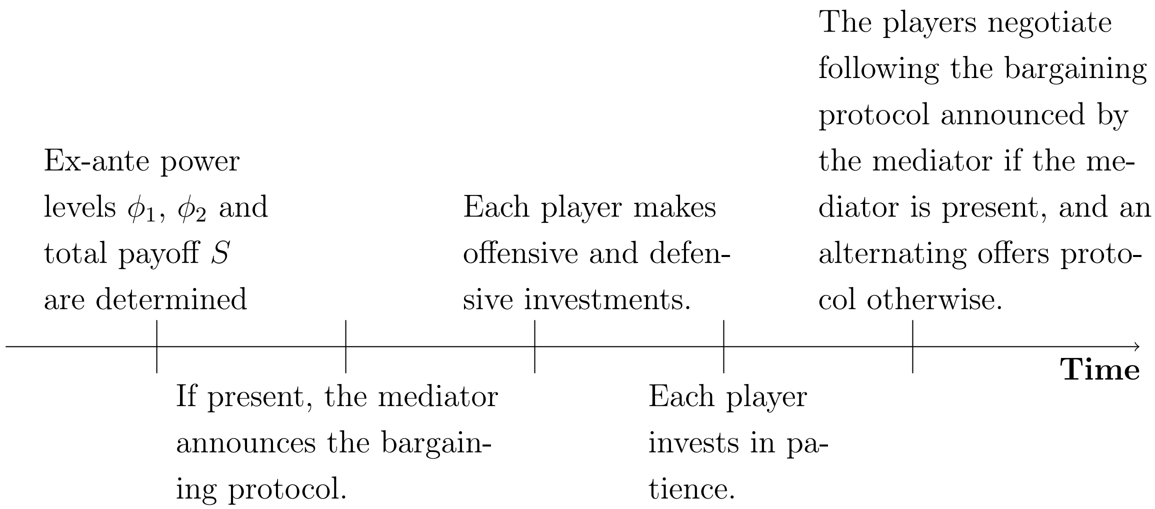

Fig. 1: Timeline

game in case no investment is made. This payoff may depend on natural elements (e.g. the presence of mountains may make one country harder to attack) or by the merit of the legal dispute. It may also depend on prior investments. The conflict payoffs can be manipulated by the players' offensive o_i and defensive investments d_i. By investing in o_i player i decreases player $-i$'s payoff in the conflict game, while by investing d_i player i can increase his own payoff in the conflict game.

As examples of offensive investment, a player may purchase ballistic missiles or collect evidence against the opponent to be used in a court case. As examples of defensive investments, a player may purchase antimissile system and bunkers, or move assets to jurisdictions where they are harder to seize in case the outcome of a lawsuit is negative. In the case of industrial conflict, a firm may invest resources to make the relocation of the factory a credible threat, which should be considered "offensive." These investments can also be interpreted as pre-bargaining costly actions aimed at appropriating part of the surplus. For example, before the start of a negotiation over a territorial dispute, a country may invade part of the disputed territory. This costly action is formally identical to a defensive investment because it increases the payoff of the invading party in case of breakdown of the negotiation over the allocation of the remaining territory. Similarly, an offensive investment is a costly action that transforms a territory formerly in the hands of a given country into a contested area to be allocated via a negotiation.[14]

An important caveat is that many investments are simultaneously offensive and defensive, in the sense that they simultaneously increase a player's utility and decrease the opponents utility in case of conflict (for example, hiring a very expensive but competent lawyer or purchasing tanks). Some other investments decrease both players utilities in case of conflicts, and are therefore mutually offensive (for example, nuclear weapons). Whether an investment is offensive, defensive (or both) depends also on the details of the conflict game.[15] But in any model of conflict, any type of investment can be expressed as a combination of offensive and defensive investments based on its effect on the players' payoffs in the Nash equilibrium of the conflict game. As a first approximation here we only consider purely offensive or purely defensive investments.

Let us denote by $\underline{u}_i$ the payoff of player i in the conflict game, taking into account her ex-ante power ϕ_i, own defensive investment d_i and the opponent's offensive investment o_{-i}, that is, $\underline{u}_i = u(\phi_i, d_i, o_{-i})$. We specify this payoff function to

$$u(\phi_i, d_i, o_{-i}) \equiv \phi_i e^{-o_j}(2 - e^{-d_i}),\ i, j = 1, 2.$$

The key feature of this expression is weak separability: the marginal rate of substitution between d_i and ϕ_i is independent of o_{-i} and the one between o_{-i} and ϕ_i is independent of d_i. This implies that the rate at which player i can compensate for being weak ex-ante (i.e., having low ϕ_i) by making an investment in defense is independent on o_{-i}. Similarly, the rate at which player i can compensate for player $-i$ being strong ex-ante (i.e., having high ϕ_{-i}) by making an investment in offense is independent on d_{-i}. This key feature is justified by the fact that ϕ_i can be the result of investments in offensive and defensive technology made in prior periods, the marginal effect of which should not depend on the current level of investments.[16]

[14] We thank Attila Ambrus for suggesting this interpretation.

[15] For example, in Dixit (1987) players pre-commit effort before entering a contest. He shows that the strongest player (the one with a higher probability of winning) may want to pre-commit effort above the one-shot Nash equilibrium level. Once the contest is reached, the strongest player will exert no additional effort, while the other player will set effort below the one-shot Nash equilibrium level. In this case, the pre-commitment of effort can be a form of "mutually defensive" investment because it reduces effort during the conflict and therefore may increase both players' payoff in case the conflict is reached. Instead, in a war of attrition, some investments may increase the length of a conflict and therefore be mutually destructive.

[16] One can alternatively assume that the marginal rate of substitution between d_i and ϕ_i is

Clearly, weak separability is satisfied if and only if $u_i = \psi[f(\phi_i)g(d_i)h(o_{-i})]$. Our assumed payoff function is a member of this class that we use for the sake of convenience, as it yields tractable closed forms. The marginal cost of investing in defensive and offensive technology are c_d and c_o, assumed constant.

Note that, if the players expect no negotiation to occur –and hence invest solely in view of influencing the payoff of the conflict game– their offensive investment will be zero and hence the joint payoff is

$$\max_{d_1} \{u(\phi_1, d_1, 0) - c_d \cdot d_1\} + \max_{d_2} \{u(\phi_2, d_2, 0) - c_d \cdot d_2\}.$$

there is a role for a mediator only when sharing S in a negotiation dominates the conflict payoff. Otherwise, the players will never reach the negotiation stage. Therefore, throughout the paper maintain the assumption that

$$S \geq \max_{d_1} \{u(\phi_1, d_1, 0) - c_d \cdot d_1\} + \max_{d_2} \{u(\phi_2, d_2, 0) - c_d \cdot d_2\} \tag{A1}$$

Investing in patience.

After setting offensive and defensive investments the players can invest in patience. Call $\bar{\beta}_i \in (0,1)$ the players' initial discount factor. Each player can increase his discount factor to any $\beta_i \in [\bar{\beta}_i, \hat{\beta}]$ at a cost $c(\beta_i - \bar{\beta}_i)$, where $0 < \bar{\beta}_1, \bar{\beta}_2 < \hat{\beta} < 1$. We assume that $\lim_{x \to 0} c'(x) = 0$ so that the investment in patience will be strictly positive whenever a player's payoff depends on his/her discount factor.

Investing in patience should be interpreted as taking costly actions that decrease the cost of protracting the negotiation. As already discussed in the Introduction, a famous example is the two-year lease for a house payed by the Vietnamese delegation at the beginning of the Paris negotiations to end the Vietnam war. After the lease was payed, the marginal cost of protracting the negotiation was greatly reduced because the cost of housing was sunk. Similarly, unions routinely set up resistance funds to support striking workers, and hence reduce the cost of protracting the negotiation (and the corresponding industrial action).

a function of o_{-i}. But then o_{-i} should affect the relative benefit not only of current defensive investments, but also of past defensive investments. It follows that ϕ_i should be a function of the investments o_{-i} and d_i, a complication that we wish to avoid. Note also that our specification abstracts away issues related to risk, that are not central to our analysis.

The negotiation.

After all investments are made, the players negotiate over how to split the ex-post surplus (also called "peace dividend"):

$$S - \underline{u}_1 - \underline{u}_2,$$

which is always positive because of (A1). We model the negotiation as an infinite-horizon bargaining game.[17] Absent the mediator, the bargaining game is one of alternating offers: in every period one player makes an offer and the other player can accept, reject and move to the next period, or reject and trigger a conflict. If the negotiation is mediated by a third party, the mediator will decide who proposes in each period. Hence, the mediator here is an independent agent with some enforcement power –such as a large country, an international organization or an international court– who can coerce players into respecting a specific bargaining protocol.[18]

The bargaining protocol imposed by the mediator can be represented by two sequences

$$\{p_{1,1}, p_{1,2}, p_{1,3}...\}$$

$$\{p_{2,1}, p_{2,2}, p_{2,3}...\}$$

where $p_{i,t}$ is the probability that player $i \in \{1, 2\}$ makes an offer in period t, also called *recognition probability.* We rule out simultaneous offers, so that $p_{1,t} \leq 1-p_{2,t}$.[19] For example, a bargaining protocol of the form

$$\{1, 0, 1, 0, ...\}$$

[17] Note that we do not allow the players to make further investments once the negotiation has started. We consider this possibility in Section 5.2, where we argue that our results remain qualitatively unchanged.

[18] The negotiations between the Colombian government and the FARC provide a good example. They have taken place in Cuba because the assumed leverage of this government on the FARC. In Germany, for many years the negotiations between unions and employers have been chaired by the government because its leverage on the parties.

[19] It is well known that if simultaneous offers are possible then the bargaining game may have multiple equilibria, some of them inefficient. For example, Chatterjee and Samuelson (1990) consider an infinitely-repeated bargaining game in which players can make simultaneous offers and show that the set of *perfect* equilibria consists of every possible individually rational outcome. We assume later that the mediator cannot commit to destroying welfare, and hence will not allow simultaneous offers if he expects the resulting equilibrium to be inefficient.

$$\{0, 1, 0, 1, ...\}$$

corresponds a game or alternating offers. We call a bargaining protocol of the form

$$\{p, p, p, ..., \}$$

$$\{q, q, q, ..., \}$$

with $q \leq 1 - p$ a bargaining protocol with *constant recognition probability*. Furthermore, any correlation between probability of proposing at different periods is possible. For example, in the bargaining protocol

$$\{p_{1,1}, p_{1,2}, ...\},\ \{p_{2,1}, p_{2,2}, ...\} = \begin{cases} \{1, 1, ...\},\ \{0, 0, ...\} & \text{with probability } \gamma \\ \{0, 0, ...\},\ \{1, 1, ...\} & \text{with probability } 1 - \gamma \end{cases}$$

the probabilities of proposing in each period are perfectly correlated. We call such bargaining protocol *random permanent proposer*.

Information structure

We assume that the two players are fully informed: they observe the power levels ϕ_1 and ϕ_2, the total payoff to be shared S, both players' offensive and defensive investments, both players' investment in patience and the resulting discount factors. We therefore abstract away from the usual role of the mediator as a filter of the information flow between the two players, and only focus on his role in determining the order of offers. We will analyze the mediator's problem under different assumptions regarding what the mediator can observe.

The mediator

We assume that the mediator is benevolent: his objective is to maximize welfare. However, the mediator lacks commitment, which implies that the only bargaining protocols that he can credibly implement are those that maximize welfare at every stage of the game, both on and off equilibrium.

This implies that the mediator will always choose bargaining protocols of the form $p_{i,t} \equiv p_t$ and $p_{2,t} = 1 - p_t$, so that costly delays in reaching an agreement are avoided. Among bargaining protocols of that form, the mediator will choose the bargaining protocol that is ex-post efficient, that is, that minimizes the investment in patience.

It follows that the solution to the mediator's problem will be the bargaining protocol that minimize expenditure in offensive and defensive technology, $c_o(o_1 + o_2) + c_d(d_1 + d_2)$, under the constraint that the bargaining protocol chosen must have the form $p_{i,t} \equiv p_t$ and $p_{2,t} = 1 - p_t$ and be ex-post efficient. Depending on what the mediator can observe, the bargaining protocol $\{p_1, p_2, p_3, ...\}$ can be contingent on the initial power ϕ_1 and ϕ_2, and/or on the investments made by each player.

