



Logic as Normative Modeling

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Received: 24 September 2025 / Accepted: 5 September 2026
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Abstract

Many philosophers understand logic as a theory of truth preservation that is value-neutral and becomes prescriptive only when combined with external, non-logical rules. In this article, we develop an alternative view informed by recent work in the philosophy of scientific modeling. We argue that certain logical systems, in certain scientific and philosophical practices, are used as normative scientific models: they represent their targets by combining descriptive idealizations with assumptions adopted—and defended—on normative grounds. We propose a practice-based account of normative models in science, according to which the normative character of a model depends on its use in the relevant research practice, and apply it to the case of classical first-order logic, in its uses as a model of correct reasoning. Finally, we show how our proposal opens up a new perspective on several contemporary debates in the philosophy of logic.

1 Introduction

Within philosophy, logic is commonly understood as a theory of correct reasoning; that is, as the study of those argument patterns in which the conclusion necessarily follows from the premises. As a consequence of this, logic is also often viewed as a normative discipline, one that sets out normative criteria for distinguishing good from bad forms of reasoning. There are, however, philosophers who resist this view and reject the idea that logic is normative in any meaningful sense. Instead, they understand logical systems as describing formal relations between abstract objects;

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relations that are entirely value-neutral and from which, consequently, no normative prescription can be derived (cf. Harman, 1986, 2002).

In a recent paper, Gillian Russell (2020) takes up this skeptical line of argument. Logic, she claims, classifies which argument patterns preserve truth much like physics describes energy conservation. Any prescriptive ‘ought’ could appear only if logical facts are combined with external norms such as ‘believe what is true’; but the calculus itself contains no normative content, and fallacious reasoning must be interpreted as the product of factual mistakes. For Russell, logic thus stands to patterns of truth-preservation as natural science stands to patterns in the physical world: it becomes epistemically or practically relevant only when an applied counterpart translates its generalizations into practical rules, much as physics acquires prescriptive force when employed in engineering.

We believe, however, that equating logic with the natural sciences is anything but straightforward. Logic does not seem to have a natural phenomenon as its subject matter; its principles are not connected to an observational language, and they presuppose no measurement procedures. Furthermore, logic—by itself—produces no testable predictions. In what sense, then, can logic plausibly be said to operate “like” a natural science?

In this article, we address the above challenge to the normative role of logic by building upon a trend in the philosophy of logic that treats logical systems as analogous to scientific models. We propose an extension of that view that includes the possibility of normative modeling uses of logical systems in certain philosophical and scientific practices. In these uses, logical systems are indeed models, yet not merely descriptive but including in their representation of the target certain normative idealizations. Drawing on recent work in philosophy of science, we advance a practice-based account of normative models, which claims that normative models are models with normative idealizations, i.e., idealizations that, within the relevant scientific practice, are adopted and defended on explicitly normative grounds.

To illustrate our practice-based account of normative modeling and normative idealizations, we apply it to the specific case of first-order classical logic in its philosophical uses as a model of correct reasoning, carefully distinguishing between the descriptive and normative idealizations embedded in this use of this important instance of logical modeling. We show that casting classical logic in these terms helps to explain its normative status in certain scientific and philosophical practices, while, at the same time, allowing for the multiple roles that logic plays in different contexts, e.g., a formal calculus, a standard of valid inference, and an indispensable methodological instrument across specialized domains of inquiry.

In Sect. 2, we will examine existing philosophical accounts of logic as a model of various phenomena, highlighting the lack of a proper account of how the intuitive normativity of logic can be embedded within a scientific model. Section 3 will instead be devoted to the analysis of the recent literature in philosophy of science on normative models in social and cognitive sciences and to the presentation of our practice-based account of normative idealizations and normative modeling. In Sect. 4, we will apply our characterization of normative modeling to the case of logic, showing how this idea adequately accounts for the normative role of logic in many of our social and cultural practices, as well as for its descriptive representation of its target domains. We will

illustrate our account of logical modeling by examining in detail the descriptive and normative idealizations involved in the use of first-order classical logic as a model of correct reasoning. In Sect. 5, we analyze the general implications of our proposal, showing how it meets Russell's skeptical challenge and how it gives a fresh perspective on several contemporary debates in the philosophy of logic. Section 6 concludes.

2 Logic as a Scientific Model

In the philosophy of science of the latter part of the twentieth century, the classical notion of a *theory* was gradually replaced by the more flexible notion of a *model* as the central theoretical unit of philosophical interest. Unlike theories, models are not logically articulated systems of propositions, but rather symbolic structures (generally mathematical) that *represent* an empirical system in such a way that it becomes more 'epistemically accessible'. In the vast majority of cases in science, the representational relationship between a model and its target is *idealized* and *descriptive*. That is, the model simplifies various aspects of the target for reasons of epistemic or computational tractability, but it preserves those structural features that are essential for describing and explaining the behavior of the original system.

In contemporary philosophy of logic, several philosophers have found in the concept of a model an opportunity to elucidate the complex relationship between logical systems and their underlying subject matter. This gave rise to what is known as the logic-as-model (LaM) view (Cook, 2010; Stei, 2023), which treats logical theories as models of certain linguistic or epistemic phenomena, in the same way that scientific models represent natural phenomena (Frigg & Nguyen, 2020).

John Corcoran (1973) was the first to propose this view. He compares logical theories to mathematical models in physics, arguing that, much like physics creates models to depict physical systems, mathematical logic constructs models to represent the "underlying logic" of different mathematical domains—e.g., geometry, number theory, or set theory—(1973, p. 23). Although Corcoran does not develop this idea in great detail, his comparison clearly relies on the conventional notion of descriptive modeling in science.

