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Logical norms as defeasible obligations: disentangling sound and feasible inferences

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ABSTRACT

This paper develops a novel approach to the question of the normativity of logic, which we reinterpret as a clash between two intuitions: the direct normativity intuition and the unfeasibility intuition. The standard response has been to dismiss the direct normativity intuition, bridging logic and reasoning via principles that relativize the normative import of logic to pragmatic and feasibility considerations. We argue that the standard response is misguided. Building upon theories of bounded rationality, our approach conceptualizes reasoning as constrained by multiple, independent normative factors, logical and non-logical ones. These different factors can conflict with one another, to the effect that logically sound inferences might not coincide with what is feasible for an agent to infer. From this perspective, we will argue that logic gives us only *prima facie*, i.e. contributive and defeasible, obligations on what to believe, but that such obligations do not always coincide with what an agent ought to believe *all things considered*. These distinctions will dissolve the alleged clash between the direct normativity and the unfeasibility intuition.

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1. Introduction

Are logical principles norms of good reasoning? Answering this question, one is pushed towards opposite intuitions. On the one hand, it appears that we reason correctly when we come to believe the logical implications of what we genuinely assert. Logic therefore appears to be directly normative for

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our reasoning. On the other hand, in real situations, it may be beyond one's cognitive capacities to draw a certain logical conclusion. Thus, logic does not seem to tell us what we ought to do in actual reasoning situations.

The standard approach to logical normativity has been to favor the second intuition to the detriment of the first one, connecting logic and reasoning by virtue of so-called 'bridge principles' (e.g. H. Field 2009; MacFarlane 2004; Steinberger 2019a, 2019b). We will show that this approach is misguided. We will demonstrate that bridge principles, by abandoning the direct normativity intuition, mix together logical and pragmatic constraints and, thus, do not offer a satisfactory account of the normative import that logic *itself* exerts on reasoning.

We propose a different picture. We will rethink the question of the normativity of logic for reasoning within a cognitive theory of rational agency. Building upon theories of bounded rationality in cognitive science (cf. Anderson 1990; Kahneman 2003; Simon 1957, 1972, 1982; Todd and Gigerenzer 2012), we understand reasoning as regulated by multiple normative factors, logical and non-logical ones. While logical factors amount to obligations pertaining to the soundness of our inferences, crucial non-logical factors include, for instance, obligations dependent on structural features of the agent's cognitive system and of her environment, such as minimizing time, effort, attention, memory, and complexity. These different normative factors might sometimes pull in different directions, to the point that what is logically sound might not always coincide with what is feasible for an agent, in a given situation, to infer.

Within this theoretical background, we will argue that logic gives us only *prima facie*, i.e. contributive and defeasible, obligations on what to believe, but that such obligations might not coincide with what an agent ought to believe *all things considered*. This distinction between *prima facie* and all-things-considered oughts will dissolve the alleged clash between the direct normativity and the unfeasibility intuition, allowing us to vindicate both intuitions at the same time.

Our approach breaks with the traditional literature on the normativity of logic in two ways. First, we maintain that the full normative import of logic for reasoning can only be understood by distinguishing the role of logical and cognitive-pragmatic factors. Second, we answer the question of logical normativity by situating it into a general picture of rationality that draws inspiration from theories of bounded rationality in cognitive science.

The paper proceeds as follows. Section 2 presents the question of logical nomativity as the clash between the direct normativity intuition and the unfeasibility intuition. In Section 3, we critically discuss the standard

approach to logical normativity, i.e. what we will call 'the bridge-principle approach'. Section 4 is devoted to presenting the theoretical framework that will constitute the background for our own approach to logical normativity. We first sketch the picture of reasoning that emerges from theories of bounded rationality, and then we employ it to reinterpret crucial cases of unfeasibility of logical norms as conflicts between logical and non-logical normative factors. In Section 5, we re-assess the problem of logical normativity from the perspective of our account. Section 6 concludes.

2. Direct normativity *vs* unfeasibility

We present a tension between two intuitions on the normativity of logic for reasoning: that logic has a direct normative connection with our reasoning, and that logic puts unfeasible demands on the reasoning of bounded agents like us. If logic tells us how we ought to reason, but we cannot reason as logic tells us, then it seems that at least one of these two intuitions must go.[1]

2.1. The direct normativity intuition

The intuition that motivates our, and seemingly many others', inquiry into the normativity of logic is that there is a direct, normative connection between logic and reasoning. Logical principles and rules give rise to norms for our reasoning, to the effect that our beliefs ought to obey logical principles and rules. Let us call this the 'direct normativity intuition'.

For the sake of concreteness, let us consider the case of logical entailments and logical consistency. In light of the direct normativity intuition, logical entailments and consistency are normative: we ought to endorse the logical implications of our beliefs and our beliefs ought to be consistent. Furthermore, this normativity is direct: it does not appeal to mediating conditions. It is logical entailments and consistency that have a normative connection with our beliefs and, thus, impose normative demands on us.

Paradigmatic instances of the direct normativity intuition are the following principles:

(IMP) If φ is a logical consequence of S's beliefs, then S ought to believe that φ.

(CON) S ought to avoid having logically inconsistent beliefs.

[1]The contrast between the two intuitions assumes, as we will see shortly, that 'ought implies can'.

IMP and CON exemplify how logical entailments and consistency, respectively, constrain an agent's beliefs. They are straightforward, normative principles. The literature often presents (versions of) IMP and CON as the starting, most plausible principles that may capture the normativity of logic for reasoning (cf. Harman 1986; Steinberger 2019b). We believe this is not by accident: under our reinterpretation of the dialectic surrounding the normativity of logic, IMP and CON are in fact instances of the direct normativity intuition. For these reasons, while other legitimate instances of the direct normativity intuition may exist, in this paper we will focus on IMP and CON.

Apart for the intuitive appeal, there is a further sense in which the direct normativity intuition as embodied by IMP and CON is important: it makes sense of our expectations towards a believer's behavior and of our social practices. Someone who, in the proximity of a dam, believes 'If it rains, then the dam will break' and 'It is raining' is expected to take appropriate actions given those beliefs and what they logically imply (Cherniak 1986, 10). In turn, no distinctive expectations can emerge towards someone who, for instance, systematically would not eliminate logical inconsistencies from their beliefs. The behavior they will have on the basis of their own beliefs will be hardly predictable (Cherniak 1986, 16). Furthermore, the direct normativity intuition straightforwardly vindicates our appraisal practices, and specifically our blame and responsibility attributions. As Steinberger (2022) puts it: 'We consider it to be a bad thing to be inconsistent. Similarly, we criticize others for failing to appreciate (at least the more obvious) logical consequences of their beliefs'. In this vein, MacFarlane (2004): 'We criticize people not merely for having inconsistent beliefs, but for failing to accept logical consequences of their beliefs'. The *oughts* in IMP and CON can easily explain these practices.

