Auctions with Tokens*

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First version: September 30, 2021. This version: February 21, 2022. Please check here for the latest version.

Abstract

In a repeated, private-value auction with risk-neutral bidders, the auctioneer can decide to accept payments in a blockchain-based token that he creates and initially owns. I show that, if the time horizon is finite, then the present-discounted value of expected revenues is the same whether payments occur in tokens or in dollars (or any other fiat currency). However, revenues will accrue earlier and be less variable in the auction with tokens. In particular, if the auctioneer commits to destroying all tokens received back as payment, then he will earn the present-discounted value of the future expected revenues at the beginning of the game with probability 1. If the time horizon is infinite, then financial bubbles on tokens are possible, in which case the auctioneer's expected revenues will be larger than in the auction with dollars. As an extension, I introduce a cash-in-advance constraint and show that the auctioneer strictly prefers using tokens to using dollars, because within each period revenues accrue earlier with tokens than with dollars (when selling tokens vs when selling the object).

JEL classification: D44, E42, L86

Keywords: Auctions, Blockchain, Cryptocurrencies, Tokens, Private Money

*I'm grateful to Sylvain Chassang, Lin William Cong and conference participants at the Crypto Asset Lab conference 2021 for their insightful comments and suggestions.

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1 Introduction

The recent invention of blockchain allows anyone to create new digital tokens and to commit to their supply.[1] These tokens can be exchanged freely on the blockchain, and can be (and often are) used as means of payment. As a result, the number and value of blockchain-based privately-issued digital tokens and currencies exploded in recent years, with thousands currently trading for an aggregate value of approximately USD 2 trillions.[2] These remarkable events also challenge some of economists' long-held practices, because issues traditionally studied in monetary economics from a *macroeconomic* perspective are now potentially relevant also from the *microeconomic* viewpoint. That is, when studying the optimal behavior of a single firm (or a single entrepreneur, or a single consumer) a researcher may need to consider the possibility that such firm may create a brand new currency with a specific monetary policy.

It this paper, I tackle the above question in the context of auctions. The study of auctions is among the most successful research lines in economics, having produced both deep theoretical results, and also very practical and useful insights—for example, we now know that auctions are often the revenue-maximizing way to sell an object.[3] This success was achieved by systematically studying auctions under a wide variety of assumptions with respect to bidders' valuations, information structure, equilibrium concept, etc (Klemperer, 1999). Yet, a common assumption is that all payments occur using fiat currency (for example, dollars). Here, instead, as a part of the auction design, the auctioneer can decide to accept payments in a blockchain-based token that he creates and initially owns. If he chooses to do so, the auctioneer also commits to a monetary policy, that is, a set of rules determining how the stock of tokens evolves over time.

[1] There are a number internet tutorials explaining how to create blockchain based tokens (I invite the reader to search "how to create an ERC-20 Token", where ERC-20 is the simplest token that can be created on the Ethereum blockchain). There are also a number of services allowing to create a blockchain-based tokens without directly coding (see, for example, https://www.tokenmakerclub.com)

[2] Source http://www.coinmarketcap.com.

[3] Indeed, the 2020 Nobel prize for economics was awarded to Paul Milgrom and Robert Wilson "for improvements to auction theory and inventions of new auction formats." The Nobel committee rightly noticed that "astronomical sums of money change hands every day in auctions".

I consider a sequence of private-value auctions in which multiple objects are sold (one per period). In every period, risk-neutral bidders draw their valuation for the object sold from an i.i.d. distribution. Then they submits bids. Given the profile of bids, the auction format determines the winning bidder and the payment of each player. The good is then consumed within the period. Each auction is therefore a very simple, static auction, repeated multiple times (that is, there is no connection between auctions in different periods).

The auctioneer could accept payments in dollars (or in any other existing currency), in which case the analysis of the equilibrium is straightforward: by the revenue equivalence theorem and under an appropriate assumption on the distribution of valuations, all common auction formats with a reservation price of zero maximize revenues. But the auctioneer could also create new blockchain-based tokens. I consider an auction with tokens which is as close as possible to the auction without tokens: bids and payments are expressed in dollars, but need to be settled using the token created by the auctioneer. The auctioneer earns revenues by selling newly created tokens to bidders, and also by re-selling tokens that he received as payment. At the onset of the game, the auctioneer also specifies a monetary policy.

Because the auctioneer is creating a new currency, a crucial assumption of the model is that he can commit to a specific monetary policy, which is implicitly achieved by regulating the supply of the token via a blockchain-based smart contract. Another important assumption is that the auctioneer can commit to the auction format, in the sense that he cannot refuse to accept tokens as means of payment after announcing that he would do so. Again, for some objects this type of commitment could be achieved via a smart contract.[4] However, for other objects this type of commitment may be difficult or impossible to achieve, something I consider in Section 6.2. Finally, the model assumes that tokens can be exchanged for dollars on a frictionless financial market. With this respect, note that blockchain-based decentralized-finance protocols allow anyone to create a financial market in

[4] A particular interesting example is the sale of Non Fungible Tokens (NFTs), which are blockchain-based tokens representing specific digital objects (could be art, or in-game items, or digital collectibles), which are usually sold via auctions. For the study of NFTs auctions, see Khezr and Mohan (2021).

which any two blockchain-based tokens can be exchanged.[5]

I show that when the auction uses tokens, these tokens might be purchased for speculation and not for bidding. This happens when the realized valuations in a given period are low (relative to the future expected valuations) and, as a consequence, the demand for tokens *for bidding* is low, therefore creating an arbitrage opportunity for bidders: they may purchase tokens, not use them for bidding in that period, and sell them (or use them) in future periods. The speculative demand for tokens increases the price for tokens in a given period and, as a consequence, drives the auctioneer's expected revenues for that period above those of the auction with dollars. At the same time, today's speculators will compete with the auctioneer on tomorrow's market for tokens, therefore lowering the auctioneer's future expected revenues relative to those of the auction with dollars. Furthermore, the speculative demand in a given period depends on the *expected* future valuations, and therefore. transforms future uncertain revenues into present certain revenues.

In the finitely-repeated version of the game, the speculative demand for tokens impacts present and future revenues equally: the present-discounted expected revenues of the auction with tokens is the same as in the auction with dollars. However, revenues will accrue earlier and will be less uncertain in the auction with tokens than in the auction with dollars. How exactly depends on the specific monetary policy announced by the auctioneer. A particularly relevant case is a policy according to which all tokens that are used to pay the auctioneer are then destroyed. In this case, the entire revenues are earned in the first period, when the auctioneer sells the initial stock of tokens. Furthermore, the present-discounted value of the expected revenues from period-2 onward is earned with probability 1.[6] In the infinitely-repeated version of the game, then an additional consideration emerges: the speculative demand

[5] For example, the auctioneer could use Uniswap to create a *decentralized financial market* for exchanging his tokens against, for example, a stablecoin (i.e., a blockchain-based token with constant value relative to the dollar). To do so, the auctioneer needs to provide the initial liquidity by "committing" some tokens and some amount of stablecoin. Doing so is costly, but also generates revenues because Uniswap's trading fees are earned by the liquidity providers. Once the two liquidity pools are created, anyone can exchange tokens for the stablecoin (or vice versa), and also contribute additional liquidity.

[6] The reason is that, in the model, the first sale of tokens happens after the period-1 valuations are drawn, which therefore is the only risk faced by the auctioneer. There is a straightforward extension of the model in which the auctioneer can sell tokens also before period-1 valuations are drawn, in which case the auctioneer is able to eliminate all risk.

for tokens could be positive also in the long run, giving rise to financial bubble, in which case the present-discounted value of the auctioneer's expected revenues is strictly greater than in the auction with dollars (in the equilibrium without bubbles, they are the same). Again, if the auctioneer commits to destroying all tokens received as payment, the present-discounted value of future revenues (which here includes the bubble component) is earned in period 1 with probability 1.

Hence, by designing an appropriate auction with tokens, the auctioneer is able to fully front-load his revenues and eliminate (almost) all risk. Interestingly, when credit and insurance markets are perfect, the auctioneer can achieve an identical outcome by holding an auction with dollars while simultaneously borrowing and purchasing insurance against the variability of his revenues. If instead either the credit market or the insurance market is imperfect, then the auctioneer strictly prefers the auction with tokens. Similarly, the auctioneer could issue equity (i.e., transferring to investors his future cash flow), in which case the auction with tokens is equivalent to an auction with dollars in which the auctioneer also sells equity.

Importantly, whereas the same outcome can be implemented via different mechanisms, each of these mechanisms rest on different assumptions. For example, front loading revenues and reducing risk via issuing equity (or purchasing insurance and issuing debt) is possible only if the auctioneer's revenues are contractible. Contractibility of revenues is not required in the auction with token, which instead requires the ability to commit to a given monetary policy. Also, whereas bubbles may emerge both on equity and on tokens, the two types of bubbles can have different dynamics. For example, in the auction with tokens in which the auctioneer commits to destroying all tokens received as payment, the entire stock of tokens is initially purchased by bidders, who then progressively use them to pay the auctioneer. A bubble on tokens emerges if the rate an which bidders use tokens is sufficiently low.

I extend the model by introducing a cash-in advance constraint. The above results are obtained under the assumption that every dollar earned can be immediately used to purchase consumption goods, which are then consumed at the end of each period, while tokens can be exchanged for dollars (and then consumption goods) only once in every period. However, we know from previous literature that sometimes people need to accumulate dollars before being able to spend them. In these cash-in-advance models, an agent is allowed to exchange dollars for consumption goods only in certain moments within a period. As a consequence, dollars that are earned earlier in the period are more valuable because they are more likely to be converted in consumption during that period (rather than in the following period With a cash-in-advance constraint, therefore, revenues (measured in the value of the consumption they generate) are higher in the auction with tokens than in the auction with dollars.