A refined version of LaM was later proposed by Stewart Shapiro (1998, 2014). According to this view, a logical system is a mathematical model of a notion of logical consequence in natural language.¹ He writes:

...a formal language is a mathematical model of a natural language in roughly the same sense as, say, a collection of point masses is a model of a system of physical objects[...]In other words, a formal language displays certain features of natural languages, while ignoring, simplifying, or idealizing other features [...] (Shapiro, 2014, p.46)

Shapiro claims that logical systems, as mathematical models that rely on various idealizations, must balance *simplicity* with *realism*, and that an exact representation of the phenomena being modeled is generally unattainable. He also acknowledges that

¹ A similar view is suggested by Burgess (1992).

scientific modeling has a pragmatic dimension, since formal systems are designed in light of specific goals, and decisions about which aspects of a phenomenon to idealize or preserve depend on the purposes for which the model is intended. In the case of logic, Shapiro claims, the goal is to study correct reasoning. Therefore, it is reasonable to concentrate on—and idealize—only those features of natural language that are relevant to reasoning.

The most recent articulation of LaM has been proposed by Michael Glanzberg (2015, 2021). Glanzberg focuses on model theory and its application to various language-related phenomena. In his view, building a logical system for a phenomenon such as natural language involves a systematic process of abstraction, identification, and idealization relative to specific empirical data. We first abstract away from ‘absolute semantics’ to allow interpretations to vary. Then we identify a restricted logical vocabulary whose meanings are held constant and, finally, we introduce idealized assumptions to regulate the resulting system. The upshot of this multi-step process is a logical model that functions as a scientific representation of an empirical phenomenon in a simplified, idealized setting.

Glanzberg suggests that such logical systems can play two different but complementary modeling roles. First, they can function as analogical models, that is, as formal systems that shed light on an empirical phenomenon by reproducing some of its relevant structural features. Generalized Quantifier Theory (Barwise & Cooper, 1981), for example, can be seen as a model of quantifier use in natural language in this sense. Although it does not capture the full richness of linguistic practice, it helps clarify some of the central patterns underlying quantificational expressions. Second, they can work analogously to model organisms in biology (cf. Weber 2004; Ankeny and Leonelli 2020), which offer controlled “micro-environments” that allow one to observe the behavior and interactions of different linguistic particles with specific logical properties.

Crucially, the models produced by this process are not merely descriptive tools, but also play an important epistemic role. According to Glanzberg, logical models aim to capture key patterns in an idealized, mathematically tractable form that facilitates analysis and explanation. This requires abstracting away from many features of the phenomenon that, though real, would make the model less tractable or less useful. As Glanzberg reminds us, “any model has limits” (2021, p. 221); logical modeling must acknowledge those limits even as it provides theoretical insight.

We can thus understand LaM as a general approach to the relation between logic and one of its intended domains: like scientific models in the natural sciences, logic represents its target phenomena by highlighting certain features while idealizing or omitting others.²

One important advantage of this perspective is that it helps address some apparent difficulties in relating logic to certain paradigmatic intended domains. Take, for example, human reasoning, the most paradigmatic domain of application of logic *qua* model of correct reasoning. In the last few decades, cognitive science has highlighted several limitations that logical models have in accounting for the full complexity of

² As the philosophy-of-science-inclined reader is likely to have already noticed, many key terms of this definition of LaM such as representation, target phenomena, and idealization are left under-determined in this literature in philosophy of logic. We will discuss this issue in the next sections.

human reasoning (cf., Perkins, 2002; Oaksford and Chater, 2007; Gärdenfors and Osta-Vélez, 2023). From LaM's perspective, these limitations can be seen as expected consequences of its status as a scientific model. The focus on formal and deductive aspects of reasoning, as well as its heavily idealized view of the resources and computational power of reasoning agents, can be understood as descriptive idealizations of logical modeling (see Yap, 2014; Russell, 2023). After all, as philosophers of science have long recognized, idealizations and abstractions are ubiquitous in scientific models, up to the point of constituting a permanent positive feature of their representation of the world (cf., Batterman, 2009; Bokulich, 2008; Elgin, 2017; Potochnik, 2017).

However, some aspects of logical modeling are not so readily explainable by the LaM view. Take for instance, the normative role that logic plays in our communal inferential practices. We often blame people for inconsistencies and for ignoring the logical consequences of their beliefs. In scientific debates, logical flaws can undermine entire theories. Because inferences are actions that can go right or wrong, reasoning does not seem to be an empirical phenomenon in the same way physical systems are. While this normative dimension of logical modeling has not gone unnoticed by the proponents of LaM (cf., Shapiro, 1998; Glanzberg, 2021), no specific proposal of how this intuitive normativity of logic can be embedded in the LaM view has been put forward in the literature.

As noted in the introduction, Gillian Russell (2020) accepts that logic should be understood as a scientific model but insists on its purely descriptive character. Countering Shapiro and Glanzberg's emphasis on the relation between logic and epistemic and linguistic phenomena, Russell characterizes logic as a descriptive model of truth-preservation analogous to physics due to its focus on structural relations. Where physics studies how properties like mass or energy are conserved under certain conditions, logic studies how truth is preserved across patterns of argument. For example, modus ponens outlines how P and $P \rightarrow Q$ guarantee Q , not as a normative prescription, but as a structural description of truth-preserving inference.³

Russell's descriptivist view challenges the idea that logic can be normative in any substantial sense. While she accepts that logic could play a role in communal practices of reasoning—like argument evaluation or holding people accountable for inconsistencies—these normative dimensions are not part of logic itself, but they arise from broader social and epistemic commitments, such as the belief that one ought to reason in ways that align with truth. For Russell, the relationship between logic and normativity is contingent, not intrinsic.

In what follows, we will employ Russell's skeptical view as a challenge to any proponent of the LaM view who believes in the inherent normativity of logic: how can we reconcile the normativity of logic with the idea that logical modeling is a kind of scientific modeling? This is the challenge that we will try to meet in the rest of the paper. Our proposal will begin by looking at how contemporary philosophy of science conceptualizes analogous issues in social and cognitive sciences.

³ For instance, she writes that "Logic has the same relation to truth as physics has to mass or heat. Objects have mass or heat. Truth-bearers like sentences are true or false. Physics studies the ways in which mass or heat are preserved or lost over time. Logic studies the way the property of truth is preserved over arguments." (Russell, 2020, p.382).