3 Bargaining without the mediator.

Suppose the mediator is absent and the bargaining game is in alternating offers. If we take the discount factors β_1, β_2 and the outside options $\underline{u}_1, \underline{u}_2$ as given, the game is fairly standard and is discussed, for example, in Binmore, Shaked, and Sutton (1989).[20] Call $v_1 \geq \underline{u}_1$ and $v_2 \geq \underline{u}_2$ the players' utilities in equilibrium. Because the game is stationary, v_1 and v_2 are also the players utilities in case period 3 of the game is reached. Suppose period 2 of the negotiation is reached and player 2 makes an offer to player 1. After receiving player 2's offer, player 1 has three options: accept the offer, reject and trigger a conflict so as to earn $\underline{u}_1$, or reject and continue the negotiation so as to earn $\beta_1 v_1$. It follows that player 2's offer will be equal to $\max\{\beta_1 v_1, \underline{u}_1\}$. Hence, if period 2 of the negotiation is reached, player 2's utility is $S - \max\{\beta_1 v_1, \underline{u}_1\}$. Consider now period 1. For the same logic, player 1 will offer $\max\{\beta_2(S - \max\{\beta_1 v_1, \underline{u}_1\}), \underline{u}_2\}$ and player 2 will accept. The equilibrium payoffs therefore solve

$$v_2 = \max\{\beta_2(S - \max\{\beta_1 v_1, \underline{u}_1\}), \underline{u}_2\}$$

$$v_1 = \max\{S - v_2, \underline{u}_1\}$$

with solution

$$v_1 = \begin{cases} S - \underline{u}_2 & \text{if } S\frac{1-\beta_2}{1-\beta_1\beta_2} > S - \underline{u}_2 \\ \underline{u}_1 & \text{if } S\frac{1-\beta_2}{1-\beta_1\beta_2} < \underline{u}_1 \\ S\frac{1-\beta_2}{1-\beta_1\beta_2} & \text{otherwise,} \end{cases}$$

$$v_2 = S - v_1.$$

See Figure 2 for an illustration. There are therefore two relevant cases to consider: the case in which the equilibrium payoffs are determined by the discount

[20] The only difference is that, here, the two players may have different discount factors.

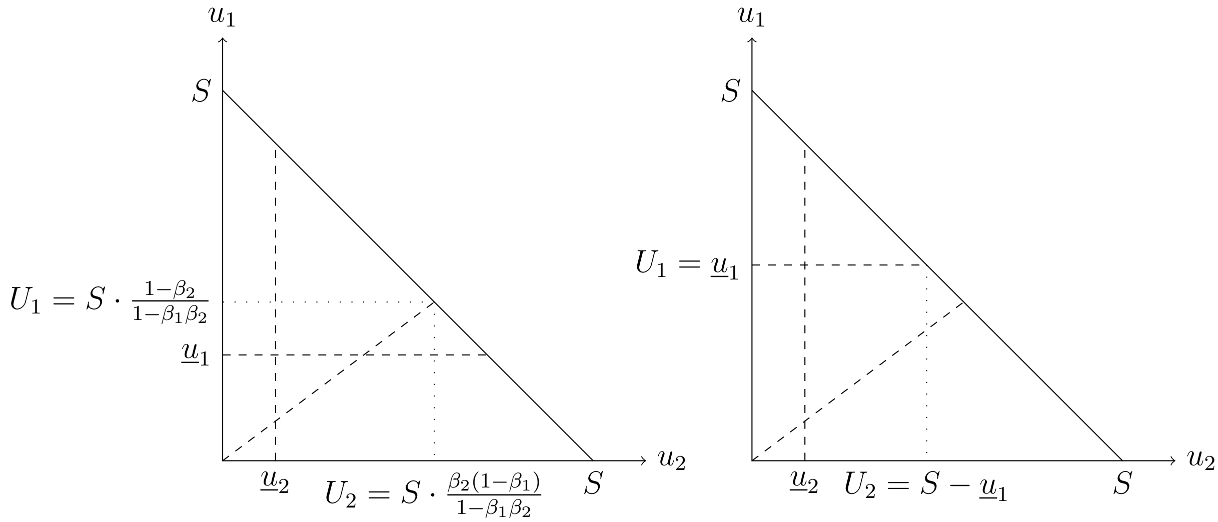

Fig. 2: Solution to the negotiation without the mediator for different values of $\underline{u}_1$.

factors, and the case in which the equilibrium payoffs are determined by one of the two outside options.[21] In each case, the players have the incentive to make wasteful pre-negotiation investments. Suppose that, absent any investment in patience, we are in the first case, that is:

$$v_1 = S\frac{1-\underline{\beta}_2}{1-\underline{\beta}_1\underline{\beta}_2} > \underline{u}_1 \qquad v_2 = S - v_1 > \underline{u}_2.$$

In this case, the marginal benefit of an arbitrarily small investment in patience is strictly positive, while its marginal cost is zero. It follows that, in equilibrium both players will invest in patience.

Suppose instead that, absent any investment in patience, the solution is such that one of the two players is kept at his outside option, that is, suppose that either

$$S\frac{1-\underline{\beta}_2}{1-\underline{\beta}_1\underline{\beta}_2} < \underline{u}_1$$

or

$$S\frac{1-\underline{\beta}_2}{1-\underline{\beta}_1\underline{\beta}_2} > S - \underline{u}_2$$

[21] Of course, there is also a limit case in which the equilibrium payoffs are determined by the discount factors, but at the same time one of the two equilibrium payoffs is equal to one of the options. For ease of exposition we ignore this case.

In this case, the player kept at his outside option can increase his payoff by making a defensive investment, while the other player can increase his payoff by making an offensive investment. If player i is kept at his outside option, he will set positive defensive investment whenever

$$\frac{\partial u_i(\phi_i, d_i, o_{-i})}{d_i}|_{d_i=0} = \phi_i e^{o_{-i}} > c_d$$

Similarly, player $-i$ will set positive offensive investment whenever:

$$\frac{\partial u_i(\phi_i, d_i, o_{-i})}{o_{-i}}|_{o_{-i}=0} = \phi_i(2 - e^{d_i}) > c_o.$$

If ether c_o and c_d are sufficiently small, one of the two players will want to invest.

The bottom line is that the solution to the negotiation is either determined by the discount factors or is determined by the outside options. In one case, the two players will have an incentive to invest in patience, while in the other case they have an incentive to manipulate their outside options by making offensive and defensive investments. The following proposition summarizes these observations and also provides a proof of the existence of the equilibrium.

Proposition 1. *Assume, without loss of generality, that $\phi_1 > \phi_2$. Whenever $\phi_2 > \min\{c_o, c_d\}$ then the equilibrium is inefficient (if multiple equilibria exist, then all equilibria are inefficient).*

Note that the conditions described in the above proposition are sufficient but by no mean necessary in order for an inefficient equilibrium to emerge.

It is also possible to characterize when there will be investment in patience or investment in manipulating the outside options. If S is sufficiently large, then for any $\{\underline{u}_1, \underline{u}_2\}$ the solution will be independent from the two outside options and the players will invest in patience with probability 1. If instead S is sufficiently small, one of the two outside options will be binding, and therefore the players may make offensive and defensive investments. The composition of the investment therefore depends on the size of the peace dividend.

4 Mediation in the shadow of conflict

We now introduce the mediator. As already discussed, the mediator cannot commit to destroying welfare. The only credible announcements are therefore bargaining protocols that minimize waste ex-post, that is, after the offensive and defensive investments are set. We therefore start by deriving the set of ex-post efficient bargaining protocols. We then analyze the full waste-minimization problem.

4.1 Ex-post efficient bargaining protocol.

We split the analysis in two cases, depending on whether or not the bargaining protocol can be contingent on the investment in patience.

Non-contingent bargaining protocol Suppose that the mediator announces that one of the two players will be a permanent proposer: he will propose with probability one in every period. In this case, in the unique Subgame Perfect Nash Equilibrium (SPNE), an agreement is reached immediately and the proposer earns the entire surplus.

It follows that by announcing that player 1 will be the permanent proposer with probability $\gamma \in [0, 1]$, the mediator can allocate a share γ of the ex-post surplus to player 1, *independently from the players discount factors.* This protocol eliminates the incentives to invest in patience and hence achieves ex-post efficiency. The next lemma shows that if the bargaining protocol cannot be contingent on the investment in patience, then all bargaining protocols that achieve ex-post efficiency must be payoff-equivalent to random permanent proposer.[22]

Lemma 1. *Suppose the bargaining protocol cannot be contingent on the investment in patience. Then all ex-post efficient bargaining protocols are payoff-equivalent to the random permanent proposer.*

Given that the negotiator will choose an ex-post efficient bargaining protocol, the players' payoffs are

$$U_1 = \underline{u}_1 + \gamma \left(S - \underline{u}_1 - \underline{u}_2 \right), \text{ and } U_2 = \underline{u}_2 + (1 - \gamma) \left(S - \underline{u}_1 - \underline{u}_2 \right),$$

[22] Clearly the following lemma relies on the players' risk neutrality. Because randomizing who is the permanent proposer exposes the bargaining parties to risk, if the players are risk-averse, the mediator may instead announce a fixed probability of proposing in each period, or transform the game into one of alternating offers. Studying how the mediator should solve the tradeoff between reducing the investments in becoming more patient and reducing players' exposure to risk is left for future work.

where γ is the probability of player 1 being the permanent proposer, which is also the share of the peace dividend accruing to each player (see Figure 3). Notice that by choosing the sharing rule the mediator can determine $\{U_1, U_2\}$ only if he observes $\underline{u}_1$ and $\underline{u}_2$, and S. When this is the case, the mediator's role is similar to that of an arbitrator. For most of the paper we will instead assume that the outside options $\underline{u}_i$ and S are not known by the mediator, who is therefore an outsider knowing less about the dispute than the players. In this case, under a random permanent proposer bargaining protocol, the players' payoffs will depend both on the probability of being the permanent proposer and on the players' actions.

Finally, remember that there is a one-to-one relationship between the share of surplus accruing to each player in the negotiation and the weight attached to a player's utility in a Generalized Nash Bargaining (GNB) problem. Hence, equivalently, we can think of the set of non-contingent ex-post efficient bargaining protocols as those leading to a bargaining solution that satisfies Pareto optimality, independence of irrelevant alternatives, invariance to rescaling of utility (which are the axioms that characterize the GNB solution).

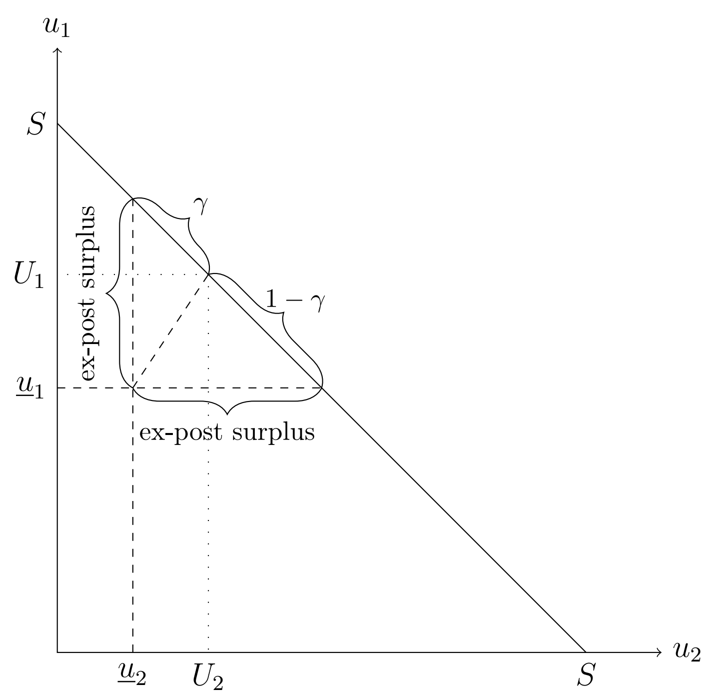

Fig. 3: Solution to the negotiation for given γ

Contingent bargaining protocol Suppose instead that the bargaining protocol can be made contingent on the investment in patience. Consider a bargaining protocol with constant recognition probability p. Call $v_1 \geq \phi_1$ and $v_2 \geq \phi_2$ the players' utilities in equilibrium. Because the game is stationary, v_1 and v_2 are also the players' utilities in case any period of the negotiation is reached.

Assume that player 1 makes an offer in period 1. His offer will be such that player 2 is indifferent between accepting the offer, and either going to the following period and earning v_2 or triggering a conflict immediately. The players' utilities are

$$S - \max\{\beta_2 v_2, \underline{u}_2\} \qquad \max\{\beta_2 v_2, \underline{u}_2\}$$

Similarly, if player 2 proposes, the players' utilities are

$$\max\{\beta_1 v_1, \underline{u}_1\} \quad S - \max\{\beta_1 v_1, \underline{u}_1\}$$

It follows that the equilibrium payoffs solve

$$v_1 = p\,(S - \max\{\beta_2 v_2, \underline{u}_2\}) + (1-p)\max\{\beta_1 v_1, \underline{u}_1\}$$

$$v_1 = (1-p)\,(S - \max\{\beta_1 v_1, \underline{u}_1\}) + p\max\{\beta_2 v_2, \underline{u}_2\}$$

Solving for v_1 and v_2 we get:[23]

$$v_1 = \begin{cases} S - \underline{u}_2 & \text{if } S\frac{p(1-\beta_2)}{1-\beta_1+p(\beta_1-\beta_2)} > S - \underline{u}_2 \\ \underline{u}_1 & \text{if } S\frac{p(1-\beta_2)}{1-\beta_1+p(\beta_1-\beta_2)} < \underline{u}_1 \\ \tau \cdot S & \text{otherwise,} \end{cases}$$

$$v_2 = S - v_1.$$

where

$$\tau \equiv \frac{p}{p + (1-p)\frac{1-\beta_1}{1-\beta_2}}.$$

See Figure 4 for an illustration. Note that τ can achieve any value between 0 and 1 (included) as a function of p for any β_1, β_2. That is, for any β_1, β_2, by adjusting the recognition probability the mediator can share *the total surplus S* in any way that satisfies the players' outside options. These observations lead to the following lemma.