I also extend the model by introducing effort: in every period, the auctioneer can take visible costly actions that increase the bidders' valuation for that period. I show that the incentives to exert effort (and hence the auctioneer's revenues) are different in the auction with dollars than in the auction with tokens. In the auction with dollars, the choice of effort is a static problem: in each period, the auctioneer compares the marginal benefit and the marginal cost of effort for that period. In the auction with tokens, instead, there are both static and dynamic incentives. The static incentives to exert effort are below that of the auction with dollars when the auctioneer does not own the entire stock of token. The dynamic incentives come from the fact that when effort today is high, today's speculative demand for tokens is low, the fraction of the total stock of tokens held by the auctioneer in the future is high, and future effort is high. It is possible to build examples in which the dynamic effect dominates and effort is higher in the auction with tokens than in the auction with dollars.

Literature review

The mechanism design literature noticed long ago that certain centralized mechanism can be implemented in a decentralized way using tokens (see, for example, Ostroy and Starr, 1974, Kocherlakota, 1998). Similarly, some of the early papers in monetary theory considered general-equilibrium models in which there is at least an equilibrium in which money emerges (see Samuelson, 1958, and Townsend, 1980). Because the equilibrium with money is Pareto superior to that without money, again, we can think of money as allowing the decentralized implementation of some (usually constrained) optimal allocation. Within this literature, the most closely related papers are those proposing models in which money has value because of exogenous reasons. For example, in Starr (1974), a government creates money and establishes that taxes need to be paid using money. Likewise, in the cash-in-advance model of Clower (1967), fiat money needs to be acquired one period in advance. In Lucas Jr and Stokey (1983), some goods do not require cash in advance, while others do. Note that in these models, the optimal monetary policy is typically time inconsistent, and hence the issue of commitment to a monetary policy arises (see, for example, Lucas Jr and Stokey, 1983).

The advent of blockchain and blockchain-based tokens provided new impulse to the above literature (for an overview, see Townsend, 2020). The reason is that blockchain-based smart contracts can be used to generate commitment, for example to perform payments based on contingencies. Several authors have therefore studied how blockchain-based smart contracts can be used to implement various types of mechanism in a decentralized way (see, Holden and Malani, 2019, Gans, 2019, Lee, Martin, and Townsend, 2021). With this respect, note that, in the model presented here, the auction itself could be a traditional, centralized auction, or a decentralized one (via a smart contract). Hence, this paper makes no contribution to the study of how auctions (or other mechanism) can be implemented using blockchain. However, the issue of implementation is relevant because the allocation that can be achieved via the creation of tokens can also be achieved via, for example, pledging future cash flow via a security. Crucially, however, the two ways of implementing the same allocation holds under different assumptions (contractible of revenues in one case, commitment to a monetary policy in the other case).

A number of papers studied theoretically firms' incentives to issue blockchain-based tokens, which can represent a pre-sale of a given unit of future output, or the only currency that the firm will accept in the future, or a claim on future revenues or profits. Some of these papers showed that, in the presence of network externalities, selling tokens helps avoiding coordination failures (Sockin and Xiong, 2018, Cong et al., 2021, Bakos and Halaburda, 2018, and Li and Mann, 2018). Other papers focused on the sale of tokens as an innovative way to raise capital and finance the development of a product or a platform (Catalini and Gans, 2018, Malinova and Park, 2018, Canidio, 2018, Bakos and Halaburda, 2019, Goldstein et al., 2019, Cong et al., 2020, Canidio, 2020, Gryglewicz et al., 2021, Garratt and van Oordt, 2021, Chod and Lyandres, 2021). In the model considered here, the auctioneer has no financing need and there are no network externalities. Hence, tokens are sold purely to earn a profit. Nonetheless, there is a connection with the above literature because by issuing tokens the auctioneer can manipulate the time-profile and riskiness of his revenues. Both these elements are important in determining the incentives to invest and create new ventures.

As mentioned in the introduction, almost all the existing literature on auctions considers the case when bidders use cash payments. A notable exception are papers studying the possibility that bids are securities representing the cash flow generated by the object sold in the auction (see Hansen, 1985, Riley, 1988, DeMarzo et al., 2005, and more recently Cong, 2019). The interesting aspect of this problem is that the value of the security may reflect a bidder's private information relative to the value (in this case, the cash flow) of the object. Here I abstract away from those consideration by restricting my attention to a very simple private-value auction in which the object sold is immediately consumed.

I will frequently refer to two important results in auction theory. The first is the revenue equivalence theorem, which states:[7]

> *Assume each of a given number of risk-neutral potential buyers of an object has a privately-known signal independently drawn from a common, strictly-increasing, atomless distribution. Then any auction mechanism in which (i) the object always goes to the buyer with the highest signal, and (ii) any bidder with the lowest-feasible signal expects zero surplus, yields the same expected revenue (and results in each bidder making the same expected payment as a function of her signal).*

For our purposes, the above statement implies that all common auction formats (i.e., first-price, second-price, all-pay, ...) generate the same expected payment from bidders and hence the same expected revenues to the auctioneer. The second result is the design of optimal auctions (see, again, Myerson, 1981, Bulow and Roberts, 1989, Bulow and Klemperer, 1996 and Klemperer, 1999). In particular, in the model

[7] Vickrey (1961) developed some special case of the revenue equivalence theorem. The statement presented here is taken from Klemperer (1999), and summarizes results in Myerson (1981) and Riley and Samuelson (1981). For a more general formulation, see Milgrom and Segal (2002).

I will assume that the distribution of valuations is such that all common auction formats with a reservation price of zero maximize the auctioneer's expected revenues.

2 The model

I consider a single-object private-value auction repeated $T \geq 1$ times, where T could be a finite number or infinity. There are $n \geq 2$ ex-ante identical bidders and an auctioneer. In period 0, the auctioneer decides whether to accept payments in fiat currency (for simplicity, dollars), or in tokens. If the auction format requires the use of tokens, then the auctioneer creates an initial stock of tokens M_1, and also announces a monetary policy, that is, how the stock of tokens will evolve over time (see below). Then, in every period $t \in \{1, ..., T\}$ the auctioneer sells a single object according to the auction format specified initially. Each object sold has zero value to the auctioneer.

Bidders are risk neutral and cash abundant, in the sense that their cash constraint is never binding. The auctioneer's per-period utility function is $U()$, assumed concave. There is a common discount factor $\beta \in (0, 1)$. Bidders and the auctioneer can also hold a risk-free asset yielding a per-period gross return $R \geq 1$. For ease of derivations, I assume that $R = \frac{1}{\beta}$.[8] The auctioneer can only save, and cannot borrow nor access insurance to smooth out the variability of the revenues earned.[9] The auctioneer's initial assets are $w_1 \geq 0$. Consumption occurs at the end of each period.

Dollars and tokens differ in how conveniently they can be converted into consumption goods. Here I assume that dollars convert into consumption goods instantaneously, so that every dollar earned in a given period can be consumed at the end of that period. Instead, the market for tokens opens once in every period, so that tokens earned in a given period may be converted into consumption only in the following period. As a consequence, tokens are maximally inconvenient relatively to dollars. In Section 4, instead, I introduce a cash-in-advance constraint, so that only

[8] Hence, R is the steady-state rate of return of the Ramsey-Cass-Koopmans growth model (with no population growth or exogenous productivity growth). This assumption is not essential for the results but simplifies the derivations.

[9] I discuss what happens when the auctioneer can borrow and also insure his risk in Section 6.1.

a fraction of the dollars earned during a period can be consumed during that period, with the rest consumed in the following period. In that version of the model, tokens are less inconvenient (relative to dollars) than in the benchmark case considered here.

I now provide a more precise timeline of each type of auction.

Auctions with dollars If the auctioneer decides to accept payments in dollars, then in each period $t \in \{1, ..., T\}$:

- First, each bidder draws a valuation $v_{i,t} > 0$ from a continuous and atomless distribution with c.d.f $F(v)$, p.d.f. $f(v)$ and support $[\underline{v}, \overline{v}]$. The auction is in private values, and hence each bidder's valuation is independent of the other bidders' valuations. Each $v_{i,t}$ is bidder i's private information, but the distribution $F(v)$ is common knowledge.

- Each bidder sends a message $m_{i,t} \in \mathbb{R}_+$ to the auctioneer, interpreted as his bid.

- As a function of the messages received and the auction format initially announced, the auctioneer determines who is the winner and a payment $b_{i,t} \leq m_{i,t}$ for each bidder (implicitly a function of all messages received).

- Each bidder then pays $b_{i,t}$ dollars to the auctioneer, and the winner receives and consumes the object. The period-t payoffs of the winning bidder is $v_{i,t} - b_{i,t}$; the period-t payoff of all other bidders is $-b_{i,t}$. The auctioneer's period-t utility is:
$$U_t \left(\sum_i^n b_{i,t} + R \cdot w_t - w_{t+1} \right),$$
where $w_{t+1} \geq 0$ are the asset carried to the following period, chosen optimally by the auctioneer at the end of the period, just before consuming.