3 Normative Models in Philosophy of Science

As we recalled at the beginning of the last section, a defining feature of scientific models is that they incorporate some degree of idealization. Idealizations involve simplifying, distorting, or omitting features of the target phenomenon to make analysis more tractable or illuminating (Batterman, 2002; Weisberg, 2007; Elgin, 2017; Potochnik, 2017; Rice, 2019).⁴ Following Weisberg's influential taxonomy of kinds of idealizations, idealizations in scientific models may be 'Galilean', exaggerating one factor while neglecting others for computational reasons (cf., McMullin, 1985; Weisberg, 2007), 'minimalist', stripping away all but the core explanatory mechanisms (Weisberg, 2007; Batterman, 2002), or 'multiple-models idealizations', in which multiple, simplified, incompatible models are used together to represent a target phenomenon (Cartwright, 1999; Weisberg, 2013).⁵ It is important to stress that taxonomies of idealizations as the one we just illustrated should not be thought as perfectly sharp and always mutually exclusive, as many idealizations in actual scientific models often serve multiple purposes at the same time and could therefore be classified in multiple kinds (see Potochnik (2017) for several examples of this phenomenon). Regardless of type(s), idealizations are increasingly considered an indispensable part of scientific models (cf., Cartwright, 1983; Bokulich, 2011; Potochnik, 2017), as they allow scientists to isolate crucial causal-inferential dynamics that might otherwise be lost in the complexity of real-world systems (cf., Cartwright, 1999; Batterman, 2002, 2009; Pincock, 2022). Moreover, only through idealization can scientists transport knowledge, techniques, and methods across scientific domains (cf., Wimsatt, 2007; Weisberg, 2013; Potochnik, 2017). In this respect, scientific models are invariably both descriptive, capturing salient aspects of what they represent, and idealized, departing from literal accuracy in order to highlight key relationships.

Although for decades the philosophical literature on scientific representation has focused almost exclusively on descriptive scientific models, in recent years, philosophers of science have increasingly paid attention to normative models in science (e.g., Titelbaum, 2013, *forthcoming*; Buchak, 2013; Beck and Jahn, 2021; Roussos, 2022b, a). This increase in interest was also prompted by the increasing attention that philosophy of science has dedicated to the social and human sciences, scientific disciplines where practitioners had to always face the inherent normativity of many of their theories, concepts, and models. Examples of normative models in science are decision theory and game theory in economics (Buchak, 2013; Gintis, 2000), Bayesianism and causal models in formal epistemology (Titelbaum, 2013; Woodward, 2021) and optimality models in biology and cognitive science (Orzack & Sober, 1994; Anderson, 1991).

Intuitively, normative models are scientific models that involve a normative component in their way of representing the target phenomenon. Philosophers have stressed

⁴ There are, of course, many different definitions of idealizations available in the literature, as well as many different taxonomies of types of idealizations. In what follows, we mostly follow Weisberg's (Weisberg, 2007, 2013) use of idealization and related taxonomy of types of idealizations for illustrative purposes, referring the reader to other accounts when necessary.

⁵ It should be noted that, in a more recent paper, Rohwer and Rice (2013) argue that we should add a fourth kind of idealization to Weisberg's taxonomy, i.e., what they call 'hypothetical pattern idealizations'.

how this normative component makes normative models differ significantly from descriptive models in their nature, scope, and role in scientific practice. Opinions vary in the literature on what is the ultimate source of this normative component of normative models, i.e., on whether, to cite some proposals, it is given by normative idealizations themselves (Colyvan, 2013), normative facts (Titelbaum, forthcoming), or normative purposes (Beck & Jahn, 2021; Roussos, 2022a). For the purpose of this paper, we follow Colyvan (2013) in defining *normative models* as all the scientific models that involve normative idealizations in their representation of their target domain. However, unlike Colyvan's account, we do not assume any substantial position on what ultimately grounds the normativity of these idealizations, e.g., whether it ultimately rests on there being inherently normative phenomena or whether it stems from the normative character of certain human purposes and goals.

This is possible because we uphold a fully practice-based understanding of *normative idealization*, and thus of normative models *tout court*. According to our understanding of normative idealizations, an idealization is normative if its justification, within the relevant scientific practice, involves reference to requirements, obligations, prescriptions, norms, and other intuitively normative phenomena describing what, in a certain context, agents ought to do. Importantly, for our account, what gives a normative status to a certain idealization is not a feature or form of the idealization itself, nor a component of the model, or of the target, but rather the way in which the idealization is used, defended, and attacked in the relevant scientific practice. A significant degree of practice-dependence in the role and scope of scientific idealization is a commonplace in pragmatist accounts of scientific models (e.g., Potchnik, 2017; Van Fraassen 2008; Weisberg, 2013) and we hold that the distinction between descriptive and normative idealizations is equally practice-dependent. As such, this definition implies that, in our account, whether a given scientific model is normative or not depends on the specific modeling practice that is taken into consideration.

More exactly, we take a *scientific practice* to be a set of epistemic activities that a given scientific community, at a given point in time, carries out to achieve certain epistemic goals, together with the necessary pragmatic, conceptual, methodological, and epistemic commitments presupposed by the community to carry out these activities.⁶ Then, we call a *modeling practice* any scientific practice in which modeling activities take center stage in the epistemic goals (and in the underlying commitments) of the community.⁷ In a modeling practice, a central set of epistemic and methodological commitments presupposed by the community activities underlies the specific interpretation of the idealizations contained in the models around which the practice is centered. This includes, for us, whether a given idealization is treated as descriptive or normative by the members of the community, and, therefore, whether a given scientific

⁶ This definition of a scientific practice is loosely based on Chang's (2011, 2014) conception. For the purposes of this paper, we simplified his fine-grained description of a practice to keep the focus on the epistemic/methodological components of the practice.

⁷ Since our focus in this paper will be exclusively on modeling practices in science, from now on, whenever we use the term scientific practice, we will refer to modeling practices specifically.

model is employed descriptively or normatively within the specific context of a given practice.