Thus, we argue, a satisfactory account of the normative role of logic for reasoning has to capture the direct normativity intuition.

2.2. The unfeasibility intuition

A second, equally appealing intuition is widely shared within the debate on the normativity of logic. Let us start by observing that, in real situations, reasoning agents face various limitations: it may be beyond one's cognitive capacities to detect an inconsistency or to draw a certain logical inference; or one might not have enough time at their disposal to invest in those tasks. More generally, there are cognitive and

environmental limitations that concern us as actual reasoners. Because of these limitations, the normative demands of logic are largely unfeasible for us. Thus, the intuition goes, it is not true that we ought to believe all the logical consequences of our beliefs nor that our beliefs ought to be always consistent. Let us call this the 'unfeasibility intuition'.

Recent literature has proposed a plethora of cases that support the unfeasibility intuition, specifically against IMP and CON (Harman 1986; MacFarlane 2004; Steinberger 2019b). The case of 'clutter avoidance' is an example (Harman 1986, 12). IMP requires that an agent endorses the logical consequences of what she already believes. Yet, each individual proposition may logically imply propositions that have no theoretical or practical relevance, and an infinite lot of them. Iterative applications of IMP would, therefore, lead to obligations to come to believe all these non-salient logical consequences. But these obligations, taken together, are unfeasible: coming to believe a non-salient logical consequence would contribute to squandering our limited cognitive storage; coming to believe an infinite lot of them would be a cognitive impossibility.

A further example supporting the unfeasibility intuition is the case of 'unavoidable inconsistencies' (Harman 1986, 15). In certain situations, the example goes, inconsistent beliefs are unavoidable. The reasons for their unavoidability might be manifold. For instance, due to the organization of our memory system, two inconsistent beliefs might never be activated simultaneously and so the inconsistency might never be detected (cf. Borgoni, Kindermann, and Onofri 2021; Cherniak 1986; Lewis 1982; Stalnaker 1984). Or, even when the inconsistency is manifested to the agent, she might not have the resources to resolve it. An example of this second kind of unavoidable inconsistencies is the 'Preface Paradox' (Makinson 1965), a situation where the writer of a book excuses herself for the mistakes that the book might contain. Based on the data she collected, she believes each individual sentence written in the book but, on inductive grounds, she does not believe the truth of the book as a whole, i.e. the conjunction of all its sentences. In fact, she believes the negation of such conjunction. There is, thus, an inconsistency in the agent's beliefs about the truth of her book. Yet, this inconsistency is unavoidable: given the evidence the agent currently possesses, the agent is not in a position to abandon either of her beliefs. These cases of unavoidable inconsistencies stand in tension with the obligation that CON issues. CON would require the agent to eliminate inconsistencies among their beliefs, but if these

inconsistencies are unavoidable, either because they cannot be detected or because they are (currently) unresolvable by the agent, then what CON demands is unfeasible to the agent. Cases of clutter avoidance and unavoidable inconsistencies, thus, support the unfeasibility intuition. In those cases, it is false that we ought to believe what IMP and CON say.

It is important to emphasize how exactly the conflict between the unfeasibility intuition and the direct normativity intuition is structured. This conflict emerges from the assumption that oughts must entail that the agent *can* fulfill what these oughts demand. In other terms, the two intuitions clash once we assume that 'ought implies can'. The relevant sense of 'can' is the one of ability or feasibility (Vranas 2007), here understood as encompassing both the cognitive limitations of the agent and of her environment. More schematically, this is the situation:

Direct Normativity: $O(\varphi)$
Ought-Can principle: $O(\varphi) \rightarrow \Diamond (\varphi)$
Empirical Fact: $\neg \Diamond (\varphi)$
Unfeasibility: $\neg O(\varphi)$

Thus, we are left with a tension: either the direct normativity intuition or the unfeasibility intuition must be rejected.

3. The bridge-principle approach

Much of the recent literature on the normativity of logic has sought to solve the tension between the direct normativity intuition and the unfeasibility intuition by abandoning the former. The most pessimistic stance has been endorsed by Harman (1986), who, from worries akin to the unfeasibility intuition, concluded that there is no special relation (*a fortiori*, no special normative relation) between logic and reasoning. Others have proposed more positive approaches. In the wake of the seminal (MacFarlane 2004), the standard response has involved two steps. First, it abandoned the direct normativity intuition embodied by IMP and CON, under the assumption that the unfeasibility intuition has shown their untenability. Second, it attempted to capture the normative connection between reasoning and logic by virtue of other principles, now weakened to account for the problematic cases that support the unfeasibility intuition. Following MacFarlane's (2004) terminology, we refer to those principles as 'bridge principles' and we will call the aforementioned standard line of response, for want of a better term, 'the

standard bridge-principle approach' or, for short, 'the bridge-principle approach'.[2]

To illustrate the core features of the bridge principle approach, let us recall the general form of bridge principles. A bridge principle is a conditional statement of the form 'if …, then…', where the antecedent ('the triggering condition') is a logical entailment or an agent's doxastic attitude towards such entailment, and the consequent is a normative claim about the agent's doxastic attitudes towards premises and conclusion of the logical entailment.

Let us begin by noticing that IMP itself can be rewritten as a bridge principle:

(IMP BP) If $A_1, \ldots, A_n \vDash B$, then if the agent believes $A_1, \ldots, A_n$ then she ought to believe B.

IMP BP is challenged by the unfeasibility intuition, as is the original IMP. As already mentioned, the first step of the bridge principle approach consists in rejecting IMP BP; the second one consists in replacing IMP BP with a weaker bridge principle (or a combination thereof) whose normative demands are feasible for the agent. For instance, in response to the unfeasibility worries raised by the case of clutter avoidance, the antecedent of the bridge principle could be revised as follows:

(BP′) If $A_1, \ldots, A_n \vDash B$ and the agent considers B, then if the agent believes $A_1, \ldots, A_n$ then she ought to believe B.