To avoid uninteresting complications, I assume that $vf(v) \geq 1 - F(v)$ for all $v \in [\underline{v}, \overline{v}]$. Under this assumption, any standard auction format with a reservation price of zero maximizes the auctioneers' revenues, and calculating the revenues from the optimal auction with dollars is straightforward (see the next subsection for more details).

Auctions with tokens If the auctioneer uses tokens, then the timeline of each period $t \in \{1, ..., T\}$ is the following:

- Again, at the start of a period, each bidder draws a valuation $v_{i,t}$ from the distribution $F(v)$. Note that, at this point, both auctioneer and bidders may own tokens that they accumulated from previous periods. Call $A_t \geq 0$ the tokens owned by the auctioneer at the beginning of the period, and $a_{i,t} \geq 0$ the tokens owned by bidder i. By assumption, $A_1 = M_1$ and $a_{i,1} = 0$ for all $i \leq n$.

- Each bidder sends a message $m_{i,t} \in \mathbb{R}_+$ to the auctioneer, interpreted as his bid *in dollars*.

- As a function of the messages received and the auction format initially announced, the auctioneer determines who is the winner and a payment $b_{i,t} \leq m_{i,t}$ for each bidder (implicitly a function of all messages received). This payment is expressed in dollars, *but needs to be settled using tokens*.

- A frictionless, anonymous financial market for tokens opens, in which both the auctioneer and bidders participate. All market participants are price takers. Call p_t the equilibrium price for tokens; call $q_{i,t}$ the equilibrium demand for tokens of bidder i; call Q_t the equilibrium demand for tokens of the auctioneer. Both $q_{i,t}$ and Q_t could be positive or negative; if negative the bidder/auctioneer is a net seller of tokens in equilibrium, otherwise he is a net buyer. Furthermore, feasibility implies $A_t + \sum_i a_{i,t} = Q_t + \sum_i q_{i,t}$. Also, because tokens will be used to pay the auctioneer (see the next point) it must be that $q_{i,t} \geq \frac{b_{i,t}}{p_t} - a_{i,t}$.

- Then, each bidder sends $\frac{b_{i,t}}{p_t}$ tokens to the auctioneer. The winner receives the object and consumes it. At this point, each bidder owns $a_{i,t} + q_{i,t} - \frac{b_{i,t}}{p_t}$ tokens, and the auctioneer owns $A_t + Q_t + \sum_i \frac{b_{i,t}}{p_t}$ tokens.

- The winning bidder enjoys a per-period payoff equal to the value of the object minus the expenditure in tokens, that is: $v_i - p_t \cdot q_{i,t}$. Similarly, the losing bidders enjoy a per-period payoff equal to $-p_t q_{i,t}$, and the auctioneer enjoys a per-period payoff equal to $U_t(-p_t Q_t + R \cdot w_t - w_{t+1})$.

- The stock of tokens changes according to the monetary policy announced by the auctioneer. Here I restrict my attention to two time-varying monetary-policy parameters: a uniform increase (or decrease) of all tokens by the same factor $\tau_t \geq -1$, and an increase (or decrease) of only the tokens used for bidding by a time-varying factor $\sigma_t \geq -1$. As a result, at the beginning of the subsequent period each bidder i owns $a_{i,t+1} = (1+\tau_t)(a_{i,t} + q_{i,t} - \frac{b_{i,t}}{p_t})$, while the auctioneer owns $A_{t+1} = (1+\tau_t)(A_t + Q_t + (1+\sigma_t)\sum_i \frac{b_{i,t}}{p_t})$. Hence, the total stock of tokens at the beginning of period $t+1$ is $M_{t+1} \equiv A_{t+1} + \sum_i^n a_{i,t+1}$.

Finally, the auction format is assumed to be such that: (i) those bidding the most tokens win, (ii) those bidding zero tokens pay zero and win with probability zero.[10]

The monetary policy considered here may sound far fetched, but is actually very simple to implement in the context of blockchain based tokens. In particular, the fact that the tokens used for bidding may grow at a different rate than the other tokens is inspired by staking. In staking, those who "lock" some tokens (or, more in general, do not use them) are rewarded with additional tokens. Here the staking reward is positive if $\sigma_t < 0$, in which case those who do not use the tokens receive an additional reward relative to those who use them.

The above auction with tokens is as close as possible to a traditional auction with dollars: bids and payments are expressed in dollars but need to be settled using tokens. However, other assumptions are possible. For example, bidders may be required to bid by submitting tokens, which could then be partially returned to the bidders after the winner is determined.[11] Also, here I consider only two possible monetary policies, but many more are possible. The bottom line is that the above is the least complex auction with tokens, and not the most general auction with tokens.

[10] Hence, if this was an auction with dollars, the revenue equivalence theorem would hold.

[11] In this case, there is an additional complication: bidders need to purchase tokens before bidding, which means that the equilibrium price for tokens may reveal some information relative to the realized distribution of valuations. Hence, the equilibrium on the market for tokens should be a rational expectation equilibrium.

3 Equilibrium

I start by deriving the solution of the auction with dollars. I then consider the auction with tokens.

3.1 Auction with dollars

If the auction is with dollars, then all the standard results from auction theory apply.[12] Quite immediately, in every period, the revenue equivalence theorem holds: all standard auction formats generate the same expected revenues. Also, given our assumption on the distribution of valuations, revenues are maximized when the reservation price is zero. By considering a second-price auction, in every period expected revenues are

$$k \equiv E[v_{Max-1,t}].$$

where $v_{Max,t} \equiv \max_i\{v_{i,t}\}$ is the realized highest valuation in period t, and $v_{Max-1,t} \equiv \max_{i \neq Max}\{v_{i,t}\}$ is the realized second-highest valuation in period t. Also, each bidder's expected payoff from the auction is

$$g \equiv E[\max\{v_i - v_t^{Max-1}, 0\}]$$

Hence, from period-1 viewpoint, the present-discounted value of the expected revenues of the auction with dollars are:

$$\Pi_{USD} = k \sum_{t=1}^{T} \beta^{t-1} = \begin{cases} \frac{1-\beta^T}{1-\beta} k & \text{if } T \text{ finite} \\ \frac{1}{1-\beta} k & \text{if } T = \infty, \end{cases}$$

and the present discounted value of participating in the auction as a bidder is

$$u_{USD} = g \sum_{t=1}^{T} \beta^{t-1} = \begin{cases} \frac{1-\beta^T}{1-\beta} g & \text{if } T \text{ finite} \\ \frac{1}{1-\beta} g & \text{if } T = \infty. \end{cases}$$

[12] See, for example, Klemperer (1999), in particular Section 4 (for the revenue equivalence theorem) and Appendix B (how to calculate the optimal reservation price).

For a given auction format, the auctioneer's utility is

$$U_{USD} = \max_{w_2 \geq 0, ..., w_T \geq 0, w_{T+1}=0} \left\{ \sum_{t=1}^{T} \beta^{t-1} EU \left(\sum_{i}^{n} b_{i,t} + Rw_t - w_{t+1} \right), \right\},$$

where $EU_t()$ is the auctioneer's expected utility in period t. Note that, if the auctioneer is risk averse (i.e. his utility function is strictly concave), not all common auction formats maximize expected utility. The reason is that the variance of the revenues also matters. Nonetheless, for what follows, it is sufficient to establish an upper bound to the auctioneer's utility, as the next lemma does (its proof is omitted).

Lemma 1. *Consider a given auction format, a given level of initial assets w_1, and realized period-1 revenues $\sum_i^n b_{i,1}$. Define $w_2^*, ..., w_T^*$ as the unconstrained optimal sequence of assets in the absence of risk, that is*

$$\{w_2^*, ..., w_T^*\} \equiv argmax_{w_2,...,w_{T+1}=0} \left\{ U \left(\sum_{i}^{n} b_{i,1} + Rw_1 - w_2 \right) + \sum_{t=2}^{T} \beta^{T-1} U(k + Rw_t - w_{t+1}) \right\}.$$

It must be that

$$U_{USD} \leq E \left[U \left(\sum_{i}^{n} b_{i,1} + Rw_1 - w_2^* \right) + \sum_{t=2}^{T} \beta^{T-1} U(k + Rw_t^* - w_{t+1}^*) \right]$$

where the expectation is taken over period-1 revenues $\sum_i^n b_{i,1}$. The above inequality is strict if $U()$ is strictly concave.

The above lemma says that, for every possible auction formats, the auctioneer's utility must be lower than the utility when all risks after period 1 are eliminated and credit constraints are removed, allowing him to borrow at the risk-free rate. The inequality must be strict if his utility is strictly concave. Quite intuitively, if the auctioneer is risk averse, he would rather receive the largest possible expected revenues k with probability 1 in each period rather than being exposed to the variability of these revenues. Furthermore, the fact that utility is strictly concave also implies that the auctioneer benefits from smoothing consumption across periods. Because achieving optimal consumption smoothing may require borrowing, the auctioneer benefits when borrowing constraint are removed (independently on whether risk is also removed).

3.2 Auction with tokens

I start by deriving a number of results that hold both in the finite and in the infinite time horizon of the game, before considering each case separately.

The next lemma derives the price of tokens in a given period as a function of the future expected price and the realization of bids.