Our practice-based account of normative idealizations and normative models allows us to explain the seemingly puzzling fact that many normative models share the mathematical structure with different, yet closely related, descriptive models in different scientific contexts. A paradigmatic example is Bayesian confirmation theory, which is used as a normative model of rationality in formal epistemology and as a descriptive model of how our mind works in Bayesian cognitive science. Although these two models are sometimes confused by practitioners and philosophers (cf., Crupi & Calzavarini, 2023), they are different, independent modeling enterprises: Bayesianism could be an excellent normative theory of rationality, while being at the same time a poor description of how our mind actually works (or *vice versa*). Thus, it is helpful to understand the notion of a normative model pragmatically, as dependent on the specific scientific practice under consideration. Indeed, our account of normative idealizations is able to distinguish also normative models from their highly idealized descriptive counterparts. Again, Bayesianism is a case in point. Bayesian cognitive science gives a highly idealized description of many of our cognitive faculties and, yet, it remains a descriptive model, which is conceptually different from Bayesianism in its uses as a normative theory of rationality.

To illustrate how our practice-based account of normative idealizations applies to a given scientific model, let us examine decision theory as a concrete example of a normative scientific model.⁸

An example: Expected Utility Theory. Several philosophers and economists argue that decision theory nowadays, when understood as a theory of human rationality, involves a normative component in how it represents human decisions. More specifically, consider the following four central assumptions of expected utility theory.

- Completeness: the agent has a weak preference between every two options.
- Transitivity: the preferences of the agent are transitive.
- Continuity: if an agent prefers A to B and B to C, there exists a probability p such that the agent is indifferent between B and a lottery where A has probability p and C has probability $1 - p$.
- Independence: if an agent prefers A to B, then she prefers also the lottery $pA, (1 - p)C$ to the lottery $pB, (1 - p)C$.

All these four assumptions are crucial to expected utility theory (at least in its classical form), and all of them involve important idealizations. Yet, these four assumptions are usually justified in very different ways in the relevant modeling practice. In fact, the assumptions of continuity and completeness are usually justified for pragmatic reasons of mathematical convenience. That is, we assume that an agent's preferences are complete and continuous because it makes our formal model of an agent's decisions far more convenient mathematically. This is, of course, an idealized way of representing people's choice, since, especially for certain kinds of real-life choices,

⁸ For two other philosophical analyses of the case of EUT as an instance of normative modeling see (Beck & Jahn, 2021; Colyvan, 2013).

it is easy to come up with pairs of options among which an agent might not have even a weak preference or with radical outcomes (e.g., death) against which an agent might behave insensitively to probability outcomes. Nevertheless, these two assumptions make the overall model easier and more tractable, and do not seem to create serious conceptual issues (indeed, there are also known ways for extending people's non-complete and non-continuous preference sets to complete and continuous sets). These two idealizations might be thus considered a case of what philosophers of science call Galilean idealizations (McMullin, 1985). The assumption of independence is instead usually treated as a "close enough" idealization of an agent's preference, i.e., a slight distortion of our decision practices that allows us to focus on the important aspects of an agent's decisions. In general, when we choose between two competing options, we focus solely on their differences. As such, this latter idealization seems best characterized as what the philosophical literature on modeling calls a "minimalist" idealization. Although they belong to different kinds of idealization, and they are therefore justified in different ways, we can note that all the idealizations contained in these three assumptions have an exclusively descriptive character. In contrast, the assumption of transitivity is usually justified on completely different grounds. The transitivity of an agent's preferences is in fact assumed to be a rationality requirement. This assumption is not justified on grounds of convenience nor for other descriptive-pragmatic reasons. Transitivity is a constraint that we impose on a normative theory of rationality because we hold that a rational agent ought to have transitive preferences. Indeed, one of the usual ways to justify this assumption, when confronted with empirical falsification of some of its instances, is a kind of money-pump argument, according to which a rational agent ought to have transitive preferences on pain of (the possibility of) an endless loss of money. The transitivity assumption involves then what we have called a normative idealization. This also explains why expected utility theory, as a whole, when it is understood as a theory of rationality, can be considered a normative model: this model is normative because it embeds in its representation of human decision-making a normative dimension, encapsulated in the idealized constraint of allowing only transitive preferences to agents.

It should be noted that this contemporary normative understanding of EUT is not the only one possible. Indeed, as several historical perspectives on the history of decision theory have highlighted (cf., Guala, 2000; Herfeld, 2018; Malecka, 2020; Briggs, 2023), virtually all models of human decision-making, EUT included, started as purely descriptive models of how humans make decisions. In the case of EUT, the descriptive usage of this model can be seen in the usages and dialectics surrounding the model assumptions before the 1950s, which did not include any significant normative talk, but rather considered the whole model as a good description of how actual human beings make choices (see Guala, 2000; Herfeld, 2018; Malecka, 2020). The difference can be appreciated by looking at the discussion surrounding the transitivity assumption, which, before the 1950s was not justified by means of normative arguments, but rather as a convenient Galilean idealization that allowed (in combination with the completeness assumption) agents' preferences to be represented by a continuous and coherent

utility function (cf., von Neumann & Morgenstern, 1944; Marschak, 1950).⁹ In the 1950s, as a consequence of the discovery of the Allais paradox and as a byproduct of a more general methodological turn in the community of human decision-making, EUT started to get used as a normative model in most of the relevant scientific practices, and its assumptions started to get justified (and also attacked by its opponents) on normative grounds (cf., Guala, 2000; Herfeld, 2018). This normative understanding of EUT as a benchmark of rationality got more and more traction in the following decades, also in connection with the rise of alternative normative models of rationality that directly challenge the norms of rationality prescribed by the normative understanding of EUT. We take this very brief historical account to constitute evidence for the practice-dependence of the normative status of a given scientific model: as we just saw, the same exact model can be used descriptively or normatively depending on how its idealizations are conceptualized and justified in the relevant scientific practice.