By relativizing the antecedent to the logical consequences the agent considers, the problem of clutter avoidance disappears: the agent ought to believe not all logical consequences, but only those that the agent is wondering about at the time of reasoning. These are salient propositions, and there are finitely many of them.

BP′, however, falls prey of other unfeasibility worries. For instance, in the scenario of the Preface Paradox, BP′ would require the writer wondering about the conjunction of the individual statements in her book to come to believe such conjunction. But this obligation appears to be

[2]The term 'bridge-principle approach' is to be understood as an umbrella term, comprising the various approaches in the literature that exemplify the standard line of response of weakening IMP and CON to meet the unfeasibility intuition. Examples include MacFarlane (2004), H. Field (2009), Steinberger (2019b), and Dutilh Novaes (2015). For a general investigation into bridge principles, see also C. Field and Jacinto (2022) and Tajer (2022).

unfeasible: the writer is not currently in a position to acquire such belief (she still has strong evidence supporting its negation). The following weakening of the consequent of BP′ could address this concern:[3]

(BP″) If $A_1, \ldots, A_n \vDash B$ and the agent considers B, then if the agent believes $A_1, \ldots, A_n$ then she has reasons to believe B.

The deontic modality appearing in the consequent of BP″ is weaker than 'ought': the fact that a salient proposition B is entailed by the agent's beliefs in $A_1, \ldots, A_n$ just counts in favor of believing B, but does not necessary lead to the obligation to believe B.

Similar weakening strategies can be employed to account for other cases related to the unfeasibility intuition as well as other objections that can be found in the literature. Steinberger (2019b), for instance, arrives at the following bridge principle:[4]

(BP‴) If the agent believes $A_1, \ldots, A_n \vDash B$ and the agent considers B or has subjective reasons to consider B, then she has reasons to (believe B, if the agent believes $A_1, \ldots, A_n$).

We are skeptical that BP‴ appropriately captures the normative connection between logic and reasoning. In fact, the approach that we will develop in this paper is conceived explicitly as alternative to the bridge-principle approach, of which BP‴ is an instance. Our first objection to BP‴, and to the bridge-principle approach more generally, is that it completely disregards the direct normativity intuition. This becomes evident once the antecedents of IMP BP and BP‴ are compared: in BP‴, what triggers the normative claim towards an agent's beliefs is not a logical entailment, but the agent's beliefs towards a logical entailment together with the contextual salience of the entailment's conclusion. In other terms, the normative connection between logic and reasoning is not direct at all, to the effect that it is not logical entailments (nor logical consistency) that have normative bearing on our beliefs. Moreover, bridge principles like BP‴ mix together, in a single general norm, logical and pragmatic factors that constrain our reasoning. As we will show in the remaining of the paper, in order to understand the full normative import of logic for our reasoning, logical and pragmatic constraints must be

[3]The same weakening could be applied to the bridge-principle version of CON:
(CON BP): If $A_1, \ldots, A_n \vDash \bot$, then the agent ought not to believe $A_1, \ldots, A_n$.
[4]Steinberger (2019b) goes on to endorse an bridge principle that is even weaker than BP‴.

distinguished. This distinction will be at the heart of our alternative approach to logical normativity.

4. A change of perspective: multiple normative factors

We saw how the bridge principle approach sacrifices the direct normativity intuition to accommodate the unfeasibility intuition, and we briefly expressed our worries about this alleged solution. In this section, we sketch an alternative picture. We first present our general framework of reasoning and of the normative role that logic plays in it, stressing how it builds upon theories of bounded rationality in cognitive science. Then, in Section 4.1, we apply this framework to the most known cases of unfeasibility of logical norms.

This is the story that we will tell. We understand logical normativity as embedded within a theory of rationality that portrays reasoning as constrained by multiple, independent normative factors. Sometimes these normative factors pull in contrasting directions. When such a situation arises, what a rational agent is prescribed to do results from a trade-off between these factors. Logic is only one of these independent normative factors. Yet, since it is not the only normative factor at play, its *actual* normative role for reasoning cannot be understood in isolation from the normative contribution of the other factors.

What are the non-logical normative factors that constrain our reasoning? Crucial cognitive-pragmatic factors depend on structural features of an agent's cognitive system and environment, such as minimization of time, effort, computational power, attention, memory, and maximization of conservativity of an agent's core beliefs. Let us be clear: there are, of course, many other normative factors that constrain our reasoning, such as, for instance, evidential, linguistic, social, and perhaps even cultural ones. Despite many of these factors arguably play a role in shaping our reasoning, in this paper, we focus our attention on the cognitive-pragmatic factors that emerge from the bounded rationality literature.[5]

The above picture, indeed, draws inspiration from theories of bounded rationality in cognitive science and their idea of human rationality.[6]

[5]This is not to deny that there are also interesting questions and challenges as to whether logic is normative for idealized, non-bounded agents. See, for instance, Christensen (2007, 2010), Smithies (2015), and Skipper (2021). We thank an anonymous reviewer for suggesting us this connection.

[6]The term 'bounded rationality' refers to a heterogeneous group of theories in psychology, economics, cognitive science that draws inspiration from Herbert Simon's (Simon 1955, 1956, 1957, 1972, 1982, 1996) seminal work in human decision making. Paradigmatic

Theories of bounded rationality (Anderson 1990; Kahneman 2003; Simon 1957, 1972, 1982; Todd and Gigerenzer 2012) conceptualize human rationality as fundamentally bounded to our human limitations, also in its normative dimension. In Simon's original words:

> The principle of bounded rationality: The capacity of the human mind for formulating and solving complex problems is very small compared with the size of the problems whose solution is required for objectively rational behavior in the real world–or even for a reasonable approximation to such objective rationality. (Simon 1957, 198).

Simon took this principle to imply the need of a complete restructure of theories of human reasoning. According to Simon, any theory of human reasoning – even a normative one – has to take human limitations into account when prescribing the rational way of approaching a given situation. In the last seventy years, several research programs in psychology, cognitive science, and economics built upon Simon's ideas. Advocates of bounded rationality often disagree on the weighting of the different factors involved in human reasoning and on the best mechanism for modeling rational behavior, but they all share a commitment to Simon's principle of bounded rationality and its call for prescribing feasible norms to actual reasoning agents.