Lemma 2. *Consider a given p^e_{t+1} and a given realization of period-t valuations (and hence given profile of bids $m_{i,t}, ..., m_{n,t}$ and payments $b_{i,t}, ..., b_{n,t}$). The demand for tokens in period t is*

$$\frac{\sum_i b_{i,t}}{p_t} + S_t$$

where $S_t \geq 0$ is the speculative demand for tokens, that is, the demand for tokens not used for paying the auctioneer, and is defined as

$$S_t = \max\left\{M_t - \frac{\sum_i^n b_{i,t}}{\beta(1+\tau_t)p^e_{t+1}}, 0\right\}.$$

The equilibrium period-t price is:

$$p_t = \max\left\{\frac{\sum_i b_{i,t}}{M_t}, \beta(1+\tau_t)p^e_{t+1}\right\}. \tag{1}$$

The most important observation is that some tokens may be purchased not for bidding but for speculative purposes. This happens in equilibrium when the realized distribution of valuations is such that total payments to the auctioneer are low. In this case, if the demand for tokens was determined exclusively by the tokens used for bidding, we would have $p_t < \beta(1+\tau_t)p^e_{t+1}$ which cannot be an equilibrium because the return on investing in tokens would be strictly greater than that of investing in the risk-free asset (remember that the risk-free asset generates a present-discounted return equal to $\beta R = 1$). The possibility of purchasing tokens for speculation implies that the period-t price for tokens has a lower bound given by $\beta(1+\tau_t)p^e_{t+1}$.

The above lemma can be used to derive the bidders' incentives to bid for given sequence of token prices. If $p_t \geq \beta(1+\tau)p^e_{t+1}$ then bidders do not want to hold any token between the two periods, hence $a_{i,t+1} = 0$. If instead $p_t = \beta(1+\tau)p^e_{t+1}$, then bidders are indifferent between holding any amount of tokens between period t and period $t+1$, including zero tokens. It follows that, in both cases, bidder i's utility as a function of p_t, p^e_{t+1} and the profile of bids is:

$$\begin{cases} v_{i,t} + p_t a_{i,t} - b_{i,t} + \beta u_{t+1}(0) & \text{if } i \text{ is the winner} \\ p_t a_{i,t} - b_{i,t} + \beta u_{t+1}(0) & \text{otherwise,} \end{cases} \tag{2}$$

where $u_{t+1}(a_{i,t+1})$ is the expected continuation utility from period $t+1$ onward, as a function of the tokens owned at the start of period $t+1$. It immediately follows that, for given auction format, the bidders' incentives to bid are the same with or without tokens. Because of this, the revenue equivalence theorem holds also here: any standard auction format with a reservation price of zero maximizes the auctioneer's expected revenues which are equal to $E[\sum_i b_{i,t}] = k$. As a consequence, also here, and the bidders' expected payoff from participating in the auction is g.

Importantly, by the above equation, the bidders' payoffs depend both on the expected payoff from participating in the auction g, and on the expected value of the tokens held at the beginning of the period $p^e_t \cdot a_{i,t}$.[13] Knowing this, I can take the expectation of 2 and write a bidder i's expected continuation utility as:

$$u_t(a_{i,t}) = p^e_t a_{i,t} + g \sum_{t=1}^{T} \beta^{t-1} = p^e_t a_{i,t} + g \cdot \begin{cases} \frac{1-\beta^T}{1-\beta} & \text{if } T \text{ finite} \\ \frac{1}{1-\beta} & \text{if } T = \infty \end{cases}$$

Note that, because $a_{i,1} = 0$, then a bidder's expected continuation utility from period 1 viewpoint is the same as in the auction with dollars. But because $a_{i,t}$ could be positive for $t \geq 2$, then the time-profile of the bidder's utility may differ in the auction with tokens from the auction with dollars.

Having determined the bidders' expected utility, I can now derive the auctioneer's revenues. I do so by considering separately the finitely- and infinitely-repeated

[13] It is useful to think of each bidder selling all their tokens and earning $p_t a_{i,t}$, while simultaneously purchasing tokens to bid $b_{i,t}$. The expected cost of the bid is part of the expected payoff from the auction.

auction. The difference between the two cases is that, in the infinitely-repeated case, financial bubbles on tokens may emerge.

***T* finite.** The next proposition derives the auctioneer's expected revenues for the case in which T is finite.

Proposition 1 (Expected revenues)**.** *Consider a given sequence of equilibrium expected prices (i.e., prices such that equation 1 holds). If T is finite, at the beginning of each period t the present-discounted value of the auctioneer's expected revenues is*

$$\Pi_{Tokens,t}(A_t) = \frac{1-\beta^T}{1-\beta}k - p_t^e(M_t - A_t).$$

The key observation is that the speculative demand for tokens in period t increases the price of tokens and the auctioneer's revenues in that period. At the same time, speculators compete against the auctioneer on the market for tokens in period $t+1$. The proposition shows that, in expected terms, the two effects cancel out. As a consequence, the present discounted value of the auctioneer's expected revenues at the beginning of the game is $\Pi_{Tokens,1}(M_1) = \frac{1-\beta^T}{1-\beta}k$, as in the auction with dollars. In any subsequent period, the auctioneers' continuation revenues may be lower than those in the auction with dollars, by an amount equal to the value of the tokens not held by the auctioneer in that period.

The above proposition highlights the effect of the speculative demand for tokens on the time-profile of the auctioneer's revenues. However, the speculative demand for tokens also affects the variability of these revenues. This is because, as discussed above, the fact that bidders can purchase tokens for speculation implies that the price of tokens cannot be below $\beta(1+\tau_t)p_{t+1}^e$. The speculative demand for tokens, therefore, transforms uncertain future revenues into certain present revenues. Crucially, the incentive to purchase tokens for speculation—and as a consequence the time profile and variability of revenues—depends on the monetary policy specified by the auctioneer.

For example, when $\sigma_t > 0$ in every period the auctioneer creates additional tokens which he keeps for himself. As a consequence, if σ_t is sufficiently large for all $t \leq T$, for every realization of valuations, holding tokens between periods yields a return that is below that of the risk-free asset. The speculative demands for tokens is therefore zero: the entire stock of tokens is sold by the auctioneer in a given period, then received back as payment, then sold again in the following period. Like in the auction with tokens, the auctioneer earns $\sum_i b_{i,t}$ dollars in every period. As σ_t decreases, the incentive to purchase tokens for speculation increases, causing revenues to accrue earlier and to be less uncertain.

The case $\sigma_t = -1$ is therefore particularly relevant: bidders purchase all tokens in period 1, and progressively use them to pay the auctioneer, who then destroys these tokens. The next proposition derives the auctioneer's revenues for this case.

Proposition 2 (Revenues when $\sigma_t = -1$ for all $t \leq T$)**.** *Suppose T is finite $\sigma_t = -1$ for all $t \leq T$ then the auctioneer's revenues are*

$$p_1 M_1 = \sum_i b_{i,t} + \beta \frac{1-\beta^{T-1}}{1-\beta} k \tag{3}$$

Quite intuitively, when the auctioneer destroys all tokens received as payment, then he earns revenues only in period 1. The revenues pertaining to period 1 are subject to risk, but the present-discounted value of the expected revenues pertaining to all following periods is earned with probability 1.[14] Note also that, because all revenues are earned in period 1 and all risk past period 1 is eliminated, the auctioneer can achieve optimal consumption smoothing by simply saving. Hence, for every possible realization of $\sum_i b_{i,t}$, the auctioneer achieves utility:

$$U\left(\sum_i^n b_{i,1} + Rw_1 - w_2^*\right) + \sum_{t=2}^{T} \beta^{T-1} U(k + Rw_t^* - w_{t+1}^*)$$

where $w_2^*, ..., w_T^*$ are defined in Lemma 1. Lemma 1 then immediately implies the following corollary:

Corollary 1. *Suppose T is finite. The auction with tokens with $\sigma = -1$ is the preferred auction with tokens, strictly so if $U()$ is strictly concave. It is also preferred*

[14] There is a straightforward extension of the model in which the auctioneer is allowed to sell tokens already in period 0. In that case, also the variability of period-1 revenues is eliminated, because the auctioneer earns the total present-discounted value of the expected revenues in period 0 with probability 1.

to the auction without tokens, strictly so $U()$ is strictly concave.

***T* infinite.** Suppose now that T is infinite. In this case, absent a transversality condition (which I do not impose) financial bubbles are possible, that is, the price of tokens may exceed their "fundamental" value, as the next proposition shows.

Proposition 3 (Expected revenues)**.** *Consider a given sequence of equilibrium expected prices (i.e., prices such that equation 1 holds). If T is infinite, at the beginning of each period t the present-discounted value of the auctioneer's expected revenues is*

$$\Pi_{Tokens,t}(A_t) = \frac{1}{1-\beta}k - p_t^e(M_t - A_t) + \lim_{s\to\infty} \beta^{s-1}\Pi_{Tokens,s}(M_s).$$

The emergence of bubbles depends on the expected long-run speculative demand for tokens: when it is high, then the speculative demand for tokens today will be high, and the current price for tokens will be high. As a consequence, from period-1 viewpoint, for any monetary policy the auctioneer may specify the present-discounted value of the expected revenues in the auction with tokens are at least as large as those in the auction with dollars.

A particularity relevant monetary policy is, again, one that specifies $\sigma_t = -1$ for all t, so that all revenues accrue in period 1.

Lemma 3. *Suppose T is infinite and $\sigma_t = -1$ for all $t \leq T$. Call Y_t the expected speculative demand in period t. The auctioneer's revenues are*

$$p_1 M_1 = \sum_i b_{i,1} + \beta\frac{1}{1-\beta}k + \lim_{t\to\infty}\beta^{t-1}p_t^e Y_t \tag{4}$$

where $p_t^e = \frac{p_1}{\beta^{t-1}\prod_{s=1}^{t-1}(1+\tau_s)}$.