[23] These derivations are adapted from Binmore, Shaked, and Sutton (1989), who consider a bargaining game that is identical to the one presented here, except that they assume alternating offers while here we have a constant recognition probability.

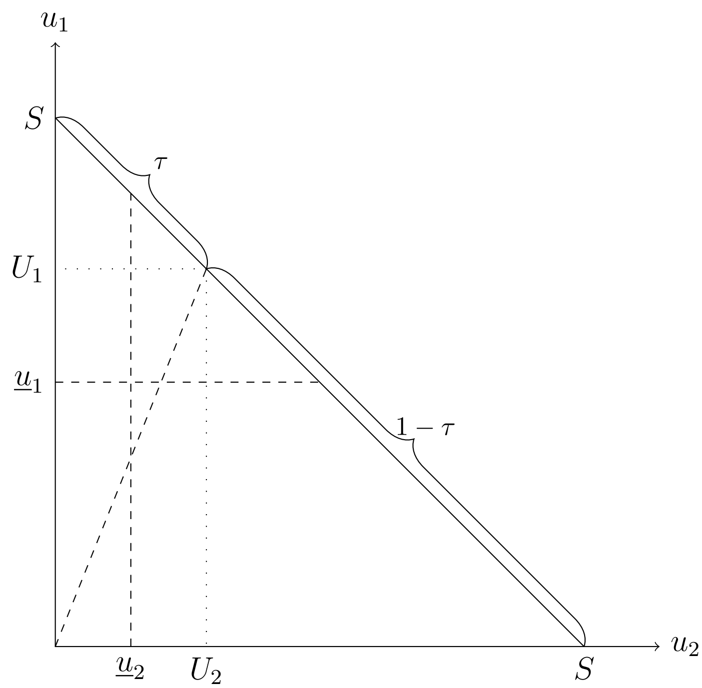

Fig. 4: Solution to the negotiation for given τ

Lemma 2. *Suppose the bargaining protocol can be contingent on the investment in patience. The mediator can always specify v_1 and $v_2 (= S - v_1)$ such that the players' payoffs are:*

$$\min\{\max\{v_1, \underline{u}_1\}, S - \underline{u}_2\} \text{ and } S - \min\{\max\{v_1, \underline{u}_1\}, S - \underline{u}_2\}$$

independently from their investments in patience. It follows that players do not invest in patience.

Proof. In the text. □

The lemma is based on the fact that, for any $\tau \in [0, 1]$, by announcing that

$$p = \frac{\tau}{\tau + (1-\tau)\frac{1-\beta_1}{1-\beta_2}}.$$

the mediator can completely offset any investment in patience and maintain the utility achieved by both players constant. This eliminates all incentives to invest in patience and achieves ex-post efficiency. Similarly, any bargaining protocol with constant recognition probability given by the above expression but with τ drawn at random before the start of the negotiation is ex-post efficient. This last observation implies that random permanent proposer is ex-post efficient here as well.[24]

4.2 The mediator's problem

We can now consider the mediator's problem: among the bargaining protocols that are ex-post efficient, choose the one that minimizes the expenditure in offensive and defensive investments. We split our analysis in different cases, depending on what the mediator can observe.

4.2.1 Observable investments

Suppose that the mediator can credibly announce that each player will be kept at a specific utility profile v_1 and $v_2 = S - v_1$, independently from their investments. Given this announcement, there is an equilibrium in which utilities are v_1 and $v_2 = S - v_1$ and there are no investments whenever no level of defensive investment delivers higher payoff to either player, that is whenever:

$$u_1 \geq \max_{d_1} \{u(\phi_1, d_1, 0) - c_d \cdot d_1\} \tag{1}$$

$$S - u_1 \geq \max_{d_2} \{u(\phi_2, d_2, 0) - c_d \cdot d_2\} \tag{2}$$

Note also that when either (1) or (2) are violated at some v_1 and $v_2 = S - v_1$, then it is not possible to implement v_1 and v_2 and have no investment in equilibrium. The reason is that $\max_{d_i} \{u(\phi_i, d_i, 0) - c_d \cdot d_i\}$ is the harshest punishment that the mediator can impose on player i in case of deviation from the equilibrium. (1) or (2) are therefore necessary and sufficient in order to achieve efficiency.

By condition (A1), the set of utility profiles that achieves zero investment is non empty and is given by

$$u_1 \in \left[\max_{d_1} \{u(\phi_1, d_1, 0) - c_d d_1\}, S - \max_{d_2} \{u(\phi_2, d_2, 0) - c_d d_2\}\right].$$

[24] Note that there may be other bargaining protocols that achieve ex-post efficiency. The next section will make clear that, when it comes to minimizing offensive and defensive investments, focusing on constant recognition bargaining protocols that achieve a specific τ is without loss of generality.

That is, if the mediator's announcement is credible, there always is a utility profile $\{u_1, S - u_1\}$ that achieves full efficiency.

The question we turn next is whether the mediator can impose a specific utility profile on the players. Suppose the mediator can observe the investments in patience but not offensive and defensive investments. By Lemma 2, if no offensive nor defensive investments were made, the mediator can impose any utility profile $u_1 \in [\phi_1, S - \phi_2]$ and $u_2 = S - u_1$ and achieve ex-post efficiency. Because $\phi_i \leq \max_{d_i} \{u(\phi_i, d_i, 0) - c_d d_i\}$, then the set of utility profiles that satisfies (1) and (2) can always be achieved via an appropriate bargaining protocol contingent on the players' investment in patience.[25] Hence, offensive and defensive investments can be fully eliminated.

Suppose instead that the mediator can observe offensive and defensive investments but not the investment in patience. The ex-post efficient bargaining protocol is random permanent proposer, in which the mediator allocates a fraction γ of the peace dividend to player 1 and the rest to player 2. Conditions (1) and (2) become:

$$\phi_1 + \gamma(S - \phi_1 - \phi_2) \geq \max_{d_1} \{u(\phi_1, d_1, 0) - c_d d_1\} \tag{3}$$

$$\phi_2 + (1 - \gamma)(S - \phi_1 - \phi_2) \geq \max_{d_2} \{u(\phi_2, d_2, 0) - c_d d_2\} \tag{4}$$

It is immediate that for any u_1 that satisfies (1) and (2) there exists a γ that satisfies (3) and (4). That is, choosing the players' utility profile or choosing the share of ex-post surplus are completely equivalent whenever the investments (and therefore the outside options $\underline{u}_1$ and $\underline{u}_2$) are observed by the mediator. The fact that there exists a u_1 that satisfies (1) and (2) therefore implies that there exists a γ that satisfies (3) and (4) and that eliminates all incentives to make offensive and defensive investments.

The following proposition summarizes the observations already made in the text.

Proposition 2. *When either the investment in patience or offensive and defensive investments are observable by the mediator, the mediator can always achieve full efficiency.*

Proof. In the text. □

[25] Going back to Lemma 2, this also implies that considering exclusively bargaining protocols with constant recognition probabilities is without loss of generality

Note also that all utility profiles that satisfy (1) and (2) (and are therefore consistent with efficiency) can be achieved both when the investment in patience is observable (but not offensive and defensive investments) and when offensive and defensive investments are observable (but not the investment in patience). There is no additional gain from observing everything (that is, offensive investment, defensive investments, and investments in patience) because the set of utility profiles that can be implemented are again given by (1) and (2). This observation implies the following corollary.

Corollary 1. *There set of outcomes that the mediator can implement under full information is the same as when only the investment in patience is observable (or only offensive and defensive investments are observable).*

We now turn to characterizing the set of utility profiles that the mediator can implement while achieving efficiency, and argue that a trade off between equity and efficiency may emerge. The set of γ that satisfies both (3) and (4) is given by

$$\gamma \in \left[\frac{\phi_1 - \min\left\{\phi_1, c_d\left(1+\log\left(\frac{\phi_1}{c_d}\right)\right)\right\}}{S-\phi_1-\phi_2}, 1 - \frac{\phi_2 - \min\left\{\phi_2, c_d\left(1+\log\left(\frac{\phi_2}{c_d}\right)\right)\right\}}{S-\phi_1-\phi_2} \right]$$

The question we are after is whether $1/2$ is an element of the above set, that is, whether the mediator can eliminate all investment *and* be fair at the same time. Using the fact that $\phi_1 > \phi_2$, simple algebra shows that

$$\frac{1}{2} < 1 - \frac{\phi_2 - \min\left\{\phi_2, c_d\left(1+\log\left(\frac{\phi_2}{c_d}\right)\right)\right\}}{S-\phi_1-\phi_2},$$

which implies that, there is a *tradeoff between efficiency and fairness* if and only if

$$\frac{\phi_1 - \min\left\{\phi_1, c_d\left(1+\log\left(\frac{\phi_1}{c_d}\right)\right)\right\}}{S-\phi_1-\phi_2} > \frac{1}{2}$$

or

$$3\phi_1 + \phi_2 > S + 2\min\left\{\phi_1, c_d\left(1+\log\left(\frac{\phi_1}{c_d}\right)\right)\right\}.$$

Hence, the tradeoff between efficiency and fairness is more likely to emerge whenever the cost c_d is small, S is small, or ϕ_2 is large. The reason is that a small S or a large ϕ_2 reduce the ex-post surplus to be shared in the negotiation and the benefit for player 1 of accepting $\gamma = \frac{1}{2}$ rather than deviating. This tradeoff is also more likely to emerge whenever ϕ_1 is large. Again, as ϕ_1 increases the ex-post surplus decreases. Furthermore, ϕ_1 large also increases the benefit for player 2 to deviate and make a large defensive investment. When this tradeoff emerges, the mediator needs to set $\gamma > 1/2$ in order to eliminate the incentives to invest and be biased in favor of player 1, who is the strongest player and therefore has the strongest incentive to deviate. The following Corollary summarizes these observations.

Corollary 2. *Suppose that either the investment in patience or offensive and defensive investments are observable by the mediator. If the distribution of power is uneven, the mediator must be biased toward the strongest player. When the distribution of power is sufficiently even, the mediator can eliminate all waste and at the same time be fair.*

4.2.2 Unobservable investments

Suppose the mediator does not observe neither offensive investments, nor defensive investments, nor investments in patience.[26] In this case, the only ex-post efficient bargaining protocol is random permanent proposer, in which a share of the peace dividend γ is allocated to player 1, with the rest allocated to player 2. The mediator's problem is choosing γ such that the pre-negotiation wasteful investments are minimized.

For given γ announced by the mediator, player 1's problem and player 2's problem are, respectively:

$$\max_{o_1,d_1} \left\{ \phi_1 e^{-o_2}(2-e^{-d_1}) + \gamma\Big[S - \phi_1 e^{-o_2}(2-e^{-d_1}) - \phi_2 e^{-o_1}(2-e^{-d_2})\Big]\right\} - c_o o_1 - c_d d_1.$$

$$\max_{o_2,d_2} \left\{ \phi_2 e^{-o_1}(2-e^{-d_2}) + (1-\gamma)\Big[S - \phi_1 e^{-o_2}(2-e^{-d_1}) - \phi_2 e^{-o_1}(2-e^{-d_2})\Big]\right\} - c_o o_2 - c_d d_2.$$

[26] Because the players observe each other's investments, the mediator may try to elicit this information from them. Note, however, that in case the players' reports on the investment levels do not match (so that at least one of the two players is lying), the mediator is unable to punish both players at the same time. The reason is that the mediator cannot commit to destroying surplus ex-post. Hence, if one player is punished by receiving a low surplus share, the other player must be rewarded. It follows that there is no equilibrium in which the players report truthfully. We do, however, show in Section 5.3 that the mediator may benefit from announcing that the bargaining protocol will depend on some costly actions taken by the players.

In this case, the four best responses are

$$o_1(\phi_2, d_2, \gamma) = \log\left(\frac{\gamma\phi_2(2 - e^{-d_2})}{c_o}\right) \text{ if } \frac{\gamma\phi_2(2 - e^{-d_2})}{c_o} \geq 1 \text{ else } o_1 = 0, \tag{5}$$

$$d_1(\phi_1, o_2, \gamma) = \log\left(\frac{(1-\gamma)\phi_1 e^{-o_2}}{c_d}\right) \text{ if } \frac{(1-\gamma)\phi_1 e^{-o_2}}{c_d} \geq 1 \text{ else } d_1 = 0, \tag{6}$$

$$o_2(\phi_1, d_1, \gamma) = \log\left(\frac{(1-\gamma)\phi_1(2 - e^{-d_1})}{c_o}\right) \text{ if } \frac{(1-\gamma)\phi_1(2 - e^{-d_1})}{c_o} \geq 1 \text{ else } o_2 = 0, \tag{7}$$

$$d_2(\phi_2, o_1, \gamma) = \log\left(\frac{\gamma\phi_2 e^{-o_1}}{c_d}\right) \text{ if } \frac{\gamma\phi_2 e^{-o_1}}{c_d} \geq 1 \text{ else } d_2 = 0. \tag{8}$$

Hence $o_1(\phi_2, d_2, \gamma)$ and $d_2(\phi_2, o_1, \gamma)$ are both increasing in γ, while $o_2(\phi_1, d_1, \gamma)$ and $d_1(\phi_1, o_2, \gamma)$ are both decreasing in γ. Intuitively, as the share of ex-post surplus received increases, a player's payoff depends more and more on the opponent's outside option rather than on his own outside option. In the limit case in which all ex-post surplus is allocated to player i, the final payoff for both players only depends on player $-i$'s outside option. As a consequence, the incentive to degrade the opponent and make an offensive investment increases with the share of ex-post surplus received. Similarly, as the share of ex-post surplus received *decreases*, a player's payoff depends more and more on his own outside option rather than on his opponent's. It follows that, as the share of ex-post surplus received decreases, the incentive to make a defensive investment increases. See Figures 5 and 6 for an illustration.