What is important for our account is Simon's lesson that the cognitive system of an agent and her environment are not mere descriptive limitations to ideal reasoning, but they are instead ineliminable components of human rationality. This is the starting assumption of our account. Accordingly, the time needed for a given reasoning task is not just a descriptive variable that can be idealized away by normative considerations, but it exemplifies an independent normative factor: agents *ought* to minimize *time* when performing a given reasoning task (cf. Gabaix et al. 2006; Lieder and Griffiths 2020; Stigler 1961; Woodford 2014, 2016). That is, they ought to choose less time-consuming reasoning chains. Similarly, agents *ought* to minimize the *effort* and the *computational power* needed for performing a given reasoning task (cf. Bossaerts and Murawski 2017; Bossaerts, Yadav, and Murawski 2018; Dickhaut, Rustichini, and Smith 2009; Niven and Laughlin 2008; Shah and Oppenheimer 2008; Sterling and Laughlin 2015), they *ought* to

examples of theories of bounded rationality include the optimization-under-constraints paradigm in economics (Stigler 1961), the bias and heuristic project (Kahneman and Tversky 1979; Tversky and Kahneman 1974), rational analysis (Anderson 1990, 2007; Icard 2014; Lieder and Griffiths 2020), the theory of rational inattention (Sims 2003, 2006), and ecological rationality (Gigerenzer 2004, 2021; Gigerenzer and Goldstein 1996; Gigerenzer and Selten 2001; Todd and Gigerenzer 2012).

minimize the *attention* and the *memory* required by a given reasoning chain (cf. Anderson and Milson 1989; Bates et al. 2019; Caplin and Dean 2015; Caplin, Dean, and Leahy 2017; Gabaix 2014, 2017; Sims 2003, 2006), and they *ought* to maximize the *conservativity* of their *core* beliefs (cf. Batson 1975; Bossaerts and Murawski 2017; Bossaerts, Yadav, and Murawski 2018; Dolan and Dayan 2013).[7]

Our picture of rationality follows Simon's lesson by considering the aforementioned structural limitations of an agent's cognitive system and her environment as independent normative factors that constrain what an agent ought to do, when confronted with a certain reasoning task. Within this bounded-rationality-inspired picture of rationality, then, the normative role of logic is seen in connection with all these other normative factors. More specifically, the direct role of logic in reasoning can be represented by an independent normative factor, i.e.what we can call the soundness factor.[8] According to this factor, agents *ought* to make sound inferences. The obligations issued by such a normative factor can be understood, for instance, as those prescribed by principles like IMP and CON (cf. Section 2). Yet, the soundness factor can be sometimes overridden or defeated, so to speak.[9] This is because, as we will see in detail in the next subsection, the soundness factor is often in tension with the other non-logical normative factors that we presented before. Making sound inferences often requires, in fact, a lot of time, effort, and computational power. It also requires an enormous amount of attention and memory and it can lead an agent to radically revise her core accepted beliefs. In this way, the soundness factor pulls often in an opposite direction than the one towards which other normative factors pull. In such situations, what a rational agent ought to do is determined by the result of a trade-off between these diverging normative factors.[10] Thus, whether a rational agent ought to actually make

[7]Note here that we do not assume any specific, psychological or epistemological, theory about the nature of these core beliefs, e.g. how they are structured or ordered. We just assume that the set of beliefs is somehow structured with a core and a periphery, i.e. it is not flat, and that agents ought to privilege these core beliefs in the process of belief revision.

[8]Note that we use " soundness" here as a general term of art, denoting deductively valid inferences.

[9]We treat the terms 'overridden' and 'defeated' as synonymous. We say that a normative factor is overridden or defeated (by another normative factor) when it does not become an actual obligation for the reasoning agent. Yet, overriding is not canceling: in our picture, the overridden normative factor does not disappear but maintains its place as a normative factor for reasoning. On the distinction between overriding and canceling, see for instance (van der Torre 1997, 117); on the formal specifications of the overriding procedure, see for instance (Goble 2013, 268); on the meta-ethical foundation of these conceptual distinctions, see Ross (1930). We return to these considerations in Section 5.

[10]We are not assuming that an agent is always capable of making such a trade-off (consciously or not) nor that the trade-offs between normative factors could always be resolved.

sound inferences depends on whether the soundness factor is defeated in such trade-offs. It is in this sense that the actual normative role of logic for reasoning can be understood only by looking at all these normative factors and their dynamics.

4.1. Re-assessing the unfeasibility examples

In what follows, we apply the theory of rationality that we just sketched to specific cases of unfeasibility of logical norms. These cases are central in the literature on logical normativity, and have been used to argue for the untenability of IMP and CON, and for the related discard of the direct normativity intuition in favor of the unfeasibility intuition. We already mentioned two cases of this alleged unfeasibility of logical norms in Section 3, i.e. clutter avoidance and unavoidable inconsistencies. In this section, we discuss these two cases in detail, together with three other familiar cases: the problem of excessive demands, the case of belief revision, and the case of logical obtuseness.

We will discuss each of those cases separately, repeating the following narrative structure: first, we will present the specific problem in its original formulation; then, we will discuss the standard analysis of this problem proposed by the bridge-principle approach. After criticizing such analysis, we will show that, thanks to the theory of rationality we proposed in the last section, an alternative lesson can be drawn from these examples. This piecemeal discussion of the specific cases of unfeasibility will show how our theory of rationality is able to draw an alternative picture of the normative role of logic for reasoning to the one proposed by the bridge-principle approach. In the next section, we will present our picture of logical normativity in full generality.

4.1.1. Clutter avoidance

The problem of clutter avoidance is part of Harman's (1986) original criticism to the normativity of logic for reasoning. It states that it a waste of someone's cognitive resources to clutter up one's mind with a high number of useless, albeit inferentially sound, beliefs. Take for instance the case of irrelevant disjunctions, i.e. inferences from a given belief (e.g. 'I am currently in my office') to a disjunction made by this belief and another arbitrary belief (e.g. 'I am currently in my office or all penguins are robots'). Inferring irrelevant disjunctions is perfectly legit according to classical logic and, as such, a straightforward conversion of

logical entailments into reasoning norms, such as the aforementioned IMP, would prescribe that we ought to make such inferences.[11] However, we do not often make these inferences, and rightly so; this is because otherwise we would be soon overloaded by an infinite number of completely useless and non-salient beliefs.

The solution of the bridge-principle approach to the problem of clutter avoidance is to weaken the connection between logical entailments and norms for reasoning by relativizing the antecedent of the bridge principle to topics under consideration. In this way, only when an agent is considering a given group of sentences, the entailments between them exert normative force on her reasoning.