The case of EUT is also a good example of how a scientific model can be used descriptively or normatively in different modeling practices, even within the same discipline at the same time. In fact, although, as we just recalled, classical EUT is nowadays employed as a normative model in most modeling practices, it is still employed, in weaker and less idealized versions, as one of the founding models of agents' behavior in several domains of behavioral economics (see Starmer, 2000; Heukelom, 2014 for historical surveys of these uses). Moreover, there are still some specific practices in which classical EUT is used descriptively, without modifications. For instance, in microeconomic models of (semi-)automated markets, such as many financial markets nowadays, EUT is still employed as a good descriptive model of the behavior of the agents (e.g., Avellaneda & Stoikov, 2008; Ho & Stoll, 1981). This is because, in this specific modeling practice, in which many agents within the market take their decisions via algorithms, the transitivity assumption functions as a good-enough description of an agent's preferences (indeed, many algorithms for trading and many automated market makers respect classical EUT axioms by design). Once again, the status of a given idealization within a scientific model cannot be studied in isolation from the specific practice(s) in which a model is used, but must be understood contextually to the specific activities and goals of the community by which it is employed.

4 Logic as a Normative Scientific Model

It is now time to put the pieces together. We first saw how it is unclear whether and how the intuitive normative dimension of logic can be accommodated within the LaM view. We then drew on recent philosophy of science to introduce the notion of a normative scientific model as a possible tool to clarify the previous problem. On this basis, we proposed a practice-based account according to which a model is normative when it represents its target by means of a normative idealization, that is, an idealization

⁹ For detailed discussions of the scientific discussions surrounding the rise of EUT before its “normative turn”, see (Herfeld 2018, pp. 15–32, and Guala (2000), pp. 61–64).

justified on intuitively normative grounds within a given scientific practice. This, in turn, allows us to expand the LaM view with the possibility of understanding logic as a normative scientific model in precisely this sense.

Before presenting the details of our view of logic as a normative scientific model, let us make explicit some of our terminological choices that will help to clarify what we mean when we say that logic is a normative model. Firstly, following some established uses in the philosophy of logic (cf., Haack, 1978; Shapiro, 2014; Stei, 2023), we take a logic to consist of a formal system composed of a language, together with a well-defined syntax and semantics, providing the foundation for a consequence relation that specifies when a conclusion follows from a set of premises. Secondly, we understand logical identity in Haack's (1978) broad sense, so that when two logics have the same theorems and validate the same inferences, we consider them two alternative versions of the same logic. In this way, when we speak of different logical systems such as classical logic, intuitionistic logic, and the like, we mean to refer to the whole class of alternative formulations of these logics, without singling out any specific notational variant, choice of primitive constants, set of axioms, and rules.¹⁰

Moreover, it is important to stress again that when we speak of logic as a normative model, our scope is limited to some uses of certain logical systems in certain scientific and philosophical practices and, as such, our proposal should not be understood as capturing all the possible mathematical and philosophical uses of logic. What we want to argue for is an existential statement, i.e., that there exist some scientific and philosophical practices in which some logical systems are used as normative scientific models, not a universal one. As such, our proposal is completely consistent with the existence of important purely descriptive uses of logical systems in many domains. For instance, in computer science, relational databases rely on logical principles to structure and query data, with many operations resembling automatic theorem proving. Similarly, equivalence checking is a process that verifies whether two systems or components (such as combinational circuits) are functionally identical. Mathematically, this is equivalent to determining the validity of a proposition in propositional logic, and algorithms based on binary decision diagrams can handle even large and complex circuits efficiently. These are clear examples of uses of logic as a purely descriptive scientific model.

After these clarifications, we can state our main thesis in this paper, i.e., that, in some important scientific and philosophical practices, logic is best characterized as a normative scientific model. That is, logical systems are used in these practices to represent their target domains through normative idealizations, i.e., idealizations that are justified solely based on rationality, obligations, prescriptions, and other intuitively normative phenomena regarding what agents ought to do. From this perspective, a given logical system projects a notion of logical consequence that (within the specific domain of the relevant practice) plays a normative role, in that it establishes the conditions under which affirming a set of formulas

¹⁰ Of course, as Haack (1978, pp. 22–23) herself makes clear in her seminal discussion, several other, stricter, notions of logical identity are possible. For our purposes, the choice between these different notions of identity is entirely conventional and nothing significant hinges on our use of Haack's broad sense.

commits the practitioners to affirming another formula. This notion of logical consequence is shaped by the normative idealizations embedded in the relevant logical practice.¹¹

To show how logical systems can function as normative scientific models, we will examine the case of classical first-order logic, discussing its descriptive and normative idealizations.¹² More specifically, we will use the case of classical logic as it is often used in contemporary philosophical practice, i.e., as a theory of correct reasoning. This is a very widespread interpretation in logic textbooks and in several philosophical debates in philosophy of language and epistemology. Consider for instance the following quote from the introduction of one of the most used logical textbooks in the world: “logic is the study of the methods and principles used to distinguish good (correct) from bad (incorrect) reasoning . . . [and] reasoning is a special kind of thinking” (Copi & Cohen, 1990, pp.3–4). We will see that, in addition to descriptive idealizations, several clearly normative assumptions are involved in this modeling practice of classical logic.

An Example: Classical Logic Let us start with some common descriptive idealizations that classical logic assumes. Among them, we can highlight:

- Bivalence: every proposition is assigned exactly one of the two truth-values.
- Truth-functionality: truth-value of compound statements depends only on the truth-value of their components.
- Meta-properties: compactness, (semi-)decidability, recursive enumerability.
- Uniqueness and fixity of interpretations of formulas
- Structural rules: Cut, weakening, contraction, exchange, ...