Note that $o_1(\phi_2, d_2, \gamma)$ and $d_2(\phi_2, o_1, \gamma)$ are best response of each other, and $o_2(\phi_1, d_1, \gamma)$ and $d_1(\phi_1, o_2, \gamma)$ are best responses to each other. There are therefore two separate games. The first one is a "fight over player 2's outside option" in which player 1 makes an offensive investment and player 2 makes a defensive investment. In this game, the two best responses are increasing in γ. The other game is a "fight over player 1's outside option" in which player 1 makes a defensive investment and player 2 makes an offensive investment. In this game, the two best responses are decreasing in γ. Putting the best responses together, we can characterize the Nash equilibrium of the game:

Lemma 3. *The Nash equilibrium of the game is as follows:*

- *If* $c_o \leq c_d$,

$$o_1 = \max\left\{\log\left(\frac{\gamma\phi_2}{c_o}\right), 0\right\}, o_2 = \max\left\{\log\left(\frac{(1-\gamma)\phi_1}{c_o}\right), 0\right\}$$

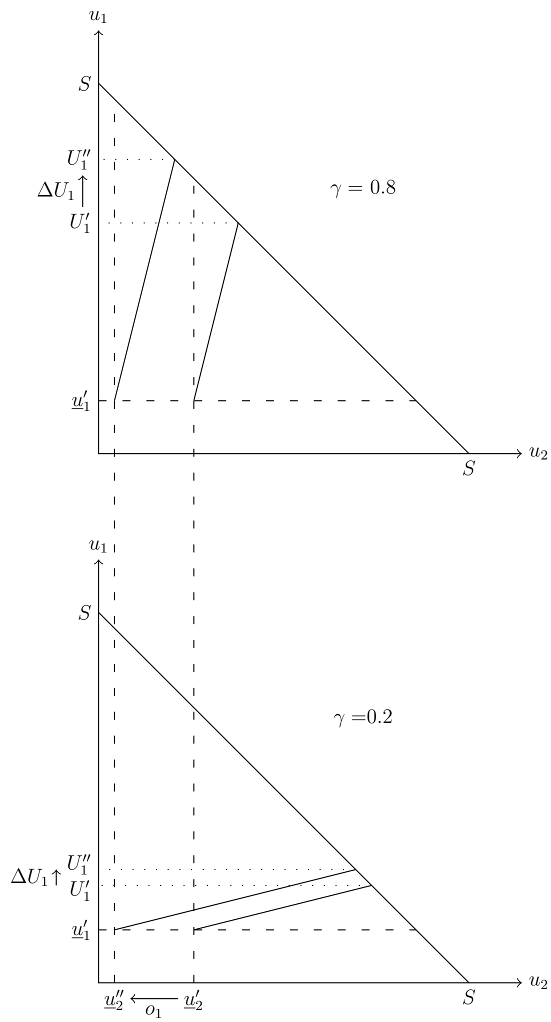

Fig. 5: Benefit of player 1's offensive investment for different values of γ.

and $d_1 = d_2 = 0$.

- *If* $c_o > c_d$,
  - *for* (o_1, d_2)
    - *for* $\gamma \geq \frac{c_o+c_d}{2\phi_2}$ *we have* $o_1 = \log\left(\frac{2\gamma\phi_2}{c_o+c_d}\right)$ *and* $d_2 = \log\left(\frac{c_o+c_d}{2c_d}\right)$;
    - *for* $\frac{c_d}{\phi_2} \leq \gamma \leq \frac{c_o+c_d}{2\phi_2}$ *we have* $o_1 = 0$ *and* $d_2 = \log\left(\frac{\gamma\phi_2}{c_d}\right)$.
    - *for* $\gamma < \frac{c_d}{\phi_2}$, *we have* $o_1 = 0$ *and* $d_2 = 0$.
  - *for* (d_1, o_2)
    - *for* $1-\gamma \geq \frac{c_o+c_d}{2\phi_1}$ *we have* $o_2 = \log\left(\frac{2(1-\gamma)\phi_1}{c_o+c_d}\right)$ *and* $d_1 = \log\left(\frac{c_o+c_d}{2c_d}\right)$;

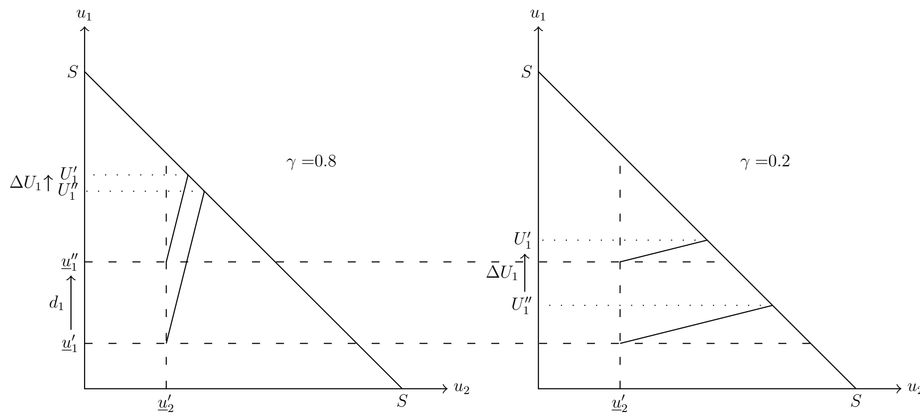

Fig. 6: Benefit of player 1's defensive investment for different values of γ.

- $*$ *for* $\frac{c_d}{\phi_1} \leq 1 - \gamma \leq \frac{c_o + c_d}{2\phi_1}$ *we have* $o_2 = 0$ *and* $d_1 = \log\left(\frac{(1-\gamma)\phi_1}{c_d}\right)$.
- $*$ *for* $1 - \gamma < \frac{c_d}{\phi_1}$, *we have* $o_2 = 0$ *and* $d_1 = 0$.

The Nash equilibrium depends on the cost of offensive and defensive investments. When $c_d \geq c_o$, in equilibrium there never is any investment in defensive technology. Remember that defensive investment is decreasing in offensive investment. When the cost of offensive investment is low relative to the cost of defensive investment, each player will make a large investment in offensive technology and, in equilibrium, drive the incentive to invest in defensive technology of the other player to zero.

If instead $c_d < c_o$, for extreme sharing rules, (i.e. $\gamma \geq \frac{c_o + c_d}{2\phi_2}$ or $\gamma \leq 1 - \frac{c_o + c_d}{2\phi_1}$), one player invests only in offensive technology while the other invests only in defensive technology. Instead, for intermediate sharing rules, players only make defensive investments and no offensive investments.[27] Intuitively, because of the cost advantage, players are more likely to make a defensive investment, the more so the larger the share of ex-post surplus going to the other player. However, the offensive investment made by player i increases with the defensive investment made by player $-i$, which implies that for extreme sharing rules one player makes a defensive investment while the other player makes an offensive investment.

[27] The reader may wonder why a player may make a defensive investment when the other player is not making any offensive investment. To understand this, remember that players may have made offensive or defensive investments before the game start. These investments are embedded into the initial power levels ϕ_1 and ϕ_2. Hence, the result here is that, for intermediate sharing rule players make additional defensive investment but no additional offensive investment.

The Nash equilibrium also depends on γ. Suppose the mediator allocates most of the ex-post surplus to player 2 by setting γ close to zero. Depending on c_o and c_d, Player 1 may make positive defensive investment but no offensive investment. Player 2 will make a positive offensive investment and no defensive investment. As γ increases, the players substitute one type of investment with the other. For γ close to one, the situation is reversed, with Player 1 making only an offensive investment, and Player 2 possibly only a defensive investment. The key observation is that the choice of γ determines whether the fight will be over Player 1's outside option (with Player 2 attacking it and Player 1 defending it) or over Player 2's outside option (with Player 1 attacking it and Player 2 defending it). The following proposition derives the solution to the mediator's problem.

Proposition 3. *Whenever*

$$\min\{c_o, c_d\} \geq \frac{\phi_1 \phi_2}{\phi_1 + \phi_2}, \tag{9}$$

then any

$$\gamma^\star \in \left[1 - \frac{\min\{c_o, c_d\}}{\phi_1}, \frac{\min\{c_o, c_d\}}{\phi_2}\right]$$

drives wasteful investment to zero. Whenever (9) *is violated, the mediator minimizes waste by setting*

$$\gamma^\star = 1 - \frac{\min\{c_o, c_d\}}{\phi_1}.$$

At the waste-minimizing γ player 2's offensive investment and player 1's defensive investments are zero. If $c_o \leq c_d$ player 1's offensive investment is positive but player 2's defensive investment is zero. If $c_d < c_o$, player 2's defensive investment is positive.

Condition (9) implies that, for given $\phi_1 + \phi_2$, the distribution of initial power is sufficiently uneven. In this case, the mediator can completely eliminate the players' incentives to invest. Instead, whenever the distribution of initial power levels is sufficiently equal so that (9) is violated, the mediator is unable to eliminate wasteful investment. As we discussed earlier, the choice of the sharing rule determines whether the fight is over player 1 or player 2 outside option. The proposition shows that total waste is minimized when the mediator sets $\gamma^\star = 1 - \frac{\min\{c_o, c_d\}}{\phi_1}$, which is the sharing rule that eliminates the fight over player 1's outside option. This sharing rule may generate a fight over player 2's outside option. However, because of the difference in initial power levels, player 1's incentive to perform offensive investment is lower than player 2, and the opposite holds for the incentives to perform defensive investments. Hence, total waste is minimized when the fight is over player 2's outside option, with player 1 attacking and player 2 defending, rather than player 1's outside option. That is, the mediator directs the fight away from the largest outside option, toward the smallest outside option.

If the distribution of power is uneven so that 9 holds, $\gamma^\star = \frac{1}{2}$ eliminates all investments if and only if $\phi_1 \leq 2\min\{c_o, c_d\}$. If instead (9) is violated simple algebra shows that $\gamma^\star$ is greater than $1/2$. Hence, there is no trade off between fairness and efficiency if and only if $\phi_1 \leq 2\min\{c_o, c_d\}$. In this case, both players' initial power levels are low (remember that $\phi_2 \leq \phi_1$), and the incentives to invest of both players are also low. If instead $\phi_1 > 2\min\{c_o, c_d\}$ fairness and efficiency are mutually exclusive, and the mediator minimizes waste by favoring the strongest player.[28]

5 Extensions

5.1 When the mediator is completely uninformed.

An interesting question is how the mediator's problem changes when he is completely uninformed, that is, he does not observe neither the investment levels, nor the ex-ante power levels, nor S.[29] In this section we answer this question for the case $c_o \leq c_d$, and for the a specific distribution of the mediator's belief over ϕ_1 and ϕ_2 (Pareto).

For given γ, the optimal investments by each player are the same as derived in Section 4.2.2. It follows that, for a given belief over the distribution of ex-ante power levels, the mediator solves:

$$\min_{\gamma}\left\{\Pr\left(\phi_2 > \frac{c_o}{\gamma}\right) E\left[\log\left(\frac{\gamma\phi_2}{c_o}\right) \Big| \phi_2 > \frac{c_o}{\gamma}\right] + \Pr\left(\phi_1 > \frac{c_o}{1-\gamma}\right) E\left[\log\left(\frac{(1-\gamma)\phi_1}{c_o}\right) \Big| \phi_1 > \frac{c_o}{1-\gamma}\right]\right\}.$$

[28] Contrary to the case of observable investments (Section 4.2.1), here S and ϕ_2 play no role in determining the emergence of the tradeoff between fairness and efficiency. The reason is that, here, the sharing rule does not change with the investments made by the players. Hence, the benefit of shifting a player's outside option depends only on γ and is independent from the other player's outside option or the total payoff S. When the sharing rule reacts to the investment as in Section 4.2.1, instead, the size of the ex-post surplus is a relevant element in determining the existence of the tradeoff.

[29] We maintain that (A1) holds for every realization of ϕ_1 and ϕ_2 and S.

That is, the mediator minimizes each player's probability of investing times the level of investment in case a player invests. The mediator's objective function can be written explicitly whenever ϕ_1 and ϕ_2 are drawn from two Pareto distributions.