However, why should this be the case? On purely logical grounds, a useless disjunction is perfectly sound as much as any salient inference. However, it is still not rational for an agent to draw useless disjunctions indefinitely. This is because of our cognitive limitations as bounded agents, equipped with limited reasoning capacities. Useless disjunctions are then perfectly sound inferences, but they are not feasible inferences for agents with limited capacities like ourselves. The reason why we ought not to draw useless disjunctions is because, as bounded agents, we ought not only to make sound inferences, but also to minimize the time and effort that we employ in reasoning. Therefore, an agent ought not to clutter her mind with useless beliefs because these would make her waste time and effort, without any practical advantage towards solving a given reasoning task. Thus, in our approach, where minimizing time and effort are independent normative factors, we need not to relativize logical normativity to topics under considerations, like the bridge-principle approach does. Instead, we understand the normative contribution that useless disjunctions get from their soundness as overridden by the time and the effort that an agent would waste in drawing them. Reasoning is a trade-off between different normative factors and a result of this trade-off is that we ought not to deduce useless and non-salient consequences of our beliefs.

4.1.2. Unavoidable inconsistencies

The second case of unfeasibility we are going to focus on is the case of unavoidable inconsistencies. This amounts to the realization that, sometimes, inconsistent beliefs seem unavoidable.

[11] Note that, throughout this paper, we work under the assumption that the logical principles and rules we are talking about are the ones of classical logic. While we feel that questions concerning logical pluralism and its relationship with logical normativity underlie important issues, we do not discuss them in the present paper. For a survey of related problems, see Steinberger (2019c).

The prominent solution of the bridge-principle approach to the problem of unavoidable inconsistencies is to weaken the consequent of the bridge principle, ending up with a weaker modality such as 'having reasons to'. In this way, despite the agent has some reasons for avoiding having inconsistent beliefs, she is under no obligation to do so.

This retreat to a weaker modality solves indeed the problem of unavoidable inconsistencies, but at a cost. In fact, such solution makes logical consistency never mandatory for agents. Yet, in certain situations, agents seem under the obligation to avoid inconsistencies altogether.[12] The key to solve this impasse and the problem of unavoidable inconsistencies, we mantain, is to distinguish between feasible and unfeasible inferences. Achieving consistent beliefs appears, indeed, sometimes unfeasible. This can be seen, for instance, in the aforementioned case of the Preface Paradox (cf. Section 2.2, 4). Consider the belief in the conjunction of all the sentences contained in a book, i.e. the belief that allegedly leads to the paradoxical situation. Drawing such conjunctive inference from the single sentences of the book might be feasible or unfeasible depending on the specific context under consideration. For instance, if the context is a book of three easy sentences, such as 'I am currently in my office', 'I am wearing black socks' and 'Penguins are birds', then believing the conjunction is indeed feasible. And, if the conjunction is feasible, no preface paradox arises. This is because, since our attention is sufficient to hold these three sentences together, it is also feasible for the agent to believe their conjunction. Due to such feasibility, provided that the agent firmly believes each of these three sentences, doubts on the truth of their conjunction are unwarranted. This is a case, then, in which the agent ought to believe such conjunction, that is, the agent's beliefs ought to be consistent. The situation is different if the book under consideration includes a thousand sentences. For such book, believing the conjunction of all its sentences involves drawing an unfeasible inference, i.e. an inference that exceeds by far our limited attention. In this situation, then, even when one believes all the individual sentences in the book, modesty seems mandatory, and no one would blame the author for not believing the conjunction of so many sentences. Seen in this way, the Preface Paradox can be given a contextual solution,

[12]This critical point is also made by MacFarlane (2004) who stressed that a bridge principle with a weak modality such as 'having reasons to' falls prey of the so-called strictness problem (Broome 1999), i.e. a meta-theoretical desideratum according to which the relationship between logic and reasoning is stricter than merely 'reasons-giving'. The tension between these two desiderata is part of Steinberger's (2019a) call for distinguishing three ways in which logic might be normative for reasoning. We will discuss Steinberger's proposal and compare it to ours in Section 5.

distinguishing situations where believing the conjunction is feasible from situations where it is not.[13] Situations of the former kind are the ones where the agent ought to believe the conjunction, while situations of the latter are the ones where the agent does not have this obligation. The general moral is that inconsistencies are unavoidable when the complexity of the task exceeds our limited attention. This is why the problem of unavoidable inconsistencies can only be solved contextually, looking at the specific situation under focus.[14] In our approach, where minimizing attention and complexity are independent normative factors, we can thus distinguish between feasible and unfeasible inconsistencies. Feasible inconsistences are the ones where the soundness factor defeats the attention and the complexity-minimization factors; these are the inconsistencies that ought to be revised by the agent. Unfeasible inconsistencies are instead the ones where the soundness factor gets overridden by the attention-minimization and the complexity-minimization factors; these are the inconsistencies that an agent is allowed to maintain. In our picture, reasoning is a trade-off between logical and non-logical factors, and a result of this trade-off is that we ought to be consistent when it is feasible.

4.1.3. Excessive demands

The problem of excessive demands revolves around the observation that there are certain logical conclusions or certain inconsistencies, the detection of which seems beyond one's cognitive capacities. For instance, take the axioms of Peano Arithmetic. Most mathematicians and philosophers of mathematics believe the truth of these axioms. However, believing all their logical consequences, i.e. the set of theorems of Peano Arithmetic, seems a task beyond the possibilities of any real agent.

The solution of the bridge-principle approach to the problem of excessive demands is, again, to weaken the bridge principle, either by relativizing the antecedent to the entailments that are believed by the agent or by weakening the consequent to a modality like 'having reasons to'. In the former case, the entailment exerts normative force on an agent's reasoning only when she believes that a certain logical entailment holds. In the latter case, instead, the normative force of logical entailment

[13]Note here that other authors have argued that the preface case fundamentally revolves around evidential considerations (cf. Kolodny 2007 on logical vs. evidential norms). Our discussion shows that the Preface Paradox can arise, and also be resolved, as a conflict between logical and cognitive-pragmatic factors.

[14]This moral is also shared by the so-called 'fragmented-belief solutions' to the preface paradox (cf. Borgoni, Kindermann, and Onofri 2021, Ch. 2).

does not issue obligations, but only reasons in favor of believing a given statement. Either way, the agent is not required to believe all theorems of Peano Arithmetic anymore.