Similarly to the finite-horizon case, also here the only risk the auctioneer faces is due to the period-1 realization of the bidders' valuations; the remaining part of his revenues are earned with probability 1. However, unlike the finite-time version of the game here the long-run speculative demand for tokens may be strictly positive, which translates in a higher period-1 price for tokens and hence revenues.

For intuition, note that when $\sigma = -1$, bidders are buying all tokens in period 1 and then progressively using them for bidding. However, how many tokens are used for bidding in each period depends on the expected future speculative demand for tokens: when it is high, then the price of tokens is high, and fewer tokens will be used for bidding (and then burned); when it is low, the price of tokens will be low and more tokens will be used for bidding (and then burned). This logic implies that there are multiple equilibria, because when fewer tokens are used for bidding, more of them are held for speculation. It is possible that the speculative demand for tokens is strictly positive also in the long-run, that is, it is possible that a financial bubble drives up the initial price of tokens.

Finally, note that p_1 appears on both sides of equation (4). The reason is that, when $\sigma_t = -1$ for all t, the speculative demand for tokens must be strictly positive in all periods (otherwise, the demand for tokens for bidding could not be satisfied). By equation 1, this implies that for all t, p_t^e can be expressed as a function of p_1. Hence, if long-run speculative demand is strictly positive, then finding the equilibrium revenues requires solving for p_1, which the next proposition does.

Proposition 4 (Revenues when $\sigma_t = -1$ for all t)**.** *If $\sigma = -1$ and $T = \infty$, then any*

$$p_1 M_1 \geq \sum b_{i,1} + \frac{\beta}{1-\beta} k$$

is an equilibrium revenue.

Hence, when $\sigma = -1$ then, depending on the specific equilibrium of the game, the auctioneer's expected revenues are greater or equal (and never smaller) than those in the auction with dollars. Also, with respect to other auctions with tokens, the auction with tokens with $\sigma_t = -1$ generates revenues that are at least as large but accrue earlier and are less variable. These observations directly imply the following corollary.

Corollary 2. *Suppose that $\sigma = -1$ and $T = \infty$. If in equilibrium there are no financial bubbles, then Corollary 1 holds here as well. If in equilibrium there are financial bubbles, the auction with tokens is always strictly preferred to the auction with dollars.*

4 Extension: Cash in Advance

As already mentioned, the above model presents a very stylized view of dollars, because every dollar earned can be immediately turned into consumption. However, converting dollars into consumption may take time. To capture this fact, here I introduce a cash in advance constraint by assuming that a fraction $\lambda \in [0,1]$ of the dollars held can be converted in consumption goods anytime, while a fraction $1-\lambda$ of the dollars held is converted in consumption goods once per period.[15] Furthermore, the market for converting dollars in consumption goods (the ones that cannot be continuously exchanged for dollars) opens simultaneously to the market for tokens. The parameter λ therefore represents the inconvenience of using tokens relative to dollars. The case $\lambda = 1$ correspond to the benchmark case presented earlier, in which tokens are maximally inconvenient relative to dollars; as λ decreases the inconvenience of tokens becomes more similar to that of dollars; when $\lambda = 0$ dollars and tokens are equally inconvenient, because both of them can be converted into consumption once per period and at the same time.

I now provide a more precise timeline of each type of auction, this time starting with the auction with tokens. In the description of the timeline, I focus on the elements that are different from the benchmark case presented earlier.

Auctions with tokens If the auctioneer uses tokens, then the timeline of each period $t \in \{1, ..., T\}$ is the following:

- Again, at the start of a period, each bidder draws a valuation $v_{i,t}$.
- Then, each bidder sends a message $m_{i,t} \in \mathbb{R}_+$ to the auctioneer. The auctioneer determines who is the winner and a payment $b_{i,t} \leq m_{i,t}$ for each bidder.
- The per-period market for goods and services opens. Bidders and the auctioneer can exchange a fraction $1-\lambda$ of their dollars for the consumption goods that cannot be purchased continuously during the period. Simultaneously, a

[15] This is inspired by the cash-in-advance model of Lucas Jr and Stokey (1983), who distinguish between credit goods for which no cash in advance is needed, and cash goods that require cash in advance.

frictionless, anonymous financial market for tokens also opens, in which both the auctioneer and bidders participate.

- Then, each bidder sends $\frac{b_{i,t}}{p_t}$ tokens to the auctioneer. The winner receives the object and consumes it.

- The winning bidder enjoys a per-period payoff equal to the value of the object minus the expenditure in tokens, that is: $v_i - p_t \cdot q_{i,t}$. Similarly, the losing bidders enjoy a per-period payoff equal to $-p_t q_{i,t}$, and the auctioneer enjoys a per-period payoff equal to $U_t(-p_t Q_t + R \cdot w_t - w_{t+1})$.

- The stock of tokens changes according to the monetary policy announced by the auctioneer.

Hence, cash in advance constrains play no role in the auction with tokens. The reason is that the market for exchanging dollars for goods and services is open simultaneously to that of tokens. Hence, independently from the cash constraints, tokens earned in a given period, can be converted into consumption only by waiting for the market for tokens to open again. All the results derived earlier continue to apply. In particular, if the auction is repeated finitely many times or if it is repeated infinitely many times with $\sigma > -1$, then the auctioneer's revenues *expressed in unit of realized consumption* are:

$$k \sum_{t=1}^{T} \beta^{t-1} = \begin{cases} \frac{1-\beta^T}{1-\beta} k & \text{if } T \text{ finite} \\ \frac{1}{1-\beta} k & \text{if } T = \infty \end{cases}$$

Auctions without tokens If the auctioneer runs an auction that does not require tokens, then in each period $t \in \{1, ..., T\}$:

- First, each bidder draws a valuation $v_{i,t} > 0$.

- Then, each bidder sends a message $m_{i,t} \in \mathbb{R}_+$ to the auctioneer and the auctioneer determines who is the winner and payments $b_{i,t} \leq m_{i,t}$ for each bidder.

- The per-period market for goods and services opens.

- Each bidder then pays $b_{i,t}$ dollars to the auctioneer, and the winner receives and consumes the object. Note that the profile of payments is announced before the market for goods and services opens. Hence, paying $b_{i,t}$ is equivalent to decreasing consumption by $b_{i,t}$ in period t. It follows that the period-t payoffs of the winning bidder is $v_{i,t} - b_{i,t}$, and the period-t payoff of all other bidders is $-b_{i,t}$. The auctioneer, however, receives the payments after the per-period market for tokens is closed. He can consume only a fraction $1 - \lambda$ of his revenues in period t, with the rest in the following period. His period-t utility is

$$U_t\left((1-\lambda)(\sum_i^n b_{i,t} + R\cdot w_t - w_{t+1}) + \lambda(\sum_i^n b_{i,t-1} + R\cdot w_{t-1} - w_t)\right),$$

  where $\lambda(\sum_i^n b_{i,t-1} + R\cdot w_{t-1} - w_t)$ is the fraction of the dollars earned in period $t-1$ which is turned into consumption in period t (assumed equal to zero if $t = 1$).

In this case, the bidders' incentives are the same as in the benchmark case above. Similarly, the auctioneer's per-period expected revenues in every period are k dollars. However, these revenues here translate into $(\lambda + \beta(1-\lambda))k$ units of consumption. Hence, the present-discounted value of the expected revenues earned (expressed in consumption units) is:[16]

$$\Pi_{USD} = \begin{cases} \frac{1-\beta^T}{1-\beta}(\lambda + \beta(1-\lambda))k & \text{if } T \text{ finite} \\ \frac{1}{1-\beta}(\lambda + \beta(1-\lambda))k & \text{if } T = \infty \end{cases}$$

These derivations immediately imply the following proposition.

Proposition 5. *If $\lambda < 1$, then the present discounted value of the expected revenues (expressed in units of consumption) is strictly greater in the auction with tokens than in the auction with dollars, both in the finitely and in the infinitely-repeated game.*

[16] I'm implicitly assuming that the revenue earned in period T will be partly consumed in period T and partly in period $T+1$, that is, after the sequence of auctions ended. This assumption is for ease of notation, because all results continue to hold even without it.

The key observation is that the auctioneer earns his revenue earlier in the auction with tokens that in the auction with dollars. This is because, when the auctioneer holds an auction with dollars, he earns revenues when the bidders pay, while in the auction with tokens he earn his revenues when selling tokens (which happens before bidders pay). As it is typically the case in cash-in-advance model, also here earlier revenues are more likely to be transformed into consumption within the period and hence are more valuable than later revenues.

5 Extension: Effort

An interesting extension to consider is one in which, in every period, the bidders' valuation depend on costly actions taken by the auctioneer during that period. In this case, because the time-profile of the auctioneer's revenues is different in the auction with tokens and the auction with dollars, we should expect the incentives to exert effort also to differ in the two types of auction. As a consequences, revenues may also be different.

Intuitively, in the auction with dollars, the choice of effort is a static problem, and it depends exclusively on the per-period marginal benefit and marginal cost of effort. Instead, the choice of effort in the auction with tokens depends on dynamic considerations. On the one hand, the static benefit of exerting effort depends on the number of tokens held by the auctioneer, which may be lower than the total stock of tokens. Hence, the static incentive to exert effort is lower in the auction with tokens than in the auction with dollars. At the same time, however, effort in a given period also determines the speculative demand for tokens in that period, the fraction of the total stock of tokens held by the auctioneer in the following period and hence future effort and future expected utility. These dynamic consideration are mediated by the monetary policy, which determines the elasticity of future utility to effort. For example, quite intuitively, in the auction with tokens with $\sigma_t = -1$ for all t, all revenues are earned in period 1 and hence effort is zero from period 2 onward. In this case, therefore, effort is lower in the auction with tokens than in the auction with dollars. However, for different monetary policies, it is possible that the dynamic incentives dominate so that the equilibrium effort in the auction with tokens is above that of the auction with dollars.