Lemma 4. *Assume that ϕ_1 and ϕ_2 are drawn from two Pareto distributions with parameters $\kappa_1 > 0$ and $\kappa_2 > 0$, and minimum values $\underline{\phi}_1 > 0$ and $\underline{\phi}_2 > 0$ respectively. Then, the mediator minimizes*

$$\begin{cases} \left(\frac{\underline{\phi}_2\gamma}{c_o}\right)^{\kappa_2}\frac{1}{\kappa_2} + \left(\log\left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right) + \frac{1}{\kappa_1}\right) & \text{if } \gamma \leq \min\{1-\frac{c_o}{\underline{\phi}_1}, \frac{c_o}{\underline{\phi}_2}\} \\ \left(\frac{\underline{\phi}_2\gamma}{c_o}\right)^{\kappa_2}\frac{1}{\kappa_2} + \left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right)^{\kappa_1}\frac{1}{\kappa_1} & \text{if } 1-\frac{c_o}{\underline{\phi}_1} \leq \gamma \leq \frac{c_o}{\underline{\phi}_2} \\ \left(\log\left(\frac{\underline{\phi}_2\gamma}{c_o}\right) + \frac{1}{\kappa_2}\right) + \left(\log\left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right) + \frac{1}{\kappa_1}\right) & \text{if } \frac{c_o}{\underline{\phi}_2} \leq \gamma \leq 1-\frac{c_o}{\underline{\phi}_1} \\ \left(\log\left(\frac{\underline{\phi}_2\gamma}{c_o}\right) + \frac{1}{\kappa_2}\right) + \left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right)^{\kappa_1}\frac{1}{\kappa_1} & \text{if } \max\{1-\frac{c_o}{\underline{\phi}_1}, \frac{c_o}{\underline{\phi}_2}\} < \gamma \end{cases} \tag{10}$$

Without loss of generality, we assume that $\underline{\phi}_1 > \underline{\phi}_2$. As the parameters κ_1 and κ_2 increase, the masses of the two distributions become more and more concentrated near their minimum value. It follows that for κ_1 and κ_2 arbitrarily large, the mediator's problem converges to the one studied in the previous section. On the other hand, as the parameters κ_1 and κ_2 decrease, the tails of the two Pareto distributions become thicker, with higher ϕ_i becoming more likely and therefore increasing the expected investment levels by the two players. In particular, when $\kappa_i < 2$ the tails of the distribution are so thick that $Var[\phi_i]$ is not well defined; when $\kappa_i < 1$ the tails of the distribution are even thicker and also $E[\phi_i]$ is not well defined. As $\kappa_1 \to 0$, the mediator's belief becomes an improper prior.

We interpret κ_i as a measure of how informed the mediator is about player i. If the mediator is well informed, then κ_i is large, the tail of the Pareto distribution is thin and the probability that player i turns out to be extremely powerful is low. On the other hand, the mediator could be completely uninformed: the only thing he may know is that player i's power is above a certain threshold. In this case κ_i is small, the tail of the Pareto distribution is thick and there is a non-negligible probability that player i turns out to be extremely powerful.

The following proposition characterizes the solution to the mediator's problem.

Proposition 4. *The waste-minimizing sharing rule is weakly increasing in $\underline{\phi}_1$, weakly decreasing in $\underline{\phi}_2$, weakly increasing in κ_2, weakly decreasing in κ_1.*[30] *Furthermore:*

[30] If the solution to the mediator's problem is not unique, then both the smallest and the largest waste-minimizing γ are weakly increasing in $\underline{\phi}_1$ and κ_2, and weakly decreasing in $\underline{\phi}_2$ and κ_1.

- *for* $\kappa_1,\ \kappa_2 \to \infty$*, the players power levels are* $\underline{\phi}_1$ *and* $\underline{\phi_2}$ *with almost certainty, and the waste minimizing sharing rule converges to the one derived in Proposition 3.*

- *for* $\kappa_1,\ \kappa_2 \leq 1$ *the waste minimizing sharing rule is*

$$\gamma^\star = \begin{cases} 1 & \textit{if } \frac{1}{\kappa_2} - \frac{1}{\kappa_1} < \log\left(\frac{\underline{\phi}_1}{\underline{\phi}_2}\right) \\ 0 & \textit{otherwise,} \end{cases} \tag{11}$$

- *for* $\underline{\phi}_1,\ \underline{\phi}_2 \to \infty$ *the waste minimizing sharing rule converges to* (11)*,*

- *for* $\underline{\phi}_1,\ \underline{\phi}_2 \leq c_o$ *(so that for every* γ *there is a positive probability that neither player invests), the waste minimizing sharing rule is*

$$\gamma^\star : \left(\frac{\underline{\phi}_2}{c_o}\right)^{\kappa_2} \gamma^{\star \kappa_2 - 1} = \left(\frac{\underline{\phi}_1}{c_o}\right)^{\kappa_1} (1-\gamma^\star)^{\kappa_1 - 1}$$

Also here, keeping κ_1 and κ_2 constant, as the expected strength of player i relative to player $-i$ increases the mediator will increase the share of ex-post surplus received by player i. Furthermore, keeping $\underline{\phi}_1$ and $\underline{\phi}_2$ constant, the surplus share received by player i decreases with κ_i and increases with κ_{-i}. That is, each player prefers when the mediator has a precise belief about his opponent's power level, but an uninformative belief about his own power level. This is again due to the fact that for given $\underline{\phi}_1$ and $\underline{\phi}_2$, the expected strength of player i relative to player $-i$ decreases with κ_i and increases with κ_{-i}.

Finally, as $\underline{\phi}_1,\ \underline{\phi}_2$ increase or $\kappa_1,\ \kappa_2$ decrease, the two players become more likely to invest. Hence, for $\underline{\phi}_1,\ \underline{\phi}_2$ sufficiently high or $\kappa_1,\ \kappa_2$ sufficiently low, the waste minimizing sharing rule becomes extreme, allocating all ex-post surplus to one of the two players. In less extreme cases, the waste-minimizing sharing rule is intermediate, because each player has a low probability of investing.

A related question is whether the players would prefer to have a more knowledgeable mediator, who has more precise information about both players' power levels. For example, the mediator may be given the ability to gather intelligence and inspect both players, leading to an increase in both κ_1 and κ_2.[31] To explore this

[31] See, for example, the inspections of IRAN's nuclear sites prior to the 2015 framework agreement.

possibility, let us assume $\kappa_1 = \kappa_2 \equiv \kappa$, meaning that the mediator's prior beliefs over ϕ_1 and ϕ_2 are equally precise.

By the previous Proposition, if $\underline{\phi}_1 \leq c_o$ and $\underline{\phi}_2 \leq c_o$, the solution to the mediator's problem is simply

$$\gamma^\star = \begin{cases} \left[\left(\frac{\underline{\phi}_2}{\underline{\phi}_1} \right)^{\frac{\kappa}{\kappa-1}} + 1 \right]^{-1} & \text{if } \kappa > 1 \\ 1 & \text{otherwise.} \end{cases}$$

Again, the player expected to be stronger receives a larger share of ex-post surplus. Note also that, as the mediator belief becomes more imprecise (lower κ), player 1 receives a larger share of ex-post surplus. Whenever $\kappa \leq 1$, the objective function is strictly concave and the mediator's problem has a corner solution $\gamma = 1$.

Hence, the player who is expected to be stronger prefers a less informative belief (in the sense of lower κ and thicker tails), while the opposite is true for the player who is expected to be weaker. This implies that, for example, the player expected to be weaker would want the mediator to have the ability to gather information and inspect both players, so to have a more precise belief about their power levels. The player expected to be stronger instead would oppose this.

5.2 Investing during the negotiation.

Suppose that the players can make investments also during the negotiation. If the bargaining protocol announced by the mediator has fixed recognition probabilities, then the game repeats identical in every period. It follows that the players initial investment are also optimal in every subgame, and hence in every subgame-perfect equilibrium the players never change their investments after period 1. Hence, when the mediator observes the investment in patience, the results are the same as in the main text.

This, however, may not be true for the case of random permanent proposer. In this case, when players make their initial investment, they may not know who the permanent proposer will be, but they will learn it at the beginning of the negotiation. If the second stage of the negotiation is ever reached, they will adjust their investment mix so to reflect this new information.

It is easy to see that, in any equilibrium, the agreement will be reached in the first period. In making his first offer, however, the permanent proposer will keep into consideration the possibility that the other player may reject, go to the next period, adjust his investment mix and therefore change the outside options of the negotiation. Importantly, the benefit for the receiver of going to the next stage of the negotiation depends on his discount factor, leading to positive investment in patience. In equilibrium, therefore, allowing the players to invest in every period of the negotiation leads to positive investment in patience.

The only random permanent proposer bargaining protocol that is therefore ex-post efficient is one in which one of the two players is the permanent proposer with probability 1. That is, if the mediator does not observe the investment in patience, the only ex-post efficient bargaining protocol allocates the entire peace dividend to one of the two players. The mediator will choose which player will receive the entire peace dividend so to minimize the expenditure in offensive and defensive investments.

If investments in patience are not observable, the problems analyzed in Section 4.2.1 and 4.2.2 remain the same, but now the mediator can only choose between allocating the entire surplus to one of the two players. The derivations in Section 4.2.2 imply that the mediator will want to allocate the entire surplus to the strongest player.

5.3 Mediation with pre-negotiation concessions

We saw that when the mediator observes either offensive and defensive investments, or investment in patience, he can always eliminate all waste. This outcome, however, may not be achievable if he does not observe the players' investments. In this section we explore whether the mediator can compensate for this lack of information by conducting a contest for γ, that is, by announcing that the probability of being the permanent proposer will depend on visible, costly actions taken by the players. We allow these costly actions to benefit the other player, and therefore interpret them as *concessions*.

Before the negotiation begins the mediator asks each player to make concessions to the other player. If player i makes concessions $b_i \geq 0$, player i bears a cost equal to b_i while player $-i$ enjoys a benefit equal to $\alpha \cdot b_i$, where $\alpha \in [0,1]$.[32] Note that

[32] Our results can be easily extended to more general expressions for the cost and benefit of concessions, including asymmetries between the two players. However, for ease of notation, here we assume a simple, common linear function.

whenever $\alpha < 1$, making a concession generates a welfare loss. The concessions are used by the mediator to set the sharing rule γ, which is now

$$\gamma = f(b_1, b_2)$$

with f continuous and differentiable in both arguments, increasing and concave in b_1, decreasing and convex in b_2. We assume that the function $f(b_1, b_2)$ and α are announced by the mediator at the beginning of the game. That is, the mediator can require the players to make specific concessions (corresponding to a given α; given costs/benefits to the player making/receiving the concession) and then use the level of concessions to determine γ in a way that is fully anticipated by the players at the beginning of the game.

We first present our argument under the assumption that the initial power levels ϕ_1 and ϕ_2, and S are observed by the mediator and can be used to design the function $f(b_1, b_2)$. We later argue that if the mediator does not observe the initial power levels, the contest for γ can be constructed in such a way to induce the players to truthfully reveal ϕ_1, ϕ_2. and S.

In the choice of concession levels, player 1 solves

$$\max_{b_1 \geq 0} \left\{ f(b_1, b_2)(S - \underline{u}_1 - \underline{u}_2) - b_1 + \alpha b_2 \right\},$$

with FOC:

$$\frac{\partial f(b_1, b_2)}{\partial b_1}(S - \underline{u}_1 - \underline{u}_2) = 1. \tag{12}$$

Similarly, player 2 solves:

$$\max_{b_2 \geq 0} \left\{ (1 - f(b_1, b_2))(S - \underline{u}_1 - \underline{u}_2) - b_2 + \alpha b_1 \right\},$$

with FOC:

$$-\frac{\partial f(b_1, b_2)}{\partial b_2}(S - \underline{u}_1 - \underline{u}_2) = 1. \tag{13}$$

We define the peace dividend as $P \equiv S - \underline{u}_1 - \underline{u}_2$, and the equilibrium level of concessions as $b_1(P)$; $b_2(P)$.[33] By the implicit function theorem, whenever the players' FOC have an internal solution we have:

$$b_1'(P)\frac{\partial^2 f(b_1, b_2)}{\partial b_1^2} + b_2'(P)\frac{\partial^2 f(b_1, b_2)}{\partial b_1 \partial b_2} = -\frac{1}{P^2}$$

[33] Remember that $f(b_1, b_2)$ is chosen by the mediator. Hence, making sure that the equilibrium exists and is unique will be part of the mediator's problem.

$$b_2'(P)\frac{\partial^2 f(b_1,b_2)}{\partial b_2^2}+b_1'(P)\frac{\partial^2 f(b_1,b_2)}{\partial b_1 \partial b_2}=\frac{1}{P^2}.$$

Note that if, similarly to a standard contest function, the mediator chooses a function $f(.,.)$ such that $\frac{\partial^2 f(b_1,b_2)}{\partial b_1 \partial b_2}>0$, then if (12) and (13) hold then the equilibrium level of concessions strictly increases with P. If instead, the equilibrium level of concessions are determined by a corner solution at zero, then changes in P do not affect the level of concessions — which are constant at zero. It follows that $b_1'(P)$ and $b_2'(P)$ have a discontinuity at the value of P such that zero concessions solve (12) and (13).