Either solution to the problem of excessive demands faces familiar problems. The first solution, i.e. relativizing the antecedent to the logical entailments believed by the agent, is structurally analogous to the alleged solution of the bridge-principle approach to the case of clutter avoidance. And just like that solution, we hold that is wrong to relativize logical norms to what the agent considers or believes, since every logically valid inference is equally sound and, therefore, it should exert normative force to our reasoning. The second solution of the bridge-principle approach to the problem of excessive demands, i.e.weakening the modality of the bridge principle, is instead structurally analogous to the one we criticized in the problem of unavoidable inconsistencies. And, just like we discussed above, weakening the modality solves the issue at a significant cost, namely, to end up with a norm that is too lax in simple situations. Most importantly, both solutions do not take into consideration the contextual nature of the excessiveness of the demands imposed by logic on the agent. Whether the demands that logic impose on an agent's reasoning are excessive depends, in fact, on the specific case under focus, i.e. on whether we are considering an inference that is feasible or unfeasible for the agent. If one believes the axioms of PA, believing all theorems of PA is perfectly sound from a purely logical point of view, but it is not feasible for bounded agents with limited computational power like ourselves. Thus, the solution to the problem of excessive demands cannot be a general one, such as the relativization of the antecedent or the weakening of the modality of the consequent of the bridge principle, but only a contextual one. It is a matter of accepting that other non-logical normative factors, such as the minimization of the computational power required to the agent, can defeat the soundness factor when logical inferences pose excessive demands to the cognitive system of a given agent.

4.1.4. Belief revision

The problem of belief revision states that we do not always have to adopt the logical implications of our beliefs, as they sometimes conflict with other beliefs that we already have. Sometimes it appears rational to revise our beliefs in the premises of an argument, the objection goes, instead of believing their logical conclusion. Harman's (1986) famous example of the problem of belief revision involves a cupboard and a

cereal box. Assume that I believe that 'if I will open the cupboard in my kitchen, I will find a box of cereals in it', because I distinctively remember to have put it there yesterday. Say that I also believe that 'I will open the cupboard in my kitchen', as I always do that every morning. Thus, according to IMP, I ought to believe also that I will find a box of cereals in the cupboard, as it follows from these two premises by *modus ponens*. Yet, argues Harman, if one opens the cupboard and there is no cereal box in it, one does not (and should not) come to believe that there is a cereal box in the cupboard. Rather, one rationally revises her belief in the premises, thinking that perhaps someone else in the house moved the cereal box.

The solution of the bridge-principle approach to the problem of belief revision is to modify the scope of the deontic operator in the bridge principle, from narrow scope to wide scope ranging over the whole set of beliefs under consideration.[15] In this way, logical norms do not force reasoners to believe or disbelieve a given proposition, but they act only on sets of beliefs (e.g. they tell that reasoners ought not to believe the premises of an argument and to disbelieve its conclusion).

Our issue with the solution of the bridge-principle approach to the case of belief revision is that, once again, it misrepresents the relationship between logical and non-logical factors in our reasoning. The soundness factor would prescribe to believe the logical consequences of our beliefs. Yet, in our picture, reasoning is always a trade-off between different normative factors. And among these other factors there is the conservativity factor, i.e. the normative factor that tells us that we ought to maximize the conservativity of our core beliefs. The conservativity factor is what, in the cereal box example of Harman, overrides the soudness factor. Instead of coming to believe that there is a cereal box in the cupboard, as it would follow from *modus ponens*, the reasoner revises her belief in the premise 'if I will open the cupboard in my kitchen, I will find a box of cereals in it'. Believing that there is a cereal box in the cupboard would, in fact, imply a revision of very entrenched beliefs in her belief sets such as 'I see things clearly' and 'If there is an object in front of me, I will see it'. Thus, coming to believe the conclusion of the *modus ponens* would imply a radical revision of her core beliefs, a revision completely at odds with the demands of the conservativity factor.

[15]An example of a narrow scope bridge principle is (IMP BP), i.e. 'If $A_1, \ldots, A_n \vDash B$, then if the agent believes $A_1, \ldots, A_n$ then she ought to believe B'. Its wide scope variant would be, instead, 'If $A_1, \ldots, A_n \vDash B$, then it ought to be the case that if the agent believes $A_1, \ldots, A_n$ then she believes B'.

4.1.5. Logical obtuseness

The problem of logical obtuseness states that there is something amiss with reasoners who refuse to take a stand towards the simplest logical consequences of their beliefs. A person who believes A and B but suspends the judgment towards their conjunction $A \wedge B$ is, as MacFarlane (2004) argues, somehow failing to obey a logical norm.

The case of logical obtuseness has been interpreted against bridge principles with negative polarity (e.g. bridge principles prescribing only what one ought not to disbelieve). Consequently, the bridge-principle approach solution to this problem is to adopt a bridge principle with positive polarity, according to which logical norms impose constraints on agents' beliefs (and not merely on their disbeliefs).

The problem of this solution is that, once again, the bridge-principle approach blurs the distinction between feasible and unfeasible inferences. We submit that there is no general problem of logical obtuseness and, as such, no general solution to it, but only a contextual matter of feasible and unfeasible inferences. In MacFarlane's example, in fact, we think that there is something amiss with the agent's reasoning only because of the simplicity of the inference and of the (implicitly assumed) simplicity of the propositions involved in the inference. Inferring the direct logical consequences of our beliefs can be very easy, as in MacFarlane's case, but also quite difficult. In certain complex cases, as the long history of inconsistent logical systems put forward by eminent logicians shows, inferring direct logical consequences of our beliefs is not trivial at all. In those cases, it could be rational for an agent to suspend judgment. Moreover, experimental results suggest that we sometimes do not recognize even the simplest logical consequences of our beliefs, because we store different beliefs in different parts of our memory (cf. Anderson and Milson 1989; Bates et al. 2019; Borgoni, Kindermann, and Onofri 2021; Cherniak 1986). Information stored in different parts of our memory is, in fact, seldom activated together and sometimes categorized in different formats. Hence, reasoners can be guilty of logical obtuseness only when the inference under focus is feasible in terms of complexity and memory-recall-effort minimization. Our solution to the problem of logical obtuseness identifies, then, complexity and memory-recall-effort minimization as independent normative factors that can override the soundness factor, when the inference involved are too complex or they involve information too diversely stored in our memory.