Let us look at these assumptions in some detail. Bivalence assures that every proposition in our logic is either true or false, so that there are no semantic gaps or gluts in the domain. As heavily discussed in the philosophical debate over vagueness, this is indeed a strong idealization, given how pervasive vagueness and ambiguity phenomena are in our everyday linguistic activities. Despite the strength of this idealization, the choice of assuming bivalence in classical logic seems to be of a non-normative character. For many logicians, bivalence is, in fact, a helpful idealization, because it makes the logicians’ job easier, allowing them to reason with a totality of propositions that are either true or false. Bivalence is therefore what in philosophy of science

¹¹ In the meta-ethical literature over the nature of normativity, philosophers usually distinguish between *robust* normativity and *formal* normativity (McPherson, 2011; Scanlon, 1998). The former binds agents as such (for example, *qua* rational agents) and imposes genuine requirements whose violation constitutes a substantive mistake, while the latter merely sets standards that are internal to a specific system of rules (such as chess or etiquette). In these terms, our account of logic as a normative model aims to capture a robust sense of normativity, one that enforces genuine normative requirements upon the agents, albeit relativized to the specific philosophical or scientific practice that employs a logical model normatively. We thank an anonymous reviewer for highlighting this connection.

¹² Note that, consistently with our aforementioned broad understanding of logical identity, we do not single out here any specific presentation or characterization of first-order classical logic (e.g., any specific presentation of the calculus, any specific choice of primitive constants, any specific presentation of the semantics). The exact instantiation of many idealizations we will treat might vary from presentation to presentation, but their cognitive-epistemological core is sufficiently stable to blur the presentation-dependent aspects for simplicity.

is usually called a Galilean idealization (see Sect. 3), i.e., a pragmatic idealization, the assumption of which is justified by mathematical convenience. The status of bivalence in classical logic is analogous to the status of the completeness and the continuity assumptions in expected utility theory (see Sect. 3): these idealizations are justified descriptively based on pragmatic reasons of convenience and fruitfulness. Similarly, we can see how assuming truth-functionality makes the logicians' modeling job easier, by allowing them to recursively define sentential truth as a function only of the truth of simple propositions and of the connectives used. It is its pragmatic usefulness, then, that justifies truth-functionality and not any normative consideration.

Equally descriptive seem the rest of the idealizations in the above list, i.e., compactness, (semi-)decidability, uniqueness and fixity of interpretations of formulas, and the structural rules we mentioned. All these assumptions of classical logic idealize from the complexity of natural language, allowing logicians to work with finite models, with (semi)-algorithmic decision procedures, with non-ambiguous interpretations, and with useful structural rules. Some of these idealizations are extremely important for classical logic, up to the point that compactness and recursive enumerability can be considered defining features of first-order classical logic in contrast to its competitors, but they nevertheless pertain to the descriptive dimension of logical modeling, in that they are usually justified in the relevant scientific practice by reasons of convenience and usefulness.

Although we could add other idealizations of classical logic to the previous list of descriptive idealizations (see, for instance, the descriptive idealizations discussed in Russell, 2023), some other idealizations of classical logic seem to have a completely different nature in their design, use, and justification in contemporary philosophical practice. Consider the following assumptions:

- Necessity: In a logically valid argument, the conclusion follows necessarily from the premises.
- Formality: The logical validity of an argument is determined by its form.
- Definitions of logical connectives: introduction and elimination rules for conjunction, implication, disjunction, and negation.
- Certain simple logical laws: non-contradiction, excluded middle.

Let us look at these assumptions in some detail. The first two conditions, sometimes also called Tarski's conditions (cf., Etchemendy, 1990; Sher, 1991; Sagi, 2014), are semantic principles that prescribe necessary conditions for the notion of logical consequence.¹³ In classical logic, a valid argument is one in which the conclusion follows necessarily from the premises. The validity of the argument is, in turn, granted by the logical forms of the premises and the conclusion. Yet, do these two properties, necessity and formality, apply to the phenomena in the paradigmatic domains of applications of classical logic, such as human reasoning and linguistic communication? They do not. Most of the arguments that we use in everyday life do not satisfy these conditions, as robustly highlighted by recent cognitive science (cf., Johnson-Laird, 2010; Oaksford

¹³ Note that Tarski conditions are often used as minimal conditions that any pre-theoretic notion of logical consequence must satisfy. Consistent with our non-committal stance on the pre-theoretic, extra-systematic notion of logical consequence, we only discuss them as idealizations of classical logic and we do not commit to their other philosophical uses.

& Chater, 2007). Indeed, contemporary cognitive science tells us that humans prefer to argue with contingent, often deductively invalid, arguments that strongly rely on the conceptual and semantic content of their elements. Then, why does classical logic represent only arguments that are necessarily valid in virtue of their form?

It is not for pragmatic reasons, as it arguably does not make our representation of reasoning any easier. Instead, a possible justification of this assumption would start with recalling that, according to a broadly rationalist tradition in philosophy, correct reasoning is general, as it is meant to apply to every content, and it is necessarily truth-preserving, as it must proceed from certain truth to certain truth (cf., Macfarlane, 2000; Proust, 1989). Normative intuitions on what correct reasoning requires are behind the assumption of necessity and formality that classical logic employs in its representation of human reasoning and language. For example, MacFarlane (2004, pp. 20–21) claims that the demand that logical validity be formal derives just from its normative role as a benchmark of correct reasoning. In his words: “...we require logical validity to be formal because we require it to be transparent, and we require it to be transparent because of the reasons and responsibilities to which it gives rise.” (Macfarlane 2004, p. 21).