To illustrate this possibility, here I consider a simplified version of the above model, in which $T = 2$ and the auctioneer's utility function is linear. Furthermore, in the auction with tokens I restrict my attention to uniform monetary policies in which all tokens grow (or shrink) by the same factor, that is, I assume that $\sigma = 0$. Effort e_t is set at the beginning of each period t before valuations are draw, and is common knowledge once set. It shifts uniformly the distribution of valuations, so that when bidder i draws $v_{i,t}$ from the distribution $F(x)$, his utility from consuming the object is $v_{i,t} + e_t$.[17] Effort has a quadratic cost equal to $\frac{e_t^2}{2}$. Also, here I restrict my attention to second price auctions, in which, both in the auction with dollars and in tokens, bidders bid their true valuation and total payments (in dollars) to the auctioneer are $\sum b_{i,t} = v_{Max-1,t} + e_t$.

I first consider the auction with dollars and then the auction with tokens.

Auction with dollars. In the auction with dollars, in every period, effort is set so to solve

$$\max_{e_t \geq 0} \left\{ v_{Max-1,t} + e_t - \frac{e^2}{2} \right\}.$$

Optimal effort is therefore

$$e_t^* = 1.$$

The present-discounted value of the auctioneer's revenues is therefore

$$\Pi_{USD} = (1+\beta)(k+1)$$

and his expected utility is

$$U_{USD} = (1+\beta)\left(k + \frac{1}{2}\right)$$

Auction with tokens. Without loss of generality, I normalize the initial stock of tokens so that $M_1 = 1$. In the second period of the auction with tokens, the speculative demand is zero for any level of effort. Hence, optimal period-2 effort

[17] Note that, because e_t is observed, the auction is still in private value despite the valuations having a "common" component.

solves:

$$\max_{e_2 \geq 0} \left\{ A_2 \frac{k + e_2}{M_2} - \frac{e_2^2}{2} \right\}$$

$$\text{s.t. } A_2 = (1 - S_1)(1 + \tau), \; M_2 = 1 + \tau.$$

Optimal effort is therefore

$$e_2^* = \frac{A_2}{M_2} = 1 - S_1,$$

and is equal to the fraction of total tokens held by the auctioneer. When the auctioneer holds all tokens, then his level of effort is equal to that in the auction with tokens. When the auctioneer holds no tokens, effort is zero.

It follows that the expected period-2 price for tokens is:

$$p_2^e = \frac{k + e_2^*}{M_2} = \frac{k + 1 - S_1}{1 + \tau},$$

and the developer's period-2 utility is

$$U_{Tokens,2} = k(1 - S_1) + \frac{(1 - S_1)^2}{2}.$$

Consider now period 1. For given period-1 effort and given bidders' valuations, the speculative demand for tokens is

$$S_1 = \max \left\{ 1 - \frac{v_{Max-1,1} + e_1}{\beta(k + 1 - S_1)}, 0 \right\}$$

By solving for e_1, I can write the speculative demand for tokens as a function of period-1 effort and realization of the bidders' valuations as:

$$S(e_1) \equiv \begin{cases} \frac{k}{2} + 1 - \sqrt{\frac{k^2}{4} + \frac{v_{Max-1} + e_1}{\beta}} & \text{if } v_{Max-1} + e_1 < \beta(1 + k) \\ 0 & \text{otherwise.} \end{cases}$$

The auctioneer's expected lifetime utility as a function of period-1 effort is therefore

$$E\left[\frac{v_{Max-1,1} + e_1}{1 - S_1(e_1)}\right] - \frac{e_1^2}{2} + \beta E\left[k(1 - S_1(e)) + \frac{(1 - S_1(e))^2}{2}\right]$$

By distinguishing between the case in which the speculative demand is positive and the one in which it is zero and using the definition of $S(e_1)$, the above expression can be rewritten as

$$\bar{F}(\beta(1+k)-e_1)E\left[\left(1-\frac{k}{2}\right)\frac{\beta k}{2}+\beta\left(1+\frac{k}{2}\right)\sqrt{\frac{k^2}{4}+\frac{v_{Max-1}+e_1}{\beta}}+\frac{v_{Max-1,1}+e_1}{2}|v_{Max-1,1}\leq\beta(1+k)-e_1\right]$$
$$(1-\bar{F}(\beta(1+k)-e_1))E\left[v_{Max-1,1}+e_1+\beta\left(k+\frac{1}{2}\right)|v_{Max-1,1}>\beta(1+k)-e_1\right]-\frac{e_1^2}{2}$$

Where $\bar{F}(x)$ is the CDF of the second-highest idiosyncratic valuation $v_{Max-1,1}$.

The important observation is that effort is valuable both when the speculative demand is expected to be positive, and when it is expected to be zero. The latter case is identical to the auction with dollars, in which effort in period 1 linearly increases period-1 revenues. The former is indirect, because higher effort implies lower speculative demand, which implies higher future effort, future price and, as a consequence, period-1 price. Finally, effort decreases the probability that the speculative demand is positive.

Taking derivatives, if optimal effort has an interior solution (i.e., one such that $\underline{v}\leq\beta(1+k)-e_1\leq\overline{v}$), then it solves

$$\bar{F}(\beta(1+k)-e_1)\underbrace{E\left[\left(1+\frac{k}{2}\right)\left(\frac{k^2}{4}+\frac{v_{Max-1}+e_1}{\beta}\right)^{-\frac{1}{2}}+\frac{1}{2}|v_{Max-1,1}\leq\beta(1+k)-e_1\right]}_{>1}+(1-\bar{F}(\beta(1+k))$$
$$+\bar{f}(\beta(1+k)-e_1)\left(\underbrace{E\left[v_{Max-1,1}+e_1+\beta\left(k+\frac{1}{2}\right)|v_{Max-1,1}>\beta(1+k)-e_1\right]}_{E[U_{Tokens}|S_1(e_1)=0]}-\right.$$
$$\left.\underbrace{E\left[\left(1-\frac{k}{2}\right)\frac{\beta k}{2}+\beta\left(1+\frac{k}{2}\right)\sqrt{\frac{k^2}{4}+\frac{v_{Max-1}+e_1}{\beta}}+\frac{v_{Max-1,1}+e_1}{2}|v_{Max-1,1}\leq\beta(1+k)-e_1\right]}_{E[U_{Tokens}|S_1(e_1)>0]}\right)=e_1$$

Where $\bar{f}(x)$ is the PDF of the second-highest idiosyncratic valuation $v_{Max-1,1}$.

It is easy to check that the first term in the square bracket is greater than 1, which implies that the marginal benefit of exerting effort is larger when the speculative demand is strictly positive than when the speculative demand is zero. Furthermore, $E\left[U_{Tokens}|S_1(e_1)=0\right]$ (i.e., the utility when the realization of $v_{Max-1,1}$ is such that there is no speculative demand) is higher than $E\left[U_{Tokens}|S_1(e_1)>0\right]$ (the utility when the realization of $v_{Max-1,1}$ is such that the speculative demand is strictly positive).[18] It follows that, whereas the incentive to exert effort in period 2 may be lower in the auction with tokens than in the auction with dollars, the incentive to exert effort in period 1 is higher in the auction with tokens than in the auction with dollars.

To conclude, note that if $\beta(1+k) > \underline{v} + 1$, then it is possible that optimal period-1 effort is a corner solution at $e_1 = \beta(1+k) - \underline{v} > 1$.[19] In this case, the auctioneer sets effort above 1 so to drive the speculative demand for tokens to zero and own the entire stock of tokens in period 2. As a consequence, effort in period 1 will be above that of the auction with dollars, while effort in period 2 will be equal to that of the auction with dollars.

6 Discussions

6.1 Borrowing, insurance and equity.

An important question is whether the equilibrium of the auction with tokens can be implemented via other means.

I assumed that the auctioneer cannot borrow and cannot insure against the variability of his revenues. Clearly, in the absence of cash-in-advance constraints and in the finitely repeated auction, if both credit and insurance markets are perfect, then the auctioneer can eliminate all risk and achieve optimal consumption smoothing by holding a traditional auction with dollars. It is however interesting to note that if only the insurance market or only the credit market is perfect, then the auctioneer still prefers the auction with tokens. If only the credit market is perfect, then the only way the auctioneer can reduce the variability of his revenues is by running an auction with tokens. If the insurance market is perfect but the credit market is not—because, for example, the rate at which the auctioneer can borrow is higher

[18] Note that, for given, $S_1(e_1) = 0$ occurs when the valuations are higher than when $S_1(e_1) > 0$. Furthermore, when the speculative demand is positive, future effort will be lower than when the speculative demand is zero.

[19] This happens when $\beta(1+k)$ is sufficiently close to $\underline{v}+1$, so that for any level of effort strictly greater than 1, then $\beta(1+k) - e_1 < \underline{v}$ and the probability that the speculative demand is positive is zero.

than the return on savings—then the auctioneer is better off by front-loading all the revenues, which can be achieved via an auction with tokens.