This is how our account of rationality applies to five central cases of unfeasibility of logical norms discussed in the literature. For each case,

our solution is to keep the distinction between feasible and unfeasible inferences and to recognize that reasoning is a trade-off between different normative factors. In such trade-off, the soundness factor is sometimes overridden by non-logical factors such as the minimization of time, effort, attention, complexity, computational power, memory recall effort, and the maximization of conservativity.

5. The problem of logical normativity re-assessed

We saw in the last section how several specific cases of unfeasibility can be explained by the interactions among the multiple, independent normative factors that constrain our reasoning. Crucially, in our picture, the soundness factor can be defeated by non-logical factors. Let us now re-assess the question of logical normativity from this perspective.

The soundness factor reflects the normative role of logic for our reasoning: we ought to make sound inferences. Yet, we saw that this ought is defeasible. Borrowing from meta-ethical terminology, we propose to understand this ought as a *prima facie* ought (e.g.Brink 1994; Chisholm 1964; Goble 2013; Ross 1930). *Prima facie* oughts are obligations that play a contributive and defeasible role in determining what an agent actually, or all-things-considered, ought to do.[16] If there are no other conflicting and more relevant obligations at play in a given situation, a *prima facie* ought to X translates to an all-things-considered ought to X; otherwise, the *prima facie* ought to X is said to be defeated and does not give rise to an all-things-considered ought to X. Thus, an all-things-considered ought is what an agent, on balance, ought to do: it is the result of taking into consideration and combining the various *prima facie* oughts in a given situation, and possibly selecting those that are contextually more relevant.

That is the kind of normative demand that logic exerts on reasoning: logical rules give rise to obligations, but these obligations do not necessarily determine what the agent, all things considered, ought to do. This is because logic is not the only relevant normative factor. All the other normative factors we recognized as emerging from cognitive and environmental features (e.g. minimization of time, effort, computational

[16]Sometimes the oughts that play a defeasible and contributive role are called '*pro tanto* oughts' (see, Reisner 2013). In this paper, we will be referring to these oughts as '*prima facie* oughts', following the terminology stemming from the work of W.D. Ross. See, e.g. Ross (1930, 18–20), and Brink (1994, 219–220).

power, …), correspond to other *prima facie* oughts. Therefore, reasoning is a conflict between many, independent *prima facie* oughts.[17]

Importantly, this picture vindicates the two intuitions that started our discussion of logical normativity: the direct normativity intuition and the unfeasibility intuition. The direct normativity intuition amounted to the fact that there is a direct normative connection between logic and reasoning. More specifically, as discussed in Section 2.1, this intuition has two core components: that our beliefs ought to obey logical principles and rules, and that it is precisely logical principles and rules that carry such normative force. Both components of this intuition are vindicated by the presence of the soundness factor as an independent ought for any reasoning agent. Thanks to this factor, beliefs ought indeed to obey logical principles and rules, albeit only as a *prima facie* obligation. Moreover, which inferences we ought, according to the soundness factor, to make is determined by nothing else than logical principles and rules. Differently from what happens in the bridge-principle approach, our approach does not relativize logic to doxastic attitudes or topics under consideration, nor it merges logical principles with other epistemic norms (cf. Section 4.1). Thus, logical normativity in our picture is not mediated. As such, our picture vindicates both components of the direct normativity intuition.

The unfeasibility intuition amounts instead to the observation that the normative demands that logic puts on our beliefs are unfeasible and thus sometimes do not reflect what we ought to believe. This intuition is vindicated, in our picture, by the fact that what the reasoning agent ought to do all things considered might contrast with her logical obligations. In fact, we saw in Section 4.1 that, in several cases, the *prima facie* ought corresponding to the soundness factor is defeated by the oughts corresponding to the other normative factors that constrain our reasoning. In this way, logical obligations often do not reflect what an agent ought, all things considered, to do.

By vindicating both the direct normativity intuition and the unfeasibility intuition, our approach dissolves the apparent tension between them.

[17] Such conflict between different *prima facie* oughts might explain what MacFarlane hinted at with his talk of conflicting epistemic obligations in MacFarlane (2004, 13–14). We do not explore in this paper the exact formal dynamics of the conflict between independent normative factors that constrain our reasoning. However, there are many logical and psychological frameworks that model these kinds of contexual situations, such as heuristics models (Gigerenzer and Goldstein 1996; Gigerenzer and Selten 2001; Shah and Oppenheimer 2008), dual-system theories of reasoning (Evans 2007), argumentation theories (Mercier and Sperber 2011), deontic logics (Goble 2013; Horty 2012), resource belief revision (Wassermann 1999).

The key to our solution is maintaining the distinction between sound and feasible inferences, which –in our account– corresponds to the normative distinction between *prima facie* and all-things-considered oughts. The direct normativity intuition highlights the fact that we ought *prima facie* to make sound inferences. Yet, as the unfeasibility intuition shows, feasible inferences do not sometimes correspond to what logical soundness demands. These distinctions identify two aspects of the normativity of logic that are conceptually distinct and that we must keep separated. As a consequence of this, we can see that the tension between the two intuitions that we identified in Section 2 was only apparent:

Direct Normativity: $O_{primafacie}(\varphi)$
Ought-Can principle: $O_{all-things-considered}(\varphi) \rightarrow \Diamond (\varphi)$
Empirical Fact: $\neg \Diamond (\varphi)$
Unfeasibility: $\neg O_{all-things-considered}(\varphi)$

We are now in a better position to determine what was wrong with the bridge-principle approach. Bridge principles do not separate the soundness and the feasibility of our inferences: by blurring the differences between what is sound and what is feasible, they seek a general one-size-fits-all solution to the problem of logical normativity. This can be seen, for instance, if we look at principles such as Steinberger's (2019b) BP‴, where worries about the soundness of our inferences, such as the fact that an agent has reasons to believe the logical implications of her beliefs, are entangled with feasibility considerations, like the agent's doxastic attitudes towards the entailment and its content. This is why the standard line of response to the unfeasibility intuition championed by the bridge-principle is problematic. The problem does not lie in constructing a principle in a bridge-like form connecting logic and reasoning, but in lumping together matters pertaining to the soundness of our inferences, i.e. logical considerations, with matters pertaining to the feasibility of our inferences, i.e.cognitive-pragmatic considerations. Mixing logical and cognitive-pragmatic considerations in this way, principles like BP‴ are not adequate to capture neither kind of considerations: they are *too weak* to capture only the soundness of our inferences and they are *too strong* to encompass only the inferences that are feasible for a given agent in a given context.[18] BP‴ and alike, thus, strike an unsatisfying middle ground between soundness and feasibility considerations.