The important observation is that by choosing the first derivatives of $f(.,.)$, the mediator can choose the level of concession in equilibrium. If the equilibrium level of concessions are such that the best responses have an internal solution, then by choosing the second derivatives of $f(.,.)$ the mediator can determine how the equilibrium level of concession reacts to changes in P. More precisely, $b_1'()$ and $b_2'()$ can be made arbitrarily large by setting $\frac{\partial^2 f(b_1,b_2)}{\partial b_1^2}$ and $\frac{\partial^2 f(b_1,b_2)}{\partial b_2^2}$ sufficiently small. Furthermore, whenever (12) and (13) hold at $b_1(P)=b_2(P)=0$, the equilibrium level of concessions react to increases in P (in a way that is determined by the second derivatives of $f(.,.)$) but is constant when P decreases.

Given this, we can analyze the players' choice of offensive and defensive investments. Player 1 solves

$$\max_{o_1,d_1}\left\{\{u(\phi_1,d_1,o_2)+\gamma P\}-c_o\cdot o_1-c_d\cdot d_1-b_1(P)+\alpha_2 b_2(P)\right\},$$
$$\text{s.t.}\quad\begin{cases}\gamma=f(b_1(P),b_2(P))\\ P=S-u(\phi_1,d_1,o_2)-u(\phi_2,d_2,o_1)\end{cases}$$

By the envelope theorem, we can ignore the effect of o_1 and d_1 on b_1. The FOC with respect to o_1 is:

$$-\frac{\partial f(.,.)}{\partial b_2}\frac{\partial u(\phi_2,d_2,o_1)}{\partial o_1}b_2'(P)P-\gamma\frac{\partial u(\phi_2,d_2,o_1)}{\partial o_1}-\alpha\frac{\partial u(\phi_2,d_2,o_1)}{\partial o_1}b_2'(P)=c_o$$

Assuming (13) holds, the above FOC becomes:

$$-\gamma\frac{\partial u(\phi_2,d_2,o_1)}{\partial o_1}+(1-\alpha)\frac{\partial u(\phi_2,d_2,o_1)}{\partial o_1}b_2'(P)=c_o$$

Therefore, player 1 anticipates that by investing in o_1, he will increase the peace dividend and therefore the concessions made by player 2. This has two effects. It directly benefits player 1 because concessions are something valuable to the player receiving them. It, however, indirectly hurts player 1 because concessions by player 2 increase the share of ex-post surplus accruing to player 2. If $\alpha < 1$ the negative effect dominates, and player 1 decreases his investment in offensive technology to reduce the intensity of the contest over γ. If instead $\alpha = 1$ the two effects cancel out and the contest for γ has no impact on o_1, in the sense that the FOC is the same derived in Section 4.2.2 for the case of a fixed γ announced at the beginning of the game.

Similarly, the FOC with respect to d_1 is

$$-\frac{\partial f(.,.)}{\partial b_2}\frac{\partial u(\phi_1, d_1, o_2)}{\partial d_1}b_2'(P)P + (1-\gamma)\frac{\partial u(\phi_1, d_1, o_2)}{\partial d_1} - \alpha\frac{\partial u(\phi_1, d_1, o_2)}{\partial d_1}b_2'(P) = c_d,$$

Assuming (12) holds, we have:

$$(1-\gamma)\frac{\partial u(\phi_1, d_1, o_2)}{\partial d_1} + (1-\alpha)\frac{\partial u(\phi_1, d_1, o_2)}{\partial d_1}b_2'(P) = c_d,$$

In this case, whenever $\alpha < 1$ the contest over γ *increases* the benefit of making a defensive investment. The intuition is the reverse of what discussed in the previous section. A defensive investment decreases the ex-post surplus to be shared in the contest and therefore the incentive of both players to perform monetary payments. Hence, by making a defensive investment, player i can decrease b_{-i} and obtain a higher surplus share during the negotiation. If instead $\alpha = 1$, the contest for γ has no impact on the investment made by player 1. Following similar steps, we can derive the two FOCs for player 2. The FOC for o_2 is

$$-(1-\gamma)\frac{\partial u(\phi_1, d_1, o_2)}{\partial o_2} + (1-\alpha)\frac{\partial u(\phi_1, d_1, o_2)}{\partial o_2}b_1'(P) = c_o,$$

and the FOC with respect to d_2 is:

$$\gamma\frac{\partial u(\phi_2, d_2, o_1)}{\partial d_2} + (1-\alpha)\frac{\partial u(\phi_2, d_2, o_1)}{\partial d_2}b_1'(P) = c_d,$$

To summarize the above observations: whenever (12) and (13) holds, the contest for γ can be used to discourage offensive investments. The reason is that offensive investments increase the peace dividend and therefore the intensity of the "fight" in the concession game. This logic however also implies that the contest for γ may increase the incentives to make defensive investments. If instead the contest for γ has corner solutions, (12) and (13) do not hold. In this case $b_1'(P) = b_2'(P) = 0$, and the contest for γ does not affect the choice of optimal investment. The FOCs are again the ones derived in Section 4.2.2 for the case of a fixed γ announced at the beginning of the game.

An important case is when (12) and (13) hold at $b_1 = b_2 = 0$. In this case, $b_1'(P)$ and $b_2'(P)$ are positive for changes in the investment mix that increase the peace dividend (i.e., either an increase in offensive investment or a decrease in offensive investments) and are zero otherwise. In this case the contest for γ can be used to discourage offensive investments, without affecting the incentives to make defensive investments.

We are now ready to derive conditions under which the mediator can achieve full efficiency. The mediator can achieve zero concessions in equilibrium by setting

$$\frac{\partial f(0,0)}{\partial b_1} = -\frac{\partial f(0,0)}{\partial b_2} = \frac{1}{S - \phi_1 - \phi_2}$$

At the same time the mediator sets both $\frac{\partial^2 f(0,0)}{\partial b_1^2}$ and $\frac{\partial^2 f(0,0)}{\partial b_2^2}$ low, so that concessions are very sensitive to increases in P. As a consequence, both players expect that if they set positive offensive investments, they will increase the peace dividend and therefore generate a large concession from the opponent. If $\alpha < 1$ this expectation draws offensive investment to zero. This strategy achieves full efficiency if the two players do not want to make defensive investments, that is, whenever

$$(1-\gamma)\phi_1 \leq c_d$$
$$\gamma\phi_2 \leq c_d,$$

Proposition 5. *Full efficiency can be achieved at a given $\gamma \in (0,1)$ if and only if*

$$\gamma \in \left[1 - \frac{c_d}{\phi_1}, \frac{c_d}{\phi_2}\right] \tag{14}$$

Proof of Proposition 5. The "if" part is shown in the text. For the "only if" part, note that the contest for γ cannot be used to achieve efficiency by discouraging defensive investments. To achieve efficiency the equilibrium level of concession must be zero. If, starting from zero concessions, a player makes a defensive investment, the peace dividend will decrease and the concession level will be unchanged and equal to zero. Hence the incentives to make defensive investments are unaffected by the presence of the contest for γ when this contest is designed so to induce no concessions in equilibrium. □

Few points are worth noting. First, despite the fact that there is no waste in equilibrium, the contest is effective only if $\alpha < 1$. That is, concessions need to be an inefficient way to transfer surplus among players. They cannot be monetary transfers, but should rather be "in kind" transfers. The proposition shows that the mediator can achieve full efficiency as long as he can require the players to make concessions that are wasteful.

Second, although there is no welfare loss in equilibrium, the contest should generate inefficiencies off equilibrium (i.e., for positive offensive investment). Interestingly, here the mediator can easily commit to destroying welfare off equilibrium. The reason is that the mediator does not observe the player's investments. Hence, following a positive offensive investment, the mediator has no incentive to modify the function $f(b_1, b_2)$ so to avoid costly concessions.

Corollary 3. *1. If $c_d < \frac{\phi_1\phi_2}{\phi_1+\phi_2}$, then the mediator cannot achieve full efficiency.*

2. If $c_d > \frac{\phi_1\phi_2}{\phi_1+\phi_2}$ and $c_d < \frac{\phi_1}{2}$ then to achieve full efficiency the mediator needs to implement $\gamma > \frac{1}{2}$.

3. If $c_d > \phi_1$ the mediator can achieve full efficiency and implement any sharing rule $\gamma \in (0, 1)$.

By comparing the above corollary with Proposition 3, we see that introducing the contest for γ helps to achieve full efficiency whenever $c_d > c_o$. If instead $c_d \leq c_o$, the set of parameters for which the mediator can achieve full efficiency is the same with and without contest for γ. Simialry, if full efficiency is achievable, then there is a potential tradeoff between faireness and efficiency. Whenever $c_d > c_o$, this tradeoff is less likely to emerge when the mediator uses a contest for γ. When instead $c_d \leq c_o$ this the set of parameters for which this tradeoff emerges is the same with and without the contest.

What happen when efficiency is not achievable? The following lemma shows that it is never profitable to generate a welfare loss in the form of positive concessions in order to reduce pre-bargaining wasteful investments.

Lemma 5. *Suppose that $\alpha < 1$. The welfare-maximizing $f(.,.)$ is such that $b_1(P) = b_2(P) = 0$.*

As already discussed, starting from zero concessions the mediator can punish any investment that increases the size of the peace dividend but is unable to discourage investments that decrease the peace dividend.[34] Together with the above lemma, this implies that the mediator will use the contest for γ to drive offensive investments to zero, and will choose $\gamma = f(0,0)$ so to minimize total defensive investments:

$$\max\left\{\log\left(\frac{(1-\gamma)\phi_1}{c_d}\right), 0\right\} + \max\left\{\log\left(\frac{\gamma\phi_2}{c_d}\right), 0\right\}$$

Again, the mediator will minimize waste by choosing

$$\gamma = 1 - \frac{c_d}{\phi_2}$$

so that only player 2 invests in defense. Note that if $c_d < \frac{\phi_1\phi_2}{\phi_1+\phi_2}$ (so that efficiency is not achievable), then $1 - \frac{c_d}{\phi_2} > \frac{1}{2}$ and the mediator, again, favors the strongest player. The following proposition summarizes these observations.

Proposition 6. *Suppose that $c_d < \frac{\phi_1\phi_2}{\phi_1+\phi_2}$, so that efficiency is not achievable. The mediator will use the contest for γ to eliminate offensive investments. The mediator minimizes defensive investments by setting*

$$\gamma = 1 - \frac{c_d}{\phi_2} > \frac{1}{2}$$

Proof. In the text. □

These results are basically the same derived in Section 4.2.2, with the only difference being that, here, the mediator chooses γ exclusively to minimize defensive investments.

Unobservable S, ϕ_1 and ϕ_2.

Suppose $c_d > \phi_1$, so that, thanks to the contest, the mediator can achieve full efficiency and implement any sharing rule $\gamma \in (0,1)$ (while, if c_o is sufficiently low, absent the contest there will be positive waste in equilibrium). We want to argue that, because of the contest for γ, if the mediator cannot observe S, ϕ_1 and ϕ_2, can elicit them from the two players, which is in sharp contrast with the case considered in Section 5.1.

The mediator can announce that $\frac{\partial f(.,.)}{\partial b_1} = -\frac{\partial f(.,.)}{\partial b_2}$ (so that the player's equilibrium concessions levels are always identical) and $f(b_1, b_2)|_{b_1=b_2} = \gamma$ (so that the sharing

[34] This is a consequence of the "only if" part of Proposition 5. See its proof for all details.

rule implemented is constant in equilibrium). It follows that the players cannot manipulate the allocation of the ex-post surplus by misreporting S, ϕ_1 or ϕ_2. The only effect of misreporting is to, potentially, cause positive concessions and positive offensive investment in equilibrium. However, it is easy to see that no player can benefit from inducing positive offensive investments and positive concession. Suppose a player expects his report to have an effect on the function $f(.,.)$. Because player i's concessions and investment are optimal given $f(.,.)$, by an envelope argument manipulating $f(.,.)$ affects player i's utility only because it may induce player $-i$ to change his behavior. It is however evident that player i cannot do better than reporting truthfully and inducing player $-i$ to set both concessions and investments to zero.

If instead $c_d < \phi_1$, ether the mediator can achieve full efficiency only at some γ, or he cannot achieve full efficiency at all. In the first case, the possibility of eliciting a truthful report about S, ϕ_1 and ϕ_2 depends, again, on the parameters. If efficiency can be achieved at the same γ for every possible S, ϕ_1 and ϕ_2, then the logic discussed above continues to hold. The mediator can ask the players to report S, ϕ_1 and ϕ_2, which are then used to determine the shape of $f(.,.)$ so to generate zero waste. Because the sharing rule can be made independent from the reports, the players have no incentives to misreport. If instead there is no γ that achieves efficiency for all possible values of S, ϕ_1 and ϕ_2, then this argument will fail, because the mediator cannot commit to maintain the same sharing rule for all possible reports, making truthful reporting impossible.

Similarly, if the mediator is unable to eliminate all waste, the the waste-minimizing sharing rule will depend on the players' report, who therefore will not report truthfully.

6 Conclusions

We have studied a bargaining problem in which players may invest/waste valuable resources in order to improve their negotiating outcome. These investments could be aimed at shifting the outside options—the equilibrium payoffs of the non-cooperative game played in case of breakdown of the negotiation—or the cost of delaying the agreement—typically interpreted as the time discount factor. In the absence of a mediator, the negotiation is conducted as a standard bargaining game in alternating offers. We show that each player benefits from performing at least one of the two types of investments.