Similarly, in the auction with tokens with $\sigma = -1$, the auctioneer earns all revenues in period 1 and no revenues afterwards. If the revenues earned by the auctioneer are contractible, then the same outcome can be achieved in an auction with dollars via the creation of equity: the auctioneer pledges all his future cash flows to investors. Also, if the game is repeated infinitely many times, financial bubbles on equity are possible. These bubbles emerge when the "bubble component" of the price grows at rate equal to the interest rate (here $R - 1 = \frac{1-\beta}{\beta}$). This is problematic in many contexts (for example, if the growth rate of the economy is below the interest rate), which is why bubbles on equity are often ruled out from the onset by applying a transversality condition (see Brunnermeier, 2016). Similarly, a bubble on tokens exists if the value of the tokens held for speculation grows at a rate equal to the interest rate. Importantly, however, the stock of tokens held for speculation decreases over time: all of tokens are purchased by speculators initially, and are progressively sold for bidding (and then burned). Financial bubbles emerge if the speed at which the speculative demand for tokens decreases is sufficiently slow.

To summarize: in the absence of financial bubbles, the equilibrium of the optimal auction with tokens can be achieved by holding an auction with dollars and either borrowing plus accessing insurance, or issuing equity. The important observation is that these alternative implementation of the same equilibrium are possible only if the credit/financial market are perfect and the auctioneer's revenues are contractible. These conditions play no role in the equilibrium of the auction with tokens, which however relies on the auctioneer's ability to commit to accept tokens as a means of payment and also to a given monetary policy. Furthermore, in the infinite horizon game, financial bubbles can emerge both when the auctioneer issues equity and when he issues tokens. Nonetheless, whereas a bubble on equity exists if fresh investment causes the "bubble" component to grow indefinitely at a constant rate, a bubble on tokens exists as long as the speculative demand for tokens decreases over time at a sufficiently slow pace. In this sense, the conditions under which a bubble on tokens emerge are less problematic than those under which a bubble on equity emerges.

6.2 Commitment.

The model rests on the ability of the auctioneer to commit to a monetary policy and also to accepting tokens in the future. The former is easily achieved via a smart contract regulating the supply of tokens; the latter instead is more problematic especially if the objects sold are physical (rather than digital). For this reason, here I consider the possibility that the auctioneer may refuse to accept tokens as payment, and instead demand to be payed in dollars.

Quite clearly, in finite time, in the auction with tokens the speculative demand for tokens is always zero, and hence the outcome is identical to the auction with dollars. To see this, consider the last period of the game. If some tokens were purchased for speculation in the previous period, then the auctioneer is better off announcing that he will only accept payments in dollar. Anticipating this, there will be no speculative demand for tokens in the second-to-last period. The same reasoning can be repeated: if at the beginning of the second-to-last period the auctioneer holds less than the total stock of tokens, he will prefer to accept payments in dollars. Anticipating this, the speculative demand for tokens on the third-to-last period is zero. It follows that the speculative demand is always zero, and the auction with tokens is identical to the auction with dollars.

If instead the time horizon is infinite, then the above logic does not apply.[20] To the extent that the auctioneer sells tokens in every period (i.e., to the extent that $\sigma_t > -1$ for all t), then there is always an equilibrium with a sufficiently large bubble component on tokens such that lower revenues today (relative to switching to dollars) are compensated by a very large continuation value. Furthermore, even in the absence of financial bubbles, the auction with tokens generates the same expected revenues as the auction with dollars, but these revenues are less risky. It is therefore possible that the auctioneer may be willing to earn less than what he could by switching to the auction with dollars in a given period, in order to enjoy a reduction in the variance of future revenues. Overall, therefore, when $T = \infty$ the auction with tokens has equilibria in which the auctioneer does not want to switch

[20] It is important to remember that an infinitely-repeated auction is equivalent to an auction that, in every period, may be repeated one additional period with some strictly positive probability. That is, it is one in which there is no preset end, not necessary one that runs infinitely many periods.

to dollars, despite the fact that the speculative demand for tokens may be strictly positive in some periods (and hence the equilibrium is different from that of the auction with dollars).

7 Conclusions.

I have shown that, if the auction is repeated finitely many times, the auction with tokens generates the same expected revenues as the auction with dollars. However, revenues accrue earlier and are less variable in the auction with tokens than in the auction with dollars. In particular, if the auctioneer holds an auction with tokens in which he destroys all the tokens he receives as payment, then the auctioneer is able to fully front-load his revenues and (almost) fully eliminate their variability. If the auction is repeated infinitely many times, then the auction with tokens can generate higher expected revenues if financial bubbles on tokens emerge.

I have considered two extensions. First, I have shown that if there is a cash-in-advance constraint, then the auction with tokens generates higher consumption than the auction with dollars. This is because, within each period, the auction with tokens generates revenues earlier than the auction with dollars. In the presence of cash-in-advance constraints, earlier revenues are preferred because they are more likely to be transformed into consumption within the period (vs in the following period). Second, I have shown that the incentives to exert effort are very different in the auction with tokens than in the auction with dollars. The main reason is that, in the auction with tokens, effort in a given period determines the speculative demand for tokens for that period, and the future marginal benefit of exerting effort. These dynamic considerations imply that, in some cases, the equilibrium effort in the auction with tokens is larger that the equilibrium effort in the auction with dollars.

I assumed that the market for tokens is frictionless. Hence, holding tokens is inconvenient only because exchanging them for consumption may require waiting one period. However, this exchange generates no cost. A more realistic view is that the market for tokens has frictions, and these frictions further reduce the benefit of using tokens. In an even more realistic model these frictions would depend on the volume of transactions, which itself is a function of the value of the object being exchanged. This extension that is left for future work.

The fact that the auction with tokens can be used to de-risk and front-load revenues may be relevant for entrepreneurial finance. That is, if the auctioneer is an entrepreneur who needs to spend resource to build the object being sold and is prevented from borrowing, can the auction with tokens be used to finance this investment? To fully answer this question, it is important to first establish how the frictions preventing the existence of the credit market affect the auction with tokens. For example, the entrepreneur/auctioneer may be unable to borrow as a consequence of a moral hazard problem—perhaps because he may disappear with the funds received. To establish whether he can raise financing by creating a token, it is necessary to first establish how moral hazard affects the outcome of the auction with tokens. The full exploration of this case is also left for future work.[21]

A Mathematical derivations

Proof of Lemma 2. Consider period t. Take the total payments to the auctioneer $\sum_i b_{i,t}$ and the expected future price p^e_{t+1} as given. Note that a bidder can spend 1 dollar to purchase $\frac{1}{p_t}$ tokens in period t, these tokens then multiply by $1+\tau_t$ and can be sold in period $t+1$, for a present-discounted return of $\frac{\beta(1+\tau_t)p^e_{t+1}}{p_t}$. Alternatively, he can invest the same amount of money in the risk-free asset, for a present-discounted return of $\beta R = 1$ in the following period. It follows that there can be an equilibrium in the market for tokens if and only if $p_t \geq \beta(1+\tau_t)p^e_{t+1}$.

If $p_t > \beta(1+\tau_t)p^e_{t+1}$, no tokens are purchased and then brought to the next period. The demand for tokens is given by the tokens used for bidding $\frac{\sum_i b_{i,t}}{p_t}$. The supply of tokens is M_t, and hence the equilibrium price is

$$p_t = \frac{\sum_i b_{i,t}}{M_t}$$

This is indeed an equilibrium if $\frac{\sum_i b_{i,t}}{M_t} > \beta(1+\tau_t)p^e_{t+1}$, that is, if the realized bids

[21] In Canidio (2018) I study a related question by considering a model in which an entrepreneur sells tokens to finance the creation of a decentralized digital platform and there is moral hazard.

are sufficiently high relative to the future expected price.

If instead $\frac{\sum_i b_{i,t}}{M_t} \leq \beta(1+\tau_t)p^e_{t+1}$, then tokens may be purchased but not used for bidding. I call this demand the speculative demand for tokens S_t. The total demand for tokens is now $S_t + \frac{\sum_i b_{i,t}}{p_t}$, and the equilibrium price is

$$p_t = \frac{\sum_i b_{i,t}}{M_t - S_t} = \beta(1+\tau_t)p^e_{t+1}$$

which pins down the speculative demand for tokens:

$$S_t = \max\left\{M_t - \frac{\sum_i^n b_{i,t}}{\beta(1+\tau_t)p^e_{t+1}}, 0\right\}$$

□

Proof of Proposition 1. To start, note the following three facts:

1. $\sigma > -1$ implies that $A_t > 0$;

2. in each period the auctioneer will liquidate all his tokens. If the price in a given period is such that $p_t > \beta(1+\tau_t)p^e_{t+1}$, then the auctioneer is better off selling tokens in period t at a higher price than in period $t+1$ at a lower price. If instead $p_t = \beta(1+\tau_t)p^e_{t+1}$ and the auctioneer is risk averse, then again he prefers to earn revenues in period t than to wait one period and earn the same expected revenues (but this time being exposed to risk). If $p_t = \beta(1+\tau_t)p^e_{t+1}$ and the auctioneer is risk neutral then his expected revenues are the same whether he holds tokens between periods (i.e., he acts as a speculator) or not. Without loss of generality, we can assume that he sells all his tokens in period t also in this case;

3. an implication of the above two facts is that, for given p^e_{t+1}, A_{t+1} is independent of A_t. That is because A_{t+1} are the tokens received for payment in period t, which depend exclusively on p^e_{t+1} and the realization of valuation in period t.