Equally normative assumptions are arguably behind the simple inferences that are often used as introduction and elimination rules for classical connectives. Again, MacFarlane (2004) stresses this point, in the context of belief, with the example of the introduction-rule for conjunction: we intuitively feel that there is something wrong with an agent that believes A and believes B , but that does not commit to believing $A \wedge B$. We feel that she is being irrational, because she ought to believe the conjunction, granted that she believes each of the conjuncts. Such an intuition could be used to argue that the introduction-rule for conjunction represents an intuitively valid reasoning instance. Analogously normative intuitions are behind the other elimination and introduction rules for classical connectives: implication, disjunction, and negation. Moreover, we can recognize as intuitively justified also simple examples of logical laws, such as, for instance, the law of non-contradiction or the law of the excluded middle. Behind these laws lie intuitive examples of forms of reasoning which we recognize as correct. Indeed, as we will see with some concrete examples in the next section, non-classical logicians who want to revise these logical laws or connective rules usually propose alternative intuitions on why a given law or rule does not represent a good norm of correct reasoning. A paradigmatic example of this normative view of simple logical inferences that make up classical logic as a model of correct reasoning is Restall’s (2005) defense of the classical multiple-conclusion sequent calculus as a system that codifies norms of assertion and denial of propositions. In particular, he claims that a valid sequent $X \vdash Y$ expresses that the combination of asserting every member of X while denying every member of Y is incoherent, and he then justifies the specific rules of the calculus on this basis. For instance, he understands identity as expressing that asserting and denying the very same proposition constitutes a self-defeating or incoherent (epistemic) state (Restall, 2005, p. 196). More generally, he writes that logical notions “are nothing if they have no applicability to regulate the cognitive states of agents like us, and the content of such states” (2005, p. 191).

This second group of idealizations of classical logic is then made of normative idealizations, because, just like the transitivity assumption that we saw at play in

expected utility theory (cf. Sect. 3), these idealizations are justified normatively in the relevant scientific practice. These assumptions enforce rationality requirements and reflect intuitive standards of sound inference.

5 Logical Practice as a Normative Modeling Practice

After having presented our picture of logic as a normative scientific model, we can now appreciate its implications for the possibility of combining the intuitive normative role of logic and an understanding of logic as a scientific model, as well as its consequences and relations with some related discussions in contemporary philosophy of logic.

First, we are now in a position to answer the skeptical question we started from in Sect. 2, i.e., how can a logical system be normative if it functions as a kind of scientific model? We think that fully descriptivist views such as Russell's (2020) focus exclusively on one aspect of logic's scientific and philosophical role, namely its descriptive use. Logical models can certainly be used descriptively, but in some modeling practices they also serve normative functions. This is true, for example, of the contemporary philosophical use of classical logic as a model of correct reasoning, discussed in the previous section. Such uses are no less legitimate than the normative uses of scientific models in the social and cognitive sciences, as illustrated by familiar cases such as decision theory and Bayesianism.

The key to defusing the skeptical challenge is then to adopt a broader conception of scientific models, one that includes normative as well as descriptive uses. To do so, we must recognize two points. On the one hand, the nature, scope, and role of a scientific model is heavily dependent on the specific modeling practice in which it is employed and cannot be therefore judged absolutely. On the other hand, some scientific practices can employ models normatively without thereby committing themselves to mysterious normative targets or to the inclusion of special normative components within the model itself. As we have argued, the normativity of scientific models lies neither in their syntax nor in their semantics, but in their pragmatics, i.e., in the ways in which their idealizations are used and justified within a particular scientific practice.

Secondly, an important implication of our view of logic as a normative scientific model is that some logics can play a normative role in certain philosophical and scientific practices, without thereby having to assume that this normativity comes from a primitive pre-theoretical normative notion of logical consequence. As we have recalled above, in fact, our practice-based account of normative models allows us to explain the normative role that logic plays in many social and cultural practices by embedding this normativity in the communal design, use, and justification of logical models and their idealizations, not in the intended target or domain of logical systems. That is, we do not assume that the intended target of logical systems is inherently normative or consists in normative facts.

The situation is analogous to normative models in social sciences or in meta-ethics, the intended targets of which (e.g., moral agency, well-being, social practices) might not necessarily consist in normative facts. The normativity of logic as a model is neither in the model nor in its target, but rather in the relevant modeling practices that use logic as a normative model. This leads to the second important consequence

of our definition of logic as a normative scientific model, namely, that our picture is neutral with respect to what the ultimate source of logical normativity is and whether the pretheoretical notion of logical consequence has a descriptive or normative nature. Our picture of logic as a normative model is consistent both with views that want the pre-theoretic notion of logical consequence to be an inherently normative notion (e.g., Beall & Restall, 2005) or with thoroughgoing descriptive accounts of it (e.g., Sher, 1991).

Thirdly, our picture of logic as a normative model is silent about whether and to what extent logic is normative for ordinary reasoners. In fact, our claim that certain logics are used as normative models in certain scientific and philosophical practices does not, by itself, imply that these models ought to norm our ordinary reasoning nor pose any constraint on how such normativity ought to be implemented by the reasoners. The extent to which logic is normative for ordinary reasoners is a very lively debated topic in contemporary philosophy of logic (cf., Macfarlane, 2004; Field, 2009; Dutilh Novaes, 2015; Steinberger, 2019; Oza, 2020; Tajer, 2022; De Benedetto & Marra, 2024). We do not take a stand on this matter here, as our account of logic as a normative model bears only on the starting point of the whole discussion, namely, that logic is used as a normative model in certain scientific and philosophical practices and this provides us with a serious reason to think that logical rules ought to norm our everyday reasoning as well. This *prima facie* normative connection between logic and reasoning is shared by all existing accounts of logical normativity, even by the most negative accounts of whether logic is normative for ordinary reasoning (e.g., Harman, 1986; Harbecke, 2021), which are therefore compatible with our account in this paper.

Fourthly and finally, our account is also *prima facie* compatible with logical pluralism. Since, on our view, the normativity of logical modeling stems from neither the model in itself nor from the target, but rather from the modeling practice of the relevant scientific community, the idea of logic as a normative scientific model seems to be unchallenged by the presence of many alternative normative models. After all, a quick glance at the diverse scientific communities that engage in logical modeling and the varied uses and goals associated with their applications would make anyone doubt the possibility of logical modeling being a monolithic affair. Indeed, the presence of competing normative models is the norm, rather than the exception, in the social and cognitive sciences.