We then examine whether a mediator can help in reducing waste. We assume that the mediator controls the bargaining protocol: the probability that, in each period of the negotiation, a given negotiating party will propose to the other. The mediator cannot commit to destroying welfare. Nonetheless, we show that if the mediator can observe either the investments aimed at manipulating the discount factors, or the investment aimed at manipulating the outside options, he can always achieve efficiency. If instead he cannot observe neither type of investment, then there could be waste in equilibrium. Furthermore, to reduce waste, the mediator may need to penalize the weakest player, who is the one with the strongest incentive to undertake wasteful investments. This result is robust to different assumptions regarding what the mediator can observe, and highlights a conflict between fairness and efficiency arising in negotiations. Relative to the existing literature on mediation in political science, our paper shows that the mediator can be biased not because of his preferences, but strategically to minimize social waste.

Our analysis suggests several additional lines for future research. For example, our framework can be used to explore the choice between mediated and unmediated negotiation. Despite the fact that the mediator may favor the strongest player, the weakest player may nevertheless prefer a mediated negotiation over an unmediated one, because of the reduction in wasteful investment. Also, we have showed that the precision of the mediator's information affects the sharing rule implemented. Our results suggest that the weakest player benefits from a more informed mediator while the opposite is true for the strongest player, but the full analyses of the strategic choice of transparency remains to be completed.

Also, we have assumed away inefficiencies arising at the negotiation stage and focused exclusively on inefficiencies arising before the start of the negotiation. However, several authors have drawn a connection between pre-bargaining wasteful investments and inefficiencies arising within the negotiation (see Powell, 1993, Kydd, 2000, Slantchev, 2005, Meirowitz and Sartori, 2008, Jackson and Morelli, 2009). For example, an arms build up prior to the negotiation may increase the chance that an agreement is found and therefore increase the efficiency of the negotiation, either because it makes war more costly or because it reduces the asymmetry of information between players. On the other hand, a military mobilization may decrease the probability of reaching an agreement and the efficiency of the negotiation because it generates a *hands-tying effect*: a decrease in the cost of starting a war that operates as a public commitment device. Extending our model to the case in which inefficiencies during the negotiation stage are present, and are affected by inefficiencies arising before the negotiation is also left for future work.

Finally, the fact that in order to reduce wasteful investment the mediator may favor the strongest player opens an additional potential problem. If the initial power levels are endogenous, then each player has the incentive to become the strongest player in order to obtain a more favorable share of the ex-post surplus, potentially leading to very high level of wasteful investment. Consider, for example, the model discussed in Section 4.2.2 in which the initial power levels are observable by players and mediator. Assume now that the investments are done in two steps: the players can invest before the mediator announces the sharing rule as well as after the announcement. The initial investments are observable by the mediator. By Proposition 3, the mediator sets $\gamma^\star = 1 - \frac{\min\{c_d, c_o\}}{\max\{\phi_1, \phi_2\}}$, where we allow for either player 1 or 2 to be the strongest depending on their initial investment.

Hence, if the players have the opportunity to make an investment before the mediator announces the sharing rule, the player who is the weakest ex-ante will benefit from making an offensive investment. This way, in the moment the mediator announces the sharing rule, the other player will be weaker than at the start of the game and therefore will receive a lower share of ex-post surplus. Possibly, the weakest player may become the strongest player and receive the majority of the ex-post surplus. The other player will anticipate this and may invest as well. The expectation of the mediator's intervention leads to wasteful investments before the intervention of the mediator.

Note that the above logic holds also when the mediator can announce the sharing rule at the beginning of the game (i.e., before $\hat{d}_i$ and $\hat{o}_i$ are set) but cannot commit to it. That is because the mediator will always revise the sharing rule after observing $\hat{d}_i$ and $\hat{o}_i$, leading to the same conclusions we have obtained above. Therefore, a benevolent mediator who lacks the power to commit to a sharing rule may cause a higher level of social waste than a mediator who simply implements an exogenously given sharing rule. The ability of the mediator to commit may therefore be key to the reduction of wasteful investments. Analyzing different ways in which the mediator can acquire this commitment remains an open problem.

Mathematical derivations

Proof of Proposition 1. The discussion in the text implies that, if an equilibrium exists and $\phi_2 > \min\{c_o, c_d\}$, then there will be positive investment in equilibrium—either in patience or offensive/defensive. What is left to show is the existence of the equilibrium

For given offensive and defensive investment, the existence of the equilibrium in the patience investment game is guaranteed by the fact that the set of actions β_i is compact. Of course, multiple equilibria are possible, but as long as

$$v_1 = S\frac{1-\underline{\beta}_2}{1-\underline{\beta}_1\underline{\beta}_2} > \underline{u}_1 \qquad v_2 = S - v_1 > \underline{u}_2.$$

the argument we made in the main text implies that in every equilibria there will be positive investment in patience.

Consider now the choice of offensive and defensive investments. The key observation is that the benefit for player i of making a defensive investment is always smaller or equal to

$$u_i(\phi_i, d_i, o_{-i}) = \phi_i(2 - e^{-d_i})e^{-o_{-i}}$$

that is, the benefit in case player i earns the entire ex-post surplus. This implies that any defensive investment larger than $\hat{d} : \phi_i(2 - e^{\hat{d}_i}) = c_o\hat{d}$ is dominated by $d = 0$. Hence, player i optimal investment in defense must be within $[0, \hat{d}]$.

Similarly, the benefit for player $-i$ of making an offensive investment is always smaller or equal to

$$u_i(\phi_i, d_i, o_{-i}) = S - \phi_i(2 - e^{-d_i})e^{-o_{-i}}$$

This implies that any offensive investment larger than $\hat{o} : S - 2\phi_i e^{-\hat{o}_{-i}} = c_o\hat{o}$ is dominated by $o_{-i} = 0$. Hence, player $-i$ optimal investment in offense must be within $[0, \hat{o}]$.

Again, the set of possible actions in the game in which players make offensive and defensive investment is compact, and therefore an equilibrium must exist.

□

Proof of Lemma 1. Consider a generic bargaining protocol in which player 1 proposes first. To start, note that for a bargaining protocol to achieve efficiency, in equilibrium the agreement must be reached immediately. Call $v_{2,t=2}$ player 2's expected payoff in period 2.

There are two possible cases. The first is $\beta_2 v_{2,t=2} \geq \underline{u}_2$, that is, the present discounted value of player 2's continuation payoff is greater than the payoff in case of conflict. In this case, in equilibrium, player 1 will offer $\beta_2 v_{2,t=2}$ immediately so that player 2 is indifferent between accepting and rejecting. The second case is $\beta_2 v_{2,t=2} < \underline{u}_2$, so that player 1 will offer $\underline{u}_2$ and player 2 is indifferent between accepting and triggering the conflict.

We want to show that v_2 must be an increasing function of β_2, so that when $\beta_2 v_{2,t=2} \geq \underline{u}_2$ then player 2 will want to invest in patience. Call $v_{2,s}$ the payoff achieved by player 2 if stage s of the negotiation is reached. It is easy to see that, for given $v_{2,s}$, all $v_{2,t}$ for $t < s$ are increasing in β_2 (this can be formally shown via an induction argument that we omit). As $s \to \infty$ its impact on $v_{2,t=2}$ becomes negligible. It follows that $\beta_2 v_{2,t=2}$ is strictly increasing in β_2. If $\beta_2 v_{2,t=2} \geq \underline{u}_2$, therefore, the players' payoffs depend on the players' discount factors, leading to an investment in patience.

Therefore, the only bargaining protocol in which player 1 proposes first that do not lead to investment in patience by player 2 are those such that player 2 is kept at its outside option. In this case the outcome is equivalent to that achieved when player 1 is the permanent proposer. The same reasoning applies to bargaining protocol that leave player 1 at his outside option, and bargaining protocol that randomize between leaving player 1 and player 2 at their outside options.

□

Proof of Proposition 3. For $c_d \geq c_o$, we argued in the text that there is no defensive investments. Also, $o_1^\star$ is zero if $\gamma \leq \frac{c_o}{\phi_2}$, and $o_2^\star$ is zero if $\gamma \geq 1 - \frac{c_o}{\phi_1}$. Hence, wasteful investment can be completely eliminated with any $\gamma \in [1 - \frac{c_o}{\phi_1}, \frac{c_o}{\phi_2}]$ whenever

$$c_o \geq \frac{\phi_1 \phi_2}{\phi_1 + \phi_2}. \tag{15}$$

Suppose now that

$$c_o < \frac{\phi_1 \phi_2}{\phi_1 + \phi_2}.$$

For $\gamma < \frac{c_o}{\phi_2}$ we have that $o_1 = 0$ and o_2 is strictly decreasing in γ. For $\gamma > 1 - \frac{c_o}{\phi_1}$ we have that $o_2 = 0$ and o_1 is strictly increasing in γ. Therefore, it has to be that the waste minimizing $\gamma \in [\frac{c_o}{\phi_2}, 1 - \frac{c_o}{\phi_1}]$.

For this range of values, the mediator solves:

$$\min_{\gamma \in [\frac{c_o}{\phi_2}, 1 - \frac{c_o}{\phi_1}]} \left\{ \log\left(\frac{\gamma \phi_2}{c_o}\right) + \log\left(\frac{(1-\gamma)\phi_1}{c_o}\right) \right\} =$$

$$\min_{\gamma \in [\frac{c_o}{\phi_2}, 1-\frac{c_o}{\phi_1}]} \left\{ \log\left(\gamma(1-\gamma)\right) + \log\left(\frac{\phi_1\phi_2}{c_o^2}\right)\right\}.$$

Hence, the mediator minimizes $\gamma(1-\gamma)$ over the relevant interval. It can be verified that when $\phi_1 \geq \phi_2$—as we assume throughout—and $c_o < \frac{\phi_1\phi_2}{\phi_1+\phi_2}$ this minimum is always reached at $\gamma^\star = 1 - \frac{c_o}{\phi_1}$.

For $c_d < c_o$, consider the total expenditure fighting over player 2's outside option, with player 1 attacking and player 2 defending:

$$c_o \cdot o_1 + c_d \cdot d_2 = \begin{cases} 0 & \text{if } \gamma \leq \frac{c_d}{\phi_2} \\ c_d\left(\log(\gamma) + \log\left(\frac{\phi_2}{c_d}\right)\right) & \text{if } \frac{c_d}{\phi_2} \leq \gamma \leq \frac{c_o+c_d}{2\phi_2} \\ c_d \log\left(\frac{c_o+c_d}{2c_d}\right) + c_o\left(\log(\gamma) + \log\left(\frac{2\phi_2}{c_o+c_d}\right)\right) & \text{otherwise,} \end{cases}$$

Similarly, consider the total expenditure fighting over player 1's outside option:

$$c_o \cdot o_2 + c_d \cdot d_1 = \begin{cases} 0 & \text{if } 1-\gamma \leq \frac{c_d}{\phi_1} \\ c_d\left(\log(1-\gamma) + \log\left(\frac{\phi_1}{c_d}\right)\right) & \text{if } \frac{c_d}{\phi_1} \leq 1-\gamma \leq \frac{c_o+c_d}{2\phi_1} \\ c_d \log\left(\frac{c_o+c_d}{2c_d}\right) + c_o\left(\log(1-\gamma) + \log\left(\frac{2\phi_1}{c_o+c_d}\right)\right) & \text{otherwise,} \end{cases}$$

It is easy to verify that whenever $c_d \geq \frac{\phi_1\phi_2}{\phi_1+\phi_2}$, then any $\gamma \in [1-\frac{c_d}{\phi_1}, \frac{c_d}{\phi_2}]$ achieves zero waste. If instead $c_d < \frac{\phi_1\phi_2}{\phi_1+\phi_2}$, then $\frac{c_d}{\phi_2} < 1 - \frac{c_d}{\phi_1}$ and we have

$$c_o \cdot (o_1+o_2) + c_d \cdot (d_1+d_2) = \begin{cases} \text{strictly decreasing} & \text{if } \gamma \leq \frac{c_d}{\phi_2} \\ \text{strictly concave} & \text{if } \frac{c_d}{\phi_2} \leq \gamma \leq 1-\frac{c_d}{\phi_1} \\ \text{strictly increasing} & \text{otherwise.} \end{cases} \tag{16}$$

Hence, total expenditure $c_o \cdot (o_1+o_2) + c_d \cdot (d_1+d_2)$ is minimized either at $\gamma = \frac{c_d}{\phi_2}$, where the expenditures fighting over 2's outside options is zero, or at $\gamma = 1 - \frac{c_d}{\phi_1}$ where the expenditures fighting over 1's outside options is zero. At these two values total expenditures are

$$[c_o \cdot (o_1+o_2) + c_d \cdot (d_1+d_2)]|_{\gamma=\frac{c_d}{\phi_2}} = c_d \log\left(\min\left\{\frac{c_o+c_d}{2c_d}, \phi_1\left(\frac{1}{c_d} - \frac{1}{\phi_2}\right)\right\}\right) + c_o \log\left(\max\left\{0, \phi_1\left(\frac{1}{c_d} - \frac{1}{\phi_2}\right)\frac{2c_d}{c_o+c_d}\right\}\right)$$

$$[c_o \cdot (o_1+o_2) + c_d \cdot (d_1+d_2)]|_{\gamma=1-\frac{c_d}{\phi_1}} = c_d \log\left(\min\left\{\frac{c_o+c_d}{2c_d}, \phi_2\left(\frac{1}{c_d} - \frac{1}{\phi_1}\right)\right\}\right) + c_o \log\left(\max\left\{0, \phi_2\left(\frac{1}{c_d} - \frac{1}{\phi_1}\right)\frac{2c_d}{c_o+c_d}\right\}\right)$$

Because $c_d < \frac{\phi_1\phi_2}{\phi_1+\phi_2}$, then $\phi_2\left(\frac{1}{c_d} - \frac{1}{\phi_1}\right) < \phi_1\left(\frac{1}{c_d} - \frac{1}{\phi_2}\right)$ and total waste is minimized whenever $\gamma = 1 - \frac{c_d}{\phi_1}$.