[18]The a-contextuality of bridge principles is also problematized, from a different standpoint, by Oza (2020).

From our account, what emerges is a context-dependent picture of logical normativity, where the full normative import of logic for reasoning is given by two separate aspects: the defeasible obligation that a certain logical entailment issues and its actual normative strength for our reasoning. These two separate aspects answer, respectively, two different questions: first, what are the normative demands that logic exerts on reasoning, and, second, what ought we to believe all things considered in a given situation. Crucially, only the latter question, and the related answer, is dependent on the structural features of an agent's cognitive system and on the environmental features of the reasoning task she is faced with. The trade-off between logical and cognitive-pragmatic factors in determining what we actually ought to believe is the moral that our account draws from the bounded rationality paradigm.[19]

The distinction between sound and feasible inferences, and the related difference between *prima facie* and all-things-considered oughts, cuts across other distinctions available in the literature on logical normativity. For instance, Steinberger's (2019a) tripartite distinction between evaluations, directives, and appraisals of logical norms appears complementary to the present discussion. To the extent that the unfeasibility intuition is considered, our approach naturally leans towards the dimension of logical directive and appraisals, but our general picture of a multiplicity of independent *prima facie* oughts could also serve as a background for an evaluative approach to logical norms. Moreover, the distinction between a constitutive role and a regulative role played by logic in reasoning (cf. Cherniak 1986; Leech 2015; Oza 2020) appears orthogonal to the present discussion and compatible with it. Finally, degrees of strength of logical norms can be represented in our framework by differently weighting the soundness factor with respect to the other cognitive-pragmatic factors, expressing in this way various requirements of deductive ability for agents (cf. Cherniak 1986).

From the perspective of our account of logical normativity, we can also re-assess three further puzzles discussed in the literature: the priority question (MacFarlane 2004, 12), the strictness test (Broome 1999, 406), and the bootstrapping objection (Broome 1999, 405). These three problems are often discussed together with the five cases of unfeasibility we presented in Section 4.1. However, these problems have a more meta-theoretical character, as they do not concern the unfeasibility of

[19]This moral is also shared by many contemporary research programs on the psychology of reasoning that discuss how often the optimality of reasoning is to be distinguished from pure accuracy considerations (cf. Gigerenzer 2004, 2021; Lieder and Griffiths 2020; Todd and Brighton 2016).

logical norms, but they highlight instead specific aspects that a good bridge principle must satisfy. Our approach does not fall prey of the priority question, i.e. MacFarlane's argument against the relativization of logical norms to the doxastic attitudes of the reasoner. We do not relativize, in fact, logical norms to any attitude, since we consider logical entailments as directly determining an independent normative factor constraining our reasoning, i.e. the soundness factor. Our approach moreover meets the strictness test. This test is supposed to tell against bridge principles with weak modalities such as 'having reasons to', on the basis that we have obligations to endorse simple logical inferences. Our approach meets this test, since the soundness factor gives rise to an ought. In fact, on the metaethical account we are endorsing, *prima facie* oughts are proper obligations that require the agent to perform a certain action, albeit defeasibly. Reasons are different. We take reasons to merely register considerations in favor or against a certain action, without specifying any obligation on the side of the agent.[20] Finally, our approach does not fall prey of the bootstrapping objection. This objection states that it is just not the case that we ought to believe all the logical implications of our beliefs, as this would result into a self-supporting justification of an unreasonably held belief. To see that, suppose that an agent happens to hold the belief in p for an arbitrary faulty justification. Then, the agent could justify her belief in p, via the reflexive inference $p \vDash p$, on the ground that p is a logical consequence of her belief in p. In our approach, however, such line of reasoning gives rise only to a *prima facie* ought, an obligation that can be defeated by other *prima facie* oughts. Thus, reflexive inferences do not, as such, give rise to actual self-supporting justifications of beliefs. This shows that *prima facie* oughts, standing in between reasons and all-things-considered oughts, are the right conceptual tool to capture the normative strength of logical norms, satisfying the demands of both the strictness test and the bootstrapping objection.

[20]The difference between *prima facie* oughts and reasons can be better appreciated with the help of an example. If visiting a friend makes this friend happy, then I might have a reason to visit them. Yet, I am under no obligation to pay them a visit. Nor there is something amiss with me if I do not visit them. The situation would be different if I had a *prima facie* ought to visit my friend. Even if this ought did not translate into an all-things-considered ought, because of some other stronger conflicting obligations, there would be a violation of a duty on my side. On the distinction between *prima facie* oughts and reasons, see Goble (2013, 266).

6. Conclusion

Let us recap the main steps of this work. We started by reframing the problem of logical normativity as the clash between two intuitions: the direct normativity intuition and the unfeasibility intuition. We recalled how the bridge principle approach responds to the clash between these two intuitions by abandoning the direct normativity intuition and by bridging logic and reasoning by virtue of weaker normative principles. We noticed how this approach, by abandoning the direct normativity intuition, falls short on offering a satisfactory account of the normative import that logic itself exerts on reasoning. We then developed an alternative picture of logical normativity inspired by theories of bounded rationality in cognitive science. According to our picture, reasoning is constrained by several independent normative factors, logical and non-logical ones. From this alternative perspective, logic gives only *prima facie* oughts, that is, defeasible obligations that can be contextually defeated by other non-logical *prima facie* oughts. This defeasible normativity of logic allows us to save both intuitions we started from. In our account, in fact, logic itself gives obligations on what we have to believe, as the direct normativity intuition requires, but these obligations do not necessarily determine what an agent ought to believe all things considered, as highlighted by the unfeasibility intuition.

At the core of our alternative picture of logical normativity lies the distinction between sound and feasible inferences, a distinction that has been blurred in the literature on logical normativity. Yet, one cannot satisfactorily understand the full normative import of logic for reasoning without keeping sound and feasible inferences conceptually separated. The lack of such a distinction was the problem of the bridge-principle approach. Instead, as our analysis has shown, there are two aspects at play in the normative import of logic for reasoning: the defeasible obligations that logic imposes and the actual contribution of logic in determining what an agent all-things-considered ought to believe, where only the latter aspect is intertwined with the structural features of an agent's cognitive system and of the environmental factors of the related reasoning task. This is the lesson that one can draw from the present work.

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