I can therefore write the auctioneer's expected continuation revenues in period t in recursive form as:

$$\Pi_t(A_t) = p^e_1 A_t + \beta E[p_{t+1}A_{t+1}] + \beta^2 E[\Pi_{t+2}(A_{t+2})]$$

where the expectations are taken at the beginning of period t, before the valuations are realized.

Consider now a given p^e_{t+1}. It is possible that the realization of valuations in period t is such that $S_t = 0$ and hence $p_t = \frac{\sum_i b_{i,t}}{M_t}$. In this case, $A_{t+1} = M_{t+1}$, so that, conditional on $S_t = 0$:

$$\begin{aligned}\Pi_t(A_t) =& E[p_t|S_t = 0]A_t + \beta E[p_{t+1}A_{t+1}|S_t = 0] + \beta^2 E[\Pi_{t+2}(A_{t+2})] \\ =& E[\sum_i b_{i,t}|S_t = 0] - E[p_t|S_t = 0](M_t - A_t) + \beta p^e_{t+1}M_{t+1} + \beta^2 E[\Pi_{t+2}(A_{t+2})]\end{aligned}$$

If instead $S_t > 0$, then $p_t = \beta(1 + \tau)p^e_{t+1}$ and $A_{t+1} = M_{t+1} - S_t(1 + \tau) > 0$. In this case, conditional on $S_t > 0$:

$$\begin{aligned}\Pi_t(A_t) =& E[p_t|S_t > 0]A_t + \beta E[p_{t+1}A_{t+1}|S_t > 0] + \beta^2 E[\Pi_{t+2}(A_{t+2})] \\ =& E[p_t|S_t > 0]A_t + \beta p^e_{t+1}(M_{t+1} - E[S_t|S_t > 0](1 + \tau)) + \beta^2 E[\Pi_{t+2}(A_{t+2})] \\ =& E[p_t|S_t > 0]A_t + \beta p^e_{t+1}M_{t+1} - \beta p^e_{t+1}(1 + \tau_t)\left(M_t - \frac{E[\sum_i b_{i,t}|S_t > 0]}{\beta p^e_{t+1}(1 + \tau_t)}\right) + \beta^2 E[\Pi_{t+2}(A_{t+2})] \\ =& E\left[\sum_i b_{i,t}|S_t > 0\right] - E[p_t|S_t = 0](M_t - A_t) + \beta p^e_{t+1}M_{t+1} + \beta^2 E[\Pi_{t+2}(A_{t+2})]\end{aligned}$$

where I used the definition of S_t as well as the fact that $E[p_t|S_t > 0] = \beta p^e_{t+1}(1+\tau_t)$.

The above derivations then imply that, for given sequence of equilibrium prices, the unconditional expected revenues are

$$\Pi_t(A_t) = k - p^e_t(M_t - A_t) + \beta p^e_{t+1}M_{t+1} + \beta^2 E[\Pi_{t+2}(A_{t+2})] = k - p^e_t(M_t - A_t) + \beta\Pi_{t+1}(M_{t+1}) \quad (5)$$

where the last equality exploits the fact that, for given equilibrium expected prices, A_{t+2} is independent of A_{t+1}. The statement then follows by iterating the above equation.

□

Proof of Proposition 2. Because all tokens earned by the auctioneer are burned, in equilibrium the speculative demand for tokens is strictly positive in every period $t \leq T$ and for every possible realization of valuations. Because of this in every period t and every realization of period $t < T$ valuations, equilibrium prices must be such that bidders are indifferent between holding tokens between any two period, that is $p_t = \beta \cdot (1+\tau_t) \cdot p^e_{t+1}$. Given this, I can write the sequence of equilibrium prices as p_1, $p^e_2 = p_1/(\beta(1+\tau_1))$, ... $p^e_t = p_1/\left(\beta^{t-1}\prod_{s=1}^{t-1}(1+\tau_s)\right)$ for $t \leq T$. For this sequence to be an equilibrium, it must be that

$$p_1 = \frac{\sum b_{i,1}}{M_1 - S_1},$$

and

$$p_t = \frac{\sum b_{i,t}}{S_{t-1}(1+\tau_{t-1}) - S_t}$$

so that

$$S_{t-1} = \frac{\sum b_{i,t}}{p_t(1+\tau_{t-1})} + \frac{S_t}{(1+\tau_{t-1})} \tag{6}$$

Take $\sum b_{i,t}$ and the expected speculative demand for tokens in a given period t as given $E\left[S_t\right] \equiv Y_t \geq 0$.[22] By (6) and taking expectations I can write

$$E[S_{t-1}] = \frac{k}{p_t(1+\tau_{t-1})} + \frac{Y_t}{(1+\tau_{t-1})}$$

$$E[S_{t-2}] = \frac{k}{p_{t-1}(1+\tau_{t-2})} + \frac{k}{p_t(1+\tau_{t-2})(1+\tau_{t-1})} + \frac{Y_t}{(1+\tau_{t-2})(1+\tau_{t-1})}$$

$$\dots$$

$$S_1 = k\sum_{s=2}^{t} \frac{1}{p_s\prod_{j=1}^{s-1}(1+\tau_j)} + \frac{Y_t}{\prod_{s=1}^{t-1}(1+\tau_s)}$$

where, again, the speculative demand in period 1 does not require an expectation because $\sum b_{i,1}$ is taken as given. Using the fact that $p^e_s = p_1/\left(\beta^{t-1}\prod_{j=1}^{s-1}(1+\tau_j)\right)$, the above expression becomes:

$$S_1\left(\sum b_{i,1}\right) = \frac{k}{p_1}\sum_{s=1}^{t-1}\beta^s + \frac{Y_t}{\prod_{s=1}^{t-1}(1+\tau_s)},$$

[22] Of course, if T is finite, then the speculative demand in period T is zero. I do not use this fact now in order to derive expressions that can be used also in the infinitely-repeated case.

so that

$$p_1 M_1 = \sum b_{i,1} + k \sum_{s=1}^{t-1} \beta^s + \frac{p_1 Y_t}{\prod_{s=1}^{t-1}(1+\tau_s)} \tag{7}$$

The statement of the proposition then follows simply by noting that the speculative demand for tokens is zero in period T. □

Proof of Proposition 3. The statement follows from Equation (5) in the proof of Proposition 1, for the case in which T is infinite. □

Proof of Lemma 3. The statement follows from Equation (7) in the proof of Proposition 2, for the case in which T is infinite. □

Proof of Proposition 4. For simplicity, the proof considers a fixed τ, that is, here $\tau_t = \tau$ for all t.

Equation (7) in the proof of Proposition 2 derives revenues for given expected speculative demand for tokens in period t $Y_t > 0$. Now I'm going to move this exogenous belief forward according to some exogenous rule (of course, every time I do this, I then solve backward for the equilibrium prices). I will then take the limit of this exogenous belief to infinity. Note that, as long as all Y_t are strictly positive, there is no need for the rule I use to construct the sequence of exogenous belief to be related in any way to the equilibrium evolution of the speculative demand for tokens.

Suppose first that $\tau \neq 0$. Write

$$Y_t = (Y_{t-1}(1+\tau) - \epsilon_{t-1}(1+\tau)^{\alpha}) = (Y_1(1+\tau)^{t-1} - \epsilon_1(1+\tau)^{\alpha(t-1)})$$

where $\epsilon_t = \epsilon_{t-1}(1+\tau)^{\alpha}$ is period t expected demand for tokens for bidding. Note that, written this way $Y_t > 0$ for all $t \geq 1$ as long as $M_1 > Y_1 > \epsilon_1 > 0$ and either $\tau > 0$ $\alpha < 1$, or $\tau < 0$ $\alpha > 1$. In either case, I can write

$$\lim_{t\to\infty}\{\beta^{t-1} p_t^e \cdot Y_t\} = p_1 \lim_{t\to\infty}\{\frac{Y_t}{(1+\tau)^{t-1}}\} = Y_1 - \epsilon_1 \lim_{t\to\infty}\{(1+\tau)^{(\alpha-1)(t-1)}\} = Y_1$$

Using the above expression with equation (4), I get

$$p_1 M_1 = \sum_i b_{i,t} + \beta \frac{1-\beta^{T-1}}{1-\beta} k + p_1 Y_1$$

by solving for p_1, I get

$$p_1 M_1 = \left(\sum_i b_{i,t} + \beta \frac{1-\beta^{T-1}}{1-\beta} k \right) \left(1 - \frac{Y_1}{M_1} \right)^{-1}$$

note that Y_1 must be smaller than M_1 but can be arbitrarily close to it. Hence, the auctioneer's revenues can take any value greater or equal to $\sum_i b_{i,t} + \beta \frac{1-\beta^{T-1}}{1-\beta} k$.

Consider now the case $\tau \neq 0$. Write

$$Y_t = Y_{t-1} - \frac{1}{2}\epsilon_{t-1} = Y_1 - \epsilon_1 \sum_{s=1}^{t} \frac{1}{2^{s-1}}$$

where here the expected demand for tokens for bidding ϵ_t halves in every period. It follows that

$$\lim_{t\to\infty} \{\beta^{t-1} p_t^e \cdot Y_t\} = p_1 \lim_{t\to\infty} \{Y_1 - \epsilon_1 \sum_{s=1}^{t} \frac{1}{2^{s-1}}\} = Y_1 - 2\epsilon_1$$

By taking $\epsilon_1 \to 0$ and $Y_1 \to M_1$ and repeating the argument above, it is possible to show that also in this case the auctioneer's revenues can take any value value greater or equal to $\sum_i b_{i,t} + \beta \frac{1-\beta^{T-1}}{1-\beta} k$.

□

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