□

Proof of Lemma 4. To start, note that if ϕ_i is Pareto-distributed with minimum x and parameter κ, then $\log\left(\frac{\phi_i}{x}\right)$ is exponentially distributed with parameter κ. To see this, consider

$$\Pr\left\{\log\left(\frac{\phi_i}{x}\right) \leq y\right\} = \Pr\left\{\phi_i \leq e^y x\right\}$$

Because ϕ_i is distributed according to a Pareto distribution, the above expression becomes

$$1 - \left(\frac{x}{xe^y}\right)^\kappa = 1 - e^{-y\kappa}$$

which is the CDF of an exponential distribution with parameter κ.

Knowing this, we can compute

$$E\left[\log\left(\frac{\gamma\phi_2}{c_o}\right)\Big|\phi_2 > \frac{c_o}{\gamma}\right] = \begin{cases} \frac{1}{\kappa_2} & \text{if } \underline{\phi_2} \leq \frac{c_o}{\gamma} \\ E\left[\log\left(\frac{\gamma\phi_2}{c_o}\right)\right] = \log\left(\frac{\underline{\phi}_2\gamma}{c_o}\right) + E\left[\frac{\phi_2}{\underline{\phi}_2}\right] = \log\left(\frac{\underline{\phi}_2\gamma}{c_o}\right) + \frac{1}{\kappa_2} & \text{otherwise} \end{cases}$$

and similarly for $E\left[\log\left(\frac{(1-\gamma)\phi_1}{c_o}\right)\Big|\phi_1 > \frac{c_o}{(1-\gamma)}\right]$. Finally, using the definition of Pareto distribution we compute

$$\Pr\left(\phi_2 > \frac{c_o}{\gamma}\right) = \begin{cases} \left(\frac{\underline{\phi}_1\gamma}{c_o}\right)^{\kappa_2} & \text{if } \underline{\phi_2} \leq \frac{c_o}{\gamma} \\ 1 & \text{otherwise} \end{cases}$$

and similarly for $\Pr\left(\phi_1 > \frac{c_o}{1-\gamma}\right)$. □

Proof of Proposition 4. The mediator minimizes

$$\begin{cases} A(\gamma) \equiv \left(\frac{\underline{\phi}_2\gamma}{c_o}\right)^{\kappa_2}\frac{1}{\kappa_2} + \left(\log\left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right) + \frac{1}{\kappa_1}\right) & \text{if } \gamma \leq \min\{1 - \frac{c_o}{\underline{\phi}_1}, \frac{c_o}{\underline{\phi}_2}\} \\ B(\gamma) \equiv \left(\frac{\underline{\phi}_2\gamma}{c_o}\right)^{\kappa_2}\frac{1}{\kappa_2} + \left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right)^{\kappa_1}\frac{1}{\kappa_1} & \text{if } 1 - \frac{c_o}{\underline{\phi}_1} \leq \gamma \leq \frac{c_o}{\underline{\phi}_2} \\ C(\gamma) \equiv \left(\log\left(\frac{\underline{\phi}_2\gamma}{c_o}\right) + \frac{1}{\kappa_2}\right) + \left(\log\left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right) + \frac{1}{\kappa_1}\right) & \text{if } \frac{c_o}{\underline{\phi}_2} \leq \gamma \leq 1 - \frac{c_o}{\underline{\phi}_1} \\ D(\gamma) \equiv \left(\log\left(\frac{\underline{\phi}_2\gamma}{c_o}\right) + \frac{1}{\kappa_2}\right) + \left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right)^{\kappa_1}\frac{1}{\kappa_1} & \text{if } \max\{1 - \frac{c_o}{\underline{\phi}_1}, \frac{c_o}{\underline{\phi}_2}\} < \gamma \end{cases}$$

Whenever $\kappa_1, \kappa_2 \to \infty$, the uncertainty about the players power level disappears. The solution to the mediator's problem is the one derived in Section 4.2.2.

Taking the derivative of the mediator's objective function with respect to γ we get:

$$\begin{cases} A'(\gamma) \equiv \left(\frac{\underline{\phi}_2\gamma}{c_o}\right)^{\kappa_2}\frac{1}{\gamma} - \frac{1}{1-\gamma} & \text{if } \gamma \leq \min\{1 - \frac{c_o}{\underline{\phi}_1}, \frac{c_o}{\underline{\phi}_2}\} \\ B'(\gamma) \equiv \left(\frac{\underline{\phi}_2\gamma}{c_o}\right)^{\kappa_2}\frac{1}{\gamma} - \left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right)^{\kappa_1}\frac{1}{1-\gamma} & \text{if } 1 - \frac{c_o}{\underline{\phi}_1} \leq \gamma \leq \frac{c_o}{\underline{\phi}_2} \\ C'(\gamma) \equiv \frac{1}{\gamma} - \frac{1}{1-\gamma} & \text{if } \frac{c_o}{\underline{\phi}_2} \leq \gamma \leq 1 - \frac{c_o}{\underline{\phi}_1} \\ D'(\gamma) \equiv \frac{1}{\gamma} - \left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right)^{\kappa_1}\frac{1}{1-\gamma} & \text{if } \max\{1 - \frac{c_o}{\underline{\phi}_1}, \frac{c_o}{\underline{\phi}_2}\} < \gamma. \end{cases} \tag{17}$$

which is continuous in γ. We solve the mediator's problem by considering few separate cases:

- $\kappa_1, \kappa_2 \leq 1$. In this case $A(\gamma)$, $B(\gamma)$, $C(\gamma)$ and $D(\gamma)$ are all concave. By continuity of 17, the solution can only be at the extremes, and hence the waste-minimizing sharing rule is

$$\gamma^* = \begin{cases} 1 & \text{if } \frac{1}{\kappa_2} - \frac{1}{\kappa_1} < \log\left(\frac{\underline{\phi}_1}{\underline{\phi}_2}\right) \\ 0 & \text{otherwise} \end{cases}$$

- $\kappa_1, \kappa_2 > 1$. In this case $A'(0) < 0$ and $D'(1) > 0$ and therefore the solution is never an extreme value. If, furthermore $c_o > \underline{\phi}_1 > \underline{\phi}_2$, then the mediator problem is to minimize $B(\gamma)$, which is convex. Hence the solution to the mediator's problem is

$$\gamma^* : B'(\gamma^*) = 0$$

  If instead $\underline{\phi}_1, \underline{\phi}_2 \to \infty$, the mediator's objective function converges to $C(\gamma)$, which is concave. By continuity, the solution to the mediator's problem converges to

$$\gamma^* = \begin{cases} 1 & \text{if } \frac{1}{\kappa_2} - \frac{1}{\kappa_1} < \log\left(\frac{\underline{\phi}_1}{\underline{\phi}_2}\right) \\ 0 & \text{otherwise} \end{cases}$$

  which is the γ minimizing $C(\gamma)$.

To characterize the solution to the mediator's problem in all other cases, we take the derivative of 17 with respect to $\underline{\phi}_1$, $\underline{\phi}_2$, κ_1, κ_2 and then invoke Topkis's theorem.

The derivative of 17 with respect to $\underline{\phi}_1$ is:

$$\begin{cases} 0 & \text{if } \gamma < 1 - \frac{c_o}{\underline{\phi_1}} \\ -\kappa_1 \left(\underline{\phi}_1 (1-\gamma)\right)^{\kappa_1 - 1} \left(\frac{1}{c_o}\right)^{\kappa_1} & \text{otherwise} \end{cases}$$

which is weakly negative. The derivative of 17 with respect to $\underline{\phi}_2$ is:

$$\begin{cases} \kappa_2 \left(\underline{\phi}_2 \gamma\right)^{\kappa_2 - 1} \left(\frac{1}{c_o}\right)^{\kappa_2} & \text{if } \gamma < \frac{c_o}{\underline{\phi_2}} \\ 0 & \text{otherwise} \end{cases}$$

which is weakly positive. The derivative of 17 with respect to κ_2 is

$$\begin{cases} \left(\frac{\underline{\phi}_2}{c_o}\right)^{\kappa_2} \gamma^{\kappa_2 - 1} \log\left(\frac{\underline{\phi}_2 \gamma}{c_o}\right) & \text{if } \gamma \leq \frac{c_o}{\underline{\phi}_2} \\ 0 & \text{otherwise.} \end{cases}$$

which is weakly negative. The derivative of 17 with respect to κ_1 is

$$\begin{cases} -\left(\frac{\underline{\phi}_1}{c_o}\right)^{\kappa_1} (1-\gamma)^{\kappa_1 - 1} \log\left(\frac{\underline{\phi}_1 (1-\gamma)}{c_o}\right) & \text{if } 1 - \frac{c_o}{\underline{\phi}_1} \leq \gamma \\ 0 & \text{otherwise.} \end{cases}$$

which is weakly positive. By Topkis's theorem, therefore, the waste-minimizing sharing rule is weakly increasing in $\underline{\phi}_1$, κ_2; weakly decreasing in $\underline{\phi}_2$, κ_1.

□

Proof of Lemma 5. Suppose $b_1(P) > 0$ and $b_2(P) > 0$. The four FOCs determining the level of offensive and defensive investments are

$$-\gamma \frac{\partial u(\phi_2, d_2, o_1)}{\partial o_1} + (1-\alpha) \frac{\partial u(\phi_2, d_2, o_1)}{\partial o_1} b_2'(P) \leq c_o$$

$$(1-\gamma) \frac{\partial u(\phi_1, d_1, o_2)}{\partial d_1} + (1-\alpha) \frac{\partial u(\phi_1, d_1, o_2)}{\partial d_1} b_2'(P) \leq c_d$$

$$-(1-\gamma) \frac{\partial u(\phi_1, d_1, o_2)}{\partial o_2} + (1-\alpha) \frac{\partial u(\phi_1, d_1, o_2)}{\partial o_2} b_1'(P) \leq c_o,$$

$$\gamma \frac{\partial u(\phi_2, d_2, o_1)}{\partial d_2} + (1-\alpha) \frac{\partial u(\phi_2, d_2, o_1)}{\partial d_2} b_1'(P) \leq c_d,$$

Note how $b_i'(P)$ decreases player i's incentive to make an offensive investment but increases player i's incentive to make defensive investments.

Supoose instead that $b_1(P) = b_2(P) = 0$. Assume furthermore that (12) and (13) hold at $b_1(P) = b_2(P) = 0$. The four FOCs determining the level of offensive and defensive investments are

$$-\gamma \frac{\partial u(\phi_2, d_2, o_1)}{\partial o_1} + (1-\alpha) \frac{\partial u(\phi_2, d_2, o_1)}{\partial o_1} b_2'(P) \leq c_o$$

$$(1-\gamma) \frac{\partial u(\phi_1, d_1, o_2)}{\partial d_1} \leq c_d'(d_1)$$

$$-(1-\gamma) \frac{\partial u(\phi_1, d_1, o_2)}{\partial o_2} + (1-\alpha) \frac{\partial u(\phi_1, d_1, o_2)}{\partial o_2} b_1'(P) \leq c_o,$$

$$\gamma \frac{\partial u(\phi_2, d_2, o_1)}{\partial d_2} \leq c_d,$$

Note how, in this case, $b_i'(P)$ decreases player i's incentive to make an offensive investment but has no impact on player i's incentive to make defensive investments.

Remember that $b_1'(P)$ and $b_2'(P)$ are determined by the mediator. The mediator can therefore always choose them so that offensive investment is lower when $b_1(P) = b_2(P) = 0$ then $b_1(P) > 0$ and $b_2(P) > 0$. Furthermore, the left hand side of the FOCs for the defensive investments are lower under $b_1(P) = b_2(P) = 0$ than $b_1(P) > 0$ and $b_2(P) > 0$ for any level of $b_1'(P)$ and $b_2'(P)$. That is, it is possible to move from $b_1(P) > 0$ and $b_2(P) > 0$ to $b_1(P) = b_2(P) = 0$, while decreasing all investments.

Finally, note that if $b_1(P) > 0$ and $b_2(P) > 0$ and all investments are already zero, it is always possible to set the equilibrium concessions to zero by manipulating $\frac{\partial f(b_1,b_2)}{\partial b_1}$ and $\frac{\partial f(b_1,b_2)}{\partial b_2}$, while at the same time maintaining $b_1'(P)$ and $b_2'(P)$ (and with it the incentives to make offensive and defensive investments) constant. This is welfare increasing because it eliminates concessions (which are socially costly) while maintaining all investments at zero. □

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