As a matter of fact, the most prominent examples of non-classical logic can plausibly be understood precisely as attempts to revise these normative assumptions by proposing alternative normative models. An intuitionistic logician, for instance, would probably deny the normative validity of the law of excluded middle based upon the (equally normative) intuition that correct reasoning in mathematics should allow for a situation in which a given proposition is neither demonstrably true nor false. Similarly, many assumptions of relevance logic are defended on explicitly *normative* grounds by their supporters. For instance, relevance logic restricts weakening (if $\Gamma \vdash \Delta$ then $\Gamma, A \vdash \Delta$) which lets you add premises that do no justificatory work, because an inference should not count as valid unless each premise contributes to the conclusion (see Mares, 2024). They also reject explosion ($A, \neg A \vdash B$), because a contradiction should not entitle one to assert anything whatsoever. Technically, this is achieved by altering the conditional and/or the structural rules that generate the explosion. These

constraints are presented by relevance logicians as requirements on how one *ought* to reason if one aims to preserve justificatory connection, rather than as mere descriptive conveniences:

The antecedent of an “if...then statement-” statement *must be relevant* to the conclusion drawn. (...) the fancy that relevance is irrelevant to validity strikes us as ludicrous, and we therefore make an attempt to explicate the notion of relevance of *A* to *B*. (Anderson & Belnap, 1975, p. 18, our emphasis).

From our point of view, then, many logical disagreements can be understood as disagreements between competing normative understandings of logical models, just like it often happens in the social and cognitive sciences. Some logical disagreements might also concern the normative status of a given logical idealization. This is possible, from our perspective, because the normative status of an idealization is dependent on the specific modeling practice in which the related model is used. As such, a logical idealization (or even a whole logical model) might be normative in a certain practice, but descriptive in another one.

Indeed, just like many disagreements over the normativity of certain models in the social and cognitive sciences, part of the disagreements between different communities of logicians can be characterized as stemming from different understandings and uses of core idealizations of their modeling practices. As such, we should expect to find, in the history of logic, alternative justifications of many of the descriptive and normative idealizations of classical logic we presented here. An example of this can be found in Quine’s discussion of the notion of bivalence (Quine, 1981). Quine acknowledges that bivalence is helpful on pragmatic grounds, citing “the simplicity of theory that it affords” (p. 91), but, very differently from most logicians, claims that we must accept bivalence for its role as a regulative constraint on scientific language that enforces an ideal of precision and conceptual hygiene. He claims that we must work “the requisite precision into” our scientific predicates (p. 92) and, if need be, “posit an exact demarcation, exact to the last molecule” (p. 94), in order to block Sorites-style paradoxes and related shortcomings that stem from the vagueness of natural-language predicates. In this sense, we can see that Quine treats bivalence as a methodological norm that any properly regimented scientific language in which one can conduct philosophical inquiry ought to have. Again, we take Quine’s deviant interpretation of bivalence to support our pragmatic characterization of normative idealizations and modeling: what determines whether an idealization is normative or descriptive is not its “content” alone, but the way the relevant modeling practice, and even the single modelers, justify and use it in scientific activity. Once again, scientific models cannot be understood solely by examining their relationship to their targets; it is also necessary to consider the relevant scientific practices in which they are embedded.

6 Conclusion

Let us recap the main steps of our argument. We began by recalling how the logic-as-model view conceives of logic as a kind of scientific modeling. We then pointed

out a blind spot in this view, exemplified by Russell's (2020) skeptical take on logical normativity: while LaM captures the descriptive dimension of logic, it has problems accounting for its normative role in many philosophical and scientific practices.

To address this blind spot, we turned to contemporary philosophy of science, where normative models are recognized in many parts of social and cognitive sciences. We saw that these models differ from descriptive ones because they represent their targets through idealizations that have a somewhat normative character. On this basis, we presented a practice-based account of normative idealization and normative modeling, according to which a scientific model is normative, relative to a given scientific practice employing it, if it contains normative idealizations, i.e., if it contains idealizations that are used, defended, or attacked in the practice by reference to rationality requirements, norms, prescriptions, obligations, or other intuitively normative phenomena concerning what agents ought to do. We then refined LaM by extending it with the possibility of certain logics being used, in certain practices, as normative scientific models. We illustrated our picture by analyzing classical first-order logic as used in philosophical debates as a model of correct reasoning. We distinguished how, in the relevant practice, some idealizations have a descriptive character (e.g., bivalence, truth-functionality, compactness) while others have a normative one (e.g., necessity, formality, introduction and elimination rules of connectives).

Our account of (some) logics as normative scientific models answers skeptical takes on the impossibility for logic to be a scientific model while retaining its traditional normative role. Our picture achieves that thanks to understanding that logical systems embed normativity not due to the nature of their targets or to some special property they possess in abstract, but in virtue of how their assumptions are adopted and defended within logical practice.

Our proposal has several advantages. It clarifies logic's prescriptive role in many philosophical and scientific practices, while remaining neutral with respect to metaphysical debates about the pre-theoretic notion of logical consequence and to debates over the normative force that logic exerts on ordinary reasoners. Our view is also *prima facie* compatible with logical pluralism. Just as in the social and cognitive sciences, multiple normative models can coexist, reflecting the variety of different scientific communities and purposes connected with the different uses of scientific models.

Future work could extend our view by examining historical cases of logical disagreement as differences in modeling practices, or by applying it to debates on logical pluralism and anti-exceptionalism. More broadly, our account invites us to rethink the philosophy of logic as the philosophy of a science that, like other human sciences, combines descriptive and normative dimensions in its modeling of the world.

Acknowledgements Both authors contributed equally to this work. We would like to thank Gustavo Cevolani, Lorenzo Sartori, and two anonymous reviewers for helpful comments on earlier drafts of this paper. We are also indebted to audiences in Buenos Aires, Paris, and Bochum for constructive feedback.

Funding Open access funding provided by Scuola IMT Alti Studi Lucca within the CRUI-CARE Agreement. During the revision of this manuscript, Matías Osta-Vélez received financial support from the MUR FIS2 project 'Generic Reasoning' (Grant No. FIS2023-01405).

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical Approval Not applicable.

Informed Consent Not applicable.

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